(Some) Relativistic Aspects of the IERS Conventions

Sergei A. Klioner

Lohrmann Observatory, Dresden Technical University

IERS Workshop, Paris, 20 September 2007
Section 1.2 Numerical Standards: “units”

Confusing wording:

“TDB units” ≠ “TT units” ≠ “SI units”

What are “units”? non-SI second?

Suggestion:

* _TDB   --- TDB-compatible value  
* _TT      --- TT-compatible values  
* _TCG   --- TCG-compatible values  
* _TCB   --- TCB-compatible values

Table 1.1 listing numerical standards is organized into 5 columns: item, value, uncertainty, reference, comment. Most of the values are given in terms of SI units (Le Système International d’Unités (SI), 1998), i.e. they are consistent with the use of Geocentric Coordinate Time TCG as a time coordinate for the geocentric system, and of Barycentric Coordinate Time TCB for the barycentric system. The values of $\tau_A$, $v_A$, and $\psi_1$, however, are given in so-called “TDB” units, having been determined previously using Barycentric Dynamical Time TDB as a time coordinate for the barycentric system. In this book some quantities are also given in so-called “TT” units, having been determined using Terrestrial Time TT as a time coordinate for the geocentric system. See Chapter 10 for further details on the transformations between time scales and Chapter 3 for a discussion of the time scale used in the ephemerides.

TDB and TCB units of time, $t$, and length, $\ell$, may be easily related by the expressions (Seidelmann and Fukushima, 1992)

$$t_{TDB} = t_{TCB} / (1 - L_B), \quad \ell_{TDB} = \ell_{TCB} / (1 - L_B),$$

where $L_B$ is given in Table 1.1. Therefore a quantity $X$ with the dimension of time or length has a numerical value $x_{TCB}$ when using “TCB” (SI) units which differs from its value $x_{TDB}$ when using “TDB” units by

$$x_{TDB} = x_{TCB} \times (1 - L_B).$$

Similarly, the numerical value $x_{TCG}$ when using “TCG” (SI) units differs from the numerical value $x_{TT}$ when using “TT” units by

$$x_{TT} = x_{TCG} \times (1 - L_G)$$

where $L_G$ is given in Table 1.1.
TDB-compatible quantities

- Three scalings in the BCRS are necessary to keep the equations of motion and light propagation invariant:

  - time
  - spatial coordinates
  - masses of each body

  \[ TDB = (1 - L_B) \cdot TCB + \text{const} \]
  \[ \mathbf{x}_{TDB} = (1 - L_B) \cdot \mathbf{x}_{TCB} \]
  \[ \mu_{TDB} = (1 - L_B) \cdot \mu_{TCB}, \quad \mu = GM \]
TT-compatible quantities

• Three scalings are necessary in the GCRS to keep the equations of motion and light propagation invariant:

  • time

  \[ TT = (1 - L_G) \cdot TCG + \text{const} \]

  • spatial coordinates

  \[ \mathbf{x}_{TT} = (1 - L_G) \cdot \mathbf{x}_{TCG} \]

  • masses of each body

  \[ \mu_{TT} = (1 - L_G) \cdot \mu_{TCG} \]

  \[ \mu_{TCG} = \mu_{TCB} \]

Concise overview of the background and a collection of reasoning:

IAU Commission 52
“Relativity in Fundamental Astronomy”

• Created by the IAU in 2006

• Present Organizing Committee:

Sergei A. Klioner, President
Gérard Petit, Vice President
Victor A. Brumberg
Nicole Capitaine
Agnès Fienga
Toshio Fukushima
Bernard Guinot
Cheng Huang
François Mignard
Ken Seidelmann
Michael Soffel
Patrick Wallace
IAU Commission 52
“Relativity in Fundamental Astronomy”

• is thought to stimulate the exchange on relativistic issues

• Current projects:
  • Compile a list of unsolved problems
  • Frequently asked questions
  • Relativistic glossary for astronomers
  • “Task teams”, an ad hoc discussion group for very well posed issues:
    • One task team is currently activated: “TDB units”

Question: “What are the units of TDB?”
IAU Commission 52
“Relativity in Fundamental Astronomy”

- All interested members of the IAU are kindly invited to join:

http://astro.geo.tu-dresden.de/RIFA
Section 1.2 Numerical Standards: AU

Table 1.1  IERS Numerical Standards

<table>
<thead>
<tr>
<th>$\tau_A$</th>
<th>$c\tau_A$</th>
<th>Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$499.0047838061s$</td>
<td>$149597870691m$</td>
<td>Astronomical unit in seconds</td>
</tr>
<tr>
<td>$0.00000002s$</td>
<td>$6m$</td>
<td>Astronomical unit in meters</td>
</tr>
</tbody>
</table>

† The values for $\tau_A$, $c\tau_A$, and $\psi_1$ are given in “TDB” units (see discussion above).

(not clear why to have both quantities: $c\tau_A$ is sufficient)

Suggestion:

Just “astronomical unit in meters”, no reference to TDB

Reasoning:

• With its current definition “astronomical unit“ should have the same value with both TDB and TCB (better choice of the convention among infinite possible)

2. If some time later, the AU in meters becomes a defining constant, nothing will have to be changed
Do we need astronomical units?

• The reason to introduce astronomical units was that angular measurements were many orders of magnitude more accurate than distance measurements.

• BUT

  • The situation has changed crucially since that time!
  • Solar mass is time-dependent just below current accuracy of ephemerides
    \[ \frac{\dot{M}_{\text{Sun}}}{M_{\text{Sun}}} \approx 10^{-13} \text{ yr}^{-1} \]
  • Very confusing situation with astronomical units in relativistic framework

• Why not to define AU conventionally as fixed number of meters?
• Do you see any good reasons for astronomical units in their current form?
Section 1.2 Numerical Standards: AU

Table 1.1  IERS Numerical Standards

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$GM_{\odot}$</td>
<td>$1.32712442076 \times 10^{20} \text{ m}^3 \text{ s}^{-2}$</td>
<td>$5 \times 10^{10} \text{ m}^3 \text{ s}^{-2}$</td>
<td>[from 3] Heliocentric gravitational constant</td>
</tr>
<tr>
<td>$\tau_A$</td>
<td>499.0047838061 s</td>
<td>$0.00000002$ s</td>
<td>[3] Astronomical unit in seconds</td>
</tr>
<tr>
<td>$c\tau_A$</td>
<td>149597870691 m</td>
<td>$6 m$</td>
<td>[3] Astronomical unit in meters</td>
</tr>
</tbody>
</table>

† The values for $\tau_A$, $c\tau_A$, and $\psi_1$ are given in “TDB” units (see discussion above).

- Any reasons to have “astronomical units” outside of the software for developing solar system ephemerides?

- As a result of the current AU definition we have errors in both AU and $GM_{\text{Sun}}$ expressed in metric units...
Section 1.2 Numerical Standards: TDB

Table 1.1  IERS Numerical Standards

<table>
<thead>
<tr>
<th></th>
<th>TDB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_B$</td>
<td>$1.55051976772 \times 10^{-8}$</td>
</tr>
<tr>
<td>$L_C$</td>
<td>$1.48082686741 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

- Where $L_C$ is used? Is it needed for any practical calculations?

- The new TDB (IAU 2006)? If it is adopted by the IERS:

  \[
  L_B = 1.550519768 \times 10^{-8}\ \text{Defining}\ [?]\ 1 - d(\text{TDB})/d(\text{TCB}) \\
  L_G = 6.969290134 \times 10^{-10}\ \text{Defining}\ [?]\ 1 - d(\text{TT})/d(\text{TCG})
  \]

- $L_C$ is no longer used to compute $L_B$
- Average value of $1 - d(\text{TCG})/d(\text{TCB})$ for a particular ephemeris is no longer $1 - (1 - L_B)/(1 - L_G)$

- If the new TDB is adopted, Section 10.1 should be also modified
Section 11.2 Laser Ranging: $L_C$

- Where $L_C$ is used? Is it needed for any practical calculations?

In Section 11.2 Laser Ranging we have

$$
\vec{r}_{TDB} = \vec{r}_{TT} \left( 1 - \frac{U}{c^2} - L_C \right) - \frac{1}{2} \left( \frac{\vec{V} \cdot \vec{r}_{TT}}{c^2} \right) \vec{V},
$$

(19)

- Suggestion:

$$
\frac{r}{r_{TDB}} = \frac{\left( 1 - L_B \right)}{\left( 1 - L_G \right)} \frac{\vec{r}_{TT}}{c^2 \vec{\varepsilon}} - \frac{U \vec{\varepsilon}}{c^2 \vec{\varepsilon}} - \frac{1}{2} \frac{\vec{V} \cdot \vec{r}_{TT}}{c^2} \vec{V}.
$$
# Section 1.2 Numerical Standards: GM_Earth

## Table 1.1 IERS Numerical Standards

| $GM_\oplus$ | $3.986004418 \times 10^{14} \text{m}^3\text{s}^{-2}$ | $8 \times 10^5 \text{m}^3\text{s}^{-2}$ | [1] Geocentric gravitational constant (EGM96 value) |

**Suggestion:**

TCG-compatible value of ...
Section 1.2 Numerical Standards: GM_Earth

Table 1.1  IERS Numerical Standards

\[ GM_\oplus = 3.986004418 \times 10^{14} \text{ m}^3 \text{s}^{-2} \quad 8 \times 10^5 \text{ m}^3 \text{s}^{-2} \quad [1] \text{ Geocentric gravitational constant} \]

\[ \text{(EGM96 value)} \]

• This number contradicts the DE405 value which can be computed from constants given in Tables 3.1 and 3.2

\[
\begin{align*}
(149597870691)^3 (86400)^2 k^2 \\
\times 328900.561400^{-1} (1 + 1/81.30056)^{-1} \\
\times (1 - L_B)^{-1} \\
= 3.986004391 \times 10^{14}
\end{align*}
\]

a deviation of 3.4\( \sigma \)

it would be better to avoid such inconsistencies…
Section 1.2 Numerical Standards: \( \psi \)

Table 1.1  IERS Numerical Standards

<table>
<thead>
<tr>
<th>( \psi_1 )†</th>
<th>( 5038.47875''/c )</th>
<th>( 0.00040''/c )</th>
<th>[6]</th>
</tr>
</thead>
</table>

IAU(1976) value of precession of the equator at J2000.0 corrected by \(-0.29965''\). See Chapter 5.

† The values for \( \tau_A \), \( c\tau_A \), and \( \psi_1 \) are given in “TDB” units (see discussion above).

• why “TDB” units?

• Suggestion:

  TT-compatible value of precession constant
Section 11.2 Laser Ranging

\[ t_2 - t_1 = \left| \frac{\vec{x}_2(t_2) - \vec{x}_1(t_1)}{c} \right| + \sum_j \frac{2GM_J}{c^3} \ln \left( \frac{r_{J1} + r_{J2} + \rho}{r_{J1} + r_{J2} - \rho} \right), \quad (17) \]

• What is the accuracy requirements for this formula?

• Why “2” and not “1 + γ” here?

• Two possible issues:

  1. the post-post-Newtonian time delay due to the Sun
     (not very relevant for IERS?)

  2. motion of the body “J” during light propagation:

     simple way to take the main term into account is to change

\[
 r_{J1} = |\vec{x}_1 - \vec{x}_J|, \quad r_{J2} = |\vec{x}_2 - \vec{x}_J| \quad \rightarrow \quad r_{J1} = |\vec{x}_1 - \vec{x}_J|, \quad r_{J2} = |\vec{x}_2 - \vec{x}_J|
\]

with \( \frac{1}{x_{J1}} = \frac{1}{x_J(t_1)}, \quad \frac{1}{x_{J2}} = \frac{1}{x_J(t_2)} \)
Post-post-Newtonian light propagation

Full post-post-Newtonian expression for the Shapiro time delay with PPN parameters (Klioner, Zschocke, 2007):

\[
c \tau = \frac{R}{c} + (1 + \gamma) \frac{m}{c} \log \left( \frac{x + x_0 + R}{x + x_0 - R} \right) \\
+ \frac{1}{8} \alpha \kappa \frac{m^2}{R} \left( \frac{x_0^2 - x^2 - R^2}{x^2} + \frac{x^2 - x_0^2 - R^2}{x_0^2} \right) \\
+ \frac{1}{4} \alpha \kappa \left( 8(1 + \gamma) - 4\beta + 3\kappa \right) \frac{m^2}{R} \arctan \frac{x^2 - x_0^2 + R^2}{2|x \times x_0|} \\
- \frac{1}{4} \alpha \kappa \left( 8(1 + \gamma) - 4\beta + 3\kappa \right) \frac{m^2}{R} \arctan \frac{x^2 - x_0^2 - R^2}{2|x \times x_0|} \\
+ \frac{1}{2} \left( 1 + \gamma \right)^2 \frac{m^2}{|x \times x_0|^2} \frac{R}{|x \times x_0|} (x - x_0 - R) (x - x_0 + R).
\]

\[
m = \frac{GM}{c^2}
\]

The higher-order terms give up to 10 meters. Are all these terms relevant?
Post-post-Newtonian light propagation

NO!

The only numerically relevant term can be written as

\[ c \tau = R + (1 + \gamma) m \log \frac{x + x_0 + R + (1 + \gamma) m}{x + x_0 - R + (1 + \gamma) m} \]

This has already been derived by Moyer (2003) in a different way.

All other terms can be estimated as

\[ c \delta \tau \leq \frac{m^2}{d} \left( \frac{3}{4} + \frac{15}{4\pi} \right) \]

This gives maximally 4 cm for Sun-grazing ray, and much less in typical cases…
Possible future developments

1. The details of relativistic models for different kinds of observations are not the same:

   the VLBI model is given in full detail
   the LLR one is just sketched

   Do we need the same level of detail?

2. Required/claimed accuracies should be stated
   (clearly stated for VLBI, but not, say, for LLR)

3. General Relativity or PPN formalism?

   IAU 2000 framework is formulated in General Relativity

   IERS Conventions 2003: a mixture of PPN formalism and General Relativity
What could be interesting in principle

1. (More) detailed description of the LLR model

2. Relativity in GPS model:
   
   adding, e.g., the influence of the quadrupole gravitational field of the Earth on the clocks on the satellites

   GPS models used in Bernese have never been seriously discussed by scientific community

   Can we risk inconsistencies between the GPS and VLBI models?

   (some of the statements in the Ashby’s description of the GPS model are not compatible with the IAU framework)