System identification and uncertainty analysis for challenging measurement applications: a case study in micro-Newton level force measurement

BIPM workshop on metrology for dynamic measurement

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Outline

- What is dynamic measurement?
  - Definition
  - Features and properties of a dynamic measurement system
  - What’s the problem?
  - How to deal with it
- Case study: micro-Newton thrust measurement
  - Requirements
  - Instrument design
  - Testing
- Summary of observations and issues
What is dynamic measurement?

- **Definition:**
  A measurement is considered dynamic if the relationship between the measurand and the sensor output is frequency dependent.

- The term dynamic does not apply to all time-varying signals

- Not necessarily limited to rapid rates of change in measurand. e.g. seismometers often have a low-frequency cut-off.
Features and properties

- **Sensor** has a frequency dependency in the relationship between the **signal** (output) and **measurand** (input)
  - Can be described by a transfer function (equation)
  - Visualised graphically e.g. Bode plot
What’s the problem?

- We want our sensor output signal to represent the measurand as closely as possible.
- The frequency response of the sensor has introduced a systematic effect.
- GUM says we must compensate all known systematic effects and associate uncertainties with these compensations.
- This requires a calibration process
  - Determine relationship between measurand and output signal
  - Establish correction factor(s)/coefficient(s)
  - Estimate uncertainty associated with corrections
Solution

Measurand $\rightarrow$ Sensor $H(s)$ $\rightarrow$ Filter $H^{-1}(s)$ $\rightarrow$ Deconvolved
Inverse filter design

- Use system ID techniques to determine the transfer function of the sensor, $H(s)$.
  - Physical model
  - “Black box” model
  - “Grey box” model
- For Black box or Grey box model, perform experiment to determine model parameters
- Invert the transfer function to obtain the inverse filter, $H^{-1}(s)$.
- But,
  - Inverse filter may be unstable, will amplify noise
    - low-pass filter
  - Uncertainties and errors associated with the model
    - Validate the model
Low-pass filter

- Design of filter is an issue:
  - Cut-off frequency
  - Filter order
- Does it matter?
  - Trade-off between error reduction and uncertainty
- Guidance needed
System ID/Model validation tools

- Easy to use software tools available, e.g.
  - LabVIEW control design and simulation toolkit
  - MATLAB system identification toolkit
- Transfer function parameter estimates provided, but not always uncertainty estimates
  - We need uncertainty estimates
  - How do we get them?
- Model validation tools available
  - More guidance on use to these tools is needed
- These tools cover the engineering aspects but the requirements for metrology are lacking
Propagate uncertainties

- Errors in the model
  - Incomplete i.e. un-modelled effects
- Uncertainties in the model/filter parameters
- How do we propagate these uncertainties (frequency domain) through to the deconvolved signal (time domain)?
  - Monte Carlo?
  - Some other technique?
  - HELP!
A subtlety: Correlation

- Inverse filter is a finite impulse response (FIR) or infinite impulse response (IIR) filter. So all output data samples are inherently correlated with all previous samples.
- GUM says we need to take correlations into account.
  - How?
- e.g. Deriving an uncertainty associated with a time averaged output signal needs to take this correlation into account otherwise uncertainty estimates could be wrong!
- Note: the above applies whenever filtered data is averaged.
Summary

- System ID to determine sensor transfer function
  - Uncertainties in model & model parameters
- Derive and apply inverse filter and accompanying LPF
  - Uncertainty in filter parameters
  - Inherent correlation
- Propagating uncertainties through to time domain is an issue.
- When calibrating a dynamic sensor, how/what do we report?
  - Inverse filter coefficients and uncertainties?
  - Include correlation coefficients?
  - On paper or electronically?
  - Guidance on use of calibration data?
  - ...
Case study: μN thrust balance
Micro-thrusters

- Thrusters operating in the range 0.1 $\mu$N to 500 mN
- Used by spacecraft, for example, for
  - fine attitude control
  - drag compensation
  - station keeping
  - formation flying
- Thrust generated by accelerating ions or gas/liquid
  - Electric Propulsion
  - Cold gas
Micro-thruster performance requirements

- LISA Pathfinder requires thruster performance:
  - Range 0.3 µN to 150 µN
  - Resolution 0.3 µN
  - Noise below $0.1 \mu N/\sqrt{\text{Hz}}$ between $10^{-2}$ Hz and 10 Hz
  - $1 \mu N/\sqrt{\text{Hz}}$ between $10^{-3}$ Hz and $10^{-2}$ Hz

- **Traceable** measurements with rigorously evaluated uncertainties are required to verify thruster performance
NPL/ESA micro-newton thrust balance requirements

- Primarily for cold-gas thrusters
- 0 µN to 500 µN thrust range
- 0 Hz to 10 Hz measurement bandwidth
- Noise floor below 1 µN/√Hz
- Traceable to international standards
- Rigorous uncertainty evaluation, target 1 µN ($k = 2$)
NPL/ESA micro-newton thrust balance requirements

- Current focus is on ‘static’ thrust measurement
- Next goal is to measure thruster dynamics…
- … and thrust noise.
Thrust balance design

MBA (Measurement balance assembly)

TCA (tilt compensation assembly)

Thruster

Force actuator

Dummy load

Pendulum

Displacement sensor (inside)

Pictures courtesy of ESA
Thrust balance principles

- Pendulum mechanism
- Null-displacement, force feedback control
  - Overcomes non-linearity in mechanism
  - Sensitivity determined by force actuator (in steady-state)
  - Traceability through force actuator calibration
- **Test input** for system ID and uncertainty verification
Thrust balance principles

- Pendulum mechanism
- Null-displacement, force feedback control
- Sounds just like a seismometer!
  - Sensitive to vibration and tilt
  - Use a ‘dummy’ matched pendulum to compensate for the vibration signal
Thrust balance principles

- Pendulum mechanism
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This is a dynamic measurement!
Dynamic compensation:

- Derive grey box model from physics
- For each pendulum (MBA and TCA):
  - Perform system ID test
  - Fit grey box model to data to obtain unknown parameters of grey box model
  - Perform validation test on both pendulum models
  - Design inverse filter for each pendulum
- Perform thrust measurements
  - Record thrust/vibration data and apply inverse filters
- Subtract deconvolved TCA signal from deconvolved MBA signal to compensate for vibration
Dynamic compensation:
1. System model

- Grey box model
  - Some parameters known (green)
  - Some unknown or known within limits (red)

![Diagram](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured thrust (balance output)</td>
<td>( T_M )</td>
<td>( \mu N )</td>
</tr>
<tr>
<td>Applied thrust (to be determined)</td>
<td>( T )</td>
<td>( \mu N )</td>
</tr>
<tr>
<td>Pendulum length</td>
<td>( l_p )</td>
<td>m</td>
</tr>
<tr>
<td>Force actuator sensitivity</td>
<td>( k_a )</td>
<td>( \mu N/V )</td>
</tr>
<tr>
<td>Displacement sensor sensitivity</td>
<td>( k_p )</td>
<td>( V/\mu N )</td>
</tr>
<tr>
<td>Proportional gain (PID)</td>
<td>( k_c )</td>
<td>( V/V )</td>
</tr>
<tr>
<td>Velocity feedback gain</td>
<td>( k_d )</td>
<td>( V \cdot s/V )</td>
</tr>
<tr>
<td>Effective mass of pendulum + load</td>
<td>( M_e )</td>
<td>kg</td>
</tr>
<tr>
<td>Integration time constant (PID)</td>
<td>( t_i )</td>
<td>s</td>
</tr>
<tr>
<td>Derivative time constant (PID)</td>
<td>( t_d )</td>
<td>s</td>
</tr>
<tr>
<td>Angular stiffness of pendulum</td>
<td>( k )</td>
<td>N \cdot m/rad</td>
</tr>
<tr>
<td>Damping coefficient of pendulum</td>
<td>( \zeta )</td>
<td>N \cdot s/m</td>
</tr>
</tbody>
</table>
Dynamic compensation:

2. System ID

- Apply known stimulus via the test input
- Record stimulus and response (output)
- Fit grey box model to data.
- But currently, no uncertainties given.
Dynamic compensation: 3. Model verification

- Apply known stimulus via the test
- Record stimulus and response
- Compute prediction error
- Autocorrelation of prediction error
- Cross correlation between prediction error and stimulus
Dynamic compensation:

- Vibration compensation
Dynamic compensation:

- TF compensation
‘Static’ thrust measurement

- Interleaved simulated and real thrust steps (deconvolved)
- Need to account for residual noise (after vibration compensation)
Thrust measurement

- Fit parallel lines through data when thrust is OFF and ON.
- Standard least-squares analysis gives us the magnitude of the thrust step (separation of parallel lines) and an uncertainty.
Thrust measurement

- Simulated thrust steps repeated 50 times.
- Plotted as errors (deviation from known thrust).
- Uncertainty shown by error bars plotted at $k = 2$.
- **Calculated uncertainty is not consistent with observed variation! Why?**
Thrust measurement

- Calculated uncertainty is not consistent with observed variation! Why?
- The standard statistical analysis assumes noise is:
  - White (broadband)
  - Random
  - Stationary
  - Un-correlated
- Remember the inverse filter? This correlates the data.
- Use empirical (AR) model to compensate for correlation
Thrust measurement

- Calculated uncertainties are more consistent with observed variation
- But this is an empirical solution based on our current best guess of the cause of inconsistency
- More work needed…
Next steps

- Thruster dynamics
- Thrust Noise
Summary & Observations

- GUM does not explicitly cover dynamic measurement
- Dynamic measurement/calibration is a complex process
- Relies on several assumptions that need to be validated, LTI, model, etc
- Potential pit-falls, correlation
- Software tools exist, e.g. for system ID, but cover engineering aspects not metrology
- Scientific literature on the subject is scarce and needs to be adapted for a wider audience