A risk analysis for the *mise en pratique* of the kilogram

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What is risk?

**Oxford dictionary:**
*(noun)* a situation involving exposure to danger
*(mass noun)* the possibility that something unpleasant or unwelcome will happen

According to *Factor Analysis of Information Risk (FAIR)*: the probable frequency *and* probable magnitude of future loss. Used mostly in IT

For *Occupational Health & Safety Advisory Services (OHSAS)*: product of the probability of a hazard resulting in an adverse event, *times* the severity of the event. Close to the definition in probability and statistics.

For *ISO* risk used to be: chance or probability of loss

A definition in probability theory

The optimal estimate $\hat{X}$ of a parameter $X$ given data $Z$ is typically based on minimization of a risk function $J$, given by

$$J = E[C(\tilde{X})] = \int_{-\infty}^{+\infty} C(x - \tilde{X}) f_{X,Z}(x, z) \, dx \, dz$$

where $E[\cdot]$ is the expectation, $C$ a loss (or cost) function of the error

$$\tilde{X} = X - \hat{X}$$

and $f_{X,Z}$ is the joint probability density function of $X$ and $Z$
Risk management

A multi-stage activity

identification

assessment

prioritization

minimization

ISO 31000
Risk in realization of the kilogram

Identification: That we are unable to realize the kilogram because of lack of primary realizations

Assessment:

A discrete case:

\[ J = \sum_{i=1}^{n} C_i p_i \]

where \( C_i, p_i \) are the losses involved in the closure of the \( i \)th independent realization and the probability of closure, respectively.
Form of $C_i$

\[ J = \sum_{i=1}^{n} C_i p_i \]

Should the discussion concern (equal) nuclear power plants, or dams, it is obvious that the loss functions $C_i$ are more or less the same, independent of $i$ (all nuclear power plants are equally dangerous). Thus, the risk $J$ increases with $n$, which makes sense.

In our case, the larger is $n$, the better, i.e. the smaller the risk (minimization!). This suggests that the $C_i$ decrease when $n$ increases, which makes sense again, as we expect that closure of one realization has an impact proportional to $1/n$. 
Value(s) of $p_i$

\[ J = \sum_{i=1}^{n} C_i p_i \]

No attempt of quantitative evaluation.

It can be expected that each $p_i$ tends to increase with time (obsolescence and ageing of experimental devices, long-term changes in policy of research, instabilities…)

However, new realization are likely to appear in the medium and long term which will be more easily available, cheaper and more reliable, thus resetting $p_i$ to lower values (like lasers for the definition of the metre).
Global risk

For a large NMI:

Reasonable confidence on the constancy of funding and staff
Comparatively small impact of the kilogram realization on the global
NMI budget

\[ p \text{ is comparatively small} \]

But

The realization is likely to have a low uncertainty, i.e., high weight

\[ C_i \text{ is supposed to be comparatively high.} \]
Global risk

For a small NMI
(or a Country with unstable economical and (or) political situation)

uncertainty about the constancy of funding and staff
the cost of the kilogram realization is a non-negligible part of the global budget

\[ p \text{ is comparatively large} \]

But

The realization is likely to have a larger uncertainty, i.e., low weight

\[ C_i \text{ is supposed to be comparatively small.} \]
Global risk

All the above seems to suggest that perhaps

$$C_i p_i$$ is (very approximately) constant?

Even with no calculations, it seems reasonable to imagine that five independent realizations at the time of the redefinition would represent a redundancy sufficient to keep reasonably low the risk that $$n < 3$$ in the first ten years or so after the redefinition.
Conclusion

All the above implies that the $n$ realizations contribute in a common effort to realize a virtual international standard, represented perhaps by an ensemble of material artefacts universally available.

This is the route to the realization of the second, in contrast to that of the metre.

We are probably free to choose at this moment either way. It is my feeling that the second is a better example than the metre.
Thank you