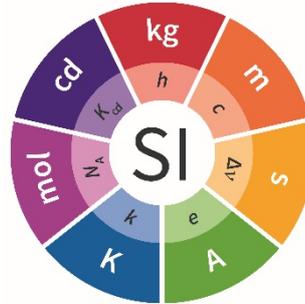# S I

A concise summary of the International System of Units, SI

Metrology is the science of measurement and its application. Metrology includes all theoretical and practical aspects of measurement, whatever the measurement uncertainty and field of application.

The International Bureau of Weights and Measures (BIPM) was established by Article 1 of the Metre Convention, which was signed on 20 May 1875. It is charged with providing the basis for a single, coherent system of measurements to be used throughout the world and it operates under the authority of the International Committee of Weights and Measures (CIPM). The decimal metric system, dating from the time of the French Revolution, was based in 1799 on the metre and the kilogram. Under the terms of the Metre Convention, new international prototypes of the metre and kilogram were manufactured and formally adopted by the first General Conference on Weights and Measures (CGPM) in 1889. In 1960 the 11th CGPM formally defined and established the International System of Units (SI). Since then the SI has been periodically updated to take account of advances in science and the need for measurements in new domains. The last major revision was adopted by the 26th CGPM (2018), which decided that the SI would be based on the fixed numerical values of a set of seven **defining constants** from which the definitions of the seven base units of the SI would be deduced**.** This document is a summary of the **SI Brochure**, a publication produced by the BIPM, which gives a detailed explanation of the current status of the SI.

The SI is the system of units in which:

- the unperturbed ground state hyperfine transition frequency of the caesium 133 atom $\Delta\nu_{Cs}$ is 9 192 631 770 Hz,

- the speed of light in vacuum c is 299 792 458 m/s,

- the Planck constant h is 6.626 070 15 $\times$ 10^{-34} J s,

- the elementary charge e is 1.602 176 634 $\times$ 10^{-19} C,

- the Boltzmann constant k is 1.380 649 $\times$ 10^{-23} J/K,

- the Avogadro constant N_A is 6.022 140 76 $\times$ 10^{23} mol^{-1},

- the luminous efficacy of monochromatic radiation of frequency 540 $\times$ 10^{12} Hz, $K_{cd,}$ is 683 lm/W,

where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, and W, respectively, are related to the units second, metre, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, and cd, respectively, according to Hz = s^{-1}, J = kg m^2 s^{-2}, C = A s, lm = cd m^2 m^{-2} = cd sr, and W = kg m^2 s^{-3}.

These definitions specify the exact numerical value of each constant when its value is expressed in the corresponding SI unit. By fixing the exact numerical value the unit becomes defined, since the product of the *numerical value* and the *unit* has to equal the *value* of the constant, which is invariant.

The defining constants have been chosen such that, when taken together, their units cover all of the units of the SI. In general, there is no one-to-one correspondence between the defining constants and the SI base units, except for the caesium frequency $\Delta\nu_{Cs}$ and the Avogadro constant N_A. Any SI unit is a product of powers of these seven constants and a dimensionless factor.

For example: using Hz = s^{-1}, one metre can be derived from the speed of light c and caesium frequency $\Delta\nu_{Cs}$,

$$1\,\text{m} = \left(\frac{c}{299\,792\,458}\right)\text{s} = \frac{9\,192\,631\,770}{299\,792\,458}\frac{c}{\Delta\nu_{Cs}} \approx 30.663\,319\frac{c}{\Delta\nu_{Cs}}$$

The concept of base units and derived units was used to define the SI until 2018. These categories, although no longer essential in the SI, are maintained in view of their convenience and widespread use. The definitions of the base units, which follow-from the definition of the SI in terms of the seven defining constants, are given in Table 1.

Table 1   *The seven base units of the SI*

Quantity	SI unit
time	The **second**, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{Cs}$, the unperturbed ground-state hyperfine transition frequency of the caesium 133 atom, to be 9 192 631 770 when expressed in the unit Hz, which is equal to s^{-1}.
length	The **metre**, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299 792 458 when expressed in the unit $m\ s^{-1}$, where the second is defined in terms of $\Delta\nu_{Cs}$.
mass	The **kilogram**, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be $6.626\ 070\ 15 \times 10^{-34}$ when expressed in the unit J s, which is equal to $kg\ m^2\ s^{-1}$, where the metre and the second are defined in terms of c and $\Delta\nu_{Cs}$.
electric current	The **ampere**, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602\ 176\ 634 \times 10^{-19}$ when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu_{Cs}$.
thermodynamic temperature	The **kelvin**, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be $1.380\ 649 \times 10^{-23}$ when expressed in the unit $J\ K^{-1}$, which is equal to $kg\ m^2\ s^{-2}\ K^{-1}$, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{Cs}$.
amount of substance	The **mole**, symbol mol, is the SI unit of amount of substance. One mole contains exactly $6.022\ 140\ 76 \times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number. The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.
luminous intensity	The **candela**, symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, K_{cd}, to be 683 when expressed in the unit $lm\ W^{-1}$, which is equal to $cd\ sr\ W^{-1}$, or $cd\ sr\ kg^{-1}\ m^{-2}\ s^3$, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{Cs}$.

All other quantities may be called "derived quantities" and are measured using derived units, which can be written as products of powers of base units. Twenty-two derived units are given a special name, as listed in Table 2.

Table 2 *Derived units with special names in the SI*

Derived quantity	Name of derived unit	Symbol for unit	Expression in terms of other units
plane angle	radian	rad	m/m
solid angle	steradian	sr	m^2/m^2
frequency	hertz	Hz	s^{-1}
force	newton	N	$kg\ m\ s^{-2}$
pressure, stress	pascal	Pa	$N/m^2 = kg\ m^{-1}\ s^{-2}$
energy, work, amount of heat	joule	J	$N\ m = kg\ m^2\ s^{-2}$
power, radiant flux	watt	W	$J/s = kg\ m^2\ s^{-3}$
electric charge	coulomb	C	$A\ s$
electric potential difference	volt	V	$W/A = kg\ m^2\ s^{-3}\ A^{-1}$
capacitance	farad	F	$C/V = kg^{-1}\ m^{-2}\ s^4\ A^2$
electric resistance	ohm	Ω	$V/A = kg\ m^2\ s^{-3}\ A^{-2}$
electric conductance	siemens	S	$A/V = kg^{-1}\ m^{-2}\ s^3\ A^2$
magnetic flux	weber	Wb	$V\ s = kg\ m^2\ s^{-2}\ A^{-1}$
magnetic flux density	tesla	T	$Wb/m^2 = kg\ s^{-2}\ A^{-1}$
inductance	henry	H	$Wb/A = kg\ m^2\ s^{-2}\ A^{-2}$
Celsius temperature	degree Celsius	^{o}C	K
luminous flux	lumen	lm	$cd\ sr = cd$
illuminance	lux	lx	$lm/m^2 = cd\ m^{-2}$
activity referred to a radionuclide	becquerel	Bq	s^{-1}
absorbed dose, kerma	gray	Gy	$J/kg = m^2\ s^{-2}$
dose equivalent	sievert	Sv	$J/kg = m^2\ s^{-2}$
catalytic activity	katal	kat	$mol\ s^{-1}$

Although the hertz and the becquerel are both equal to the reciprocal second, hertz is used only for periodic phenomena, and becquerel is used only for stochastic processes in radioactive decay.

The unit of Celsius temperature is the degree Celsius, ^{o}C, which is equal in magnitude to the kelvin, K, the unit of thermodynamic temperature. The quantity Celsius temperature t is related to thermodynamic temperature T by the equation $t/^{o}C = T/K - 273.15$.

The sievert is also used for the quantities 'directional dose equivalent' and 'personal dose equivalent'.

There are many more quantities than units. For each quantity, there is only one SI unit (although this may often be expressed in different ways by using the special names), while the same SI unit may be used to express the values of several different quantities (for example, the SI unit J/K may be used to express the value of both heat capacity and entropy). It is therefore important not to use the unit alone to specify the quantity. This applies both to scientific texts and also to measuring instruments (i.e. an instrument read-out should indicate both the quantity concerned and the unit).

There are quantities with the unit one, 1, i.e. ratios of two quantities of the same kind. For example, refractive index is the ratio of two speeds, and relative permittivity is the ratio of the permittivity of a dielectric medium to that of free space. There are also quantities with the character of a count, for example, the number of cellular or biomolecular entities. These quantities also have the unit one. The unit one is by nature an element of any system of units. Quantities with the unit one can therefore be considered as traceable to the SI. However, when expressing the values of dimensionless quantities, the unit 1 is not written.

Decimal multiples and sub-multiples of SI units

A set of prefixes have been adopted for use with the SI units in order to express the values of quantities that are either much larger than, or much smaller than, the SI unit when used without any prefix. They can be used with any SI unit. The SI prefixes are listed in Table 3.

Table 3 *The SI prefixes*

Factor	Name	Symbol	Factor	Name	Symbol
10^1	deca	da	10^{-1}	deci	d
10^2	hecto	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a
10^{21}	zetta	Z	10^{-21}	zepto	z
10^{24}	yotta	Y	10^{-24}	yocto	y

When the prefixes are used, the prefix name and the unit name are combined to form a single word. Similarly, the prefix symbol and the unit symbol are written without any space to form a single symbol, which may itself be raised to any power. For example, we may write: kilometre, km; microvolt, μV; or femtosecond, fs.

When the SI units are used without any prefixes, the resulting set of units is described as being **coherent** in the following sense: when only coherent units are used, equations between the numerical values of quantities take exactly the same form as the equations between the quantities themselves. The use of a coherent set of units has technical advantages, for example in algebraic calculus (see the SI Brochure).

The kilogram, kg, is problematic because the name already includes a prefix, for historical reasons. Multiples and sub-multiples of the kilogram are written by combining prefixes with the gram: thus we write milligram, mg, not microkilogram, μkg.

Units outside the SI

The SI is the only system of units that is universally recognized, so it has a distinct advantage for establishing international dialogue. The use of the SI, as a standard system of units, simplifies the teaching of science. For these reasons, the use of SI units is recommended in all fields of science and technology. Other units, i.e. non-SI units, are generally defined in terms of SI units using conversion factors.

Nonetheless, some non-SI units are still widely used. A few, such as the minute, hour and day as units of time, will always be used because they are part of our culture. Others are used for historical reasons, to meet the needs of special interest groups, or because there is no convenient SI alternative. It will always remain the prerogative of a scientist to use the units that are considered to be best suited to the purpose. However, when non-SI units are used, the correspondence to the SI should always be quoted. A selection of non-SI units is listed in Table 4 with their conversion factors to the SI. For a more comprehensive list, see the SI Brochure.

Table 4 *A selection of non-SI units*

Quantity	Unit	Symbol	Relation to SI
time	minute	min	1 min = 60 s
time	hour	h	1 h = 3600 s
time	day	d	1 d = 86 400 s
volume	litre	L or l	1 L = 1 dm^3
mass	tonne	t	1 t = 1000 kg
energy	electronvolt (e/C) J	eV	1 eV = 1.602 176 634 × 10^{-19} J

When units are named after an individual their symbol should begin with a capital letter (for example: ampere, A; kelvin, K; hertz, Hz; or coulomb, C). In all other cases, except the litre, they begin with a lower case letter (for example: metre, m; second, s; or mole, mol). The symbol for the litre is an exception; either a lower case letter 'l' or a capital 'L' may be used, the capital is allowed in this case to avoid confusion between the lower case letter l and the number one, 1.

The language of science: using the SI to express the values of quantities

The value of a quantity is written as the product of a number and a unit. The number multiplying the unit is the numerical value of the quantity in that unit. A single space is always left between the number and the unit. The numerical value depends on the choice of unit, so that the same value of a quantity may have different numerical values when expressed in different units, as in the examples provided below.

The speed of a bicycle is approximately
$v = 5.0$ m/s $= 18$ km/h.
The wavelength of one of the yellow lines of the sodium doublet is
$\lambda = 5.896 \times 10^{-7}$ m $= 589.6$ nm.

Quantity symbols are written using italic (slanting) type, and they are generally single letters of the Latin or Greek alphabet. Either capital or lower case letters may be used, and additional information on the quantity may be added as a subscript or as information in brackets.

Authorities such as the International Organization for Standardization (ISO) and international scientific unions such as the International Union of Pure and Applied Physics (IUPAP) and the International Union of Pure and Applied Chemistry (IUPAC) have specified recommended symbols for many quantities. Examples include:

T for thermodynamic temperature
C_p for heat capacity at constant pressure
x_i for the mole fraction (amount fraction) of species i
μ_r for relative permeability.

Unit symbols are written using roman (upright) type, regardless of the type used in the surrounding text. They are mathematical entities and not abbreviations; unit symbols are never followed by a full stop (except at the end of a sentence) nor by an 's' for the plural. The use of the correct form for unit symbols is mandatory, and is illustrated by examples in the SI Brochure. Unit symbols may be more than a single letter. They are written in lower case letters, the exception being that the first letter is a capital when the unit is named after a person. However, when the name of a unit is spelled in full, it should begin with a lower case letter (except at the beginning of a sentence), to distinguish the unit from the person (for example a temperature of 293 kelvin).

When writing the value of a quantity as the product of a numerical value and a unit, both the number and the unit may be treated by the ordinary rules of algebra. For example, the equation $T = 293$ K may equally be written as $T/K = 293$. This procedure is described as the use of quantity calculus, or the algebra of quantities. It is often useful to use the ratio of a quantity to its unit as the heading for columns in a table, or for labelling the axes of a graph, so that the entries in the table or the labels for the tick marks on the axes are all simply denoted by numbers. The example (Table 5) shows a table of the velocity squared versus pressure, with the columns labelled in this way.

Table 5 *Example of column heading in a table of the velocity squared versus pressure*

p/kPa	v^2/(m/s)^2
48.73	94766
72.87	94771
135.42	94784

In forming products or quotients of unit symbols the normal rules of algebra apply. In forming products of unit symbols, a space should be left between units (or alternatively a half-high centred dot can be used as a multiplication symbol). The importance of the space should be noted: the product of a metre and a second is denoted by m s (with a space), but ms (without a space) is used to denote a millisecond. In addition, when forming complicated products of units, brackets or negative exponents should be used to avoid ambiguities. For example, the molar gas constant R is given by:

$$pV_m/T = R = 8.314 \text{ Pa m}^3 \text{ mol}^{-1} \text{ K}^{-1}$$
$$= 8.314 \text{ Pa m}^3/(\text{mol K}).$$

When formatting numbers, the decimal marker may be either a point (i.e. a full stop) or a comma, depending on the circumstances. For documents in the English language a point is usual, but for many languages and in many countries a comma is usual.

When a number has many digits, it is customary to group the digits into threes about the decimal point to aid readability. This is not essential, but it is often done and is generally helpful. When this format is used, the groups of three digits should be

separated only by a space; neither a point nor a comma should be used. The uncertainty in the numerical value of a quantity may often be conveniently shown by giving the uncertainty in the least significant digits in brackets after the number.

For example: The value of the electron mass is given in the 2014 CODATA listing of fundamental constants as

$$m_e = 9.109\ 383\ 56\ (11) \times 10^{-31}\ \text{kg},$$

where 11 is the standard uncertainty in the final digits quoted for the numerical value.

For further information, see the BIPM website, or the **SI Brochure** 9th edition, which is available at

www.bipm.org

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