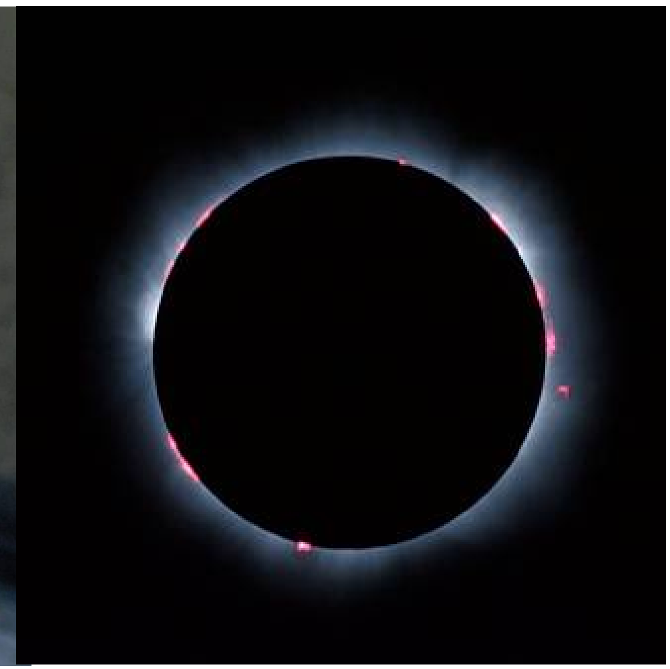
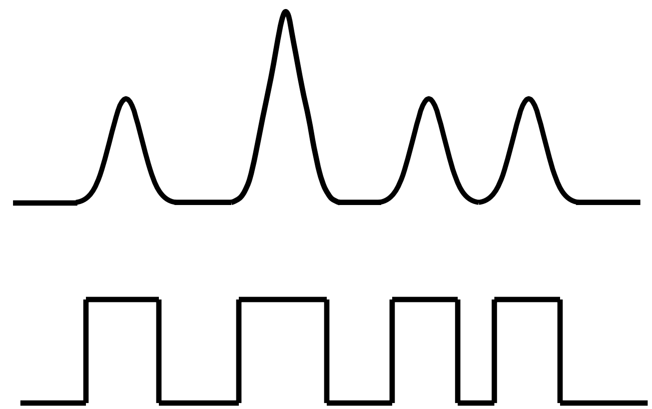
Pile-up, dead time and counting statistics

Stefaan Pommé

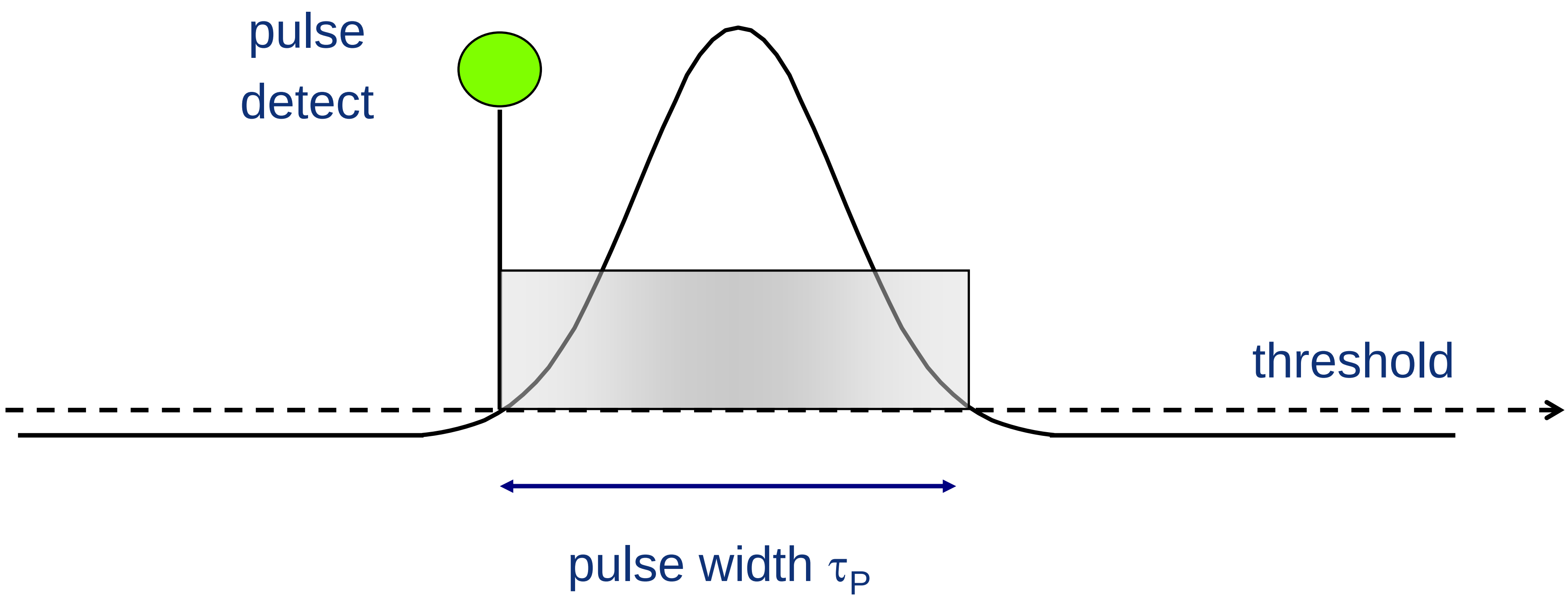


Nuclear counting in pulse mode

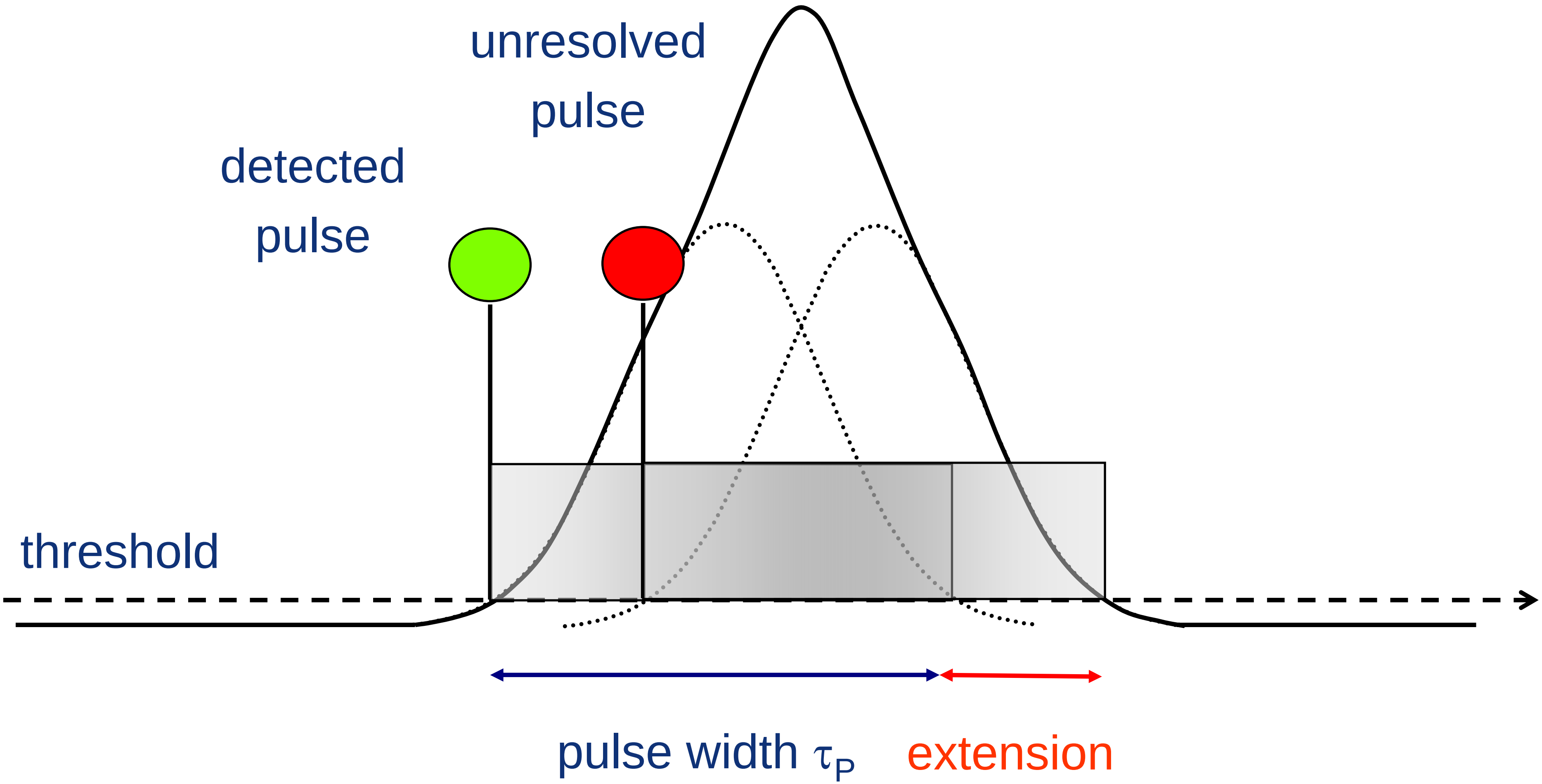
nuclear source ⇨ measuring device ⇨ electronic pulses / discriminator



Schematic representation of detector pulse



Count loss due to pile-up



One example

count rate ρ = 10,000 events per second [s^{-1}]

pulse width τ_p = 10 microseconds [s]

→ normalised rate $\rho\tau_p$ = 0.1 []

25 measurements with 1 million counts each

Random part of uncertainty

25 measurements with **1 million counts** each

Poisson statisitics

one measurement: $u/N = 1/\sqrt{N} = 0.1\%$

25 measurements: $u/N = 1/\sqrt{25N} = 0.02\%$

(random) statistical uncertainty is **insignificant**

→ focus on ‘**systematic**’ sources of error

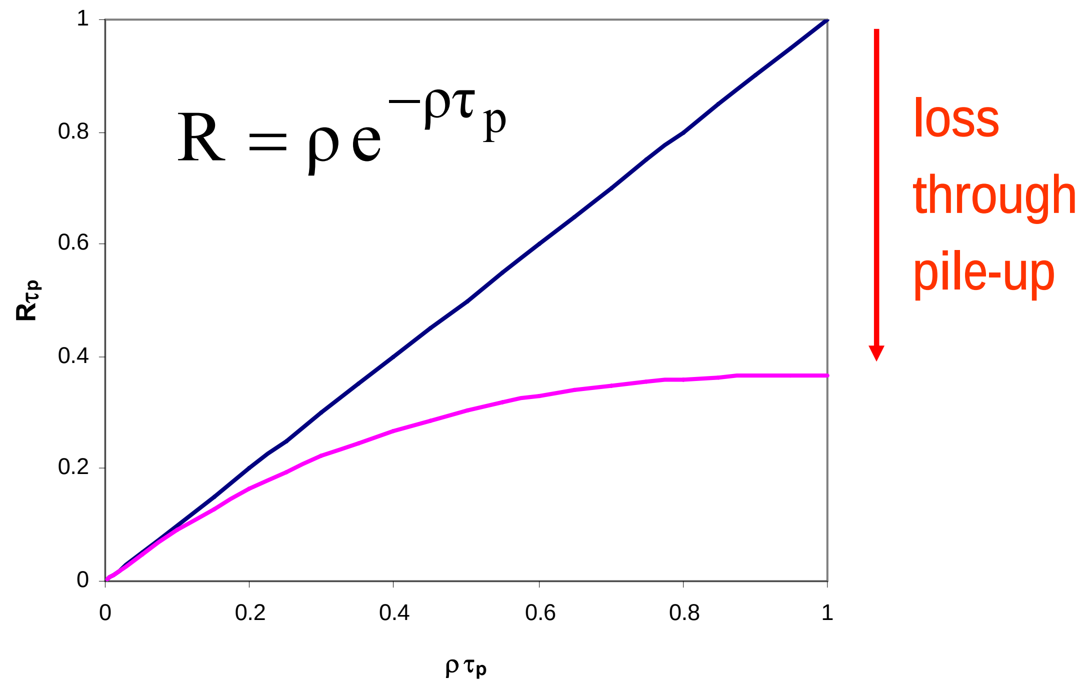
Three cases

- no correction for count loss due to dead time
- correction by inversion of throughput curve
- live-time counting

Count loss through pile-up

$\rho = 10000\ s^{-1}$

$\tau_p = 10\ \mu s$



no correction => error $= 1 - e^{-0.1}$

$= 9.5\%$

$$\frac{\varepsilon(\rho t)}{\rho t} = 1 - e^{-\rho \tau_p}$$

Count loss correction by inversion of throughput curve

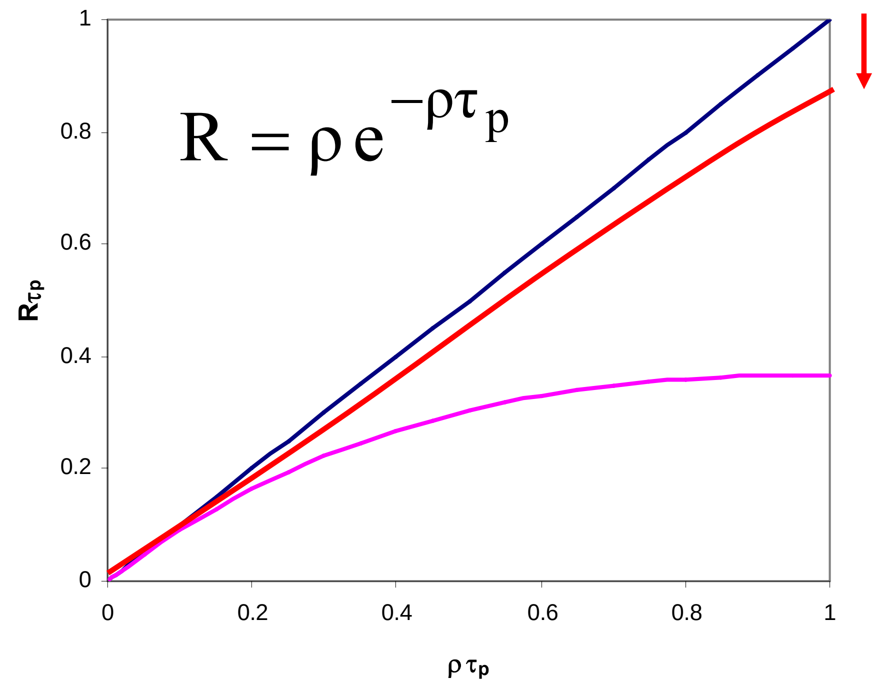
$\rho = R \exp(\rho\tau_p)$

τ_p = 10 μs ± 1 μs

τ_p = not well known

not constant

(pulse-height dependent)

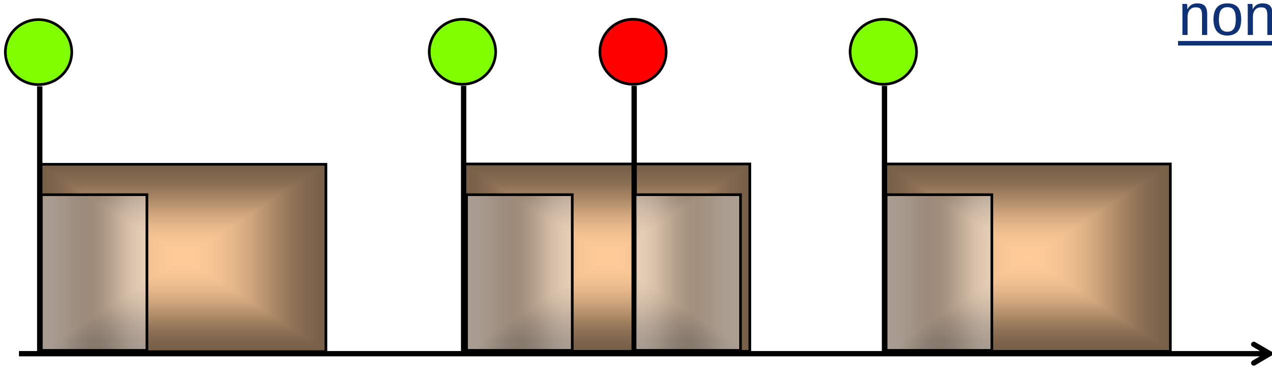


=> error = 1- $e^{-0.01}$

= 1%

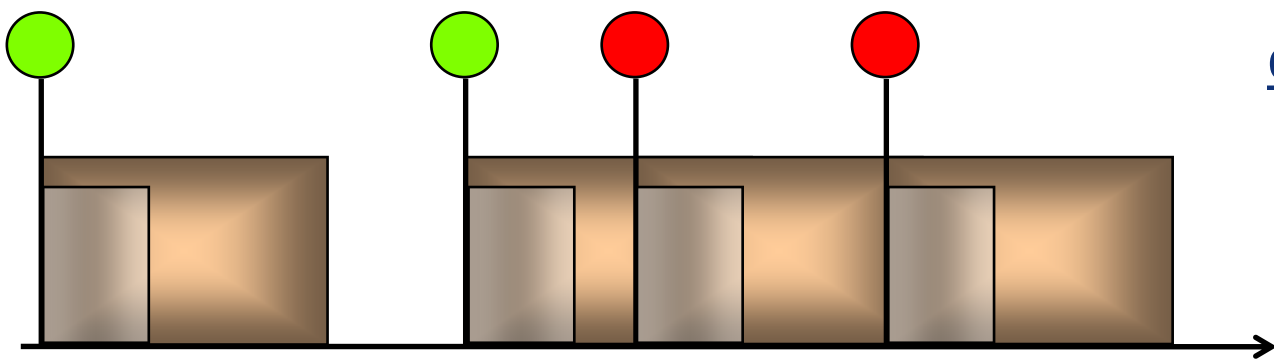
$$\frac{\sigma(\rho t)}{\rho t} \approx 1 - e^{-\rho\sigma(\tau_p)}$$

Imposition of artificial dead time



non-extending dead time

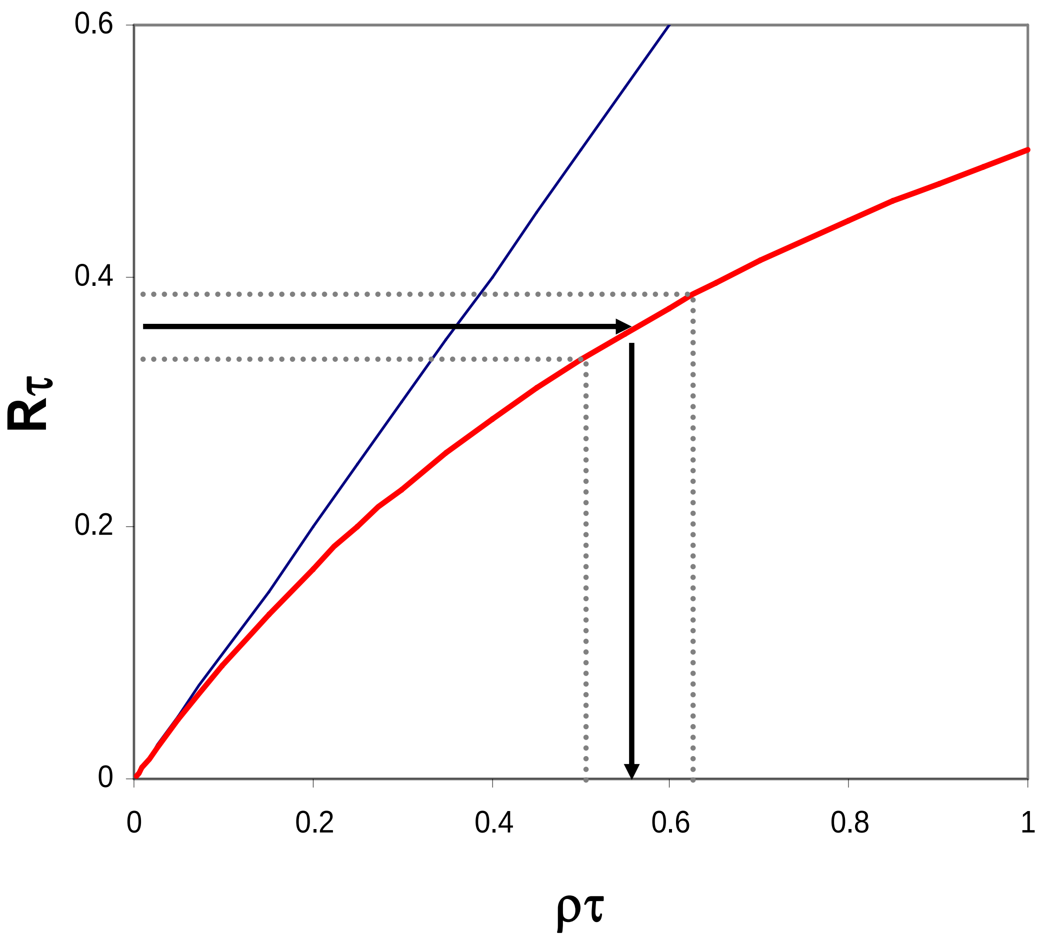
$$R = \frac{\rho}{1 + \rho\tau_{ne}}$$



extending dead time

$$R = \rho\, e^{-\rho\tau_e}$$

Real-Time Mode : inversion of throughput curve



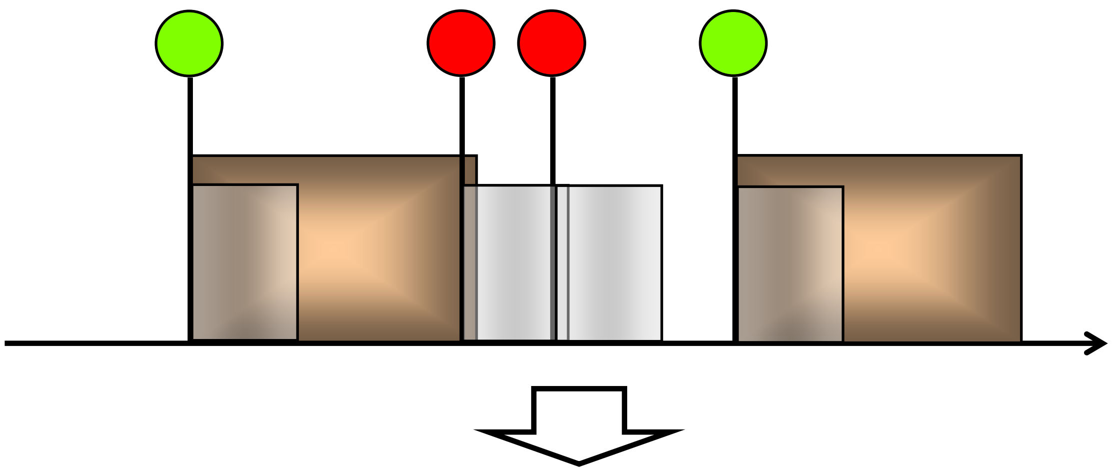
non-extending dead time

$$\rho = \frac{R}{1 - R\tau_{ne}}$$

+ random component
+ unc. propagation $\sigma(\tau_{ne})$
+ cascade effect

Cascade pile-up + non-extending dead time

Pile-up 'prolongs' dead time



additional count loss
through pile-up

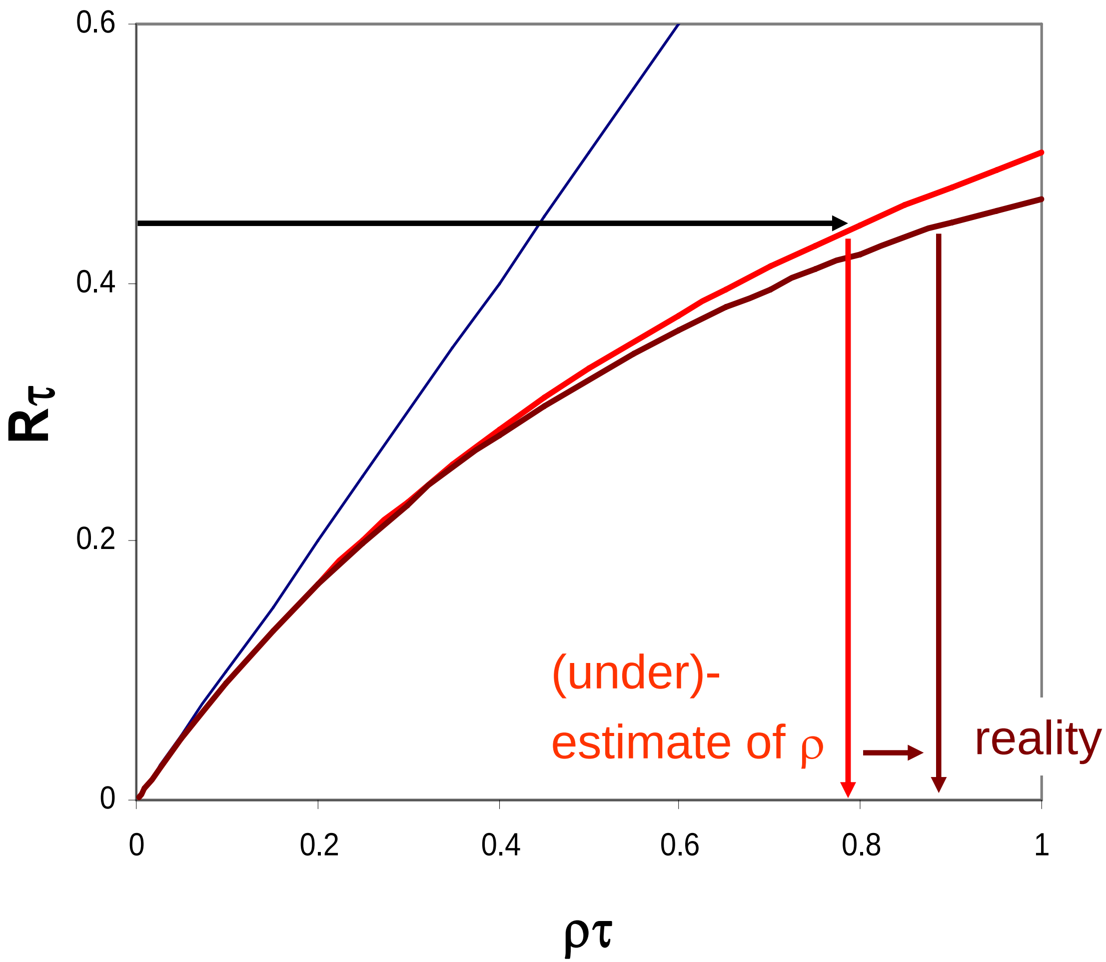
$$R = \frac{\rho}{e^{\rho\tau_p} + \rho(\tau_{ne} - \tau_p)}$$

instead of

$$R = \frac{\rho}{1 + \rho\tau_{ne}}$$

non-extending dead time

Real-Time Mode : error by neglecting cascade effect



non-extending dead time

$$\rho = \frac{R}{1 - R\tau_{ne}}\left[e^{\rho\tau_p} - \rho\tau_p\right]$$

by iteration

Count loss correction by inversion of throughput curve

non-extending dead time

τ_p = 10 μs ± 1 μs

τ_{ne} = 15 μs ± 0.15 μs

+ random component = 0.107%

$$\sigma(\rho t) = \frac{1}{\sqrt{Rt}}$$

+ propagation unc. τ_{ne} = 0.15%

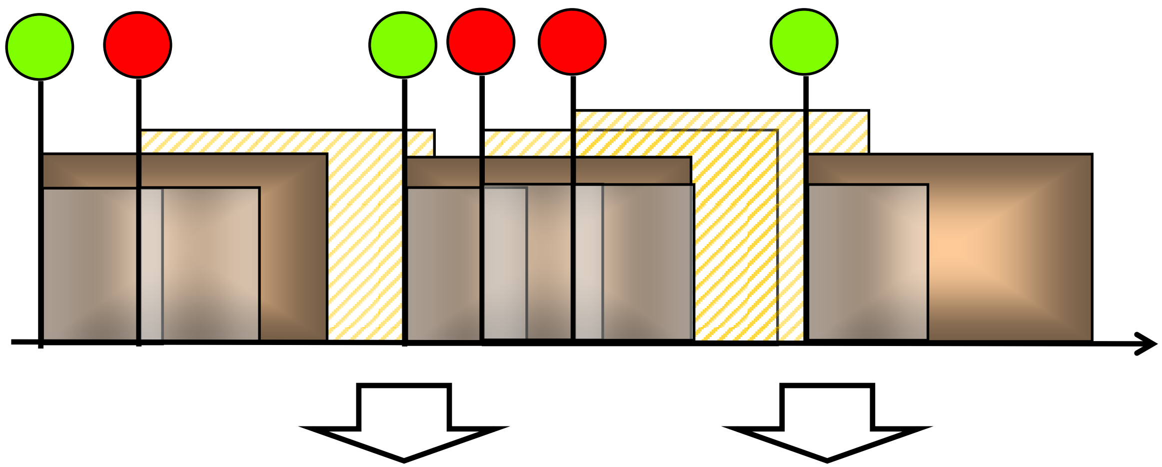
$$\frac{\sigma(\rho t)}{\rho t} = \rho\tau_{ne}\frac{\sigma(\tau_{ne})}{\tau_{ne}}$$

+ cascade effect = 0.51%

$$\frac{\varepsilon(\rho t)}{\rho t} \approx \left[e^{\rho\tau_p} - \rho\tau_p\right]^{-1} - 1$$

Cascade pile-up + extending dead time

Pile-up eliminates closely spaced events and ‘reduces dead time’



count gain
through pile-up

$$R = \rho e^{-\rho \tau_p} (1 - P_{loss})$$

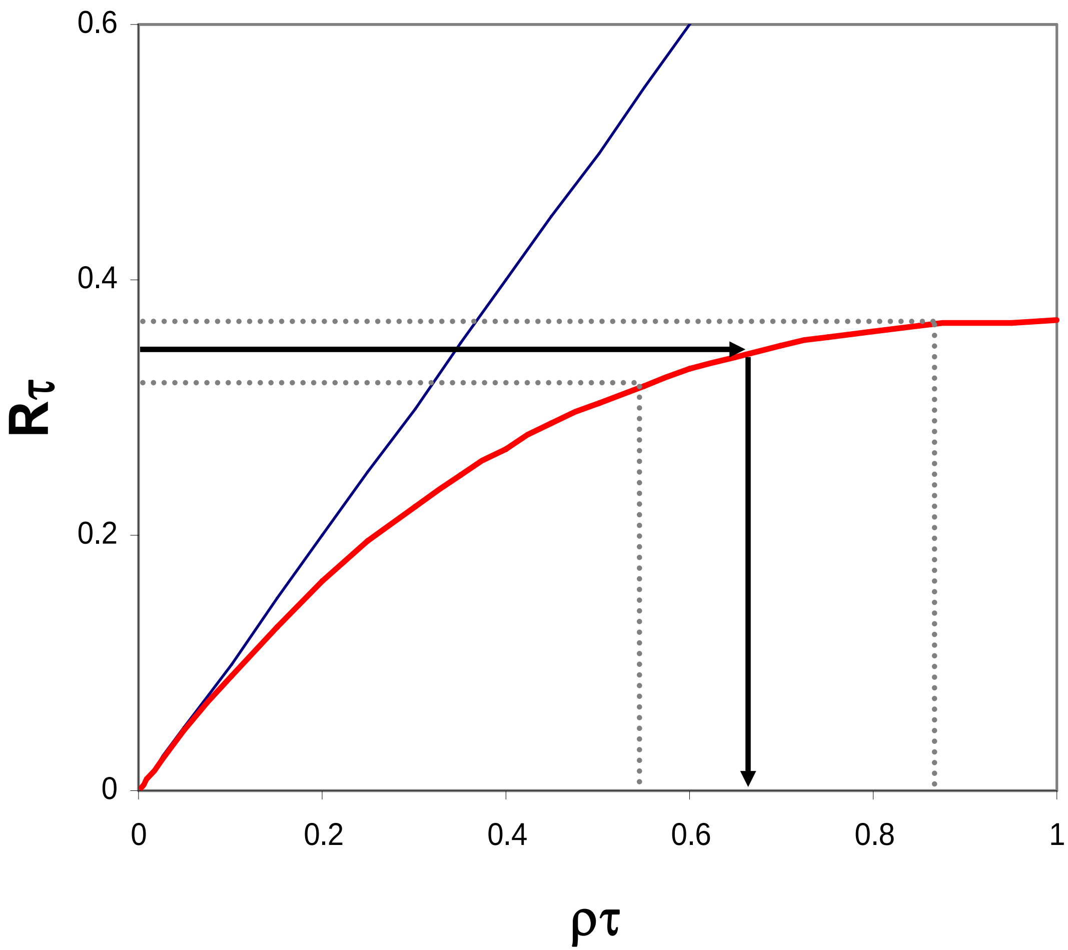
P_{loss}= probability count loss by extending dead time

instead of

$$R = \rho e^{-\rho \tau_e}$$

extending dead time

Real-Time Mode : inversion of throughput curve



extending dead time

$$\rho = R e^{\rho \tau_e}$$

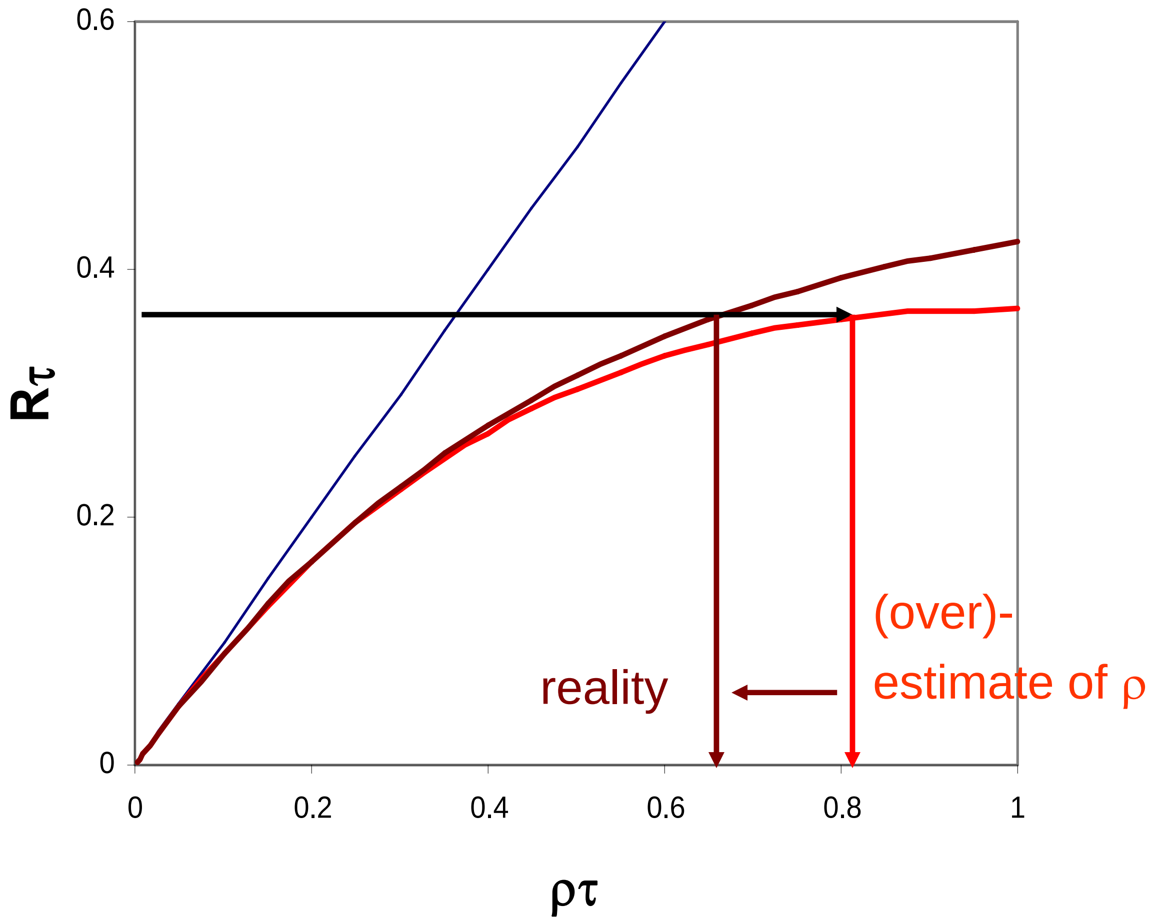
by iteration

+ random component

+ unc. propagation $\sigma(\tau_e)$

+ cascade effect

Real-Time Mode : error by neglecting cascade effect



extending dead time

$$\rho = \frac{R e^{\rho \tau_e}}{1 - P_{loss}}$$

by iteration

$$P_{loss} = -\sum_{j=1}^{J} \frac{[-\rho(\tau_e - j\tau_p) e^{-\rho\tau_p}]^j}{j!}$$

Count loss correction by inversion of throughput curve

extending dead time

τ_p = 10 μs ± 1 μs

τ_e = 15 μs ± 0.15 μs

+ random component = 0.11%

$$\sigma(\rho t) = \frac{1}{\sqrt{Rt}} \sqrt{\frac{1 - 2\rho\tau_e e^{-\rho\tau_e}}{(1-\rho\tau_e)^2}}$$

+ propagation unc. τ_e = 0.17%

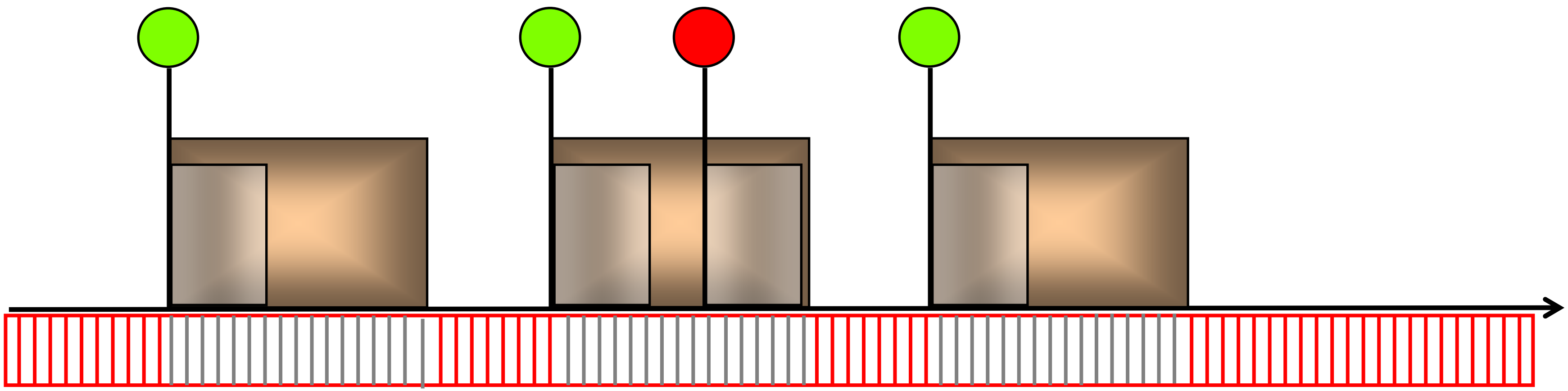
$$\frac{\sigma(\rho t)}{\rho t} = \frac{\rho\tau_e}{1-\rho\tau_e} \frac{\sigma(\tau_e)}{\tau_e}$$

+ cascade effect = 0.44%

$$\frac{\varepsilon(\rho t)}{\rho t} \approx \left[\frac{e^{\rho(\tau_e - \tau_p)}(1 - P_{loss}) - e^{\rho\tau_e}}{1 - \rho\tau_e} \right]$$

Live-time technique

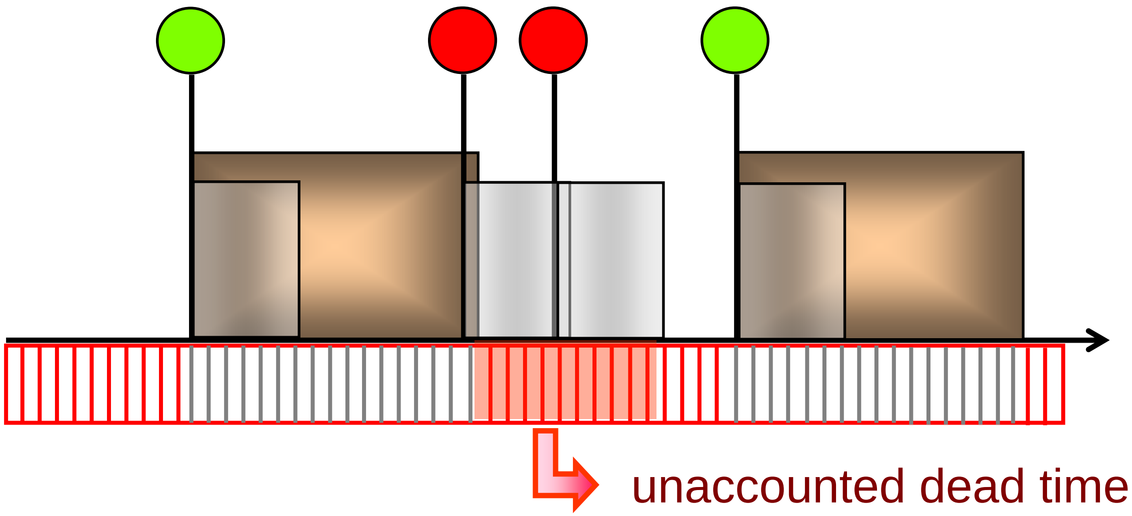
Keep track of live time of system at fixed frequency



$$R = \rho \frac{\text{Live Time}}{\text{Real Time}}$$

Cascade pile-up + non-extending dead time

Live-time technique underestimates count rate ρ



$$\rho = R \frac{\text{Real Time}}{\text{Live Time}} \left[e^{\rho\tau_p} - \rho\tau_p \right]$$

by iteration

instead of

$$\rho = R \frac{\text{Real Time}}{\text{Live Time}}$$

non-extending dead time

Count loss correction by live-time technique

non-extending dead time

τ_p = 10 μs ± 1 μs

τ_{ne} = 15 μs

+ random component = 0.107%

$$\sigma(\rho t) = \frac{1}{\sqrt{Rt}}$$

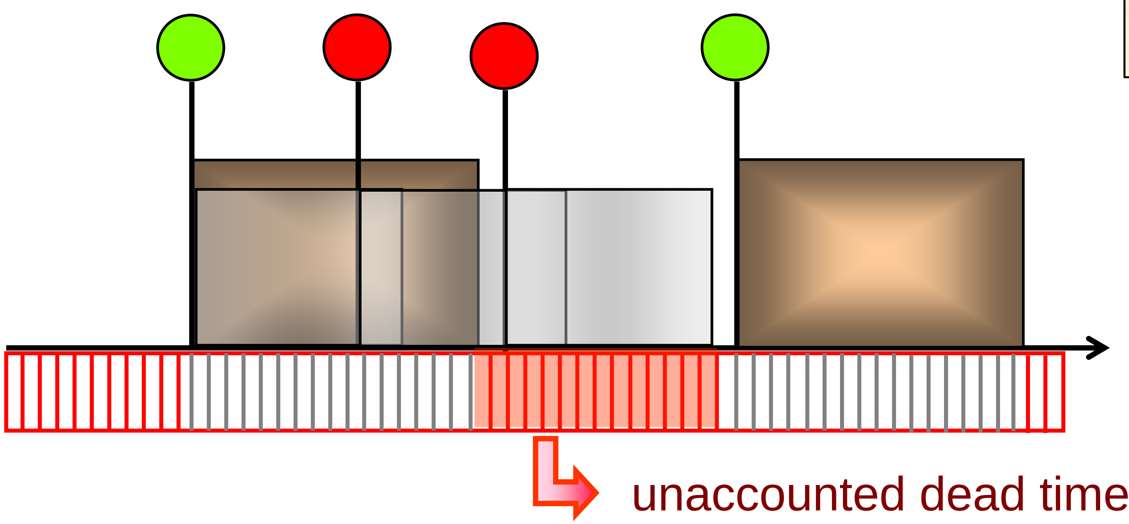
+ propagation unc. τ_{ne} ≈ 0%

+ cascade effect = 0.51%

$$\frac{\varepsilon(\rho t)}{\rho t} \approx \left[e^{\rho\tau_p} - \rho\tau_p\right]^{-1} - 1$$

Cascade pile-up + extending dead time

Live-time technique (slightly) underestimates count rate ρ



$$\rho \approx \frac{R}{1 - P_{loss}} \exp\left(\frac{\text{Real Time}}{\text{Live Time}} R \tau_p \right)$$

by iteration

instead of

$$\rho = R \frac{\text{Real Time}}{\text{Live Time}}$$

extending dead time

Count loss correction by live-time technique

extending dead time

τ_p = 10 µs ± 1 µs

τ_e = 15 µs

+ random component = 0.11%

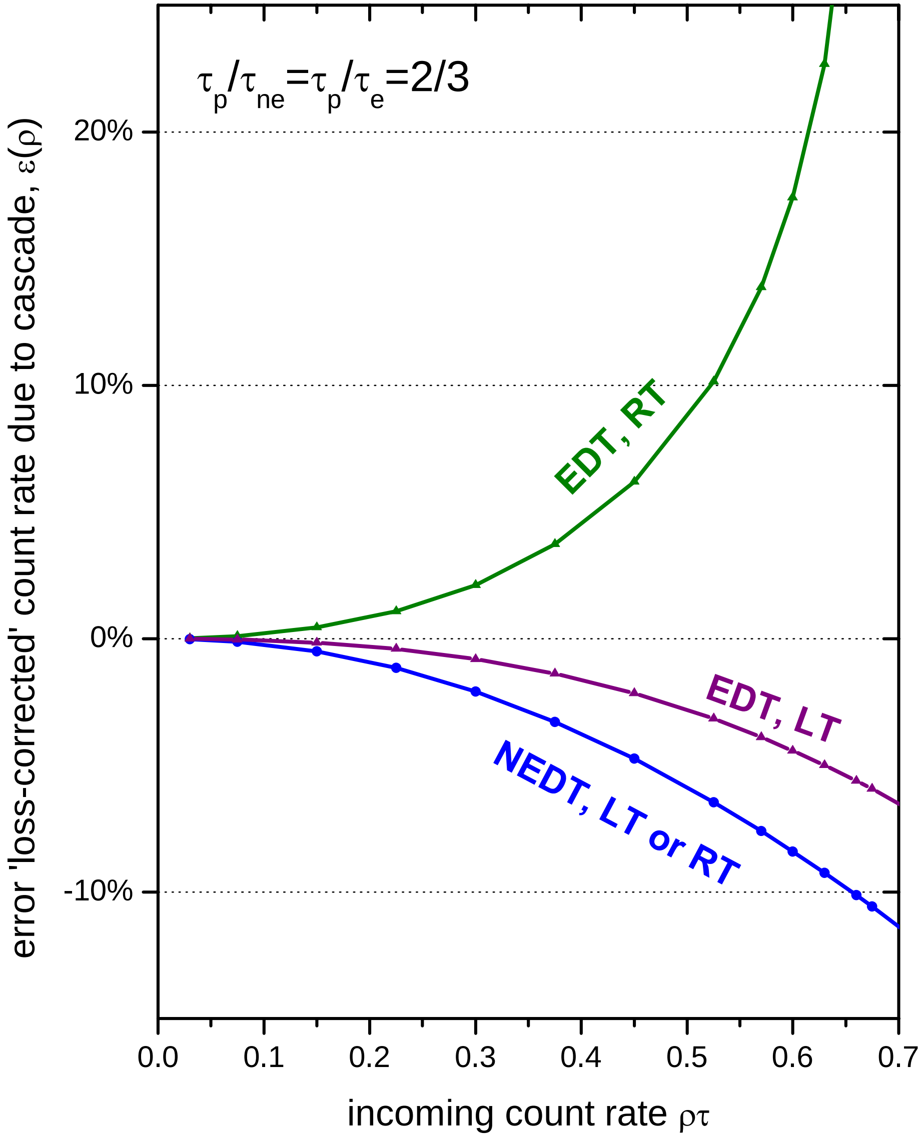
$$\sigma(\rho t) = \frac{1}{\sqrt{Rt}}$$

+ propagation unc. τ_e ≈ 0%

+ cascade effect = 0.17%

$$\frac{\varepsilon(\rho t)}{\rho t} \approx \left[1 + \rho\tau_p - e^{\rho\tau_p}\right]\left(\frac{\tau_p}{\tau_e}\right)$$

influence of cascade on corrected count rate



Uncertainty after mathematical correction for cascade effect

propagation of uncertainty on pulse width

τ_p = 10 μs ± 1 μs

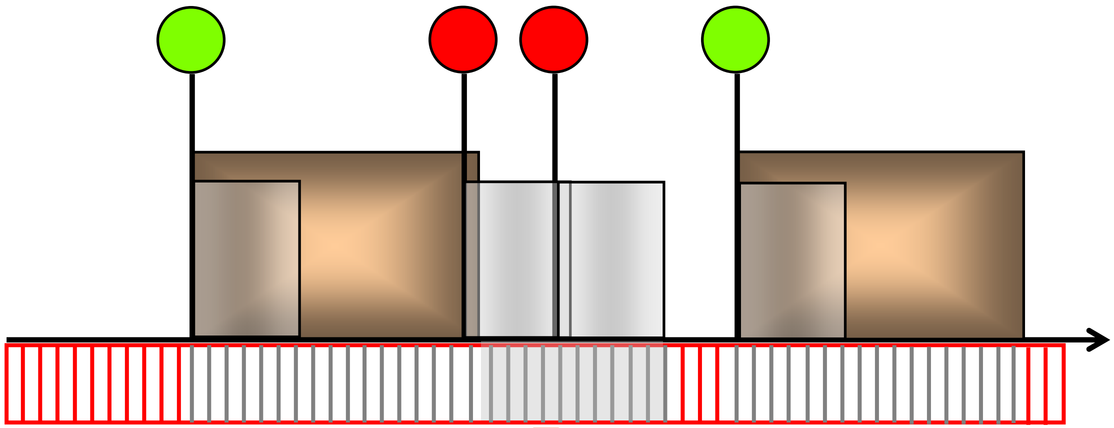
τ_e = 15 μs

non-extending dead time ≈ 0.1%

extending dead time ≈ 0.01%

Electronic solution for live-time technique

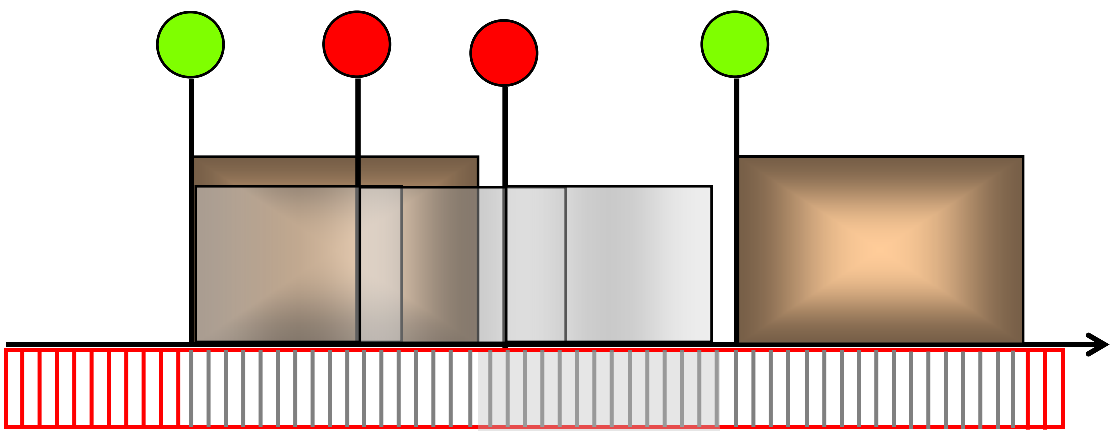
Logical OR of dead time and pile-up



non-extending dead time

account for extra ‘dead time’

$$\rho = R\,\frac{\text{Real Time}}{\text{Live Time}}$$



extending dead time

Homework ☺

Does the electronic solution eliminate all systematic errors of nuclear counting?

What is the influence of the finite resolution of the internal live-time clock on the dead-time correction?

Conclusion: How to reduce cascade effect?

* Apply live-time technique, using a logical OR between generated dead time and pulse width

or

* Apply mathematical correction factors on observed count rates

or

* Keep count rate below $0.04/\tau_p$ (e.g. $\rho<4000\ s^{-1}$ for $\tau_p=10\ \mu s$)