Considerations on *mise en pratique for the kilogram* based on work of CCM WGM TG2

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Terms of reference for TG2

TG2 should report to the WGM and the CCM on the following:

+ The present uncertainty to which the unit of mass can be disseminated from the international to the national prototypes;

+ Methods for evaluating the correlation between the measured mass values of the prototypes of the kilogram;

+ Recommendations for additional measurements which would allow an improved uncertainty evaluation. These measurements may involve use of the international prototype or its official copies; and

+ In coordination with TG1, identification and evaluation of the uncertainty components inherent in the *mise en pratique* for the kilogram when a new definition is proposed to the CGPM.
Prototypes involved in study

Prototypes/working standards (period of use):

+ The international prototype K (1889-1992)
+ The six temoins K1, 7, 8(41), 32, 43 and 47 (1889-1992)
+ Prototype 25 for special use (1889-2009)
+ Prototype 9 and 31 for routine use (1889-2009)
+ Working standard 42’ (1990-2009)
+ Prototypes 63 and 73 (1990-2009)
+ Prototype 67 (1990-1999)
+ Prototype 77 (1997-2009)
+ Working standard 650 (1993-2009)
+ Prototypes 88 and 91 (2003-2009)
Modelling change in mass over time

Model:

\[ m(t) = m_0 + \alpha(t - t_0) + \gamma \sqrt{t - t_C} + \delta m(t) \]

- \( m(t) \): Mass of weight at time \( t \)
- \( m_0 \): Mass of weight at reference time \( t_0 \)
- \( \alpha \): Rate of change in mass of cleaned weight
- \( \gamma \): Removable dirt collection coefficient
- \( t_C \): Time of last cleaning
- \( \delta m(t) \): Random variable with expectation 0 and variance \( \sigma^2 \)
Modelling change in mass over time

Model:

\[ m(t) = m_0 + \alpha(t - t_0) + \gamma \sqrt{t - t_C} + \delta m(t) \]
Least squares adjustment

+ The quantities in the model have been adjusted by the method of least squares to explain the mass differences of observed among 18 prototypes at the BIPM in the period 1889-2009.

+ The standard uncertainty of an observed mass difference $\Delta m$ is assumed to be $u(\Delta m) = 0.001$ mg.

+ The best estimate of the random mass change $\delta m(t)$ is 0 mg with standard uncertainty $\sigma = 0.025$ mg before 1946 and $\sigma = 0.006$ mg after 1946.

+ For the International Prototype $m_0 = 1$ kg, $\alpha = 0$ mg/year (exact) and $\sigma = 0.001$ mg.

+ The least squares adjustment provides best estimates of all model quantities (including random mass changes) and also the associated covariance matrix.
Calculation of mass values

+ For each prototype the expectation value of the mass

\[ \mu(t) = m_0 + \alpha(t - t_0) + \gamma \sqrt{t - t_C} \]

and the associated standard uncertainty can be calculated as a function of any time in the past or future

+ If the prototype is used as a reference standard at time \( t \), the predicted mass value ("Prediction") is

\[ m(t) = \mu(t) + \delta m \]

where \( \delta m = 0 \) mg, but \( u(\delta m) = \sigma \) if we have no specific information about the random mass change \( \delta m(t) \) at that time
Calculation of mass values

Each time $t_i$ the prototype has been compared with another one, the best estimate of its mass at that time ("measured" values)

$$m(t_i) = \mu(t_i) + \delta m(t_i)$$

and the associated standard uncertainty can be calculated

**Note:** After the least squares adjustment, the best estimate of $\delta m(t_i)$ is no longer 0, and the associated standard uncertainty is smaller than $\sigma$

Like standard uncertainties, correlation coefficients between all calculated mass values can be calculated using the covariance matrix of the output of the least squares adjustment.
Calculation of mass values

+ The most important prototype

International prototype K

[Graph showing mass correction Δm/mg over time from Jan-85 to Jan-94 with predicted values, measured values, and BIPM values indicated.]
One of the two prototypes for routine use at the BIPM:
Calculation of mass values

+ The prototype for special use at the BIPM:

![Graph showing mass correction over time for BIPM prototype no. 25]
Calculation of mass values

The other of the two prototypes for routine use at the BIPM:

BIPM prototype no. 31

Mass correction $\Delta m / \text{mg}$
Calculation of mass values

One of two working standards used for calibration of national prototypes:

BIPM working standard no. 42'
Calculation of mass values

- The other of two working standards used for calibration of national prototypes:
Lessons learned

+ A stable mass standard does not exist, nor does a stable group of mass
+ Modelling the change in mass over time is crucial; it reduces uncertainties and abrupt mass changes
+ Regular comparisons with a primary realisation is necessary in order to monitor the change in the average mass of a group of weights (we have a problem here in the transition phase!)
+ Frequent comparisons are useful for monitoring individual weights in a group relative to the average mass and for validating the assumed mass change model
+ Cleaning of weights should be reduced to a minimum; the mass will not be reset anyway
+ Discrepancies between model and measurement will occur from time to time; proper corrective actions have to be taken