Improved Determination of G Using Two Methods

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This Letter describes new work on the determination of the Newtonian constant of gravitation, G, carried out at the BIPM since publication of the first results in 2001. The apparatus has been completely rebuilt and extensive tests carried out on the key parameters needed to produce a new value for G. The basic principles of the experiment remain the same, namely a torsion balance suspended from a wide, thin Cu-Be strip with two modes of operation, free deflection (Cavendish) and electrostatic servo control. The result from the new work is: $G = 6.67545(18) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ with a standard uncertainty of 27 ppm. This is 21 ppm below our 2001 result but 241 ppm above The CODATA 2010 value, which has an assigned uncertainty of 120 ppm. This confirms the discrepancy of our results with the CODATA value and highlights the wide divergence that now exists in recent values of G. The many changes made to the apparatus lead to the formal correlation between our two results being close to zero. Being statistically independent and statistically consistent, the two results taken together provide a unique contribution to determinations of G.

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The 2010 CODATA evaluation of the fundamental constants [1] shows a spread in the recent values of the Newtonian constant of gravitation G of some 400 ppm, more than ten times the estimated uncertainties of most of the contributing values. The aim of the BIPM work from the beginning has been to design an experiment that includes at least two methods substantially independent of each other but which are carried out at the same time with the same apparatus so that unknown systematic errors in one method would be unlikely to exist in the other. Our result published in 2001 [2] had a standard uncertainty of 41 parts per million (ppm) but the value was about 200 ppm above the then new value of Gundlach and Merkowitz [3] which had an uncertainty of 14 ppm. With the aim of identifying possible sources of error and reducing uncertainties, we decided to make a new determination, rebuilding or replacing most of the apparatus.

The principles of operation of this new G experiment using two methods remain the same as described in our 2001 paper [2] and need not be repeated in detail here but in essence they are as follows. The torsion balance suspension remains a Cu-1.8% Be dispersion-hardened ribbon 30 μm thick, 2.5 mm wide, and 160 mm long from which hangs a torsion disk of aluminum alloy 295 mm in diameter and 8 mm thick in which holes with a range of diameters are symmetrically placed to reduce both its mass and coupling to the source masses. Symmetrically placed around the periphery of the disk at a radius, r, of approximately 120 mm are the four test masses of Cu-0.7% Te free-machining alloy, of diameter and height 55 mm, each with a mass, m, 1.2 kg, see Fig. 1. The total mass of the suspended torsion balance is about 6 kg with a period of 120 s and a typical Q, in the vacuum available, of about 1×10^5. The restoring torque c of the loaded torsion strip is mostly due to gravity, which is lossless, and only 4% of the total stiffness is due to the elasticity of the strip. This essentially eliminates the problem of frequency dependent stiffness due to anelasticity. Specially designed electrodes that produce a negligible effect on the restoring torque when voltages are applied are located alongside each test mass for electrostatic servo control. A central pillar with mirrors is attached to the torsion balance to allow an external autocollimator to observe the position of the torsion disk.

Just outside the vacuum chamber which houses all this is a circular carousel upon which are placed four source masses of the same material as the test masses but 115 mm in height and 120 mm in diameter, each having a mass, M, of some 11 kg. The carousel and source masses were the only parts of the original apparatus that were not replaced although the source masses were reduced in height and remeasured. The distance of the centers of the source masses from the axis of the torsion balance, R, is approximately 214 mm. The carousel can be rotated to place the set of four masses in different positions with respect to the test masses on the torsion balance. When aligned radially with the test masses, the source masses produce no torque on the balance. When turned in either direction by about 18.9 degrees, the gravitational torque is at its maximum.

In the servo method, the gravitational torque is balanced by an electrostatic torque applied to the test masses so that the torsion balance does not rotate. The electrostatic torque comes from a 1 kHz voltage applied to the electrodes alongside each test mass. The torque constant is obtained directly in SI units from the change in total electrostatic energy as a function of angle:

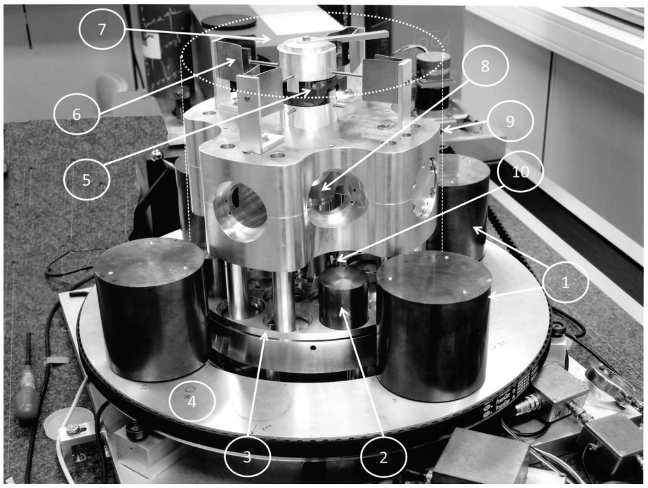

FIG. 1. The BIPM Mk II G apparatus: 1, Cu-Te 11 kg source masses; 2, Cu-Te 1.2 kg test mass resting on torsion disk (3); 4, aluminum alloy carousel; 5, gimbal from which torsion strip and torsion balance are suspended; 6, eddy-current dampers for gimbal; 7, autocollimator; 8, central mirror tower on torsion balance; 9, position of vacuum can; 10, electrodes for servo control.

$dU/d\theta = (1/2)\sum_{\mathrm{i,j}=1}^{3}(dC_{\mathrm{ij}}/d\theta)(V_{\mathrm{i}}\text{-}V_{\mathrm{j}})^2$, where the sum is over the mutual capacitances C_{ij} between the two electrodes and between each electrode and the rest of the apparatus. The quantities $dC_{\mathrm{ij}}/d\theta$ are found by measuring each capacitance as a function of angle. Then, we find $G = \tau/\Gamma$, where τ is the measured torque and Γ is the gravitational coupling between the torsion balance and source masses.

In the free-deflection (Cavendish) method, the torsion balance is allowed to rotate in response to the gravitational torque of the source masses. At equilibrium, the gravitational torque is balanced by the stiffness of the suspension. The angular deflection θ measured by an autocollimator is related to the torque by Hooke's law, $\tau = c\theta$, where again c is the stiffness of the suspension. Also, $\tau = G\Gamma$, where Γ is nominally the same function of the mass distribution as in the servo method. We obtain c from measurements of the period of free oscillation T, with the source masses in their maximum torque position, and the moment of inertia I of the suspended system using the relation $c = I(2\pi/T)^2$. Common to both methods is the calculation of Γ, the gravitational interaction between the source masses and the whole torsion balance assembly, principally, the test masses.

The principal sources of uncertainty in the Mk I result in 2001 were: (1) test and source mass coordinates, (2) angle calibration, (3) calibration of electrical instruments, ac voltage and capacitance, (4) timing, for the measurement of the period of oscillation of the torsion balance, and (5) precision of torque and deflection angle measurements.

Of these, Nos. 1 to 4 were largely type B uncertainties and No. 5 type A. It is a characteristic of the method that the positions of the four source masses must be known to high accuracy relative to each other and those of the four test masses also relative to each other, but the position of the set of source masses relative to the set of test masses is much less critical.

In the new Mk II apparatus, improvements were made that reduced the uncertainties in all five of the principal sources. The torsion balance was completely rebuilt with a new disk, new torsion strip, and a new, much more rigid, top assembly. This included an eddy-current damped cross-knife gimbal from which the torsion strip was suspended to reduce possible effects of tilt. The whole assembly was placed on the base plate of a new coordinate measuring machine, (CMM), (Brown and Sharpe, Mistral) in a different laboratory at the BIPM. Considerable efforts were made to improve the dimensional metrology. The moment of inertia of the whole suspended system was determined in two ways: first, by calculation, as before, using the measured masses and positions of all the components and second, by a separate experiment in which it was measured.

In contrast to the 2001 apparatus in which a $\times 6$ multiplying optics was used, here, we use a single mirror of much improved flatness and mounting. A new high-resolution autocollimator, a Möller-Wedel Elcomat HR, was calibrated for us by the PTB (Braunschweig) in 2003 and again in 2006. New test masses were made of the same material as before but with slightly different dimensions to fit the new torsion disk. The source masses were the original ones but with a 3 mm slice removed from the base of each mass to allow new V grooves to be cut. The density inhomogeneity of each source mass was then rechecked by determinations of its center of gravity from measurements of the period of free oscillation when supported on an air bearing, as before [2]. In addition, x-ray examination showed the absence of significant voids along the central vertical axis of the masses, the only place such voids would otherwise not have been detected by these and other earlier tests. The calculation of Γ, the gravitational interaction between the source masses and the whole torsion balance assembly was undertaken again from first principles and included a new calculation of the effects of small density gradients in the source masses inferred from the free oscillation measurements and witness samples from the original billet. New ac voltmeters, Fluke 5790A, were calibrated by the LNE (Trappes). The time base of the whole experiment was linked to an atomic frequency standard of the BIPM Time Department.

Uncertainties due to errors in length metrology were calculated using an approximate expression for Γ that assumed point masses [4] and agreed with the computed value with an accuracy of about 1%; $\Gamma = 70Mmr^4/R^5$. Thus, overall scaling errors in dimensional metrology common to both test and source mass positions lead to errors in G proportional to $\Delta\ell/\ell$, where ℓ is the distance from the balance axis. However, uncorrelated errors in the measurement of the relative positions of the test masses contribute as $4\Delta\ell/\ell$ and for the source masses, as $5\Delta\ell/\ell$ and can, thus, lead to much larger errors in G.

The uncertainty in the calibration of the CMM was established in two steps, first, the x and y scales of the CMM were calibrated using a laser interferometer and second, by using the CMM to measure 300 and 500 mm calibrated end gauges placed at various orientations across and on top of the source masses and also, for the 300 mm gauges placed alongside the source masses at their mid height. From the overall consistency of these measurements, we estimated the uncertainty of the calibration of the CMM to be 0.4 μm; i.e., the common errors that affect both test and source mass positions can be taken to be about 0.4 μm. This is equivalent to type B uncertainties of 3 ppm in G for the test masses and 2 ppm for the source masses.

The coordinates of the centers of both the test and source masses were obtained using the CMM software, which calculates the center from sets of measurements around the circumference. This software was checked by an independent calculation, which also allowed the x and y scale corrections of the CMM to be included in all subsequent measurements.

An estimate of the uncertainty in the measured source and test mass coordinates was established by multiple measurements of the relative source and test mass positions compared with measurements of the lengths of the calibrated end gauges placed on or close to the masses. From the spread of these data, we estimate the type A uncertainty in the source mass positions as 0.5 μm, equivalent to an uncertainty in the measured value of G of 15 ppm. For the test mass coordinates, a similar uncertainty was found leading to a type A uncertainty of 18 ppm in the value of G from the servo and 9 ppm from the Cavendish method.

The moment of inertia of the suspended assembly was calculated by summing the moments of inertia of each of its many components using computer-aided design software where possible. The resulting value, about, 0.076 kg m^2, had an estimated uncertainty of 13 ppm. An experimental confirmation was made after all G data had been taken. The moment of inertia of the disk assembly without the test masses, which contributed 90% of the total inertia, was measured using a pair of identical stainless steel spheres of known mass that could be placed first, near the outer edge of the disk and second, close to its central axis. From measurements of the periods of free oscillation of the balance with the spheres in these two carefully measured positions, we could determine the change in inertia of the disk assembly. By assuming that the restoring torque remained unchanged in the two configurations, the moment of inertia of the disk assembly was deduced. From this, a value for the total inertia was calculated, having an estimated uncertainty of 25 ppm that differed from the calculated value by only 9 ppm.

The balance deflected by 31.5 arcseconds (as) or 153 μrad when the source masses were moved between the positions of maximum gravitational torque on the balance, ± 18.897 deg. This was equivalent to a gravitational torque of about 3×10^{-8} N m calculated from the measured stiffness of the suspension, $c = I\omega^2$, equal to about 2.06×10^{-4} N m rad^{-1}. This is within 1% of that calculated from the nominal dimensions of the strip, the load, and elastic properties of the alloy [2].

The new autocollimator provided much improved angle resolution, the PTB calibration of the deviations from nominal was given over a range of ± 100 as at 1 as intervals with a resolution of 0.1 mas. The deviations did not exceed 3 mas over about 40 as near the center of its range, within which almost all the measurements were made. The absolute, type B, uncertainty of the calibration, common to all measurements, was 1.5 mas with relative errors within this range being at the level of the resolution.

As in the 2001 experiment, the calculated effects of density inhomogeneities in the source masses were checked by taking measurements at the three orientations of the masses. These were consistent with the calculated differences of torque between the three orientations of -32, -0.4, and 36 ppm.

The Cavendish value is based on the set of 10 data points of angular deflection shown in Fig. 2, all data points in the run being used. Each one is the average of 34 values of angular deflection obtained from successive 30 minute data collection when the source masses were permuted from angular positions of $+18.898$ deg. to -18.898 deg. The period of 30 min was chosen after an analysis of the Allen variance of the data over a longer period indicated that this was the limit at which white noise dominated. The time taken to produce one of the data points shown in Fig. 2 was, thus, about 17 hours. The servo value was also based on a set of 10 data points with all data points of the run being used. Each of these was similarly the result of some 17 hours of measurements but this time including determinations of the electrostatic torque constant $dU/d\theta$ and the servo voltages necessary to keep the torsion balance in a fixed position during the permutation of the source masses as in the Cavendish method.

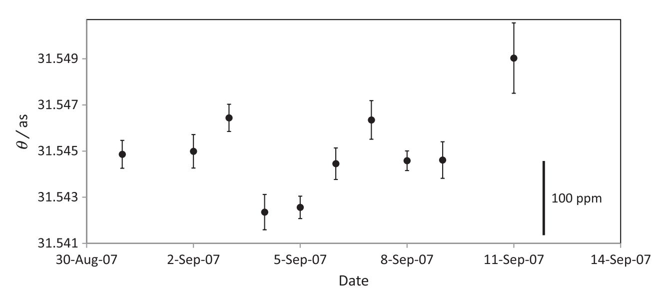

FIG. 2. For the free-deflection experiment, run-to-run dispersion of the observed angular deflection θ measured in arcseconds.

TABLE I. Uncertainty budget (types A and B in ppm of G), with correlation coefficients.

Parameter	Servo	Cavendish	Correlation coefficients
Source mass value (A)	1	1	3×10^{-4}
Test mass value (A)	1	0	0
Source mass position (A)	15	15	0
Test mass position (A)	18	9	5×10^{-2}
Dimensional metrology (B)	4	3	-3×10^{-3}
Moment of inertia (A)	0	13	0
Capacitance (B)	6	0	0
Voltages (B)	12	0	0
Balance period (B)	0	1	0
Angle (B)	47	47	−0.64
Anelasticity (B)	0	4	0
Torque (A)	30	19	0
Totals	61	56	−0.59

As in the Mk I experiment, the overall temperature stability was crucial for good measurements. A separate inner cabin was built in which the whole experiment was placed. During the periods when data were being taken, the temperature in the enclosure containing the apparatus remained stable to within 0.1 °C, the zero of the torsion balance drifted by about 0.01 as. We include in quadrature with the type A uncertainties of the source and test mass positions an uncertainty due to temperature variations of 2 ppm.

Among the other possible systematic effects examined were those due to tilt resulting from rotation of the carousel carrying the source masses, magnetic interactions between the torsion balance and external magnetic fields and contact potentials. With the source masses removed, measurements of the torque produced by the carousel and source mass kinematic mounts were found to be consistent with the predicted calculated value. This eliminated possible errors due to tilt and magnetic effects. Careful analysis of possible effects due to contact potentials led to the conclusion that the effects in our design would be negligible. Note that the relatively large gravitational signal, 3×10^{-8} N m, renders such effects less likely to be significant.

The uncertainty budget is shown in Table I. The correlation coefficients of the various components of the uncertainty are given and lead to an overall correlation coefficient of the two methods of −0.59. The coefficient is defined as the covariance of the two measurements divided by the geometric mean of their variances and the negative sign in our case shows that the mean of the two methods has significantly lower uncertainty than if the two methods were completely uncorrelated. Note that (a) the servo-control method relies essentially on electrical measurements while the Cavendish method relies on timing, (b) the mass of the test masses is eliminated in the Cavendish method since it appears in both τ and Γ, and (c) the same relative angle error produces an equal but opposite effect in the two methods, and this is essentially eliminated in the average of the two.

The servo and Cavendish methods, respectively, gave values of G of $6.67520(41) \times 10^{-11}$ m^3 kg^{-1} s^{-2} and $6.67566(37) \times 10^{11}$ m^3 kg^{-1} s^{-2} with standard uncertainties of 61 ppm and 56 ppm, respectively. The weighted mean value, taking account of correlations [5], is thus: $G = 6.67545(18) \times 10^{-11}$ m^3 kg^{-1} s^{-2} with a standard uncertainty of 27 ppm. This new value is 21 ppm below our 2001 result which had an uncertainty of 41 ppm but 241 ppm above the CODATA 2010 value (see Fig. 3).

There is a correlation between the 2001 and 2013 determinations using the servo method due to the type B uncertainty in the capacitance values (6 ppm). There is also a correlation between the Cavendish methods due to the uncertainty in the residual anelastic component of the flexure stiffness (4 ppm). The coefficients for these cross correlations amounts to about 1% and 0.5%, respectively. Further, as neither of these sources of uncertainty are common to both methods, they have no influence the final values of G. There are no other significant correlations. The value of G reported here and the value reported in 2001 are, therefore, statistically independent. They are also statistically consistent. Noting that in each, the result is based on the average of two largely independent methods, taken

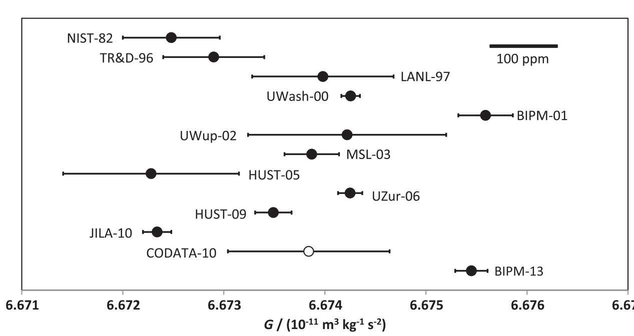

FIG. 3. The present result (BIPM-13) compared with recent measurements of G [6].

together, our two results represent a unique contribution to G determinations.

We are pleased to acknowledge the splendid work of José Sanjaime and his staff of the BIPM mechanical workshop in constructing the new apparatus, the PTB (Braunschweig) for the calibration of the autocollimator, the LNE (Trappes and Paris) for calibration of the electrical instruments, source masses, and the end gauges, and one of us (TJQ) thanks the then Director of the BIPM for permission to continue working in the laboratory after retirement from January 2004 to May 2008.

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