CCTF
WGMRA Guideline 4
(201209)
Uncertainty in frequency

The CCTF has declared the calculation of [*UTC - UTC(k)*] as published in *BIPM Circular T* to be the unique Key Comparison in the field of Time and Frequency. *Circular T* gives the deviation for each contributing laboratory in the form [*UTC - UTC(k)*] with a given combined uncertainty for intervals of 5 days. From this, the corresponding deviation for frequency and its corresponding uncertainty are therefore available for time intervals of 5 days. If the laboratories need to know the deviations for intervals and averaging times shorter than 5 days, these can be obtained by extrapolation following Guideline 3 of the CCTF WGMRA (CCTF Working Group on the CIPM MRA).

The uncertainty in the frequency can be obtained from the difference between UTC and UTC(*k*) as published monthly by the BIPM in *Circular T*. Thus the frequency uncertainties are linked to the uncertainties in [*UTC - UTC(k)*] as reported in *Circular T* (Section 1). As explained in [1], these uncertainties are linked to the link uncertainties reported also in Section 6 of the same *Circular T*.

The uncertainty in the frequency can be obtained by applying the law of uncertainty propagation [2] to the definition of the mean frequency,:

$$\bar{y} = \frac{[UTC - UTC(k)]_t - [UTC - UTC(k)]_{t-\tau}}{\tau}$$

The uncertainty in the mean frequency is given by:

$$u_{\bar{y}}^2 = \frac{u_{[UTC-UTC(k)]_t}^2 + u_{[UTC-UTC(k)]_{t-\tau}}^2 - 2Cov\left([UTC - UTC(k)]_t, [UTC - UTC(k)]_{t-\tau}\right)}{\tau^2}.$$

We assume that the uncertainty in the difference [*UTC - UTC(k)*] reported in *Circular T* is constant for the whole period so the previous relation becomes:

$$u_{\bar{y}}^2 = \frac{2(u_A)^2}{\tau^2}.$$

where τ is the integration time, minimum 5 days. In this case the main component of the uncertainty is the u_A component, considering that the calibration uncertainty (u_B) is insignificant in the frequency measurements.

However, the laboratories receive the results of [*UTC - UTC(k)*] after a delay of about 45 days, during which they do not know the difference between UTC and their local time scale UTC(*k*) and, as a consequence, they do not know the uncertainty that should be declared. In this case, if they do not wish to wait for publication of the information in the next edition of *Circular T*, they need to include a "prediction component" to the global budget of their frequency uncertainty. If the laboratories wait for the *Circular T* results then they will automatically receive the updated uncertainty information.

References

[1] Lewandowski W., Matsakis D., Panfilo G., Tavella P. The evaluation of uncertainties in [*UTC – UTC(k)*] *Metrologia*, 2006, **43**, n°3, 278-286.
[2] JCGM 100:2008 Guide to the Expression of Uncertainty in Measurement (GUM 1995 with minor corrections).