Uncertainty in frequency

The CCTF has declared the calculation of \([\text{UTC} - \text{UTC}(k)]\) as published in BIPM Circular \(T\) to be the unique Key Comparison in the field of Time and Frequency. Circular \(T\) gives the deviation for each contributing laboratory in the form \([\text{UTC} - \text{UTC}(k)]\) with a given combined uncertainty for intervals of 5 days. From this, the corresponding deviation for frequency and its corresponding uncertainty are therefore available for time intervals of 5 days. If the laboratories need to know the deviations for intervals and averaging times shorter than 5 days, these can be obtained by extrapolation following Guideline 3 of the CCTF WGMRA (CCTF Working Group on the CIPM MRA).

The uncertainty in the frequency can be obtained from the difference between UTC and UTC\((k)\) as published monthly by the BIPM in Circular \(T\). This uncertainty is linked to the uncertainties in \([\text{UTC} - \text{UTC}(k)]\) as reported in Circular \(T\) (Section 1). As explained in [1], these uncertainties are linked to the link uncertainties reported also in Section 6 of the same Circular \(T\).

The uncertainty in the frequency can be obtained by applying the law of uncertainty propagation [2] to the definition of the mean frequency:

\[
\overline{y} = \frac{[\text{UTC} - \text{UTC}(k)] - [\text{UTC} - \text{UTC}(k)]_t}{\tau}
\]

The uncertainty in the mean frequency is given by:

\[
u^2_{\overline{y}} = \frac{u^2_{[\text{UTC} - \text{UTC}(k)]_t} + u^2_{[\text{UTC} - \text{UTC}(k)]_t} - 2\text{Cov}([\text{UTC} - \text{UTC}(k)]_t, [\text{UTC} - \text{UTC}(k)]_{t-\tau})}{\tau^2}.
\]

We assume that the uncertainty in the difference \([\text{UTC} - \text{UTC}(k)]\) reported in Circular \(T\) is constant for the whole period so the previous relation becomes:

\[
u^2_{\overline{y}} = \frac{2(u_A)^2}{\tau^2}.
\]

where \(\tau\) is the integration time, minimum 5 days. In this case the main component of the uncertainty is the \(u_A\) component, considering that the calibration uncertainty \((u_B)\) is insignificant in the frequency measurements.

However, the laboratories receive the results of \([\text{UTC} - \text{UTC}(k)]\) after a delay of about 45 days, during which they do not know the difference between UTC and their local time scale UTC\((k)\) and, as a consequence, they do not know the uncertainty that should be declared. In this case, if they do not wish to wait for publication of the information in the next edition of Circular \(T\), they need to include a “prediction component” to the global budget of their frequency uncertainty. If the laboratories wait for the Circular \(T\) results they will automatically receive the updated uncertainty information.

References