1 Introduction

1.1 Quantities and units

The value of a quantity is generally expressed as the product of a number and a unit. The unit is simply a particular example of the quantity concerned which is used as a reference, and the number is the ratio of the value of the quantity to the unit. For a particular quantity many different units may be used. For example the speed v of a particle may be expressed in the form $v = 25$ m/s = 90 km/h, where metre per second and kilometre per hour are alternative units for expressing the same value of the quantity speed. However, because of the importance of a set of well defined and easily accessible units universally agreed for the multitude of measurements that support today's complex society, units should be chosen and defined so that they are readily available to all, are constant throughout time and space, and are easy to realise with high accuracy.

When an experimental measurement of a quantity is reported, two results are required: the ***estimated value*** of the measurand (the quantity being measured), and the ***estimated uncertainty*** of that value. Both are expressed in the same unit. The uncertainty is a measure of the accuracy of the measured value, in the sense that a lower uncertainty corresponds to a more accurate and more precise measurement. A simple measure of the uncertainty in a measurement result may sometimes be provided by the width of the probability distribution of repeated measurements.

In order to establish a system of units, such as the International System of units, the SI, it is necessary first to establish a system of quantities, including a set of equations defining the relations between the quantities. This is necessary because the equations between the quantities determine the equations relating the units, as described below. Thus the establishment of a system of units, which is the subject of this brochure, is intimately connected with the algebraic equations relating the corresponding quantities.

For example the quantity speed, v, may be expressed in terms of distance x and time t by the equation $v = \mathrm{d}x/\mathrm{d}t$. If the metre m and second s are used for distance and time, then the unit used for speed v might be metre per second, m/s.

As new fields of science develop, new quantities are devised by researchers to represent the interests of the fields. With these new quantities come new equations relating them to the quantities that were previously familiar, and these new relations allow us to establish units for the new quantities that are related to the units previously established. In this way the units to be used with the new quantities may always be defined as products of powers of the previously established units.

As a further example, in electrochemistry the electric mobility of an ion u is defined as the ratio of its velocity v to the electric field strength E: $u = v/E$. The unit of electric mobility is then given as (m/s)/(V/m) = $\mathrm{m^2\ V^{-1}\ s^{-1}}$, where the volt per metre V/m is used for the quantity E. Thus the relation between the units is built on the underlying relation between the quantities.

The definition of the units is established in terms of a set of defining constants, which are chosen from the fundamental constants of physics, taken in the broadest sense, which are used as reference constants to define the units. In the SI there are seven such defining constants. From the units of these defining constants the complete system of units may then be constructed. These seven defining constants are the most fundamental feature of the definition of the entire system of units.

Historically the units have always previously been presented in terms of a set of *seven base units*, all other units then being constructed as products of powers of the base units which are described as *derived units*. The choice of the base units was never unique, but grew historically and became familiar to users of the SI. This description in terms of base and derived units remains valid, although the seven defining constants provide a more fundamental definition of the SI. It is tempting to think that there is a one-to-one correspondence between the base units and the defining constants, but that is an oversimplification which is not strictly true. However these two approaches to defining the SI are fully consistent with each other.

1.2 The International System of units, SI, and the corresponding system of quantities

This brochure is concerned with presenting the information necessary to define and use the International System of Units, universally known as the SI (from the French *Système International d'Unités*). The SI was established by and is defined by the General Conference on Weights and Measures, CGPM, as described in section 1.8 below[*].

The system of quantities used with the SI, including the equations relating the quantities, is just the set of quantities and equations that are familiar to all scientists, technologists, and engineers. They are listed in many textbooks and in many references, but any such list can only be a selection of the possible quantities and equations, which is without limit. Many of the quantities, with their corresponding names and symbols, and the equations relating them, were listed in the international standards ISO 31 and IEC 60027 produced by Technical Committee 12 of the International Organization for Standardization, ISO/TC 12, and by Technical Committee 25 of the International Electrotechnical Commission, IEC/TC 25. These standards have been revised by the two organizations in collaboration, and are known as the ISO/IEC 80000 Standards, Quantities and Units, in which the corresponding quantities and equations are described as the International System of Quantities.[♣]

The base quantities used in the SI are time, length, mass, electric current, thermodynamic temperature, amount of substance, and luminous intensity. The corresponding base units of the SI were chosen by the CGPM to be the second, metre, kilogram, ampere, kelvin, mole, and candela. The history of the development of the SI is summarized in section 1.8 below.

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[*] Acronyms used in this brochure are listed with their meaning on p. XX.

[♣] In these equations the electric constant ε_0 (the permittivity of vacuum) and the magnetic constant μ_0 (the permeability of vacuum) have dimensions and values such that $\varepsilon_0\mu_0 = 1/c^2$, where c is the speed of light in vacuum. Note that the electromagnetic equations in the CGS-EMU, CGS-ESU and Gaussian systems are based on a different set of quantities and equations in which the magnetic constant μ_0 and the electric constant ε_0 have different dimensions, and may be dimensionless.

1.3 Dimensions of quantities

By convention physical quantities are organised in a system of dimensions. Each of the seven base quantities used in the SI is regarded as having its own dimension, which is symbolically represented by a single roman capital letter. The symbols used for the base quantities, and the symbols used to denote their dimension, are as follows.

Table 1. Base quantities and dimensions used in the SI

Base quantity	*Symbol for quantity*	*Symbol for dimension*
length	$l, x, r,$ etc.	L
mass	m	M
time, duration	t	T
electric current	I, i	I
thermodynamic temperature	T	Θ
amount of substance	n	N
luminous intensity	I_v	J

Quantity symbols are always written in an italic font, symbols for units in a roman (upright) font, and symbols for dimensions in sans-serif roman capitals. For some quantities a variety of alternative symbols may be used (as for length and electric current in the table).

Symbols for quantities are *recommendations*, in contrast to symbols for units (which appear elsewhere in this Brochure) which are *mandatory*, and independent of the language.

All other quantities are derived quantities, which may be written in terms of base quantities by the equations of physics. The dimensions of the derived quantities are written as products of powers of the dimensions of the base quantities using the equations that relate the derived quantities to the base quantities. In general the dimension of any quantity Q is written in the form of a dimensional product,

$$\dim Q = \mathsf{L}^{\alpha}\,\mathsf{M}^{\beta}\,\mathsf{T}^{\gamma}\,\mathsf{I}^{\delta}\,\Theta^{\varepsilon}\,\mathsf{N}^{\zeta}\,\mathsf{J}^{\eta}$$

where the exponents α, β, γ, δ, ε, ζ, and η, which are generally small integers which can be positive, negative, or zero, are called the dimensional exponents. The dimension of a derived quantity provides the same information about the relation of that quantity to the base quantities as is provided by the SI unit of the derived quantity as a product of powers of the SI base units.

There are some derived quantities Q for which the defining equation is such that all of the dimensional exponents in the equation for the dimension of Q are zero. This is true in particular for any quantity that is defined as the ratio of two quantities of the same kind. Such quantities are described as being *dimensionless*, and are simply numbers. However the coherent derived unit for such dimensionless quantities is always the number one, 1, since it is the ratio of two identical units for two quantities of the same kind. For that reason dimensionless quantities are sometimes described as being of dimension one.

There are also some quantities that cannot be described in terms of the seven base quantities of the SI at all, but have the nature of a count. Examples are a number of molecules, degeneracy in quantum mechanics (the number of independent states of the same energy), and the partition function in chemical thermodynamics (the number of thermally accessible states). Such counting quantities are usually regarded as dimensionless quantities, or quantities with the dimension one, with the unit one, 1.

1.4 Coherent units, derived units with special names, and the SI prefixes

Derived units are defined as products of powers of the base units. When this product includes no numerical factors other than one, the derived units are called *coherent derived units*. The base and coherent derived units of the SI form a coherent set, designated the *set of coherent SI units*. The word coherent is used here in the following sense: when only coherent units are used, equations between the numerical values of quantities take exactly the same form as the equations between the quantities themselves. Thus if only units from a coherent set are used, conversion factors between units are never required, since they are always equal to one.

The metre per second, m/s, is the coherent SI unit of speed. The kilometre per second, km/s, the centimetre per second, cm/s, and the nanometre per second, nm/s, are also SI units, but they are not coherent SI units.

The expression for the coherent unit of a derived quantity may be obtained from the dimensional product of that quantity by replacing the symbol for each dimension by the symbol for the corresponding base unit.

Some of the coherent derived units in the SI are given special names, to simplify their expression (see section 2.6.3, p. XXX). It is important to emphasise that each physical quantity has only one coherent SI unit, even though this unit can be expressed in different forms by using some of the special names and symbols. The inverse, however is not true: in some cases the same SI unit can be used to express the values of several different quantities (see p. XXX).

As an example of a special name, the particular combination of base units m^2 kg s^{-2} for energy is given the special name joule, symbol J, where by definition J = m^2 kg s^{-2}.

The CGPM has, in addition, adopted a series of prefixes for use in forming the decimal multiples and submultiples of the coherent SI units (see section 3.1, p.XXX, where the prefix names and symbols are listed). These are convenient for expressing the values of quantities that are much larger than or much smaller than the coherent unit. Following the CIPM Recommendation 1 (1969, p.XXX) these are given the name SI Prefixes. These prefixes are also sometimes used with non-SI units, as described in Chapter 3 below. However when prefixes are used with SI units, the resulting units are no longer coherent, because the prefix on a derived unit effectively introduces a numerical factor in the expression for the derived unit in terms of base units.

The length of a chemical bond is more conveniently given in nanometres, nm, than metres, m, and the distance from London to Paris is more conveniently given in kilometres, km, than in metres, m.

With one exception, the base units do not involve any prefixes. The exception is the kilogram, which is the base unit of mass, but which includes the prefix kilo for historical reasons. The multiples and submultiples of the kilogram are formed by attaching prefix names to the unit name “gram”, and prefix symbols to the unit symbol “g” (see section 3.2, p. XXX). Thus 10^{-6} kg is written as milligram, mg, not as microkilogram, µkg.

The complete set of SI units, including both the coherent set and the multiples and submultiples formed by using the SI prefixes, are designated the complete set of SI units, or simply the SI units, or the units of the SI. Note however that the decimal multiples and submultiples of the SI units do not form a coherent set.

1.5 SI units in the framework of general relativity

The definitions of the base units of the SI were adopted in a context that takes no account of relativistic effects. When such account is taken, it is clear that the definitions apply only in a small spatial domain sharing the motion of the standards that realise them. These units are known as *proper units*; they are realised from local experiments in which the relativistic effects that need to be taken into account are those of special relativity. The defining constants are local quantities with their values expressed in proper units.

The question of proper units is addressed in Resolution A4 adopted by the International Astronomical Union (IAU) in 1991, and by the report of the CCDS Working Group on the application of general relativity to metrology (Metrologia, 1997, **34**, 261 – 290).

Physical realisations of the definition of a unit are usually compared locally. For frequency standards, however, it is possible to make such comparisons at a distance by means of electromagnetic signals. To interpret the results the theory of general relativity is required, since it predicts, among other things, a relative frequency shift between standards of about 1 part in 10^{16} per metre of altitude difference at the surface of the earth. Effects of this magnitude cannot be neglected when comparing the best frequency standards.

1.6 Units for quantities that describe biological effects

Units for quantities that describe biological effects are often difficult to relate to the SI because they typically involve weighting factors that may not be precisely known or defined, and which may be both energy and frequency dependent. These units, which are not SI units, are described briefly in this section.

Optical radiation may cause chemical changes in living or non-living materials: this property is called *actinism* and radiation capable of causing such changes is referred to as *actinic radiation*. In some cases the results of measurements of photochemical and photobiological quantities of this kind can be expressed in terms of SI units. This is discussed briefly in Appendix 3.

Sound causes small pressure fluctuations in the air, superimposed on the normal atmospheric pressure, that are sensed by the human ear. The sensitivity of the ear depends on the frequency of the sound, but it is not a simple function of either the pressure changes or the frequency. Therefore frequency weighted quantities are used in acoustics to approximate the way in which sound is perceived. They are used, for example, in work to protect against hearing damage. The effect of ultrasonic acoustic waves poses similar concerns in medical diagnosis and therapy.

Ionizing radiation deposits energy in irradiated matter. The ratio of deposited energy to mass is termed absorbed dose. High doses of ionizing radiation kill cells, and this is used in radiation therapy. Appropriate biological weighting functions are used to compare therapeutic effects of different radiation treatments. Low sub-lethal doses can cause damage to living organisms, for instance by inducing cancer. Appropriate risk-weighted functions are used at low doses as the basis of radiation protection regulations.

There is a class of units for quantifying the biological activity of certain substances used in medical diagnosis and therapy that cannot yet be defined in terms of the units of the SI. This is because the mechanism of the specific biological effect that gives these substances their medical use is not yet sufficiently well understood for it to be quantifiable in terms of physico-chemical parameters. In view of their importance for human health and safety, the World Health Organization (WHO) has taken responsibility for defining WHO International Units (IU) for the biological activity of such substances.

1.7 Legislation on units

By legislation, individual countries have established rules concerning the use of units on a national basis either for general use or for specific areas such as commerce, health, public safety, and education. In almost all countries this legislation is based on the International System of Units.

The International Organisation for Legal Metrology (OIML), founded in 1955, is charged with the international harmonization of this legislation.

1.8 Historical note

The previous paragraphs give a brief overview of the way in which a system of units, and the International System of Units in particular, is established. This note gives a brief account of the historical development of the International System.

The 9th CGPM (1948, Resolution 6; CR 64) instructed the CIPM:

- to study the establishment of a complete set of rules for units of measurement;
- to find out for this purpose, by official enquiry, the opinion prevailing in scientific, technical and educational circles in all countries;
- to make recommendations on the establishment of a *practical system of units of measurement* suitable for adoption by all signatories to the *Convention of the Metre.*

The same CGPM also laid down, in Resolution 7 (CR 70), general principles for the writing of unit symbols, and listed some coherent derived units which were assigned special names.

The 10th CGPM (1954, Resolution 6; CR 80) and in the 14th CGPM (1971, Resolution 3, CR 78, and *Metrologia* 1972, **8**, 36) adopted as base quantities and units for this practical system the following seven quantities: length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity, and the seven corresponding base units: metre, kilogram, second, ampere, kelvin, mole, and candela.

The 11th CGPM (1960, Resolution 12; CR 87) adopted the name ***Système International d'Unités***, with the international abbreviation ***SI***, for this practical system of units and laid down rules for prefixes, derived units, and the former supplementary units, and other matters; it thus established a comprehensive specification for units of measurement. Subsequent meetings of the CGPM and the CIPM have added to and modified as necessary, the original structure of the SI to take account of advances in science and of the needs of users.

The historical sequence that led to these important decisions may be summarized as follows.

- The creation of the decimal metric system at the time of the French Revolution and the subsequent deposition of two platinum standards representing the metre and the kilogram, on 22 June 1799, in the Archives de la République in Paris, can be seen as the first step in the development of the present International System of Units.
- In 1832, Gauss strongly promoted the application of this metric system, together with the second defined in astronomy, as a coherent system of units for the physical sciences. Gauss was the first to make absolute measurements of the earth's magnetic field in terms of a decimal system based on the *three mechanical units* millimetre, gram and second for, respectively, the quantities length, mass, and time. In later years Gauss and Weber extended these measurements to include other electrical phenomena.
- These applications in the field of electricity and magnetism were further extended in the 1860s under the active leadership of Maxwell and Thompson through the British Association for the Advancement of Science (BAAS). They formulated the requirement for a *coherent system of units* with *base units* and *derived units*. In 1874 the BAAS introduced the *CGS system*, a three dimensional coherent unit system based on the three mechanical units centimetre, gram and second, using prefixes ranging from micro to mega to express decimal submultiples and multiples. The subsequent development of physics as an experimental science was largely based on this system.
- The sizes of the coherent CGS units in the fields of electricity and magnetism proved to be inconvenient, so in the 1880s the BAAS and the International Electrical Congress, predecessor of the International Electrotechnical Commission (IEC), approved a mutually coherent set of *practical units*. Among them were the ohm for electrical resistance, the volt for electromotive force, and the ampere for electric current.
- After the signing of the Convention of the Metre on 20 May 1875, which created the BIPM and established the CGPM and the CIPM, work began on establishing new international prototypes for the metre and the kilogram. In 1889 the first CGPM sanctioned the international prototypes for the metre and the kilogram. Together with the astronomical second as the unit of time, these units constituted a three-dimensional mechanical

unit system similar to the CGS system, but with the base units metre, kilogram, and second, the *MKS system*.

- In 1901 Giorgi showed that it is possible to combine the mechanical units of this MKS system with the practical electrical units to form a coherent four dimensional system by adding to the three base units a fourth unit, of an electrical nature such as the ampere or the ohm, and rewriting the equations occurring in electromagnetism in the so-called rationalized form. Giorgi's proposal opened the path to a number of new developments.
- After the revision of the Convention of the Metre by the 6th CGPM in 1921, which extended the scope and responsibilities of the BIPM to other fields in physics, and the subsequent creation of the Consultative Committee for Electricity (CCE) by the 7th CGPM in 1927, the Giorgi proposal was thoroughly discussed by the IEC, the International Union of Pure and Applied Physics (IUPAP), and other international organisations. This led the CCE to propose in 1939 the adoption of a four-dimensional system based on the metre, kilogram, second and ampere, the MKSA system, a proposal approved by the CIPM in 1946.
- Following an international enquiry by the BIPM, which began in 1948, the 10th CGPM, in 1954, approved the further introduction of the *kelvin*, and the *candela*, as base units respectively, for thermodynamic temperature and luminous intensity. The name ***Système International d'Unités***, with the abbreviation ***SI***, was given to the system by the 11th CGPM in 1960. Rules for prefixes, derived units, and the former supplementary units, and other matters, were established, thus providing a comprehensive specification for all units of measurement.
- At the 14th CGPM in 1971, after lengthy discussion between physicists and chemists, the mole was added as the base unit for amount of substance, bringing the total number of base units to seven. Subsequent meetings of the CGPM and the CIPM have added to and modified as necessary the original structure of the SI to take account of advances in science and of the needs of users.

- Since the 14th CGPM in 1971, extraordinary advances have been made in relating SI units to truly invariant quantities such as the fundamental constants of physics and the properties of atoms. Recognising the importance of linking SI units to such invariant quantities, the XXth CGPM, in 20XX, adopted a new definition of the SI based on using a set of seven such constants as references for the definitions. They chose defining constants for which there are well established experiments to determine their values, as is necessary for their use in realizing the definitions of the SI units. This is the basis of the definition presented in this Brochure, and is the simplest and most fundamental way of defining the SI.

- The SI has previously been defined in terms of seven base units, and derived units defined as products of powers of the base units. This is still a useful alternative to the definition in terms of the seven defining constants, to which it is equivalent. The seven base units were chosen for historical reasons, as the metric system evolved and the SI developed over the last 130 years. Their choice is not unique, but it has become established and familiar over the years by providing a framework for describing the SI. The definitions of the seven base units can be related to the values of the seven defining constants, which demonstrate the equivalence of the two alternative descriptions. However there is not a one-to-one correspondence between the seven defining constants and the seven base units.

- The XXth CGPM, in 20XX, also chose new definitions for four of the original base units, the kilogram, ampere, kelvin, and mole. These new definitions involve the use of the Planck constant h, elementary charge e, Boltzmann constant k, and Avogadro constant N_A as defining constants. The remaining three of the original base units, the second, metre, and candela, remained unchanged; their definitions involve the caesium hyperfine frequency $\Delta\nu(^{133}\mathrm{Cs})_\mathrm{hfs}$, the speed of light in vacuum c, and the luminous efficacy of a defined radiation K_cd as defining constants.

2 SI units

The International System of Units, the SI, is a coherent system of units for use throughout science and technology. Formal definitions of the SI units are adopted by the CGPM. These definitions are modified from time to time as science advances. The first definitions were adopted in 1889, and the most recent in 20XX.

2.1 Definitions of the SI units

The formal definitions of the SI units are presented in sections 2.2 and 2.3.

- The SI units are defined by a set of statements that explicitly specify the exact numerical values for each of seven reference constants when they are expressed in SI units. These ***defining constants*** are the frequency of the ground state hyperfine splitting of the caesium 133 atom $\Delta\nu(^{133}\mathrm{Cs})_\mathrm{hfs}$, the speed of light in vacuum c, the Planck constant h, the elementary charge (charge of a proton) e, the Boltzmann constant k, the Avogadro constant N_A, and the luminous efficacy of monochromatic radiation of frequency 540×10^{12} hertz, K_cd.
- The SI may alternatively be defined by statements that explicitly define ***seven individual base units,*** the second, metre, kilogram, ampere, kelvin, mole, and candela. These correspond to the seven base quantities time, length, mass, electric current, thermodynamic temperature, amount of substance, and luminous intensity. All other units are then obtained as products of powers of the seven base units, which involve no numerical factors; these are called ***coherent derived units***. This approach to defining the SI has previously been followed, and is described in section 2.4 below.
- The use of seven defining constants is the simplest and most fundamental way to define the SI, as described in section 2.2. In this way no distinction is made between base units and derived units; all units are simply described as SI units. This also effectively decouples the definition and practical realization of the units. While the definitions may remain unchanged over a long period of time, the practical realizations can be

established by many different experiments, including totally new experiments not yet devised. This allows for more rigorous intercomparisons of the practical realizations and a lower uncertainty, as the technologies evolve.

- Defining the entire system in this way is a new feature of the SI, adopted in 20XX by the XXth CGPM (Resolution XX, CR, XX and *Metrologia*, 20XX, **XX**, XX**)**. It thus appears for the first time in this edition of the SI Brochure.
- The names and symbols for the SI units are summarized in the Tables below. Table 2 in section 2.3 lists the seven defining constants. Tables 3, 4, 5 and 6 in section 2.6 list the base and derived units and the relations between them.
- Preserving continuity is an essential feature of any changes to the International System of Units, and this has always been assured in all changes to the definitions by choosing the numerical values of the constants that appear in the definitions to be consistent with the earlier definitions in so far as advances in science and knowledge allow.

2.2 The SI in terms of seven defining constants

All numbers for the defining constants that appear in this draft are based on the 2010 CODATA adjustment of the values of the fundamental constants. The final version will use the numbers chosen by the CGPM at the time the new definitions are adopted.

The international system of units, the SI, is the system of units in which

- **the unperturbed ground state hyperfine splitting frequency of the caesium 133 atom $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ is exactly 9 192 631 770 hertz,**
- **the speed of light in vacuum *c* is exactly 299 792 458 metre per second,**
- **the Planck constant *h* is exactly 6.626 069 57 $\times10^{-34}$ joule second,**
- **the elementary charge *e* is exactly 1.602 176 565 $\times10^{-19}$ coulomb,**
- **the Boltzmann constant *k* is exactly 1.380 648 8 $\times10^{-23}$ joule per kelvin,**
- **the Avogadro constant N_A is exactly 6.022 141 29 $\times10^{23}$ reciprocal mole,**
- **the luminous efficacy K_cd of monochromatic radiation of frequency 540 $\times10^{12}$ hertz is exactly 683 lumen per watt,**

where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, and W, respectively, are related to the units second, metre, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, and cd, respectively, according to the relations Hz = s^{-1} (for periodic phenomena), J = kg $\mathrm{m}^2\,\mathrm{s}^{-2}$, C = A s, lm = cd sr, and W = kg $\mathrm{m}^2\,\mathrm{s}^{-3}$. The steradian, symbol sr, is the SI unit of solid angle and is a special name and symbol for the number 1, so that sr = $\mathrm{m}^2\,\mathrm{m}^{-2}$ = 1.

2.3 Realising the definitions of the SI units

The seven defining constants listed in section 2.2 are summarised in Table 2. They are chosen from the fundamental constants of physics (broadly interpreted) that may be called constants of nature, because the values of these constants are regarded as invariants throughout time and space. The value of any one of these seven constants is written as the product of a numerical coefficient and a unit as

$$Q = \{Q\}\,[Q]$$

where Q denotes the value of the constant, and $\{Q\}$ denotes its numerical value when it is expressed in the unit $[Q]$. The same value, Q, may be expressed using different numerical values $\{Q\}$ depending on the unit $[Q]$, and it is sometimes convenient to use the notation $\{Q\}_{[Q]}$ for the numerical value to emphasize its dependence on the choice of the unit $[Q]$.

For example, the speed of light in vacuum is a constant of nature, denoted c, whose value in SI units is given by the equation
$c = 299\,792\,458$ m/s = $\{c\}[c]$
where the numerical value $\{c\} = 299\,792\,458$ and the unit $[c]$ = m/s.

The definitions in section 2.2 specify the exact numerical value of each constant when its value is expressed in the corresponding SI unit. By fixing the exact numerical value the unit becomes defined, since the product of the ***numerical value*** and the ***unit*** has to equal the ***value*** of the constant, which is invariant. The seven SI units defined in this way by each of the seven constants are also listed in Table 2. The seven constants are chosen in such a way that any of the other units of the International System can always be written as a product of these seven constants. Thus the specified numerical values in section 2.2 define the units of the seven defining constants, and indirectly define all the units of the SI.

Finally, to use the units to make measurements requires that we perform experiments to compare the measurand (i.e. the quantity to be measured) with the appropriate unit (i.e. the appropriate combination of the defining constants). This may be done by many different experimental methods. Descriptions of some of these methods may be found in the *mises en pratique* that are presented in Appendix 2 of this Brochure.

Table 2. The seven defining constants of the SI, and the seven corresponding units that they define

Defining constant	*Symbol*	*Numerical value*	*Unit*
hyperfine splitting of Cs	$\Delta\nu(^{133}\text{Cs})_{\text{hfs}}$	9 192 631 770	$\text{Hz} = \text{s}^{-1}$
speed of light in vacuum	c	299 792 458	m/s
Planck constant	h	$6.626\ 069\ 57 \times 10^{-34}$	$\text{J s} = \text{kg m}^2\ \text{s}^{-1}$
elementary charge	e	$1.602\ 176\ 565 \times 10^{-19}$	C = A s
Boltzman constant	k	$1.380\ 648\ 8 \times 10^{-23}$	J/K
Avogadro constant	N_{A}	$6.022\ 141\ 29 \times 10^{23}$	mol^{-1}
luminous efficacy	K_{cd}	683	lm/W

2.4 Base units and derived units

Previous definitions of the SI have been based on the concept of identifying seven base units, the second s, metre m, kilogram kg, ampere A, kelvin K, mole mol, and candela cd, corresponding to the seven quantities time, length, mass, electric current, thermodynamic temperature, amount of substance, and luminous intensity. All derived units are then defined as products of powers of the base units. In this way all SI units are defined. The definitions of the seven base units are presented in turn below.

2.4.1 The SI unit of time, the second

The second, symbol s, is the SI unit of time; its magnitude is set by fixing the numerical value of the unperturbed ground state hyperfine splitting frequency of the caesium 133 atom to be exactly 9 192 631 770 when it is expressed in the SI unit $\mathbf{s^{-1}}$, which for periodic phenomena is equal to Hz.

Thus we have the exact relation $\Delta\nu(^{133}\text{Cs})_{\text{hfs}} = 9\ 192\ 631\ 770$ Hz. Inverting this relation gives an expression for the unit second in terms of the value of the defining constant $\Delta\nu(^{133}\text{Cs})_{\text{hfs}}$:

$$\text{Hz} = \frac{\Delta\nu(^{133}\text{Cs})_{\text{hfs}}}{9\ 192\ 631\ 770} \quad \text{or} \quad \text{s} = \frac{9\ 192\ 631\ 770}{\Delta\nu(^{133}\text{Cs})_{\text{hfs}}}$$

The symbol $\Delta\nu(^{133}\text{Cs})_{\text{hfs}}$ is used to denote the value of the frequency of the hyperfine transition in the unperturbed ground state of the caesium 133 atom.

The effect of this definition is that the second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the unperturbed ground state of the caesium 133 atom.

The reference to an unperturbed atom is intended to make it clear that the definition of the SI second is based on a caesium atom unperturbed by any external field such as ambient black body radiation.. The frequencies of all primary frequency standards should therefore be corrected for the shift due to ambient radiation, as stated at the meeting of the Consultative Committee for Time and Frequency in 1999.

The second so defined is a proper time in the sense of General Relativity. A non-local time scale is a coordinate time scale. However, generally, the unit of such a scale is also called "second". Whenever this is the case, the word "second" must be followed by the name of the time scale: e.g. second of TCB (barycentric coordinate time used within the solar system). The scale unit of International Atomic Time TAI and of Coordinated Universal Time UTC (differing from TAI by a variable integral number of seconds), established by the BIPM, namely the second of TAI and UTC, is the second as realized on a rotating equipotential surface close to the geoid. Only on this surface does it coincide with the second as defined above.

The CIPM has adopted various secondary representations of the second, based on a selected number of spectral lines of atoms, ions or molecules. The unperturbed frequencies of these lines can be determined with a relative uncertainty equal to that of the definition of the second based on the ^{133}Cs hyperfine splitting, but some can be reproduced with a significantly smaller uncertainty.

2.4.2 The SI unit of length, the metre

The metre, symbol m, is the SI unit of length; its magnitude is set by fixing the numerical value of the speed of light in vacuum to be exactly 299 792 458 when it is expressed in the SI unit for speed m s^{-1}.

The symbol c (or sometimes c_0) is the conventional symbol for the value of the speed of light in vacuum.

Thus we have the exact relation c = 299 792 458 m/s. Inverting this relation gives an exact expression for the unit metre in terms of the defining constants c and $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$:

$$\mathrm{m} = \left(\frac{c}{299\,792\,458}\right)\mathrm{s} = 30.663\,318...\ \frac{c}{\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}}$$

Here and elsewhere, the three dots (ellipsis) indicate the missing digits of an exactly known rational number with an unending number of digits.

The effect of this definition is that the metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.

2.4.3 The SI unit of mass, the kilogram

The kilogram, symbol kg, is the SI unit of mass; its magnitude is set by fixing the numerical value of the Planck constant to be exactly 6.626 069 57 ×10^{-34} when it is expressed in the SI unit for action J s = kg m^2 s^{-1}.

Thus we have the exact relation h = 6.626 069 57 ×10^{-34} kg m^2 s^{-1} = 6.626 069 57 ×10^{-34} J s. Inverting this equation gives an exact expression for the kilogram in terms of the three defining constants h, $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ and c:

$$\mathrm{kg} = \left(\frac{h}{6.626\,069\,57\times10^{-34}}\right)\mathrm{m}^{-2}\,\mathrm{s} = 1.475\,521...\times10^{40}\ \frac{h\,\Delta\nu\ \ {}^{133}\mathrm{Cs}\ {}_{\mathrm{hfs}}}{c^2}$$

The Planck constant is a constant of nature, whose value may be expressed as the product of a number and the unit joule second, where J s = kg m^2 s^{-1}. The effect of this definition is to define the unit $\mathrm{kg\,m^2\,s^{-1}}$ (the unit of both the physical quantities action and angular momentum), and thus together with the definitions of the second and the metre this leads to a definition of the unit of mass expressed in terms of the value of the Planck constant h.

Note that macroscopic masses can be measured in terms of h, using the Josephson and quantum-Hall effects together with the watt balance apparatus, or in terms of the mass of a silicon atom, which is accurately known in terms of h using the x-ray crystal density approach.

The number chosen for the numerical value of the Planck constant in the definition is such that at the time of adopting this definition, the kilogram was equal to the mass of the international prototype, $m(\mathcal{K}) = 1$ kg, within a few parts in 10^8, which was the uncertainty of the combined best estimates of the value of the Planck constant at that time. Subsequently, the mass of the international prototype is now a quantity to be determined experimentally.

The symbol $m(\mathcal{K})$ is used to denote the mass of the international prototype of the kilogram, $\mathcal{K}$.

2.4.4 The SI unit of electric current, the ampere

The ampere, symbol A, is the SI unit of electric current; its magnitude is set by fixing the numerical value of the elementary charge to be exactly 1.602 176 565 ×10^–19^ when it is expressed in the SI unit for electric charge C = A s.

Thus we have the exact relation $e = 1.602\,176\,565 \times 10^{-19}$ C = $1.602\,176\,565 \times 10^{-19}$ A s. Inverting this equation gives an exact expression for the unit ampere in terms of the defining constants e and $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$:

The symbol e is used to denote the value of the elementary charge, which is the charge of a proton.

$$\mathrm{A} = \left(\frac{e}{1.602\,176\,565 \times 10^{-19}}\right)\mathrm{s}^{-1} = 6.789\,687... \times 10^{8}\ \Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}\, e$$

The effect of this definition is that the ampere is the electric current corresponding to the flow of $1/(1.602\,176\,565 \times 10^{-19})$ elementary charges per second. The previous definition of the ampere based on the force between current carrying conductors had the effect of fixing the value of the magnetic constant μ_0 to be exactly $4\pi \times 10^{-7}$ H m^{-1} = $4\pi \times 10^{-7}$ N A^{-2}, where H and N denote the coherent derived units henry and newton, respectively. The new definition of the ampere fixes the value of e instead of μ_0, and as a result μ_0 is no longer exactly known but must be determined experimentally. It also follows that since the electric constant ε_0 (also known as the permittivity of vacuum), the characteristic impedance of vacuum Z_0, and the admittance of vacuum Y_0 are equal to $1/\mu_0 c^2$, $\mu_0 c$, and $1/\mu_0 c$, respectively, the values of ε_0, Z_0, and Y_0 must also be determined experimentally, and will be subject to the same relative standard uncertainty as μ_0 since c is exactly known. The product $\varepsilon_0\mu_0 = 1/c^2$ and quotient $Z_0/\mu_0 = c$ remain exactly known. At the time of adopting the new definition of the ampere, μ_0 was equal to $4\pi \times 10^{-7}$ H/m with a relative standard uncertainty less than 1×10^{-9}.

2.4.5 The SI unit of thermodynamic temperature, the kelvin

The kelvin, symbol K, is the SI unit of thermodynamic temperature; its magnitude is set by fixing the numerical value of the Boltzmann constant to be exactly 1.380 648 8 $\times 10^{-23}$ when it is expressed in the SI unit for energy per thermodynamic temperature J K^{-1} = kg m^2 s^{-2} K^{-1}.

Thus we have the exact relation $k = 1.380\ 648\ 8 \times 10^{-23}$ J/K = $1.380\ 648\ 8 \times 10^{-23}$ kg m^2 s^{-2} K^{-1}. Inverting this equation gives an exact expression for the kelvin in terms of the defining constants k, h and $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$:

The symbol k is used to denote the Boltzmann constant.

$$\mathrm{K} = \left(\frac{1.380\ 648\ 8}{k}\right) \mathrm{kg\,m^2\,s^{-2}} = 2.266\ 665... \frac{\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}\, h}{k}$$

The effect of this definition is that the kelvin is equal to the change of thermodynamic temperature that results in a change of thermal energy kT by $1.380\ 648\ 8 \times 10^{-23}$ J.

The previous definition of the kelvin was based on an exact value assigned to the triple point of water T_{TPW}, namely 273.16 K (see section 2.5.5). Because the new definition of the kelvin fixes the value of k instead of T_{TPW}, the latter must be determined experimentally, but at the time of adopting the new definition T_{TPW} was equal to 273.16 K with a relative standard uncertainty of less than 1×10^{-6} based on measurements of k made prior to the redefinition.

Because of the manner in which temperature scales used to be defined, it remains common practice to express a thermodynamic temperature, symbol T, in terms of its difference from the reference temperature $T_0 = 273.15$ K, the ice point. This difference is called the Celsius temperature, symbol t, which is defined by the quantity equation

$$t = T - T_0$$

The unit of Celsius temperature is the degree Celsius, symbol °C, which is by definition equal in magnitude to the kelvin. A difference or interval of temperature may be expressed in kelvins or in degrees Celsius, the numerical value of the temperature difference being the same. However, the numerical value of a Celsius temperature expressed in degrees Celsius is related to the numerical value of the thermodynamic temperature expressed in kelvins by the relation

$$t/^{\circ}\mathrm{C} = T/\mathrm{K} - 273.15$$

The kelvin and the degree Celsius are also units of the International Temperature Scale of 1990 (ITS-90) adopted by the CIPM in 1989 in its Recommendation 5 (CI-1989, PV, **57**, 115 and *Metrologia*, 1990, **27**, 13). Note that the ITS-90 defines two quantities T_{90} and t_{90} which are close approximations to the corresponding thermodynamic and Celsius temperatures.

Note also that with the new definition, it becomes much clearer that thermodynamic temperature can be measured directly at any point in the scale.

2.4.6 The SI unit of amount of substance, the mole

> **The mole, symbol mol, is the SI unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles; its magnitude is set by fixing the numerical value of the Avogadro constant to be exactly 6.022 141 29 $\times 10^{23}$ when it is expressed in the SI unit mol^{-1}.**

Thus we have the exact relation $N_A = 6.022\,141\,29 \times 10^{23}\ \text{mol}^{-1}$. Inverting this equation gives an exact expression for the mole in terms of the defining constant N_A:

The symbol N_A is used to denote the value of the Avogadro constant.

$$\text{mol} = \frac{6.022\,141\,29 \times 10^{23}}{N_A}$$

The effect of this definition is that the mole is the amount of substance of a system that contains 6.022 141 29 $\times 10^{23}$ specified elementary entities.

The previous definition of the mole fixed the value of the molar mass of carbon 12, $M(^{12}\text{C})$, to be exactly 0.012 kg/mol, but now $M(^{12}\text{C})$ is no longer known exactly and must be determined experimentally. However, the value chosen for N_A is such that at the time of adopting the new definition of the mole, $M(^{12}\text{C})$ was equal to 0.012 kg/mol with a relative standard uncertainty of less than 1×10^{-9}.

The molar mass of any atom or molecule X may still be obtained from its relative atomic mass from the equation

$$M(X) = A_r(X)\,[M(^{12}\text{C})/12] = A_r(X)\,M_u$$

and the molar mass of any atom or molecule X is also related to the mass of the elementary entity $m(X)$ by the relation

$$M(X) = N_A\,m(X) = N_A\,A_r(X)\,m_u$$

In these equations M_u is the molar mass constant, equal to $M(^{12}\text{C})/12$, and m_u is the unified atomic mass constant, equal to $m(^{12}\text{C})/12$. They are related by the Avogadro constant through the relation

$$M_u = N_A\,m_u$$

In the name “amount of substance”, the words “of substance” could for simplicity be replaced by words to specify the substance concerned in any particular application, so that one may for example talk of “amount of hydrogen chloride, HCl”, or “amount of benzene, C_6H_6”. It is important to always give a precise specification of the entity involved (as emphasized in the definition of the mole); this should preferably be done by giving the molecular chemical formula of the material involved. Although the word “amount” has a more general dictionary definition, the abbreviation of the full name “amount of substance” to “amount” may often be used for brevity. This also applies to derived quantities such as “amount-of-substance concentration”, which may simply be called “amount concentration”. In the field of clinical chemistry, however, the name “amount-of-substance concentration” is generally abbreviated to “substance concentration”.

2.4.7 The unit of luminous intensity, the candela

> **The candela, symbol cd, is the unit of luminous intensity in a given direction; its magnitude is set by fixing the numerical value of the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz to be exactly 683 when it is expressed in the SI unit $\mathrm{kg^{-1}\,m^{-2}\,s^{3}\,cd\,sr = lm\,W^{-1} = cd\,sr\,W^{-1}}$.**

Thus we have the exact relation $K_\mathrm{cd} = 683\ \mathrm{lm/W} = 683\ \mathrm{cd\,sr\,W^{-1}} = 683\ \mathrm{kg^{-1}\,m^{-2}\,s^{3}\,cd\,sr}$ for monochromatic radiation of frequency $\nu = 540 \times 10^{12}$ Hz. This relation may be inverted to give an exact expression for the candela in terms of the defining constants K_cd, h and $\Delta\nu(^{133}\mathrm{Cs})_\mathrm{hfs}$:

The symbol K_cd is used to denote the value of the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz.

$$\mathrm{cd} = \left(\frac{K_\mathrm{cd}}{683}\right)\ \mathrm{kg\,m^{2}\ s^{-3}\,sr^{-1}}\ =\ 2.614\,830... \times 10^{10} \Delta\nu\,(^{133}\mathrm{Cs})^2_\mathrm{hfs}\ h\,K_\mathrm{cd}$$

The effect of this definition is that the candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} Hz and that has a radiant intensity in that direction of (1/683) W/sr.

2.4.8 Relations between the definitions of the base units

Sections 2.4.1 to 2.4.7 present individual definitions of the seven base units of the SI expressed in terms of the seven defining constants specified in section 2.2. Of these definitions only the first (for the second), and the sixth (for the mole), are independent of the other definitions. In 2.4.2 fixing the numerical value of the speed of light in vacuum actually defines the unit of speed, m/s, so that the definition of the second is required to complete the definition of the metre. In 2.4.3 fixing the numerical value of the Planck constant actually defines the unit of action, $\mathrm{J\,s = kg\,m^{2}\,s^{-1}}$, so that the definitions of the metre and second are required to complete the definition of the kilogram. In 2.4.4 fixing the numerical value of the elementary charge actually defines the unit of charge, the coulomb, C = A s, so that the definition of the second is required to complete the definition of the ampere. In 2.4.5 fixing the numerical value of the Boltzmann constant actually fixes the value of the unit of energy per thermodynamic temperature interval, $\mathrm{J\,K^{-1} = kg\,m^{2}\,s^{-2}\,K^{-1}}$, so that the definitions of the, metre, kilogram, and second are required to complete the definition of the kelvin. And finally, in 2.4.7 fixing the numerical value of the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz actually defines the unit of luminous efficacy, the lumen per watt, $\mathrm{lm\,W^{-1} = cd\,sr\,W^{-1} = kg^{-1}\,m^{-2}\,s^{2}\,cd\,sr}$, so that the definitions of the metre, kilogram, and second are required to complete the definition of the candela.

It follows that the definitions in 2.4.1 to 2.4.7 must be taken together as a coherent group of statements for the definitions of the base units of the SI, and should not be regarded as independent definitions of the individual base units. The same was true in all previous editions of the SI Brochure. Also, each of the seven definitions of the base units in 2.4 is followed by the expression implied by the definition when the unit is expressed in terms of the seven defining constants listed in 2.2. This demonstrates that the individual definitions of the base units in 2.4 are equivalent to the more fundamental definition of the entire system in 2.2

2.4.9 Definitions for coherent derived SI units in terms of defining constants

As indicated in Chapter 1, coherent derived SI units are defined as appropriate products of powers of SI base units with no numerical factors other than one. Thus the definition of any derived unit can be represented as a number multiplied by the appropriate combination of the seven defining constants by combining the corresponding equations for the base units in terms of the defining constants given above.

2.4.10 The nature of the seven defining constants

The seven defining constants have been chosen for practical reasons. These constants are believed to be invariant throughout time and space, at least for all foreseeable epochs and measurement ranges, and they allow for straightforward practical realisations.

Both the Planck constant h and the speed of light in vacuum c, are properly described as fundamental. They determine quantum effects and space-time properties, respectively, and affect all particles and fields equally on all scales and in all environments.

The elementary charge e, in contrast, corresponds to a coupling strength of the electromagnetic force via the fine-structure constant α. It is only dimensionless constants such as α for which any experimental evidence can be obtained as to its stability in time. The experimental limits of the maximum possible variation in α are so low, however, that any effect on foreseeable measurements can be excluded.

The ground state hyperfine splitting of the caesium 133 atom $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$, has the character of an atomic parameter, which may be affected by the environment, such as by electromagnetic fields. However, this transition parameter is well understood and is stable under the laws of quantum mechanics. It is also a good choice as a reference transition for practical realisations.

The Boltzmann constant k and the Avogadro constant N_{A}, have the character of conversion factors to convert the unit joule into kelvin for practical thermometry and the mole into the counting unit 1 for measurements of amount of substance.

The luminous efficacy K_{cd} is a technical constant related to a conventional spectral response of the human eye.

2.5 Historical perspective on the base units

2.5.1 Unit of time, second

The unit of time, the second, was at one time considered to be the fraction 1/86 400 of the mean solar day. The exact definition of “mean solar day” was left to astronomers. However measurements showed that irregularities in the rotation of the Earth made this an unsatisfactory definition. In order to define the unit of time more precisely, the 11th CGPM (1960, Resolution 9, CR, 86) adopted a definition given by the International Astronomical Union based on the tropical year 1900. Experimental work, however, had already shown that an atomic standard of time, based on a transition between two energy levels of an atom or a molecule, could be realized and reproduced much more accurately. Considering that a very precise definition of the unit of time is indispensable for science and technology, the 13th CGPM (1967-1968, Resolution 1, CR, 103 and *Metrologia*, 1968, **4**, 43) chose a new definition of the second referenced to the frequency of the ground state hyperfine transition in the caesium 133 atom, as presented in 2.4.1.

2.5.2 Unit of length, metre

The 1889 definition of the metre, based on the international prototype of platinum-iridium, was replaced by the 11th CGPM (1960) using a definition based on the wavelength of the radiation corresponding to a particular transition in krypton 86. This change was adopted in order to improve the accuracy with which the definition of the metre could be realized, this being achieved using an interferometer with a travelling microscope to measure the optical path difference as the fringes were counted. In turn, this was replaced in 1983 by the 17th CGPM (Resolution 1, CR, 97, and *Metrologia*, 1984, **20**, 25) with a definition referenced to the distance that light travels in vacuum in a specified interval of time, as presented in 2.4.2. The original international prototype of the metre, which was sanctioned by the first meeting of the CGPM in 1889 (CR, 34-38), is still kept at the BIPM under conditions specified in 1889.

2.5.3 Unit of mass, kilogram

The 1889 definition of the kilogram was in terms of the mass of the international prototype of the kilogram, an artefact made of platinum-iridium. This is still kept at the BIPM under the conditions specified by the 1st CGPM in 1889 (CR, 34-38) when it sanctioned the prototype and declared that “this prototype shall henceforth be considered to be the unit of mass”. Forty similar prototypes were made at about the same time, and these were all machined and polished to have closely the same mass as the international prototype. At the CGPM in 1889, after calibration against the international prototype, most of these were individually assigned to Member States of the Metre Convention, and some also to the BIPM itself. The 3rd CGPM (1901, CR, 70), in a declaration intended to end the ambiguity in popular usage concerning the use of the word “weight”, confirmed that “the kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram”. The complete version of these declarations appears on p. XXX.

By the time of the second verification of national prototypes in 1946, however, it was found that on average the masses of these prototypes were diverging from that of the international prototype. This was confirmed by the third verification from 1989 to 1991, the median difference being about 25 micrograms for the set of original prototypes sanctioned by the first CGPM in 1889. In order to assure the long-term stability of the unit of mass, to take full advantage of quantum electrical standards, and to be of more utility to modern science, it was therefore decided to adopt a new definition for the kilogram referenced to the value of a fundamental constant, for which purpose the Planck constant h was chosen, as presented in 2.4.3.

2.5.4 Unit of electric current, ampere

Electric units, called "international units", for current and resistance were introduced by the International Electrical Congress held in Chicago in 1893, and definitions of the "international ampere" and "international ohm" were confirmed by the International Conference in London in 1908.

It was already obvious on the occasion of the 8th CGPM (1933) that there was a unanimous desire to replace those "international units" by so-called "absolute units". However because some laboratories had not yet completed experiments needed to determine the ratios between the international and absolute units the Conference gave authority to the CIPM to decide at an appropriate time both these ratios and the date at which the new absolute units would go into effect. This the CIPM did in 1946 (1946, Resolution 2, PV, 20, 129-137), when it decided that the new units would come into force on 1 January 1948. In October 1948 the 9th CGPM approved the decisions taken by the CIPM. The definition of the ampere chosen by the CIPM was referenced to the force between wires carrying an electric current, and it had the effect of fixing the value of the magnetic constant μ_0 (the permeability of vacuum). The value of the electric constant ε_0 (the permittivity of vacuum) then became fixed as a consequence of the new definition of the metre adopted in 1983.

However the 1948 definition of the ampere proved difficult to realise, and practical quantum standards based on the Josephson and quantum-Hall effects, which link the volt and the ohm to particular combinations of the Planck constant h and elementary charge e, have become almost universally used as a practical realisation of the ampere through Ohm's law (18th CGPM, 1987, Resolution 6, CR 100). As a consequence, it became natural not only to fix the numerical value of h to redefine the kilogram, but to fix the numerical value of e to redefine the ampere as presented in 2.4.4, in order to bring the practical quantum electrical standards into exact agreement with the SI.

2.5.5 Unit of thermodynamic temperature, kelvin

The definition of the unit of thermodynamic temperature was given in essence by the 10th CGPM (1954, Resolution 3; CR 79) which selected the triple point of water, T_{TPW}, as a fundamental fixed point and assigned to it the temperature 273.16 K, so defining the unit kelvin. The 13th CGPM (1967-1968, Resolution 3; CR, 104 and *Metrologia*, 1968, **4**, 43) adopted the name kelvin, symbol K, instead of "degree kelvin", symbol °K, for the unit defined in this way. However the difficulties in realising this definition, requiring a sample of pure water of well-defined isotopic composition, and the development of new primary methods of thermometry that are difficult to link directly to the triple point of water, led to the adoption of a new definition for the kelvin referenced to the value of the Boltzmann constant k, as presented in 2.4.5.

2.5.6 Unit of amount of substance, mole

Following the discovery of the fundamental laws of chemistry, units called, for example, “gram-atom” and “gram molecule”, were used to specify amounts of chemical elements or compounds. These units had a direct connection with “atomic weights” and “molecular weights”, which are in fact relative atomic and molecular masses. “Atomic weights” were originally referred to the atomic weight of oxygen, by general agreement taken as 16. But whereas physicists separated the isotopes in a mass spectrometer and attributed the value 16 to one of the isotopes of oxygen, chemists attributed the same value to the (slightly variable) mixture of isotopes 16, 17 and 18, which was for them the naturally occurring element oxygen. Finally an agreement between the International Union of Pure and Applied Physics (IUPAP) and the International Union of Pure and Applied Chemistry (IUPAC) brought this duality to an end in 1959-1960. Physicists and chemists have ever since agreed to assign the value 12, exactly, to the so-called atomic weight, correctly called the relative atomic mass A_r, of the isotope of carbon with mass number 12 (carbon 12, ^{12}C). The unified scale thus obtained gives the relative atomic and molecular masses, also known as the atomic and molecular weights, respectively.

The quantity used by chemists to specify the amount of chemical elements or compounds is now called “amount of substance”. Amount of substance, symbol n, is defined to be proportional to the number of specified elementary entities N in a sample, the proportionality constant being a universal constant which is the same for all entities. The proportionality constant is the reciprocal of the Avogadro constant N_A, so that $n = N/N_A$. The unit of amount of substance is called the *mole*, symbol mol. Following proposals by the IUPAP, the IUPAC, and the ISO, the CIPM gave a definition of the mole in 1967 and confirmed it in 1969, by specifying that the molar mass of carbon 12 should be exactly 0.012 kg/mol. This allowed the amount of substance $n_S(X)$ of any pure sample S of entity X to be determined directly from the mass of the sample m_S and the molar mass $M(X)$ of entity X, the molar mass being determined from its relative atomic mass A_r (atomic or molecular weight) without the need for a precise knowledge of the Avogadro constant, by using the relations

$$n_S(X) = m_S/M(X), \text{ and } M(X) = A_r(X) \text{ g/mol}$$

Nevertheless, this definition of the mole was dependent on the artefact definition of the kilogram, with the consequence that the uncertainty in the mass of the international prototype was reproduced in the definition of the mole.

The numerical value of the Avogadro constant defined in this way was equal to the number of atoms in 12 grams of carbon 12. However, because of recent technological advances, this number is now known with such precision that a simpler definition of the mole has become possible, namely, by specifying exactly the number of entities in one mole of any substance, thus specifying exactly the value of the Avogadro constant. This has the further advantage that this new definition of the mole and the value of the Avogadro constant is no longer dependent on the definition of the kilogram. Also the distinction between the fundamentally different quantities amount of substance and mass is thereby emphasised.

2.5.7 Unit of luminous intensity, candela

The units of luminous intensity based on flame or incandescent filament standards in use in various countries before 1948 were replaced initially by the “new candle” based on the luminance of a Planckian radiator (a black body) at the temperature of freezing platinum. This modification had been prepared by the International Commission on Illumination (CIE) and by the CIPM before 1937, and the decision was promulgated by the CIPM in 1946. It was then ratified in 1948 by the 9th CGPM which adopted a new international name for this unit, the *candela*, symbol cd; in 1967 the 13th CGPM (Resolution 5, CR, 104 and *Metrologia*, 1968, **4**, 43-44) gave an amended version of this definition.

In 1979, because of the difficulties in realizing a Planck radiator at high temperatures, and the new possibilities offered by radiometry, i.e. the measurement of optical radiation power, the 16th CGPM (1979, Resolution 3, CR, 100 and *Metrologia*,1980, **16**, 56) adopted a new definition of the candela.

2.6 Names and symbols for SI units

The symbols for SI units are listed in Tables 3 through 6. They are internationally agreed, and the same symbols are used in all languages, although the names of the units are language dependent. They are formatted as described in Chapter 5.

2.6.1 The seven base units

The names and symbols for the seven base units of the SI are listed in Table 3, along with the names and symbols for the corresponding quantities (10th CGPM (1954, Resolution 6, CR 80); 11th CGPM (1960, Resolution 12; CR 87); 13th CGPM (1967/68, Resolution 3; CR 104 and *Metrologia*, 1969, **4**, 43); 14th CGPM (1971, Resolution 3; CR 78 and *Metrologia*, 1972, **8**, 36)).

2.6.2 Derived units of the SI

Derived units are products of powers of the base units. Coherent derived units are products of powers of the base units that include no numerical factor other than 1. The base and coherent derived units of the SI form a coherent set, designated the set of coherent SI units (see section 1.4, p. XX).

Since the number of quantities in science is without limit, it is not possible to provide a complete list of derived quantities and derived units. However Table 4 lists some examples of derived quantities and the corresponding coherent derived units expressed in terms of base units.

2.6.3 Units with special names and symbols; units that incorporate special names and symbols

For convenience, certain coherent derived units have been given special names and symbols. There are 22 such units, as listed in Table 5. These special names and symbols may themselves be used in combination with the names and symbols for base units and for other derived units to express the units of other derived quantities. Some examples are given in Table 6. The special names and symbols are simply a compact form for the expression of combinations of base units that are used frequently, but in many cases they also serve to remind the reader of the quantity involved. The SI prefixes (see Chapter 3) may be used with any of the special names and symbols, but when prefixes are used the resulting set of units will no longer be coherent. Tables 4 and 5 illustrate the fact that there may be several alternative ways of writing the same derived unit.

Among the special names and symbols the last four entries in Table 4 are of particular note since they were adopted by the 15th CGPM (1975, Resolutions 8 and 9, CR 105 and *Metrologia* 1975, **11**, 180); 16th CGPM (1979, Resolution 5, CR 100 and *Metrologia* 1980, **16**, 56); and the 21st CGPM (1999, Resolution 12, CR 334-335 and *Metrologia* 2000, **37**, 95) specifically with a view to safeguarding human health.

In both Tables 5 and 6 the final column shows how the SI units concerned may be expressed using only the SI base units. In this column factors such as m^0, kg^0, etc., which are all equal to 1, are suppressed.

Table 3. SI base units

Base quantity		SI base unit	
Name	Symbol	Name	Symbol
length	l, x, r, etc.	metre	m
mass	m	kilogram	kg
time, duration	t	second	s
electric current	I, i	ampere	A
thermodynamic temperature	T	kelvin	K
amount of substance	n	mole	mol
luminous intensity	I_v	candela	cd

The symbols for quantities are generally single letters of the Latin or Greek alphabets, printed in an italic font, and are *recommendations*.

The symbols for units are *mandatory*, see chapter 5.

Table 4. Examples of coherent derived units in the SI expressed in terms of base units

Derived quantity		SI coherent derived unit	
Name	Symbol	Name	Symbol
area	A	square metre	m^2
volume	V	cubic metre	m^3
speed, velocity	υ	metre per second	m/s
acceleration	a	metre per second squared	$\mathrm{m/s}^2$
wavenumber	σ, $\tilde{\nu}$	reciprocal metre	m^{-1}
density, mass density	ρ	kilogram per cubic metre	$\mathrm{kg/m}^3$
surface density	ρ_{A}	kilogram per square metre	$\mathrm{kg/m}^2$
specific volume	υ	cubic metre per kilogram	$\mathrm{m}^3\mathrm{/kg}$
current density	j	ampere per square metre	$\mathrm{A/m}^2$
magnetic field strength	H	ampere per metre	A/m
amount concentration (*a*), concentration	c	mole per cubic metre	$\mathrm{mol/m}^3$
mass concentration	ρ, γ	kilogram per cubic metre	$\mathrm{kg/m}^3$
luminance	L_{v}	candela per square metre	$\mathrm{cd/m}^2$
refractive index (*b*)	n	one	1
relative permeability (*b*)	μ_{r}	one	1

(*a*) In the field of clinical chemistry this quantity is also called “substance concentration.”

(*b*) These are dimensionless quantities, or quantities of dimension one, and the symbol “1” for the unit (the number “one”) is generally omitted in specifying the values of dimensionless quantities.

Table 5. The 22 coherent derived units in the SI with special names and symbols

	SI coherent derived unit (*a*)			
Derived quantity	Base-unit symbol of derived unit (*b*)	Special name	Special symbol	Expressed in terms of other SI units
plane angle	m/m = 1	radian (*c*)	rad	1 (*c*)
solid angle	m^2/m^2 = 1	steradian (*c*)	sr (*d*)	1 (*c*)
frequency	s^{-1}	hertz (*e*)	Hz	
force	kg m s^{-2}	newton	N	
pressure, stress	kg m^{-1} s^{-2}	pascal	Pa	N/m^2
energy, work, amount of heat	kg m^2 s^{-2}	joule	J	N m
power, radiant flux	kg m^2 s^{-3}	watt	W	J/s
electric charge, amount of electricity	A s	coulomb	C	
electric potential difference, (*f*) electromotive force	kg m^2 s^{-3} A^{-1}	volt	V	W/A
capacitance	kg^{-1} m^{-2} s^4 A^2	farad	F	C/V
electric resistance	kg m^2 s^{-3} A^{-2}	ohm	Ω	V/A
electric conductance	kg^{-1} m^{-2} s^3 A^2	siemens	S	A/V
magnetic flux	kg m^2 s^{-2} A^{-1}	weber	Wb	V s
magnetic flux density	kg s^{-2} A^{-1}	tesla	T	Wb/m^2
inductance	kg m^2 s^{-2} A^{-2}	henry	H	Wb/A
Celsius temperature	K	degree Celsius (*g*)	$^{\circ}$C	
luminous flux	cd sr (*d*)	lumen	lm	cd sr (*d*)
illuminance	cd sr m^{-2}	lux	lx	lm/m^2
activity referred to a radionuclide (*h*)	s^{-1}	becquerel (*e*)	Bq	
absorbed dose, specific energy (imparted), kerma	m^2 s^{-2}	gray	Gy	J/kg
dose equivalent, ambient dose equivalent, directional dose equivalent, personal dose equivalent	m^2 s^{-2}	sievert (*i*)	Sv	J/kg
catalytic activity	mol s^{-1}	katal	kat	

(*a*) The SI prefixes may be used with any of the special names and symbols, but when this is done the resulting unit will no longer be coherent.

(*b*) For simplicity and because they are straightforward, the names of these units are omitted. Two examples are the unit of energy, kilogram metre squared per second squared, kg m^2 s^{-2}; and the unit of inductance, kilogram metre squared per second squared per ampere squared, kg m^2 s^{-2} A^{-2}. The order of the base units reflects the order of the base quantities in the equation that relates the derived quantity to the base quantities on which it depends.

(*c*) The radian and steradian are special names for the number one that may be used to convey information about the quantity concerned. In practice the symbols rad and sr are used where appropriate, but the symbol for the derived unit one is generally omitted in specifying the values of dimensionless quantities.

(*d*) In photometry the name steradian and the symbol sr are usually retained in expressions for units.

(*e*) The hertz is used only for periodic phenomena, and the becquerel is used only for stochastic processes in activity referred to a radionuclide.

(*f*) Electric potential difference is also called “voltage” in many countries, as well as “electric tension” or simply “tension” in some countries.

(*g*) The degree Celsius is the special name for the kelvin used to express Celsius temperatures. The degree Celsius and the kelvin are equal in size, so that the numerical value of a temperature difference or temperature interval is the same when expressed in either degrees Celsius or in kelvins.

(*h*) Activity referred to a radionuclide is sometimes incorrectly called radioactivity.

(*i*) See CIPM Recommendation 2 (CI-2002), p. XX, on the use of the sievert (PV, 2002, **70**, 205).

Table 6. Examples of SI coherent derived units whose names and symbols include SI coherent derived units with special names and symbols

	SI coherent derived unit		
Derived quantity	Name	Symbol	Expressed in terms of SI base units
dynamic viscosity	pascal second	Pa s	kg m^{-1} s^{-1}
moment of force	newton metre	N m	kg m^{2} s^{-2}
surface tension	newton per metre	N/m	kg s^{-2}
angular velocity	radian per second	rad/s	m m^{-1} s^{-1} = s^{-1}
angular acceleration	radian per second squared	rad/s^{2}	m m^{-1} s^{-2} = s^{-2}
heat flux density, irradiance	watt per square metre	W/m^{2}	kg s^{-3}
heat capacity, entropy	joule per kelvin	J/K	kg m^{2} s^{-2} K^{-1}
specific heat capacity, specific entropy	joule per kilogram kelvin	J/(kg K)	m^{2} s^{-2} K^{-1}
specific energy	joule per kilogram	J/kg	m^{2} s^{-2}
thermal conductivity	watt per metre kelvin	W/(m K)	kg m s^{-3} K^{-1}
energy density	joule per cubic metre	J/m^{3}	kg m^{-1} s^{-2}
electric field strength	volt per metre	V/m	kg m s^{-3} A^{-1}
electric charge density	coulomb per cubic metre	C/m^{3}	A s m^{-3}
surface charge density	coulomb per square metre	C/m^{2}	A s m^{-2}
electric flux density, electric displacement	coulomb per square metre	C/m^{2}	A s m^{-2}
permittivity	farad per metre	F/m	kg^{-1} m^{-3} s^{4} A^{2}
permeability	henry per metre	H/m	kg m s^{-2} A^{-2}
molar energy	joule per mole	J/mol	kg m^{2} s^{-2} mol^{-1}
molar entropy, molar heat capacity	joule per mole kelvin	J/(mol K)	kg m^{2} s^{-2} mol^{-1} K^{-1}
exposure (x- and γ-rays)	coulomb per kilogram	C/kg	A s kg^{-1}
absorbed dose rate	gray per second	Gy/s	m^{2} s^{-3}
radiant intensity	watt per steradian	W/sr	kg m^{2} s^{-3} m^{-2} m^{2} = kg m^{2} s^{-3}
radiance	watt per square metre steradian	W/(sr m^{2})	kg s^{-3} m^{-2} m^{2} = kg s^{-3}
catalytic activity concentration	katal per cubic metre	kat/m^{3}	mol s^{-1} m^{-3}

It will be seen from these tables that several different quantities may be expressed using the same SI unit. Thus for the quantity heat capacity as well as the quantity entropy the SI unit is the joule per kelvin. Similarly for the base quantity electric current as well as the derived quantity magnetomotive force the SI unit is the ampere. It is therefore important not to use the unit alone to specify the quantity. This applies not only to technical texts, but also, for example, to measuring instruments (i.e. the instrument read-out should indicate both the unit and the quantity measured).

In practice, with certain quantities, preference is given to the use of certain special unit names, to facilitate the distinction between different quantities having the same dimension. When using this freedom one may recall the process by which this quantity is defined. For example the quantity torque may be thought of as the cross product of force and distance, suggesting the unit newton metre, or it may be thought of as energy per angle, suggesting the unit joule per radian. The SI unit of frequency is given as the hertz, implying the unit cycles per second; the SI unit of angular velocity is given as the radian per second; and the SI unit of activity is designated the becquerel, implying the unit counts per second. Although it would be formally correct to write all three of these units as the reciprocal second, the use of the different names emphasises the different nature of the quantities concerned. Using the unit radian per second for angular velocity, and hertz for frequency, also emphasises that the numerical value of the angular velocity in radians per second is 2π times the corresponding frequency in hertz.

In the field of ionizing radiation, the SI unit is designated the becquerel rather than the reciprocal second, and the SI units of absorbed dose and dose equivalent are designated the gray and the sievert respectively, rather than the joule per kilogram. The special names becquerel, gray and sievert were specifically introduced because of the dangers to human health that might arise from mistakes involving the units reciprocal second and joule per kilogram, in case the latter units were incorrectly taken to identify the different quantities involved.

[A new section on units for dimensionless quantities is in preparation, which might be introduced here, or possibly as part of Chapter 4, or a completely new chapter; this is still to be decided.]

3 Decimal multiples and sub-multiples of SI units

3.1 SI prefixes

The 11th CGPM (1960, Resolution 12, CR 87) adopted a series of prefix names and prefix symbols to form the names and symbols of decimal multiples and submultiples of SI units, ranging from 10^{12} to 10^{-12}. These were extended to cover 15, 18, 21 and 24 powers of ten, positive and negative, by the 12th, 15th and 19th meetings of the CGPM, as detailed in Appendix 1, to give the complete list of all approved SI prefix names and symbols presented in Table 6 below.

Prefix symbols are printed in roman (upright) type, as are unit symbols, regardless of the type used in the surrounding text, and are attached to unit symbols without a space between the prefix symbol and the unit symbol. With the exception of da (deca), h (hecto), and k (kilo), all multiple prefix symbols are capital (upper case) letters, and all submultiple prefix symbols are lowercase letters. All prefix names are printed in lowercase letters, except at the beginning of a sentence.

The SI prefixes refer strictly to powers of 10. They should not be used to indicate powers of 2 (for example, one kilobit represents 1000 bits and not 1024 bits). The IEC has adopted prefixes for binary powers in the international standard IEC 60027-2, 2005, third edition: *Letter symbols to be used in electrical technology – Part 2, Telecommunications and electronics.* The names and symbols for prefixes to be used with powers of 2 recommended there are

kibi	Ki	2^{10}
mebi	Mi	2^{20}
gibi	Gi	2^{30}
tebi	Ti	2^{40}
pebi	Pi	2^{50}
exbi	Ei	2^{60}

where B denotes a byte. These prefixes are used in the field of information technology to avoid the incorrect usage of the SI prefixes.

Table 7. SI prefixes

Factor	Name	Symbol	Factor	Name	Symbol
10^{1}	deca	da	10^{-1}	deci	d
10^{2}	hecto	h	10^{-2}	centi	c
10^{3}	kilo	k	10^{-3}	milli	m
10^{6}	mega	M	10^{-6}	micro	µ
10^{9}	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a
10^{21}	zetta	Z	10^{-21}	zepto	z
10^{24}	yotta	Y	10^{-24}	yocto	y

The grouping formed by a prefix symbol attached to a unit symbol constitutes a new inseparable unit symbol (forming a multiple or submultiple of the unit concerned) that can be raised to a positive or negative power and that can be combined with other unit symbols to form compound unit symbols.

Examples: pm (picometre), mmol (millimole), GΩ (gigaohm), THz (terahertz)

$$2.3\ \mathrm{cm}^3 = 2.3\ (\mathrm{cm})^3 = 2.3\ (10^{-2}\ \mathrm{m})^3 = 2.3 \times 10^{-6}\ \mathrm{m}^3$$

$$1\ \mathrm{cm}^{-1} = 1\ (\mathrm{cm})^{-1} = 1\ (10^{-2}\ \mathrm{m})^{-1} = 10^{2}\ \mathrm{m}^{-1} = 100\ \mathrm{m}^{-1}$$

$$5000\ \mu\mathrm{s}^{-1} = 5000\ (\mu\mathrm{s})^{-1} = 5000\ (10^{-6}\ \mathrm{s})^{-1} = 5 \times 10^{9}\ \mathrm{s}^{-1}$$

Similarly prefix names are also inseparable from the unit names to which they are attached. Thus, for example, millimetre, micropascal, and meganewton are single words.

Compound prefix symbols, that is prefix symbols formed by the juxtaposition of two or more prefix symbols, are not permitted. This rule also applies to two or more compound prefix names.

Prefix symbols can neither stand alone nor be attached to the number 1, the symbol for the unit one. Similarly prefix names cannot be attached to the name of the unit one, that is, to the word “one”.

Prefix names and symbols are used with a number of non-SI units (see Chapterr 5), but they are never used with the units of time: minute, min; hour, h; day, d. However astronomers use milliarcsecond, which they denote mas, and microarcsecond, µas, which they use as units for measuring very small angles.

3.2 The kilogram

Among the base units of the International System, the kilogram is the only one whose name and symbol, for historical reason, include a prefix. Names and symbols for decimal multiples and sub-multiples of the unit of mass are formed by attaching prefix names and symbols to the unit name “gram” and the unit symbol “g” respectively (CIPM 1967, Recommendation 2; PV, **35**, 29 and *Metrologia*, 1968, **4**, 45).

4 Units outside the SI

The International System of Units, the SI, is a system of units adopted by the CGPM, which provides the internationally agreed reference in terms of which all other units are now defined. It is recommended for use throughout science, technology, engineering and commerce.

(essentially chapter 3 of the current Brochure)

[All later sections of the Brochure are still being revised at present. However there will be few changes in the remaining sections.

Except that we have still to draft a section on dimensionless quantities. This is still being developed at the time of preparing this draft, December 2013.]