Processing Multi-Stations Data on two-way Satellite Time Transfer
To Obtain Greeter Precision
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ABSTRACT

The Two-Way satellite Time Transfer (TWSTT) can achieve time synchronization with high precision and accuracy. With application of new generation of time transfer modem and Very Small Aperture Terminals (VSAT) it becomes a method suitable for time transfer of high precision and accuracy. Now data of measurements on TWSTT are contributed to construction of International Atomic time (TAI). A two-channel modem for TWSTFT has been already presented and tested with three stations link TWSTT experiment. The Communications Research Laboratory in Japan (CRL) is developing a new modem for multi-stations [1]. Since multi-stations can simultaneously carry out TWSTT experiments with it, more information can be extracted. So, it has advantage over the signal channel modem. Based on the multi-channel modem, a method processing multi-stations data on TWSTT is provided and it will be to obtain greeter precision.

The TWSTT is one of the most precise and accurate methods for time synchronization [2][3]. Some of time institutes in Europe and northern America have carried out TWSTT experiments regularly. The four links from above area have contributed to construction of TAI. Also, in the Pacific Rim Region the TWSTT is rapidly developing. The time transfer links between CRL and NML (National Measurement Laboratory, Australia), CRL and CSAO (Shaanxi Astronomical Observatory, China), CRL and NRLM (National Research Laboratory of Metrology, Japan), CRL and TL (Telecommunication laboratories in Tai Pei) are put in real operations. Links of CRL with KRISS (Korea research institute of standards and science in South Korea) and PSB (productivity and standards board in Singapore) are being set up and will join the TWSTT network in Pacific Rim Region [4].

The most of links in Pacific Rim region work with help of satellite JCSAT-IB and each two station TWSTT experiments carry out time transfer individually. They are not in much harmony for the time differences in the network consisted by Pacific Time institutes. Now a two-channel modem has been already presented and tested. The CRL is developing a kind of new modem for the multi-stations. It can simultaneously carry out time transfer with maximum of eight institutes. Based on such kind of new modem, a process of data on TWSTT with multi-points is presented. A simple system with three stations is discussed in the section 1 and a system with multi-stations is discussed in the section 2. The summary is presented in section 3.

1. TWSTT for three stations

The two-way with two stations can not determine separately the exact distance from each station to the satellite. It only eliminates the time delay on the path that both time signals pass through. The two-way with three stations is different from that with two stations. We will discuss it.

The TWSTT with two stations is show in Fig.1. By means of the modem, the one pulse per second is modulated onto intermediate frequency, which is usually at 70 MHz. It is converted to the up link frequency and transmitted to the satellite. In the satellite transponder it is converted to down link frequency and retransmitted. At receiving station the time signal is down-converted to intermediate frequency again. By means of a modem it is demodulated. The time difference of two master clocks
at both of stations is determined by the time interval counters. The equation is as follows:

\[
\begin{align*}
R_{21} &= T_2 - T_1 + \tau_1^U + \tau_2^D + \tau_s \\
R_{12} &= T_1 - T_2 + \tau_1^D + \tau_2^U + \tau_s
\end{align*}
\]  

(1)

Where

- Fig.1 TWSTT with two stations

\[
\begin{align*}
\text{counter reading of station I for the time signal from station j;}
\end{align*}
\]

\[
\begin{align*}
\text{time of master clock at station I;}
\end{align*}
\]

\[
\begin{align*}
\text{the time delay of time signal from station I to satellite in up link;}
\end{align*}
\]

\[
\begin{align*}
\text{the time delay of time signal from satellite to station I in down link;}
\end{align*}
\]

\[
\begin{align*}
\text{the time delay of signal for the satellite transponder. (Above I or j is 1 for station 1, 2 for station 2 and so on)}
\end{align*}
\]

The instrument delay and the correction for relativistic effects are neglected because they can be measured and easily accounted for. If nonreciprocity caused by the difference for up link and down link frequency is neglected, then following relationship is given by

\[
\begin{align*}
\tau_1 &= \tau_1^U = \tau_1^D \\
\tau_2 &= \tau_2^U = \tau_2^D
\end{align*}
\]

The equation for computation of time difference of clocks between station 1 and

\[
T_i - T_j = (R_{12} - R_{21}) / 2
\]

We define: \( T_y = T_i - T_j \)  

(2)

Then above equation is written as

\[
T_{12} = (R_{12} - R_{21}) / 2
\]

(3)

It is obvious that the signal delay on the way can not be determined but is eliminated (if the nonreciprocity is neglected).

A system with three stations is shown in Fig.2. We assume that station 1 is master

\[
\begin{align*}
\text{station 1 is master station. We define that all of time differences are related to station 1. This means that the time differences which are defined are } T_{12} \text{ and } T_{13}, \text{ and others can be calculated by defined relationship of equation (2). According to equation (1) and (2), the observation equations with matrix representation are:}
\end{align*}
\]
The unknown number above is six and the observing equations are also six. In principle we can get unique results. But the rank above the matrix of coefficients is 5. It is not solvable. It is obvious that the unknowns $\tau_i$ and $\tau_s$ are related. If half of $\tau_s$ is added to $\tau_i$, that is:

$$\tau_i + \frac{1}{2} \tau_s = \tau_i^*$$

The observing equations with matrix representation can express as

$$A \cdot B = R$$

(4)

Where

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ -1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 & 1 & 1 \\ 1 & -1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} T_{12} \\ T_{13} \\ \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_s \end{bmatrix}^T$$

And

$$R = \begin{bmatrix} R_{12} \\ R_{21} \\ R_{13} \\ R_{31} \\ R_{23} \\ R_{32} \end{bmatrix}^T$$

The equation (4) can be solved by method of least squares to give

$$A^T \cdot A \cdot B = A^T \cdot R$$

(5)

Where $A^T \cdot A$ and $A^T \cdot R$ are respectively,
\[ A^T \cdot A = \begin{bmatrix} 4 & -2 & 0 & 0 & 0 \\ -2 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 2 \\ 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 2 & 2 & 4 \end{bmatrix} \]

\[ A^T \cdot R = \begin{bmatrix} (R_{12} - R_{21}) + (R_{32} - R_{33}) \\ (R_{13} - R_{31}) + (R_{23} - R_{22}) \\ (R_{12} + R_{21}) + (R_{13} + R_{31}) \\ (R_{12} + R_{21}) + (R_{23} + R_{32}) \\ (R_{13} + R_{31}) + (R_{13} + R_{32}) \end{bmatrix} \]

The matrix of coefficient, \( A^T \cdot A \) can be divided into the black to give

\[ \begin{bmatrix} 4 & -2 & 0 & 0 & 0 \\ -2 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 2 \\ 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 2 & 2 & 4 \end{bmatrix} \]

\[ \begin{bmatrix} (R_{12} - R_{21}) + (R_{32} - R_{33}) \\ (R_{13} - R_{31}) + (R_{23} - R_{22}) \\ (R_{12} + R_{21}) + (R_{13} + R_{31}) \\ (R_{12} + R_{21}) + (R_{23} + R_{32}) \\ (R_{13} + R_{31}) + (R_{13} + R_{32}) \end{bmatrix} \]

The final solutions are:

\[ \begin{bmatrix} T_{12} \\ T_{13} \end{bmatrix} = \begin{bmatrix} R_{12} - R_{21} + (R_{32} - R_{33}) \\ R_{13} - R_{31} + (R_{23} - R_{22}) \end{bmatrix} \]

\[ \begin{bmatrix} \tau_1^e \\ \tau_2^e \end{bmatrix} = \begin{bmatrix} (R_{12} + R_{21}) + (R_{13} + R_{31}) \\ (R_{12} + R_{21}) + (R_{23} + R_{32}) \end{bmatrix} \]

\[ \begin{bmatrix} \tau_3^e \end{bmatrix} = \begin{bmatrix} (R_{13} + R_{31}) + (R_{23} + R_{32}) \end{bmatrix} \]

The Value of \( T_{23} \), which is related to \( T_{12} \) and \( T_{13} \), is by the relationship:

\[ T_{23} = T_{13} - T_{12} = 1/3 \left[ 1/2(R_{23} - R_{12}) + 1/2(R_{13} - R_{12}) + (R_{23} - R_{32}) \right] \]

2. TWSTT with multi-stations

Each station in the system of n-stations has (n-1) independent observed quantities and total observed quantities for n-stations are n(n-1) independent quantities. Now we count the number of unknowns. As we define above \( T_{ij} = T_i - T_j = T_{ij} - T_{ij} \), and station 1 is the master station. The time difference between each station, and master station is determined uniquely. The number of time differences for n stations is n-1 independent quantities and the number of time delay quantities from each station to satellite is n. So number of unknowns in the total is 2n-1. While n>3, then n(n-1)>2n-1. This means that observation equations have a group of unique solutions. Now observation equations with matrix representation for the n-stations are given by

\[ A \cdot B = R \]

Where
\[ A = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 & 0 & 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\
-1 & 0 & 0 & \cdots & 0 & 0 & 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\
0 & 1 & 0 & \cdots & 0 & 0 & 1 & 0 & 1 & 0 & \cdots & 0 & 0 \\
0 & -1 & 0 & \cdots & 0 & 0 & 1 & 0 & 1 & 0 & \cdots & 0 & 0 \\
0 & 0 & 1 & \cdots & 0 & 0 & 1 & 0 & 0 & 1 & \cdots & 0 & 0 \\
0 & 0 & -1 & \cdots & 0 & 0 & 1 & 0 & 0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
-1 & 1 & 0 & \cdots & 0 & 0 & 0 & 1 & 1 & 0 & \cdots & 0 & 0 \\
1 & -1 & 0 & \cdots & 0 & 0 & 0 & 1 & 1 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & -1 & 1 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 \\
0 & 0 & 0 & \cdots & 1 & -1 & 0 & 0 & 0 & 0 & \cdots & 1 & 1
\end{bmatrix} \]

\[ B = \begin{bmatrix}
T_{i_1} & T_{i_3} & \cdots & T_{i_n} & \tau_1^i & \cdots & \tau_n^i
\end{bmatrix}^T \]

And

\[ R = \begin{bmatrix}
R_{i_1} & R_{i_2} & R_{i_3} & \cdots & R_{i_n} & R_{t_1} & R_{t_2} & R_{t_3} & \cdots & R_{(n-1)n} & R_{n(n-1)}
\end{bmatrix}^T \]

The observation equations above can be solved by method of least squares:

\[ A^T \cdot A \cdot B = A^T \cdot R \]

The matrix A is very special. To take out \((2k)\)th and \((2k-1)\)th lines, we found that the values after \(n\)th column are equal and before have opposite sign. It is obvious that matrix, \(A^T \cdot A\) and \(A^T \cdot R\) are

\[ A^T \cdot A = \begin{bmatrix}
2(n-1) & -2 & \cdots & -2 & -2 \\
-2 & 2(n-1) & \cdots & -2 & -2 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
-2 & -2 & \cdots & 2(n-1) & -2 \\
-2 & -2 & \cdots & -2 & 2(n-1) \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0
\end{bmatrix} \begin{bmatrix}
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0
\end{bmatrix} \]

We assume \(R_{i_j} = 0\), then \(A^T \cdot R\) is:
\[ A^T \cdot R = \sum_{j=1}^{n} \begin{bmatrix} R_{j2} - R_{2j} \\ R_{j3} - R_{3j} \\ \vdots \\ R_{jn} - R_{nj} \\ R_{j1} + R_{1j} \\ R_{j2} + R_{2j} \\ \vdots \\ R_{jn} + R_{nj} \end{bmatrix} \]

Above equations with matrix representation may be divided into two groups of equation with matrix representation, (13) and (14) that are:

\[
\begin{bmatrix} 2(n-1) & -2 & \cdots & -2 & -2 \\ -2 & 2(n-1) & \cdots & -2 & -2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -2 & -2 & \cdots & 2(n-1) & -2 \\ 2(n-1) & 2 & \cdots & 2 & 2 \\ 2 & 2(n-1) & \cdots & 2 & 2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & \cdots & 2(n-1) & 2 \end{bmatrix} \begin{bmatrix} T_{12} \\ T_{13} \\ \vdots \\ T_{1n} \end{bmatrix} = \sum_{j=1}^{n} \begin{bmatrix} R_{j2} - R_{2j} \\ R_{j3} - R_{3j} \\ \vdots \\ R_{jn} - R_{nj} \end{bmatrix} \]

(13)

\[
\begin{bmatrix} 2(n-1) & -2 & \cdots & -2 & -2 \\ -2 & 2(n-1) & \cdots & -2 & -2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -2 & -2 & \cdots & 2(n-1) & -2 \\ 2 & 2 & \cdots & 2 & 2 \\ 2 & 2 & \cdots & 2 & 2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & \cdots & 2(n-1) & 2 \end{bmatrix} \begin{bmatrix} \tau_k' \\ \tau_k' \\ \vdots \\ \tau_k' \end{bmatrix} = \sum_{j=1}^{n} \begin{bmatrix} R_{j1} + R_{1j} \\ R_{j2} + R_{2j} \\ \vdots \\ R_{jn} + R_{nj} \end{bmatrix} \]

(14)

The solutions are obtained:

\[
T_{ik} = 1/n \sum_{j=1}^{n} \left[ 1/2(R_{jk} - R_{kj}) + 1/2(R_{ij} - R_{ji}) \right] \\
\tau_k^* = 1/(2(n-2)) \left[ \sum_{j=1}^{n} (R_{jk} + R_{kj}) - 1/(n-1) \sum_{j=1}^{n} \sum_{j'=1, j' \neq j}^{n} R_{ij} \right] \]

(15)

Using the relationship \( T_{ik} = T_{ik} - T_{ij} \), the \( T_{ik} \) is:

\[
T_{ik} = 1/n \sum_{j=1}^{n} \left[ 1/2(R_{jk} - R_{kj}) + 1/2(R_{ij} - R_{ji}) \right] \]

(16)

3. Summary

The TWSTT with multi-stations can simultaneously be carried out with the new modem. It reduces the cost for satellite link time. Since the time transfer is made at same time for all stations in the network, the time delay of signal on the way between each station and satellite can be determined. This is not case for the time transfer with two stations. The quantity determined is very useful. It may be used to monitor the variations of ionosphere and to determine the satellite position. Also by using the method for processing data with multi-stations, a more precise solution than with two-station can be obtained. If the error in observing quantity is same as that with two stations, the relationship between precision and number of stations is given in table 1. It is obvious that the precision with number of stations is improved. This is a benefit from the method processing multi-stations data on Two-way time transfer.

Table 1. Variation with the number of stations
<table>
<thead>
<tr>
<th>Number of stations</th>
<th>Precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>3</td>
<td>0.816 $\sigma$</td>
</tr>
<tr>
<td>4</td>
<td>0.707 $\sigma$</td>
</tr>
<tr>
<td>5</td>
<td>0.632 $\sigma$</td>
</tr>
<tr>
<td>6</td>
<td>0.577 $\sigma$</td>
</tr>
</tbody>
</table>

References