A precise CC bridge for QHR standard in NIM

Zhang Zhonghua and He Qing, NIM

Mutual comparison of resistance standard between NIM and ETL

M. Nakanishi, J. Kinoshita T. Endo, ETL and
Z. Zhang, H. Shao, Q. He, B. Liang, NIM
A Precise CCC Bridge for QHR Standard in NIM

Zhang Zhonghua and He Qing

(National Institute of Metrology, 100013 Beijing, China)

Abstract - A precise ccc bridge for QHR standard was completed at NIM in 2000. Applying a series of new technique, this bridge has got very precise ratio and good tracking ability. A ratio of 8002/62 is used for the comparison of QHR(i=2) and 100Ω. A bilateral comparison of 1Ω resistors with ETL of Japan was carried out in October of 2000. The difference between the values measured by NIM and ETL is within the uncertainty limit and only a difference of 1.3nΩ was found.

Introduction

The QHR standard has been applied worldwide since 01/01/1990. The comparison of non-integer value of QHR to ordinary decimal resistance can be carried out satisfactorily with a cryogenic current comparator (ccc) bridge. This kind of bridges was developed in different laboratories and comparison uncertainties of 10^-8 to 10^-9 were obtained [1-6].

A QHR standard equipment was built at NIM in the early of nineties and the ccc work began in 1996. A ccc bridge was completed in 2000 with uncertainties of several parts in 10^-9 for comparison of QHR to 100Ω or 100Ω to 1Ω. Then a bilateral comparison of 1Ω resistors with ETL of Japan was carried out in October of 2000. The difference between the values measured by NIM and ETL is within the uncertainty limit and only a difference of 1.3nΩ was found.

1. The QHR standard

The effort to establish a QHR standard system began in 1987 at NIM. In 1988 the first result of measuring QHR(i=2) by means of the cross-capacitor method was obtained and then was submitted to BIPM for determining the SI value of von Klitzing constant[7]. The magnet system is from Oxford Instrument Ltd.. Its highest field is 12T and may reach to 13.5T by the lambda plate technique. QHR samples being used were kindly offered by Dr. Delahaye (BIPM). Samples from Dr. Endo (ETL), Dr. Bachmair (PTB) and Dr. Bruus (DFM) were also tested. The difference of their QHR values was found below several parts in 10^-9.
2. The ccc windings

In the early time the comparison of QHR with the decimal resistance was carried out at NIM with a special resistor network [8]. Later a ccc bridge was constructed for the same purpose. An essential part of this bridge is the ratio windings surrounded by the super-conducting shielding. The turn numbers of windings are designed as a binary system: 4096, 2048, 1024, 512, 256, 128, 64, 32, 16, 8, 8, 4, 4, 2, 2, 1, 1. The advantage of such an arrangement is the ratio accuracy can be self-calibrated by connecting the windings in series. For example, according to a calibration procedure as $1 : 1, \quad (1+1) : 2, \quad (1+1+2) : 4, \quad \cdots$, etc. every winding can be calibrated. The necessary special ratio may be obtained by the combination of windings. Especially, for example, the comparison of QHR to $100\Omega$ is performed by a ratio

$$8002 : 62 = \frac{(4096+2048+1024+512+256+64+2)}{(32+16+8+4+1+1)} = 129.0645161 = 129.064035 \times (1+3.728\times10^{-6}). \quad (2.1)$$

This ratio is very close to the necessary value and the difference from the nominal value is only 3.728ppm. It is much smaller than the difference for other ratio in published papers. Hence the compensation for the resistance balancing of the bridge is easier. The structure of the windings and the super-conducting shielding is similar to the Sullivan-type[1], i.e. three layers of 0.1mm lead foil form a structure like a snake swallowing its tail.

3. The ccc bridge

The ccc bridge is shown in Fig. 3.1. A 19MHz rf-SQUID of Quantum Design is used for detecting the unbalancing flux from the ccc windings. The unbalancing voltage on resistors to be measured is detected by a N1a nano-voltmeter. The output signal of N1a is fed into a V/F converter and turned into a frequency output, which is transmitted to a computer through an optical fiber. This computer also sends a series of controlling commands operating four relays to change the polarity of the primary current source.

The principle scheme of the primary and secondary current sources is shown in Fig. 3.2. In this quite simple circuit the instrument amplifier or other additional devices are avoided, thus a better dynamic character is obtained and no parasitical high frequency oscillation was found. At the same time, very good tracking ability is also observed with this design and only 10-20 second is needed for the current direction changing. A problem of this circuit is the grounded terminal of the sampling resistor cannot be distinguished into current-terminal and potential-terminal. For a low value sampling resistor it may become a troublesome problem. Therefore, for the largest current range of 50mA the value of sampling resistor is chosen as a not very low value $100\Omega$ and for other ranges the value will be higher. Grounded conductors on PCB are thickened. The input reference voltage of the current sources is enlarged to 5V instead 0.5V, which is used in other published papers. Using a higher reference voltage will decrease the noise component in output current obviously. But, at the same time, a rather larger dissipation power on the sampling resistors has to be considered. In fact, in Fig. 3.2 low value sampling resistors are of several elements connected in parallel. Especially, for the 50mA range there are 20 pieces of $2k\Omega$ resistors in parallel to give the necessary sampling resistance value of $100\Omega$. Thus no obvious self-heating effect of the sampling resistors was observed in practical operation. In addition, the material of the PCB is specially selected.
having very good insulation to avoid the leakage problem. The diagram of the whole system is shown in Fig. 3.3.

To get a better S/N ratio for the feedback loop including the SQUID, in principle larger turn numbers of the ratio windings are preferable. But in actual operation it is found that with large turn number a current jumping corresponding to integer flux quanta could appear during the current polarity changing. Based on an analysis using the nonlinear theory, it is clarified that the reason for this kind of current jumping is due to that the SQUID output shows a periodical character vs. its input flux. A dynamic compensation on the current output may eliminate this kind of jumping. Fig. 3.4 shows the effect of the dynamic compensation. For a ratio of 2065/16, which is frequently used in published papers, the feedback circuit can be operated properly without any dynamic compensation as shown in the first case in Fig. 3.4. The curve in the figure describes the difference output voltage on resistors to be compared. Positive or negative pulses appearing on the curve during the current direction changing is caused by parasitic parameters in the circuit. If a ratio with larger turn number, e.g. a ratio of 4002/31, is used as shown in the second case of this figure, a less noise component on the curve may be observed. But sometime the current in the secondary loop could jump to another value and the bridge balancing is destroyed. It can be seen that the curve is broken with the current jumping and sampling points run away towards outside of the figure. Thus the comparison measurement has to stop. When a dynamic compensation is introduced into the bridge circuit, this kind of current jumping can be avoided. The third case in Fig. 3.4 shows that even the turn number is increased further to 8002/62, owing to the dynamic compensation the bridge still operates properly without any current jumping. Of course, in this case a much less noise component in the bridge output can be observed as shown in the figure due to the large turn number of ratio winding and a comparison result with less uncertainty may be obtained. The dynamic compensation can be designed with the help of a dynamic analyzer. In practice some capacitors connected at the output of current source may also act as the dynamic compensation. A more complicated design will give better compensation.

It is also observed that the pressure fluctuation of the helium gas in the cryostat may also affect the output of the SQUID. The pressure of the exhaust helium gas in the recovering system is not very stable. Therefore, a gas pressure low-passing filter is inserted between the cryostat and the recovering system. This filter consists of a gas container (30 liter) and two arms of thin tube with very small inner diameter (about 0.2mm) and smoothes the pressure in the cryostat effectively.

### 4. The uncertainty analysis

<table>
<thead>
<tr>
<th>Factors</th>
<th>Sort</th>
<th>Uncertainty (×10^-6, k=1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The imperfectness of the super-conductive shielding of ratio winding</td>
<td>B</td>
<td>0.001</td>
</tr>
<tr>
<td>The insulation of the circuit</td>
<td>B</td>
<td>0.0013</td>
</tr>
<tr>
<td>The accuracy of the resistance compensation circuit</td>
<td>B</td>
<td>0.001</td>
</tr>
<tr>
<td>The random deviation of the readings</td>
<td>A</td>
<td>0.003</td>
</tr>
<tr>
<td><strong>Summary</strong></td>
<td></td>
<td><strong>0.004</strong></td>
</tr>
</tbody>
</table>
5. The comparison with ETL

It is interested to compare the experiment result with other laboratory with a similar QHR and ccc system. A comparison with ETL was completed in October of 2000. Dr. Nakanishi of ETL brought three pieces of 1Ω resistors to NIM. Their values were determined previously by the QHR standard of ETL. Then these resistors were measured in NIM by the QHR and ccc system described above. When these resistors return to ETL, the values were determined again with QHR standard in ETL. The result in μΩ is within the uncertainty limit and shown as follows.

<table>
<thead>
<tr>
<th>Serial number</th>
<th>64145 unc.</th>
<th>64155 unc.</th>
<th>64162 unc.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>At ETL</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sep. 27,2000</td>
<td>-12.7319 ± 0.0050</td>
<td>-6.5035 ± 0.0051</td>
<td>-0.0312 ± 0.0061</td>
</tr>
<tr>
<td>Nov. 2,2000</td>
<td>-12.7183 ± 0.0043</td>
<td>-6.4850 ± 0.0043</td>
<td>-0.0058 ± 0.0042</td>
</tr>
<tr>
<td><strong>At NIM</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oct. 25,2000</td>
<td>-12.7169 ± 0.0072</td>
<td>-6.4881 ± 0.0084</td>
<td>-0.0126 ± 0.0070</td>
</tr>
<tr>
<td><strong>Reference value</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From ETL</td>
<td>-12.7208</td>
<td>-6.4891</td>
<td>-0.0117</td>
</tr>
<tr>
<td>Difference</td>
<td>+0.0039</td>
<td>+0.0010</td>
<td>-0.0009</td>
</tr>
<tr>
<td>Average</td>
<td>+0.0013</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

References

Fig 3.1  The CCC bridge

Fig 3.2  The principle scheme of the current source
Fig. 3.3  The diagram of the whole system
Without compensation, noise is less but current jumping appears.

With compensation, noise is less further and no current jumping.

Fig. 3.4 The effect of dynamic compensation
Mutual comparison of resistance standard between NIM and ETL

Masakazu Nakazishi, Joji Kinoshita, Tadashi Endo,
Electrotechnical Laboratory
I-1-4, Umecno, Tsukuba-shi, Ibaraki, 305-8568, JAPAN
Zhanghua Zhang, Haiming Shao, Qin He and Bo Liang
National Institute of Metrology
No. 18, Bei San Huan Dong Lu, Beijing, 100013, CHINA

Abstract—The resistance standard maintained at the National Institute of Metrology (NIM) was compared to that at the Electrotechnical Laboratory (ETL) by bringing three 1-$\Omega$ resistors used as transfer resistors between both laboratories. The difference was measured to be smaller than a few parts in $10^5$, which was smaller than the combined standard uncertainty of the resistance measurements by the ETL's resistance standard and that by the NIM's resistance standard.

I. INTRODUCTION

The quantum Hall effect standard of resistance at the Electrotechnical Laboratory (ETL's resistance standard) was developed at 1980 [1] and improved at 1997 [2], in which the second or the forth plateau of the quantized Hall resistance ($R_H(2)$ or $R_H(4)$) was used as the standard of resistance. The quantum Hall effect standard of resistance at the National Institute of Metrology (NIM's resistance standard) was developed at 1991 [3]. However, the artificial 1-$\Omega$ resistors had been used as the standards of resistance because of the inconvenience of the bridge used to compare the resistors to the $R_H(2)$. Two of the authors (Z. Zhang and Q. He) finished to develop a resistance ratio bridge using a cryogenic current comparator (CCC bridge) at the middle of 2000 in order to replace the bridge with it and to improve the performance of the NIM's resistance standard.

The NIM's resistance standard was compared to the ETL's resistance standard by bringing three 1-$\Omega$ resistors from the ETL to the NIM in October 2000, which was the first trial to check its performance including the brandnew CCC bridge. The differences of the resistances measured by the NIM's resistance standard from those by the ETL's resistance standard were smaller than a few parts in $10^5$, which was smaller than the combined standard uncertainty of the resistance measurements by the ETL's resistance standard and that by the NIM's resistance standard. This paper reports the procedure and the results of the comparison.

The resistances of the artificial 1-$\Omega$ resistors, which had been used as the standard of resistance at the NIM, were measured by the ETL's resistance standard at November 1992, October 1996 and October 2000 by bringing transfer resistors between both laboratories. The results of these measurements were also reported.

II. PROPERTIES OF 1-$\Omega$ RESISTORS

The three 1-$\Omega$ resistors used as the transfer resistors were fabricated by the Commonwealth Science and Industrial Research Organization-National Measurement Laboratory (CSIRO-NML). We called them 64145, 64155 and 64162, respectively, after their serial numbers. Their temperature coefficients are so small (Table 1) that we use them as the transfer resistors. The resistors were used in the oil baths. The temperature difference of the oil bath used in the NIM from that used in the ETL were measured by using a thermistor whose resistance was measured by using digital multi meters (model HP3458A, manufactured by Hewlett-Packard). The resistance of the thermistor was measured by using the ETL's HP3458A at the ETL and by using the NIM's HP3458A at the NIM. Accuracy of the resistance measurement by using the HP3458A is not high but its linearity is high enough. Therefore, a 10 k$\Omega$ resistor was used as a reference. The temperature coefficient of the resistance ratio of the thermistor to the 10 k$\Omega$ resistor was measured to be $0.0170$ K$^{-1}$.

The resistances of the 64145 and 64155 have been measured by the ETL's resistance standard since 1990 and that of the 64162 since 1992 (Fig.1). Their resistances seemed to change during every spring, which was most clearly shown in the 64162, although all these resistors have been usually installed in a temperature stabilized oil bath at 20 $^\circ$C to reduce both thermal and mechanical shocks. We fixed our air conditioner at the summer of 1997 to control the humidity not to exceed 55 %, which can dehumidify but not humidify the air supplied from it. We have been monitored the humidity in the experimental room since the beginning of 1997. The humidity goes down below 30 % in winters but is kept at about 55 % in the rest of the year. Their resistance became more stable after the summer of 1997 except winter (Fig.2). The relatively large changes of their resistances during the winters were roughly correlated to the low humidity, even though the moisture in the oil, in which the resistors were immersed, was not monitored.

The comparison between the NIM's and the ETL's resistance standards was carried out from the end of September to the beginning of November 2000. The resistance drifts in this season were usually so small and smooth that we could correct them by the linear

<table>
<thead>
<tr>
<th>serial number</th>
<th>$\alpha_{00}/(\mu\Omega/0K)$</th>
<th>$\beta/(\mu\Omega/0K^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>64145</td>
<td>0.00265</td>
<td>0.0023</td>
</tr>
<tr>
<td>64155</td>
<td>0.0010</td>
<td>0.0028</td>
</tr>
<tr>
<td>64162</td>
<td>0.0078</td>
<td>0.0028</td>
</tr>
</tbody>
</table>

TABLE 1

Temperature coefficients of the CSIRO-NML made 1 $\Omega$ resistors at 20 $^\circ$C.
interpolation with high accuracy.

III. RESISTANCE MEASUREMENTS

A. Resistances measured by the ETL’s resistance standard

The 64145, 64155 and 64162 were measured by the ETL’s resistance standard at September 27 and November 2, 2000 (Table 2). Procedure of its resistance measurement is briefly explained hereafter [2]. We used the $R_H(2)$ as a standard. A 100-$\Omega$ resistor was directly compared to the $R_H(2)$ by using a CCC bridge which was developed in the ETL [4]. It was manufactured by the TEGAM, its model number was SR102/DC and its serial number was A2010397SR102D. It was installed in a temperature stabilized oil bath at 20 °C. The 1-$\Omega$ resistors were compared to the 100-$\Omega$ resistor by using the CCC bridge.

B. Resistances measured by the NIM’s resistance standard

The 64145, 64155 and 64162 were measured by the NIM’s resistance standard at October 25, 2000 (Table 2). The procedure of the resistance measurement is briefly explained hereafter. A 100-$\Omega$ resistor was compared to the $R_H(2)$ at October 24, 2000. It was manufactured by the TEGAM, its model number was SR102 and its serial number was A2011198. The stability of its temperature is not high enough although it is housed in a temperature controlled air bath. Its temperature was measured by a thermometer installed in it. Effect of its temperature change was corrected.
by using its temperature dependence, which was measured to be \( \Delta(T) = \Delta(23) - 2.23 \times (R_T - 100.127)^2 \times 10^{-8} \), where \( \Delta(T) \) expresses deviation of the resistance from its nominal value of 100 \( \Omega \) at \( T \) \( ^\circ \)C and \( R_T \) expresses the resistance of the thermometer. Its temperature dependence peaks at 23 \( ^\circ \)C. Therefore, no first order term was given in the dependence.

The resistance of the 100-\( \Omega \) resistor was measured to be \( 100 \times (1 - 0.0031 \times 10^{-6} \pm 2.2 \times 10^{-9}) \Omega \) at October 24, where \( \pm \) expresses the root sum squares of the measured type-A standard uncertainty of 12 \( \times \) 10\( ^{-9} \) and the estimated type-B standard uncertainty of 1.9 \( \times \) 10\( ^{-9} \). The \( R_T \) was measured to be 100.0370 \( \Omega \).

(a) 64145

The resistance ratio of the 100-\( \Omega \) resistor to the 64145 was measured to be \( 100 \times (1 + 12.727 \times 10^{-8} \pm 6.9 \times 10^{-9}) \) at October 25, where \( \pm \) expresses the root sum squares of the measured type-A standard uncertainty of 6.4 \( \times \) 10\( ^{-9} \) and the estimated type-B standard uncertainty of 1.9 \( \times \) 10\( ^{-9} \). The resistance of the 64145 was given to be \( 1 \times (1 - 12.7159 \times 10^{-8} \pm 7.2 \times 10^{-9}) \Omega \) after compensating the temperature change of the 100-\( \Omega \) resistor (\( R_T = 100.0371 \Omega \)). The temperature of the oil bath, in which the \( \Omega \)-resistor was installed, was measured to be 19 mK higher than that of the ETL's oil bath. Its effect was calculated to be \( +5.0 \times 10^{-10} \) \( \Omega \).

(b) 64155

The resistance ratio of the 100-\( \Omega \) resistor to the 64155 was measured to be \( 100 \times (1 + 6.5023 \times 10^{-9} \pm 8.1 \times 10^{-9}) \) at October 25, where the type-A standard uncertainty was measured to be 7.9 \( \times \) 10\( ^{-9} \) and the type-B standard uncertainty was estimated to be 1.9 \( \times \) 10\( ^{-9} \). The resistance of the 64155 was given to be \( 1 \times (1 - 6.4881 \times 10^{-6} \pm 8.4 \times 10^{-9}) \Omega \) after compensating the temperature change of the 100-\( \Omega \) resistor (\( R_T = 100.0001 \Omega \)). The temperature of the oil bath was measured to be +7 mK higher than that of the ETL's oil bath. Its effect was calculated to be \( +0.7 \times 10^{-10} \) \( \Omega \).

(c) 64162

The resistance ratio of the 100-\( \Omega \) resistor to the 64162 was measured to be \( 100 \times (1 + 0.0283 \times 10^{-6} \pm 6.6 \times 10^{-9}) \) at October 25, where the type-A standard uncertainty was measured to be 6.3 \( \times \) 10\( ^{-9} \) and the type-B standard uncertainty was estimated to be 1.9 \( \times \) 10\( ^{-9} \). The resistance of the 64162 was given to be \( 1 \times (1 - 0.0126 \times 10^{-6} \pm 0.9 \times 10^{-9}) \Omega \) after compensating the temperature change of the 100-\( \Omega \) resistor (\( R_T = 100.127 \Omega \)). The temperature of the oil bath was measured to be 166 mK lower than that of the ETL's oil bath. Its effect was calculated to be \( -1.66 \times 10^{-10} \) \( \Omega \).

### IV. Discussion and Conclusion

Performance of the 64145, 64155 and 64162 have been measured by the ETL's resistance standard more than 8 or 10 years. Therefore, we decided to use their resistances measured by the ETL's resistance standard as the reference values. Their drifts were corrected by the linear interpolation and the effect of the temperature difference of the NIM's oil bath from the ETL's oil bath were also corrected.

Differences of the resistances, measured by the NIM's resistance standard at October 25, 2000, from the reference values were given as +3.9 m\( \Omega \), +1.0 m\( \Omega \) and -0.9 m\( \Omega \), respectively (Table 2), all of which were smaller than the combined standard uncertainty of the ETL's resistance standard and smaller than that of the NIM's resistance standard (Fig. 3). We concluded that there was no difference between the resistance measurements by the NIM's resistance standard and by the ETL's resistance standard.

Six 1-\( \Omega \) resistors had been used to maintain the standard of resistance of the NIM's resistance standard instead of the quantized Hall resistance (QHR). They were manufactured in the Union of Soviet Socialist Republics (USSR) and their serial numbers were 00713, 00806, 001006, 001961, 002250 and 002254, respectively. They were compared to the 64145, 64155 and 64162 at October 20, 2000, by using a resistance ratio bridge using a direct current comparator (DCC bridge) manufactured by the Measurement International, whose model number was 60108. The temperature of the NIM's oil bath was measured to be 6.5 mK lower than that of the ETL's oil bath. Its effect was smaller than 2 \( \times \) 10\( ^{-10} \) \( \Omega \) and was corrected. The \( R_{\text{NIM}} \) given as the average of their resistances was measured to be -0.0166 m\( \Omega \) with respect to the ETL's resistance standard based on the QHR. The \( R_{\text{NIM}} \) due to
The artificial 1-Ω resistors had been used as the standard of resistance at NIM. The $\Omega_{NIM}$ was given as their average. The closed rectangles show the measured $\Omega_{NIM}$ with respect to the ETL's resistance standard via transfer resistors brought between both laboratories at November 1, 1992, October 23, 1996 and October 20, 2000. The broken line shows its drift whose rate was given to be 0.024 $\mu\Omega$/year.

### REFERENCES


### APPENDIX

The quantum Hall effect (QHE) system for the NIM's resistance standard is briefly explained hereafter. A 13 T superconducting magnet with a $^4$He insert was manufactured by by the Oxford. Temperature of a QHE sample was cooled down to about 1.5 K. The QHE sample manufactured by the Laboratoires d'Electronique Philips (LEP, Limoges-Brevannes, France) was used. Magnetic field was fixed to about 10.5 T, which was the center magnetic field of the second plateau. Electron density of its two dimensional layer was given to be $5.1 \times 10^{15}$ m$^{-2}$. Its longitudinal resistivity at 10.5 T was measured to be 0.22 m$\Omega$. The longitudinal resistivity is given to be 0.0352 m$\Omega$ as its aspect ratio is 0.25 (0.5mm/0.8mm). Its longitudinal resistivity with no magnetic field was measured to be 77.8 $\Omega$. The longitudinal resistivity was given to be 124 $\Omega$ and electron mobility was given to be 98.7 T$^{-1}$.