TIME SCALES

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Abstract

From the relativistic point of view, a time scale is considered as one of the coordinates of a four-dimensional space-time reference system. In practice, the generation of an accurate, stable, and reliable time scale calls for the design of an algorithm able to efficiently combine timing data from an ensemble of clocks. For most algorithms in use, the time scale is basically defined as a weighted average, but the weight determination and the specific actions for minimizing instabilities due to changes of weights, are of different types because they are adapted to different needs. These different problems are discussed and illustrated taking examples among the best time scales computed in the world.

Résumé

Dans le cadre relativiste, on considère qu'une échelle de temps est l'une des coordonnées d'un système de référence spatio-temporel à quatre dimensions. Dans la pratique, générer une échelle de temps exacte, stable et fiable demande que l'on conçoive un algorithme capable de combiner efficacement les données temporelles provenant d'un ensemble d'horloges. Pour la plupart des algorithmes utilisés, l'échelle de temps est fondamentalement définie comme étant une moyenne pondérée. Cependant, déterminer comment pondérer individuellement les horloges et comment agir afin de minimiser les instabilités provenant des changement de poids, n'a pas de solution unique car l'on doit s'adapter aux différents besoins des utilisateurs. Dans ce texte, ces problèmes sont discutés et illustrés grâce à des exemples choisis parmi les meilleures échelles de temps du monde.

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Introduction

The true nature of time has no rational explanation; we simply feel that time never stops or reverses. But, apart from philosophical considerations, experience indicates that any event can be localized by specifying three space coordinates and one temporal coordinate, generally denoted (x, y, z, t). So far, no experiment has ever called for more independent parameters. Intuitively, a time scale is thus defined as the time axis of a system of coordinates. Officially the International Radio Consultative Committee, the CCIR, defines a time scale as an *ordered set of scale markers with an associated numbering* [CCIR Recommendations 1990].

At first sight, establishing a time scale seems a simple task, as any evolving system allows the transformation of the measure of time variation into the measure of a dimensional quantity. But scientists ask for metrological properties, and reliable, stable, and accurate reference time scales are required. In addition, there is no absolute time as conceived by Newton in classical mechanics. In reality, all temporal phenomena are affected by gravitational fields and velocities with respect to the observer. Time scales must thus be defined in the framework of general relativity, as explained in section 1.

Conventionally, one distinguishes two different types of time scale: integrated time scales and dynamic time scales:

* For integrated time scales, the primary data is a unit of duration, that is, of time interval, defined from a physical phenomenon. The time scale is constructed by fixing a conventional origin and by accumulating units of duration without dead-time and without interruption. This concept for the construction of a time axis was applied to the duration of the day, leading to the definition of Mean Solar Time. The present worldwide reference time scale, International Atomic Time, TAI, is an integrated time scale; it is obtained by the accumulation of atomic seconds defined as a number of periods of the radiation corresponding to a given transition of the caesium atom.
* For dynamic time scales, the primary data results from the observation of a dynamic physical system, described by a mathematical model in which time is a parameter that unambiguously identifies the configurations of the system. The time measurement thus becomes a position measurement, and the unit of time is defined as a particular duration. Universal Time, UT1, and Ephemeris Time, ET, are dynamic time scales, based respectively on the rotation of the Earth on its axis and around the Sun.

In the past, a number of time scales, either dynamic or integrated, have been defined. The associated unit of duration was then used to define the second of the International System of Units (SI). The change from one definition to another has been motivated by the desire to improve accuracy. A brief summary is given in the following sections.

Universal Time

Universal Time, UT1, is a dynamic time, derived from the observation of the Earth's rotation: it is proportional to the angle of rotation of the Earth on its axis. The coefficient of proportionality is chosen so that 24 hours of UT1 are close to the mean duration of the day, and the phase is chosen so that 0 h UT1 corresponds, on average, to midnight in Greenwich [Guinot 1994].

The associated unit of time is the second of mean solar time. Its definition is not very precise: it is *the fraction 1/86400 of the mean solar day*. This was the SI second till 1960. Astronomers estimated that it could be realised with an uncertainty of a few parts in 10^8, this level of accuracy being achieved after a decade of astronomical observations, analysis and averaging.

The UT1 was the reference time scale, and thus the basis of legal time, until 1972. It still provides a record of the Earth's rotation for geophysics, astronomy... *etc.*

Ephemeris Time

Ephemeris Time, ET, is a dynamic time, derived from the theory of the Earth rotation around the Sun: it is provided through an expression for the mean longitude of the Sun. This expression and the initial phase of ET were chosen so that ET and UT1 were in approximate coincidence in 1900. Since then they have slowly diverged (ET - UT1 $\approx$ 56 s in 1988) [Guinot 1994].

The associated unit of time is the ephemeris second, defined as *the fraction 1/31556925,9747 of the tropical year for 1900 January 0 at 12 hours ephemeris time*. This was the SI second from 1960 to 1967.

Through observations of planets and of the moon, it has been possible to obtain the time differences between ET and UT1 since 1630, with a precision of several tens of seconds for the 17th century, and several seconds for the 19th century; ET thus constitutes a reference for studying the Earth's rotation in the past.

International Atomic Time

International Atomic Time, TAI, is an integrated time available since 1955. The unit of time is the atomic second, which became the SI second in 1967 and is still in use. Its definition, adopted by the 13th Conférence Générale des Poids et Mesures in 1967, is as follows:

> *The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.*

The atomic second can be realized in a laboratory. The best commercial caesium clocks, now widely used in timing centres, produce it with a stated accuracy of order 1 part in 10^{12}. Some laboratories maintain a number of primary frequency standards; these give an ultimate accuracy of realization of order 1 part in 10^{14} [Lee *et al.* 1994].

The function of a clock is to provide a continuous, ordered set of markers with an associated numbering. This constitutes a time scale. The number associated with a given marker is designated as the "reading of the clock". Since physical devices can fail, laboratories are inevitably led to keep several clocks. Clock data are collected and treated together through a combination of their readings in order to generate an ensemble time. On a worldwide scale, such an ensemble time is International Atomic Time, TAI.

The definition of TAI was approved by the Comité International des Poids et Mesures in 1970, and recognized by the Conférence Générale des Poids et Mesures in 1971. It reads as follows:

> *International Atomic time (TAI) is the time reference coordinate established by the Bureau International de l'Heure on the basis of the readings of atomic clocks operating in various establishments in accordance with the definition of the second, the unit of time of the International System of Units.* [In 1988, the responsibility for TAI was

transferred to the Time Section of the Bureau International des Poids et Mesures, BIPM].

The origin of TAI has been agreed officially to coincide with UT1 on 1st January 1958.

An important consequence of improved accuracies in the realization of the atomic second is that relativistic effects are significant. In this context, the definition of the second must be understood as the definition of proper time, *i.e.*, strictly speaking, the user must be in the neighbourhood of the clock and at rest with respect to it. When comparing two realizations of the SI second, differences of a few parts in 10^{13} may appear due to the different gravitational fields to which the clocks are subjected. The definition of TAI was thus completed as follows, in a declaration of the Comité Consultatif pour la Définition de la Seconde [CCDS Report, 1980] during its 9th Session held in 1980:

> TAI *is a coordinate time scale defined in a geocentric reference frame* (origin of the frame at the centre of the Earth) *with the SI second as realized on the rotating geoid as the scale unit.*

Hence a new situation (unlike UT1 and ET) arose in which the relation between the TAI scale unit and a given realization of the SI second depends on the position of the clock which produces it. For all clocks fixed on the Earth and situated at sea level, the scale unit of TAI is equal to the unit of time as realized locally; but the scale unit of TAI appears to be longer by $1{,}1 \times 10^{-13}$ s when compared with a clock at a 1000 m altitude, due to the gravitational red frequency shift [Misner *et al.* 1973]. A complete theoretical definition of TAI, in the framework of general relativity, was given for the first time in 1991 by the International Astronomical Union, IAU, (see section 1).

Coordinated Universal Time

Coordinated Universal Time, UTC, was defined in 1972. It represents a clever combination of the two time scales TAI and UT1, and is defined by the following system of equations valid for any date t:

$$UTC(t) - TAI(t) = n \text{ seconds } (n \text{ integer})$$

and

$$|UTC(t) - UT1(t)| < 0{,}9 \text{ s}. \tag{1}$$

The quantities $UTC(t)$ and $TAI(t)$ differ, for any date t, by an integer number of seconds, equal to 29 from 1st July 1994. The International Earth Rotation Service, IERS, which is responsible for the publication of UT1, decides on the adjustment of seconds by reference to the predicted divergence between the time scales UT1 and TAI. Leap seconds are introduced at the end of a month, normally at the end of June or December.

By definition, UTC has the same metrological properties as TAI, which is an atomic time. In addition, it follows the rotation of the Earth to within 1 second. This is sufficient for the purposes of astronomical navigation, where UT1 is required in real time.

The UTC is the general basis for the distribution of time around the world. Local times are derived from UTC by a shift of an integer number of hours (which can change from winter time to summer time), decided by the administration of individual countries, or regional groups. All time signals, at whatever level, including signals distributed by TV, radio, or speaking clocks, are synchronized on these local times, and thus to UTC.

The reference time scales TAI and UTC are calculated and distributed by the Time Section of the BIPM. These are deferred-time time scales, for which to achieve ultimate metrological quality requires months of data collection and treatment. National time laboratories thus keep other time scales, for more immediate use, which are carefully compared *a posteriori* with TAI and UTC at each new issue. These are the independent local time scales, TA(k), and the local representations of UTC, UTC(k), where k designates the acronym of the laboratory.

In 1994, 17 independent TA(k) are maintained. They are generated from small ensembles of clocks carefully kept on one site, such as is the American A.1(MEAN) from the USNO and AT1 from the NIST, or on several different sites within the country, such as is done for the French TA(F) and the Swiss TA(CH). The basic measurement cycle is much shorter than for TAI (1 hour to 1 day against 10 days), the time scale is updated much more often (every day or week against every 2 months), and the update may be calculated *a posteriori* or in almost real time. These time scales are free-running and have no physical representations. They are known through values of time differences with respect to a physical clock which is also kept in the laboratory. Their scientific purpose is to provide a stable local reference.

In 1994, 45 UTC(k) are in operation throughout the world. They are generally linked to the output of a commercial clock, with or without frequency correction, and thus correspond to a physical signal accessible in real time. They are not free-running but are closely steered to follow UTC. The recommended maximum time difference between the time scales UTC and UTC(k) is $\pm$ 1 μs [CCIR 1990], and the goal is to reach $\pm$ 100 ns [CCDS 1993]. The UTC(k) provide real-time synchronization, in particular they are used as references for broadcast time signals.

Other time scales support applications in navigation and timing, through satellite global navigation systems. The two principal ones are GPS time, for the American Global Positioning System, and GLONASS time for the Russian GLObal NAvigation Satellite System. Both are generated with a high update rate (of order several minutes) from an ensemble of clocks kept in the master control station of the system, and steered on a local representation of UTC: UTC(USNO) for GPS time and UTC(SU) for GLONASS time.

For most demanding applications, such as millisecond pulsar timing, the BIPM issues atomic time scales retrospectively. These are designated TT(BIPMxx) where 1900 + xx is the year of computation [Guinot 1988]. The successive versions of TT(BIPMxx) are both updates and revisions: they may differ for common dates.

Before dealing with the practice of computing and disseminating time scales such as TAI, UTC, TA(k), UTC(k), GPS time, GLONASS time, and TT(BIPMxx), in section 2 and 3, we return to the theoretical definitions of time scales in general relativity.

1. Time scales in general relativity

1.1. Coordinate systems in general relativity

In general relativity, time scales are considered as one of the coordinates of four-dimensional space-time reference systems.

Due to the curvature of space-time, the scale units of these coordinates do not, in general, have a globally constant relation to locally measurable (proper) quantities [Damour 1988, Misner *et* *al.* 1973, Brumberg 1991]. In the framework of Newtonian mechanics, it is always possible to define coordinates in such a way that their scale units are everywhere equal to measured distances and durations. This is impossible in general relativity, where the relation between measured quantities and coordinate scale units depends on the position in space-time of the measuring observer. For time scales, this implies that the relation between a coordinate time interval and the locally realized second, using an atomic clock for example, depends on the position of the clock.

In principle, one is free to use any set of coordinates for the description of space-time. However, it turns out that by defining several overlapping systems of coordinates, each valid in a restricted region, the treatment of practical problems and the relationship between coordinates and measurable quantities can be greatly simplified [Damour 1989, Soffel 1989, IAU 1992]. Such definitions provide several time coordinates, each valid in a particular region of space-time, with the relation between them given by relativistic coordinate transformations.

A system of coordinates in general relativity is defined by its metric tensor $g_{\alpha\beta}(x^{\lambda})$ (Greek indices run from 0 to 3) which is position- and time- dependent and must be known for the whole region of space-time within which the coordinate system is used.

The need to define several relativistic systems of space-time coordinates, in particular barycentric and geocentric ones, was recognized by the International Astronomical Union IAU in its 1991 Resolution A4 [IAU 1991, 1992]. This Resolution includes definitions of barycentric and geocentric coordinate time scales, and so provides the theoretical basis for the definition of TAI.

1.2. The 1991 IAU Resolution A4

The International Astronomical Union approved Resolution A4 at its General Assembly held in Buenos Aires in August 1991. The complete text of this Resolution is in the 1992 IAU Information Bulletin 67. It contains several Recommendations of importance for the definition and realization of coordinate time scales. They are explained in the following sections.

Recommendation I

Recommendation I explicitly introduces the general theory of relativity as the theoretical background for the definition of space-time reference frames. It provides the form of the metric to be used for coordinate systems centred at the barycentre of an ensemble of masses:

$$ds^2 = -c^2 d\tau^2 = g_{\alpha\beta}(x^{\lambda}) dx^{\alpha} dx^{\beta}, \tag{2}$$

$$ds^2 = -\left(1 - 2\frac{U}{c^2}\right)(dx^0)^2 + \left(1 + 2\frac{U}{c^2}\right)\left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2\right], \tag{3}$$

where ds is an infinitesimal space-time line element, τ is proper time as realized by an ideal clock, c is the velocity of light in vacuum, U is the sum of the gravitational potentials of the ensemble of masses and of a tidal potential generated by masses external to the ensemble, the latter potential vanishing at the barycentre. The four space-time coordinates are defined to be ($x^0 = ct$, x^1, x^2, x^3). The Einstein summation convention is used, implying summation over repeated indices. It should be noted that (3) gives only the first terms of a series, which is sufficient for the present level of observational accuracy. Higher order terms may be added as necessary. For time and frequency applications, this will be the case when clock stabilities reach some parts in 10^{19}.

Recommendation II

Recommendation II states that the space coordinate grid with its origin at the centre of mass of the Earth should show no global rotation with respect to a set of distant extragalactic objects, that the time coordinates for all coordinate systems should be derived from a time scale realized by atomic clocks operating on the Earth, and that the basic physical units of space-time are the second of the International System of units (SI) for proper time and the SI metre for proper length. This Recommendation should also apply to clocks on board terrestrial satellites.

Recommendation III

Recommendation III defines the scale units and origins of all time coordinates, and designates the solar system barycentric time coordinate and the geocentric time coordinate as Barycentric Coordinate Time, TCB, and Geocentric Coordinate Time, TCG, respectively. It should be noted that these time coordinates exhibit secular differences between themselves and with respect to TAI.

Recommendation IV

Recommendation IV defines Terrestrial Time, TT, a geocentric coordinate time scale differing from TCG by a constant rate, the scale unit of TT being chosen so that it agrees with the SI second on the rotating geoid. This constant rate is presently estimated to 6,9692904 parts in 10^{10} with an uncertainty of 1 part in 10^{17} (1 σ).

As the theoretical time scales TCB, TCG, and TT are completely defined by the IAU Resolution A4, the path to realized time scales is immediate.

1.3. International Atomic Time

According to its definition, International Atomic Time, TAI, is simply a realization of TT, apart from an offset of 32,184 s introduced for historical reasons [Guinot 1994]. It is obtained by combining the data from an ensemble of about two hundred atomic clocks spread world-wide. To achieve this it is necessary to compare those clocks using a convention of coordinate synchronization [Ashby and Allan 1979, Allan and Ashby 1986]. This is defined as follows:

> *Two events fixed in some reference system by the values of their coordinates (t_1,x_1,y_1,z_1) and (t_2,x_2,y_2,z_2) are considered to be simultaneous with respect to this reference system if the values of the corresponding time coordinates are equal: $t_1 = t_2$. Two clocks are considered to be synchronized with respect to some reference frame if they simultaneously (in the above sense) produce the same time markers.*

In the vicinity of the Earth a geocentric, non-rotating reference frame, as defined in Resolution A4, Recommendation II of the IAU, is used for the synchronization of clocks, and in particular for the calculation and dissemination of TAI (see also section 2.2.).

To conform with the definition of TT, the scale unit of TAI is defined to be equal to the SI second as realized on the rotating geoid [Le Système International d'Unités 1991]. To do this, data from the most accurate primary standards are individually corrected for the gravitational frequency shift arising from the elevation of the laboratory above the geoid and are then combined to form the scale unit of TAI.

1.4. Other coordinate time scales

Atomic coordinate time scales like TA(k), UTC, UTC(k), GPS time, GLONASS time, TT(BIPMxx)... *etc.* are time coordinates closely related to TAI and provided for different purposes. The scales TCG and TCB are related to TT and hence TAI by relativistic transformations [notes to Recommendations III and IV of the IAU Resolution A4, IAU 1992].

2. Generation of time scales

The practical problem is to generate a time scale from an ensemble of atomic clocks maintained in one or several laboratories. The efficient combination of the readings of the participating clocks requires [Tavella and Thomas 1990a]:

* the definition of the expected qualities of the time scale,
* the characterization of the available timing data,
* the design of an algorithm for data treatment.

2.1. Expected qualities

In general, the requirement is to generate a time scale which is as close as possible to an ideal time scale. The departure from an accumulation of ideal SI seconds on the rotating geoid can be estimated through computation of its "normalized frequency departure" at date t, commonly referred to as "frequency", defined as:

$$y(t) = \frac{\nu(t) - \nu_0}{\nu_0}, \qquad (4)$$

where $\nu_0 = 1$ Hz, and $\nu(t)$ is the reciprocal of the scale unit, expressed in SI second, of the time scale for date t.

Actual physical clocks have defaults that are minimized by combining their data to obtain a reliable, stable and accurate time scale. A separate, but important, point is the delay of access to the scale. For some purposes access must be immediate, for others a significant delay may be tolerable.

Reliability

Individual physical clocks may fail with immediate interruption of the time scales they deliver. Reliability thus calls for redundancy and eventually for national or international collaboration between laboratories maintaining atomic clocks.

The simplest solution to this problem is to replace the clock which has failed by another one. This is what is normally done in time laboratories which generate a UTC(k). Usually the UTC(k) is directly linked to the output of a physical clock, generally the best one of the ensemble on site, this being designated the "master clock" [CCIR 1990]. Its output is controlled by small, predetermined frequency and time steps, often through a microphase stepper, so as to steer UTC(k) to UTC. A change of master clock thus does not influence the output time scale if the microphase stepper is suitably programmed to handle the change (see examples in section 2.5.).

More often, reliability is ensured by using an ensemble of clocks and computing an ensemble time. This time rarely has a physical realization. In computing such an ensemble time, it is necessary to minimize the perturbations that result as clocks enter and leave the ensemble. Clearly the more clocks at the disposal of an ensemble, the less is the effect as one clock enters or leaves. For this reason there has been a general increase in the number of clocks in a given ensemble. For example, the number of clocks contributing to TAI was around 180 for some 10 years. Since beginning of 1993, however, the commercialization of the new HP 5071A clock, from the firm Hewlett-Packard, has led to a steady increase in the number of clocks, which reached 237 in March 1994. A major consequence is a gain in reliability for TAI.

Stability

The stability of a time scale may be defined as its ability to maintain a constant scale interval, even if it differs from the ideal one. A measure of stability thus consists in the estimation of the dispersion of the frequency values $y(t)$ with time. Some statistical tools have been developed to estimate stability (see Annex I). They are efficient for the characterization of the usual types of random noise which affect clock signals. The most common stability estimation tool is the two-sample, or Allan variance $\sigma_y^2(\tau)$ which depends upon the observational, or sampling, time τ.

The stability of an ensemble time scale depends upon the stabilities of the contributing clocks and the design of the algorithm used to generate it. The algorithm must, in particular, correctly handle any change in clock behaviour. The general considerations are detailed in 2.3., but the central idea is to generate a time scale more stable that any of the contributing clocks. This can be realized, but generally only for a given range of averaging times τ.

In principle, the concept of stability applies only to free-running time scales. A UTC(k) is, by definition, steered so it is affected by intentional frequency steps and its short- and middle-term stability is inevitably degraded. In addition, a crucial problem comes from the fact that time scale frequency values are always estimated or measured with respect to the frequency of another time scale or physical clock. The stability analysis of such comparative measures leads to evaluation of the coupled stability of the two time scales. Two cases may arise:

* The frequency of the time scale under test is evaluated by comparison with a time scale of better quality, such as those realized by primary frequency standards. The observed instability then may be ascribed totally to the time scale under test.
* The two time scales which are compared are supposed to be of similar quality. A technique for noise decoupling is then necessary. If the time scales involved in the comparison may be assumed to be completely independent, the N-cornered-hat technique [Barnes 1982, Allan 1987] produces an estimate of the intrinsic stability of each element. If the independence is not verified, variances and covariances should be handled together for a complete analysis [Premoli and Tavella 1993, Tavella and Premoli 1994].

Accuracy

The accuracy of a time scale may be defined as its ability to maintain a mean scale interval as close as possible to its definition. For time scales which realize TT, the mean scale interval should be as close as possible to the SI second on the rotating geoid.

For primary frequency standards, accuracy is given by an uncertainty budget obtained through the evaluation of the physical effects that modify the output frequency with respect to the definition. When it is not possible to constitute such an uncertainty budget, accuracy is evaluated by comparison of the duration of the scale interval with the best realization of the SI second provided by primary frequency standards. It is of course necessary to take into account the effect of the gravitational red shift on the primary standard frequency results in order to convert its realized SI second onto the geoid (null height). The accuracy of a time scale is generally given by a frequency difference between the time scale and the primary frequency standard, evaluated for averaging times corresponding to the best stability of the time scale, and by taking into account the uncertainty of the primary frequency standard.

Improvement in the accuracy of a time scale is generally carried out outside the main algorithm, which deals only with stability optimization. This may be done by steering the frequency of the time scale on the frequency of a primary standard or of a more stable reference time scale. For this to be effective, the frequency corrections must be smaller than the frequency fluctuations of the time scale in order to avoid a degradation of its stability.

Delay of access

The delay of access to a time scale is linked to the quality of raw timing data and to the scientific purposes the time scale is supposed to fulfil.

Raw timing data is acquired according to a basic measurement cycle, whose duration ranges from several minutes to several hours, and is affected by measurement noise. Depending on the level of this noise, it may be necessary to smooth out raw measurements by accumulating data over several successive basic samples of measurement (see section 2.2.). This delays access to the resulting time scale. In addition, it is useful to observe the behaviour of contributing clocks for a long period, both before and after the moment to which the data applies, in order to make the best use of their data. This also delays access.

What constitutes an acceptable delay of access to a time scale depends on its use. For a reference time scale, such as TAI, the requirement is extreme reliability and long-term stability. To match this purpose, the reference time scale relies on a large number of clocks of different types, located in different parts of the world. Data must therefore be collected and handled correctly, which takes time. The delay is thus considerable but is acceptable because of the ultimate quality obtained. For scientific studies inside a laboratory, however, it may be necessary to produce the time scale in near real time, immediately after the clock measurements, even if this impairs the long-term qualities of the scale.

2.2. Timing data

Timing data takes the form of time differences between clocks. An atomic clock delivers a series of physical electric pulses separated from one another by a duration of 1 second, often designated as "series of 1 pps". Each pulse is an event with an associated number, a kind of label which is tied to it. This associated number is the reading of the clock for that particular event: for example, it may read as 1994 June 13 11 h 27 min 13 s. It can also be designated as the date of the event; its origin is arbitrary and is chosen to be convenient, but it is incremented by 1 second at each new pulse. Clock readings vary continuously and rapidly, so they can only be "caught in flight". However, counters are available: they can be started with a given pulse coming from one clock and stopped with the pulse with the same label coming from another

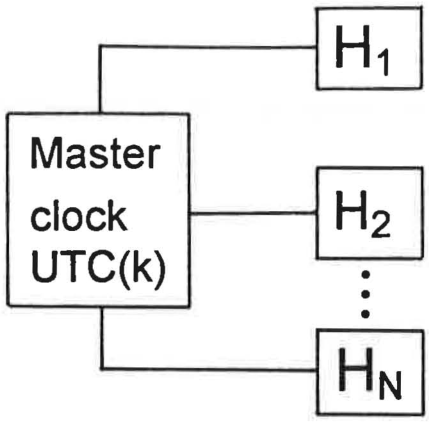


Figure 1. Comparison between clocks located on the same site.

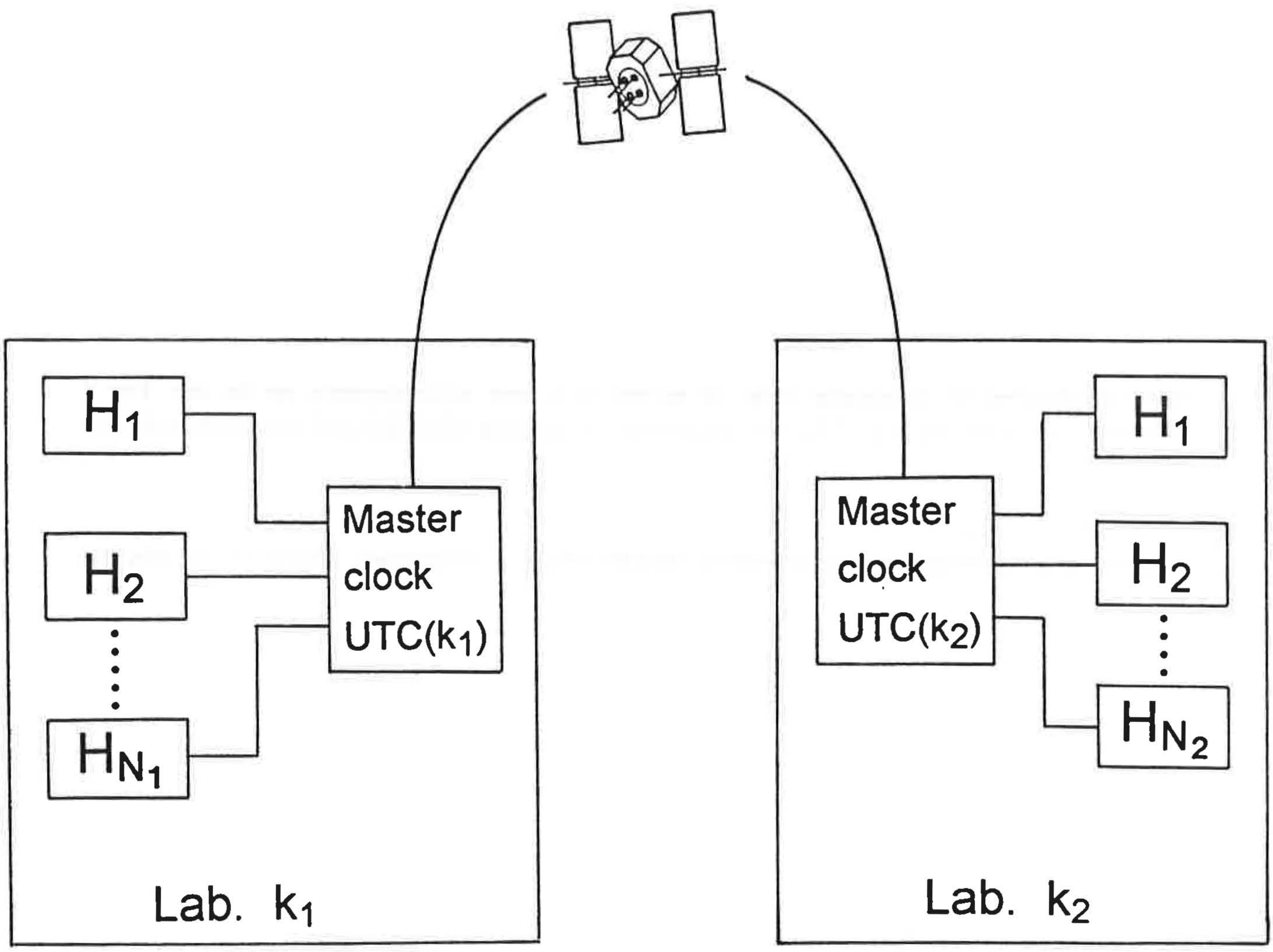


Figure 2. Comparison between clocks located on two different sites.

clock. A counter thus measures time differences which are proper quantities. These are thus measurable and expressed in SI units.

Denote $h_i(t)$ and $h_j(t)$ the time coordinates of the pulse labelled t, delivered by clock H_i, and of the pulse, with the same label, delivered by clock H_j, in a given reference frame. The interval of coordinate times:

$$x_{ij}(t) = h_j(t) - h_i(t), \tag{5}$$

is needed for the generation and dissemination of coordinate time scales.

At the present level of accuracy in clock comparisons, the coordinate quantity $x_{ij}(t)$ can be approximated using the interval of proper time issued from a counter, taking into account signal propagation delays for clocks separated by large distances [Petit and Wolf 1994]; it thus can be expressed in SI units. In addition, in current practice, one does not specify a reference frame, and one designates the time coordinate $h_i(t)$ as the "reading of clock H_i at date t", which, strictly, is not correct. For sake of conformity with the existing literature, we use the same designation in this text. However, the actual meaning of (5), which involves only coordinate quantities, should not be forgotten.

The quantities $x_{ij}(t)$ are the basic measurements used for time scale generation. They are obtained by time transfer methods, implemented between clocks located on the same site or in remote locations. Generally, a non-redundant network of time links is used, a given clock being compared only once with all the others at each date.

Comparison of clocks located on the same site

For computation of some of the TA(k) kept by national timing centres, all the contributing clocks are located on the same site. This is the case for the NIST (about 10 caesium clocks and 1 hydrogen maser), the SU (4 to 6 hydrogen masers), and the USNO (about 50 caesium clocks and 14 hydrogen masers). On each site, one clock is designated the master clock. Its output usually provides UTC(k), the local realization of UTC. It serves also as a reference clock, to which the other clocks are compared in a star pattern as shown in Fig. 1.
The measurements obtained take the form:

$$x_{ik}(t) = UTC(k)(t) - h_i(t), \text{ with } i = 1, \ldots, N \tag{6}$$

at date t, N being the number of clocks. The counters or time-interval meters normally used in timing laboratories provide measurements every second, or even more frequently, with an accuracy of order 50 ps (1 σ) for one individual measurement. Averaging on a period ranging from several tens of seconds to several minutes is sufficient to obtain a value of the quantity x_{ik} for which the measurement noise is negligible with respect to that from the clocks themselves. Such an averaging process is repeated with a basic measurement cycle τ_0 of order several hours; for example $\tau_0 = 2$ h for the generation of AT1 at the NIST.

Comparison of clocks located on remote sites

For the computation of some independent time scales TA(k), the contributing clocks are located in more than one laboratory. This is the case for the French TA(F), computed from 24 caesium clocks kept in 11 laboratories in France, for TA(CH) which includes data from 13 clocks kept in 3 Swiss laboratories, and for TAI computed from data reported by 45 national timing centres, maintaining between them about 230 atomic clocks [Annual Report of the BIPM Time Section 1993].

In addition to the pattern of Fig. 1, used within the contributing laboratories, links, of a more elaborate nature, are required between distant UTC(k). This corresponds to Fig. 2, and leads to measurements expressed in the form:

$$\begin{aligned} x_{ik_1}(t) &= UTC(k_1)(t) - h_i(t), \text{ with } i = 1, \ldots, N_1 \\ x_{jk_2}(t) &= UTC(k_2)(t) - h_j(t), \text{ with } j = 1, \ldots, N_2 \\ x_{k_1k_2}(t) &= UTC(k_2)(t) - UTC(k_1)(t), \end{aligned} \tag{7}$$

where t is the date, k_1 and k_2 the acronyms of the two laboratories being compared, and N_1 and N_2 are the number of clocks in each laboratory. Basic quantities $x_{ij}(t)$ defined in (5) are obtained by linear combination of the differences in (7).

There exist several methods for making distant time comparisons. Among the least accurate is that based on the reception of time signals emitted at radio frequencies, for example DCF77 emitted from Germany on 77,5 kHz [Annual Report of the BIPM Time Section, 1993]. Terrestrial navigation signals such as the Loran-C were also widely used until about 1985. These had a precision on a single comparison of order 0,5 µs. In addition to this noise, huge seasonal variations were observed. Calibration of the equipment, receivers and emitters, was very difficult, and the accuracy obtained was characterized by an uncertainty (1 σ) of order several microseconds. The introduction of the Global Positioning System, GPS, led to a major improvement in the precision, accuracy, and coverage of world-wide time metrology.

GPS time transfer

The GPS is a military satellite navigation system based on satellite ranging using on-board atomic clocks. Since GPS was declared operational, in December 1993, it has been able to provide position, velocity and time instantaneously and continuously anywhere on or above the Earth. In particular, the observation of any GPS satellite gives access to the time scale known as GPS time.

The GPS time is a time scale established by its Control Segment, based in Colorado Springs, Colorado, USA, from a small ensemble of clocks, and related to UTC(USNO). It is a continuous time, not corrected for leap seconds, and measured in weeks from the zero point of GPS time defined as 0 h UTC(USNO), 5 January 1980. At the end of 1993, GPS time was ahead of UTC(USNO) by 9 s. According to official documents [ICD of the US DoD, NATO Standardization Agreement (STANAG), annual issues], the Control Segment should control GPS time to within ± 1µs of UTC(USNO) (modulo 1 s). In fact, since 22 January 1992, GPS time has been subject to automatic time steering at a rate of $\pm\, 2\times10^{-19}$ s/s^2 [USNO series 4], so that it is maintained in agreement with UTC(USNO) to within ± 100 ns, modulo 1 s [UTC(USNO) is itself kept in agreement with UTC within ± 100 ns]. In addition, the navigation message contains a correction which is added to the readings of the on-board satellite clocks to give GPS time. It also allows real-time access to UTC(USNO) since it contains time corrections [GPS time - UTC(USNO)] known with an accuracy of 100 ns.

Timing laboratories have GPS time receivers designed especially for time metrology. Short-term measurements taken every second are treated by the receiver software throughout the duration of each satellite track, chosen equal to 780 s. For date t, corresponding to the mid-point of the track of satellite SV_α, the timing data is:

$$x_{\alpha k}(t) = UTC(k)(t) - GPS\ time(SV_\alpha)(t). \tag{8}$$

Quantities $x_{\alpha k}(t)$ are reported in the GPS data file saved by the receiver. They give individual users direct comparisons between the local reference clock and GPS time, as obtained by observation of satellite SV_α.

The most important error sources affecting the quantities $x_{\alpha k}(t)$ are [Lewandowski and Thomas 1991]:

* errors in the position of the local antenna used for reception,
* errors in the position of the satellite, broadcast in the GPS message,
* errors in the estimation of the signal delay when crossing the ionosphere,
* errors in the estimation of the signal delay when crossing the troposphere,
* effect of the implementation of intentional degradations of the GPS signal: Selective Availability, SA, and Anti-Spoofing, AS.

The SA brings a phase dither noise to the satellite clocks and a rapidly varying bias in the ephemerides broadcast by the satellite. According to official documents [ICD of the US DoD, NATO Standardization Agreement (STANAG), annual issues], SA has an (2 σ) accuracy of 250 ns for time. Since November 1991, only the clock dither noise has been currently implemented. It gives an observed error of about 50 ns (1 σ) on the measurements $x_{\alpha k}(t)$. The AS suppresses access to the precise P-code, but this does not affect time metrology for which only the civilian C/A-code is generally used.

Accurate determinations of GPS antenna positions [Guinot and Lewandowski 1989], the use of post-processed precise satellite ephemerides [Petit *et al.* 1991], and the measurement of ionospheric delays [Imae *et al*, 1989] greatly reduce the first three error sources [Lewandowski *et al.* 1991]. The use of semi-empirical models for the troposphere is under investigation [Kirchner *et al.* 1993]. The SA clock dither noise can be overcome only by the observation of GPS satellites in common view [Allan *et al.* 1990].

The GPS common-view method [Allan and Weiss 1980] calls for simultaneous observation of a satellite from two, or several, sites located on the Earth (see Fig. 3.). For laboratories k_1 and k_2, the data recorded takes the form:

$$x_{\alpha k_1}(t) = UTC(k_1)(t) - GPS\ time(SV_\alpha)(t), \tag{9}$$

and

$$x_{\alpha k_2}(t) = UTC(k_2)(t) - GPS\ time(SV_\alpha)(t), \tag{10}$$

which leads to:

$$x_{\alpha k_1 k_2}(t) = x_{\alpha k_1}(t) - x_{\alpha k_2}(t). \tag{11}$$

An individual value $x_{\alpha k_1 k_2}(t)$ is the time comparison value between the two remote master clocks representing $UTC(k_1)$ and $UTC(k_2)$, at date t, and through satellite SV_α. Individual values are thus affected by measurement noise coming from the observation of the specific satellite SV_α. The sequence of common-view observations of different satellites gives a time series which is treated by appropriate averaging techniques in deferred time to obtain a value $x_{k_1k_2}(t)$, valid at date t, for which the noise due to the comparison method is smoothed out.

Using the GPS common-view method for time transfer supposes a global organization for GPS reception and data exchange within the time community. The specification of GPS satellite orbits makes it possible to draw up schedules of observation valid for a given date and automatically updated by the receiver software for the following months. The Bureau International des Poids et Mesures, BIPM, is in charge of this task. It issues an International Common-View GPS Tracking Schedule, twice a year, which is followed by nearly all the laboratories contributing to TAI and by many others for national purposes. The BIPM also collects GPS data files and carries out pair comparisons between laboratories following a unified procedure. The resulting clock comparison values are used in the generation of TAI. The ultimate precision of a single measurement is of order 2 ns for short-distance links ($\leq$ 1000 km) and about 4 ns for long-distance links ($\approx$ 6000 km). Taking into account all uncertainty sources, the accuracy of GPS common-views is characterized by an uncertainty (1 σ) of 3 ns for short-distance links and 5 ns for long-distance links.

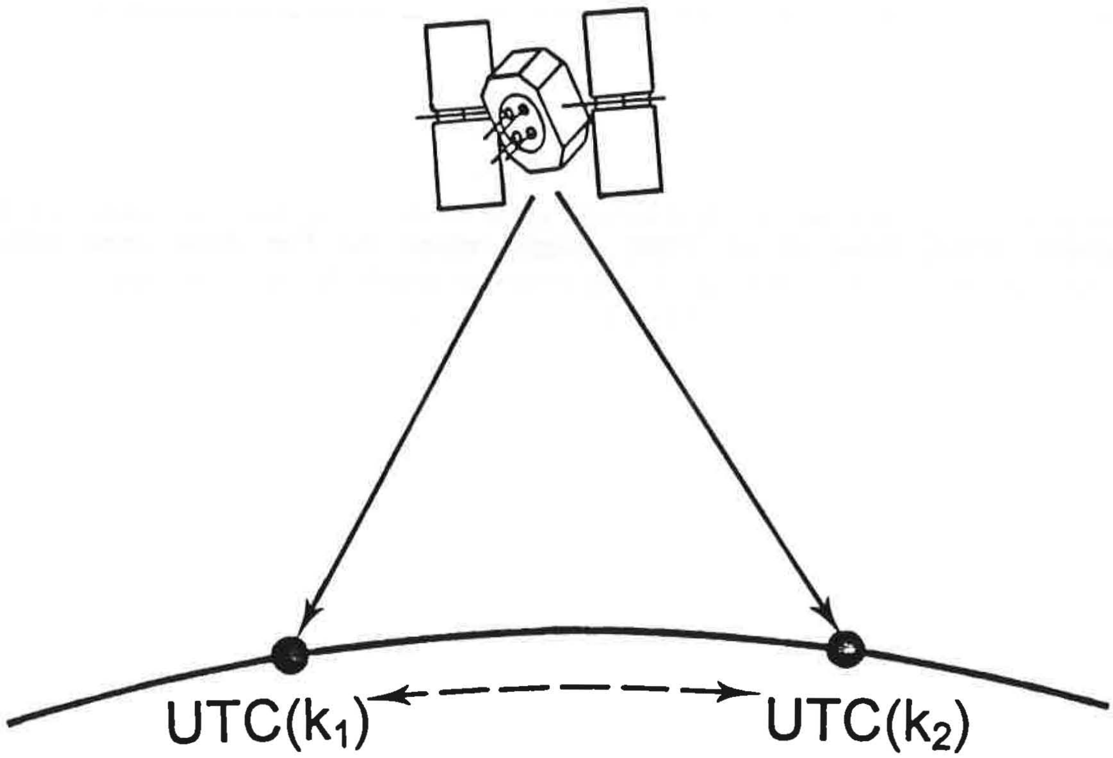


Figure 3. GPS common-view technique.

Other accurate time transfer methods

There exist other time transfer methods besides the GPS common-view technique. None are used regularly to generate a time scale, except for experimental purposes. All have a potential accuracy of order 1 ns (1 σ) or even better. At the beginning of 1994, four methods were in use or at an advanced planning stage:

* GLObal NAvigation Satellite System, GLONASS [Daly *et al.* 1992, Lewandowski *et al.* 1993].

  The GLONASS is the Russian equivalent to GPS but has no intentional signal degradation. Commercial time receivers are not yet available, so the system is not used widely.

* Two-Way Satellite Time Transfer via a geostationary satellite, TWSTT [Kirchner *et al.* 1991, De Jong 1993].

  The TWSTT requires, on site, a station for emission and reception of microwave signals in the telecommunication band, and a satellite channel for signal repetition on board.

* LAser Synchronization from Satellite Orbit, LASSO [Baumont *et al.* 1993].

  The LASSO requires, on site, a laser shooting station and a satellite equipped with stable oscillators, counters, and light retro-reflectors.

* Experiment on Timing, Ranging, and Atmospheric Soundings, ExTRAS [Thomas and Uhrich, 1994].

  The ExTRAS uses ultra-stable hydrogen masers flown on board a low orbit meteorological Russian satellite. Communication with the satellite is ensured by two different two-way techniques: the Precise RAnge and Range-rate Experiment (PRARE) system, in the microwave domain, and the Time Transfer by Laser Light (T2L2) technique, in the optical domain. The launch of this experiment is scheduled for early 1997.

Smoothing of data measurement noise

Data of comparison between remote clocks exhibits a measurement noise whose origin is the time transfer method. It is necessary to remove this noise in order:

* to take advantage of the full quality of the clocks being compared, and
* to avoid the injection of measurement noise into the time scale itself, which would degrade its short-term stability.

Efficient smoothing of measurement noise requires its statistical analysis. Here we illustrate this smoothing for the frequent case of GPS common-view time transfer between two laboratories distant by less than 1000 km. The example chosen here corresponds to the European time link between the OP, Paris, France, and the PTB, Braunschweig, Germany. The baseline OP-PTB is 700 km long. Figure 4.a shows the raw common-view values, obtained for a thirty-day period in May 1994. They correspond to about 24 daily tracks of the international GPS common-view schedule No 22. This data is first treated by the computation of the Allan standard deviation (see Annex I), with the hypothesis of equally-spaced data, distant by τ_0 = 1/24 day. In the log-log plot of Fig. 4.b, the Allan standard deviation values $\sigma_y(\tau)$ lie on a straight line of slope -1 for averaging times $\tau_0 \leq \tau \leq 1$ day. This indicates the presence of phase noise for τ smaller than one day. The actual performance of the master clocks in the OP and the PTB is not dominated by phase noise for such averaging times, and thus becomes accessible as soon as the phase noise, whose origin is the time comparison method, is smoothed out. For this purpose it is sufficient to take average values on consecutive raw data

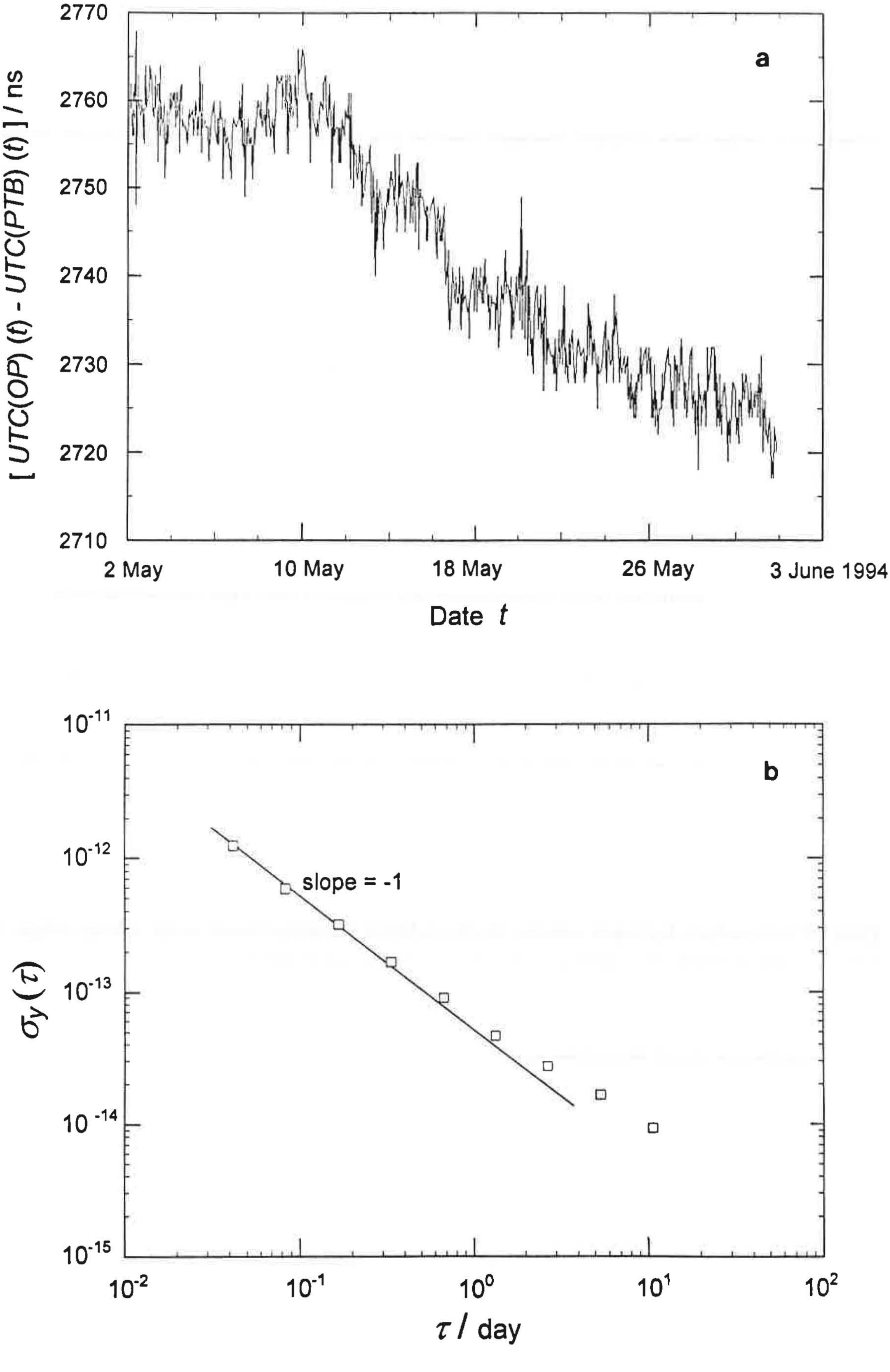


Figure 4. Common-view GPS time transfer between the OP and the PTB.
4.a. Raw timing data for month of May 1994.
4.b. Corresponding Allan standard deviation.

$x_{\alpha OP\ PTB}(t)$ covering one day. This leads to time link values $x_{OP\ PTB}(t')$ reported at the dates t' corresponding to the middle of successive days, with smoothed random noise arising from the GPS common-view method. The level of white phase noise is estimated from the Modified Allan standard deviation $Mod.\sigma_y(\tau_0)$, using (see Annex I):

$$\sigma_x = \tau_0 \frac{Mod.\sigma_y(\tau_0)}{\sqrt{3}}, \tag{12}$$

which gives $\sigma_x = 2{,}6$ ns for this example. The residual white phase noise on daily average is of order $\sigma_x/\sqrt{24}$, which is below 1 ns, and thus negligible compared with the daily clock performance.

The above example shows that, although GPS time data are taken with a rather short measurement cycle τ_0, equal to 1 hour in this particular case, measurements of interest, *i.e.* ones actually representing the quality of the clocks themselves, are available only with a basic period T_0 of order 1 day. For long-distance GPS links, T_0 ranges from 2 to 3 days in the best cases, when using measured ionospheric delays and precise satellite ephemerides. Before the introduction of GPS, durations T_0 as long as 50 days were necessary to remove the Loran-C measurement noise.

2.3. Stability algorithm

Suppose an ensemble of N clocks: at date t, corresponding timing data are the (N-1) measurements $x_{ij}(t)$, with respect to clock H_j, chosen to be non-redundant, and given by (5):

$$x_{ij}(t) = h_j(t) - h_i(t),\ i = 1, \ldots, N,\ i \neq j. \tag{5}$$

Suppose TA is the resulting software time scale; it should be computed for date t from the optimal combination of the $x_{ij}(t)$. The N time differences:

$$x_i(t) = TA(t) - h_i(t),\ i = 1, \ldots, N, \tag{13}$$

give access to TA at date t. The $x_i(t)$ are the unknowns.

Suppose TA is known for a given date t_0 where the measurements $x_{ij}(t_0)$ were available and treated. Measurements $x_{ij}(t)$ are now taken for a following date t, $t > t_0$. The dates t and t_0 are usually separated by a duration T greater than T_0. The problem is to design an algorithm, able to handle the timing data $x_{ij}(t)$ for the generation of TA at date t.

A time-scale algorithm is generally designed to ensure the best stability of the time scale, accuracy being treated externally as explained in 2.1. It is important to stress that there is no general best solution in the design of time scale algorithms. Rather, good design represents a series of choices matched to the purpose for which the time scale is to be used. An algorithm designed to provide a time-reference standard is unlikely to satisfy the requirements of those whose interest is the provision of a service for research. One critical choice is, for example, whether the algorithm must supply the time scale in real-time, or close to it, or whether a delayed scale is acceptable. In all cases, however, the statistical treatment of clock data requires at least [Tavella and Thomas 1991a, Tavella 1992]:

- the definition of an average time scale,

* the choice of a duration between two updatings of the time scale,
* the specification of a procedure to optimize the contribution of each clock,
* the implementation of a filter on each clock frequency to provide a means of frequency prediction.

The time scale algorithms which are in use in timing centres rely upon two basic assumptions:

* Measurement results $x_{ij}(t)$, given in (5), are affected by intrinsic noise which is negligible with respect to the clock noise.
* Clocks are independent and the corresponding data series are uncorrelated. This assumption is conceptually true as each clock is an independent box in which atoms run and "lock" the frequency generated inside. But, in 1989, the Comité Consultatif pour la Définition de la Seconde recommended that a study be made of possible correlations among clocks. Through a survey on the behaviour of the clocks contributing to TAI [Tavella and Thomas 1990b, 1991b], some correlations between clock frequencies were detected. These correspond mainly to responses to changes in the environmental conditions experienced by the clocks. Since several years, efforts have been pursued to improve clock independence either through better control of the environment or through the realization of less sensitive atomic clocks [De Marchi 1988].

In the following, we refer to examples for which extensive documentation can be found. These are, in particular, the algorithm ALGOS(BIPM) [Guinot and Thomas 1988, Tavella and Thomas 1991a], which produces the international reference TAI at the BIPM, and the algorithm AT1(NIST) [Varnum *et al.* 1987, Weiss *et al.* 1989], which produces the real-time time scale AT1 at the NIST. The ALGOS(BIPM) treats data from a large number of clocks spread world-wide. It is designed for extreme long-term stability and a delay of access of several weeks is acceptable for TAI delivery. The AT1(NIST) treats data from about 10 clocks kept on the same site. It is designed for scientific experiments requiring real-time access to AT1.

<u>Definition of an average time scale</u>

To match the definitions of time scales given in the introduction and in section 1, the reading of an atomic time scale TA may be theoretically written, at date t, as the weighted average of the readings of the contributing clocks:

$$TA(t) = \sum_{i=1}^{N} \omega_i(t) h_i(t). \tag{14}$$

With the hypothesis of clock independence, this simple weighting procedure makes it possible to optimize the stability of the ensemble time.

The basic mathematical definition given in (14) plays a fundamental role in the development about time scale algorithms which is detailed in the following . Some ensemble times, such as the old TA(NIST) [Jones and Tryon 1983, 1987], stopped in 1993, or GPS time [Feess *et al.* 1991], do not make use of a similar average definition. These cases are not considered here.

Relative weights $\omega_i(t)$, i = 1, ..., N, are introduced in order to discriminate between clocks according to their intrinsic qualities. They satisfy the following relation:

$$\sum_{i=1}^{N} \omega_i(t) = 1. \tag{15}$$

Suppose that one observes, between dates t_0 and t, exits, entries or changes in the characteristics of the clocks which contribute to TA. Weights $\omega_i(t)$ are thus necessarily different from the weights $\omega_i(t_0)$ used for computation of TA at date t_0. Equation (14) indicates that the time scale TA must show a time and frequency discontinuity at date t_0 [Guinot 1987] which is not acceptable because of the stability requirement. It follows that (14) is not satisfactory for actual clock ensembles, for which changes of weights are unavoidable. Equation (14) must thus be completed as follows:

$$TA(t) = \sum_{i=1}^{N} \omega_i(t)\left[h_i(t) + h_i'(t)\right], \tag{16}$$

where $h'_i(t)$ is a time correction added at date t to the reading of clock H_i, and designed to ensure time and frequency continuity of TA at date t_0 [Guinot and Thomas 1988]. The correction $h'_i(t)$ is written as:

$$h_i'(t) = x_i(t_0) + y_{ip}(t).(t - t_0), \tag{17}$$

where $x_i(t_0) = TA(t_0) - h_i(t_0)$ is known, since it results from the computation of TA at date t_0, and where $y_{ip}(t)$ is the predicted frequency of clock H_i, relative to TA, over the interval $[t_0, t]$. The frequency $y_i(t)$ of clock H_i, relative to TA, over the interval $[t_0, t]$ can be estimated from:

$$y_i(t) = \frac{\left[TA(t) - h_i(t)\right] - \left[TA(t_0) - h_i(t_0)\right]}{t - t_0}. \tag{18}$$

Until TA is computed at date t, $y_i(t)$ is unknown. It is thus necessary to predict it according to the past behaviour of clock H_i. This predicted frequency is denoted $y_{ip}(t)$ and appears in (17).

Equations (5) and (13) and (16) lead to the following system of equations, assuming no measurement noise:

$$\begin{aligned} &\sum_{i=1}^{N} \omega_i(t) x_i(t) = \sum_{i=1}^{N} \omega_i(t) x_i(t_0) + \sum_{i=1}^{N} \omega_i(t) y_{ip}(t).(t - t_0), \\ &x_i(t) - x_j(t) = x_{ij}(t). \end{aligned} \tag{19}$$

System (19) is deterministic with N equations and N unknowns. The solution is unique and the results are the time differences $x_i(t)$, $i = 1, ..., N$, which give access to TA for date t. The difference between clock H_j and TA is explicitly given by:

$$x_j(t) = \sum_{i=1}^{N} \omega_i(t)\left[h_i'(t) - x_{ij}(t)\right]. \tag{20}$$

System (19) can be found in most of the algorithms used around the world, for instance in the algorithms used for computing AT1 at the NIST [Varnum *et al.* 1987, Tavella and Thomas 1991a], TA(F) [Granveaud 1986] at the OP, TAI [Guinot and Thomas 1988] at the BIPM, TA(AUS) at the ORR [Luck 1979], TA(CRL) at the CRL [Yoshimura 1980], and A.1(MEAN) at the USNO [Percival 1978].

Recently, new developments of time-scale algorithms have been proposed, in particular the possibility of using different sets of weights to optimize both short-term and long-term stability [Wei Guo 1992, Stein 1992]. At the USNO a new algorithm is used in which the ensemble time is re-evaluated every hour from the past 75 days: the weights are modified according to a quadratic variation with time in order to match the short-term and long-term qualities of different clock types (caesium standards and hydrogen masers). The update for the last hour is used to steer the master clock [Breakiron 1991].

For some algorithms the definition of the time scale is used together with specific filters which act on the raw timing data, which were not smoothed before. This is the case of KAS-1 [Stein 1988, Stein *et al.* 1989] for which a Kalman filter is implemented. In other cases the Kalman formalism is used for the resolution of (19) in order to improve the stability of the time scale, as is done in KAS-2 [Stein 1992], or to evaluate uncertainty of the estimates and detect abnormal behaviours, as is done in TA2(NIST) [Weiss and Weissert 1994]. In the following we restrict our discussion to the "classical" and well established ensemble algorithms which rely on (19), referring the interested reader to the literature for further developments.

Since the definition of the time scale, and thus the resulting system of equations, nearly always takes the same form, the specificity of a given algorithm lies in the choices made for:

* the length of the time interval $[t_0, t]$,
* the weights attributed to clocks,
* the way clock frequencies are predicted.

These choices are closely related to the purposes for which the time scale is designed.

Length of the basic interval of computation

In previous sections, two basic durations have already been defined:

* τ_0, duration of the basic measurement cycle.
* T_0, minimum duration over which raw data should be averaged in order to smooth out the measurement noise sufficiently to reach the intrinsic qualities of the clocks being compared. Orders of magnitude for T_0 are several minutes to several hours inside a laboratory, 12 hours to 1 day between two laboratories linked via short-distance GPS common views, and several days between two laboratories linked via long-distance GPS common views.

Also defined are two dates:

* t_0, date for which TA is known.
* t, a following date ($t > t_0$) for which smoothed timing measurements are available and for which TA is to be computed by solving (19).

The update interval $T = t - t_0$ is, in general, of the same order of magnitude and slightly longer than T_0. Its length is thus directly linked to the quality of timing data. It is for example:

* $T = 2$ hours for AT1(NIST), which only uses timing data taken on site,

* T = 1 day for TA(F), which uses timing data from all over France, the maximum baseline between laboratories being of order 1000 km,
* T = 10 days for TAI, which uses timing data from all over the world, the maximum baseline between laboratories being of order 6000 km.

Another requirement is efficient characterization of the behaviour of the participating clocks in order to weight them correctly and efficiently predict their frequencies relative to TA (see the following sections). It is thus often necessary to observe clocks over a duration longer than T. One then has two possibilities:

* Update TA every interval of duration T, while retaining a memory of the last n intervals (n integer, greater than 1) of duration T. The time scale is delivered in near real time, with a delay no greater than T, but it is based only on the past behaviour of contributing clocks. There is no re-processing and no post-processing.
  The weights and the frequency prediction are valid for an interval of duration T. They are changed for the following interval of duration T. The last n intervals either constitute a moving memory buffer shifted by T at each computation, or contribute to a recursive procedure. The resulting algorithm is thus dynamic and adaptive at intervals of T.
  The advantage of this approach is that the time scale is accessible in real time. The disadvantage is that it is not possible to take into account the abnormal behaviour of one clock before it registers on the scale. A stable clock which suddenly presents a frequency jump can thus sweep along the time scale before the anomaly is detected.
  This approach is used for AT1, for which T = 2 hours and $nT \approx$ 10 days ($n \approx 120$). The problem of detecting abnormal behaviour is partly solved in an updated algorithm AT2, conceived and tested at the NIST [Weiss and Weissert 1991].
* Update TA only when the interval of duration nT is ended, treating as a whole the (n + 1) dates included in the interval. This delivers a deferred-time time scale, computed in post-processing.
  The weight and the frequency prediction of a given clock are valid for an interval of duration nT and may be based on the clock behaviour over a number of past, contiguous, and non-overlapping intervals each of duration nT. They are changed for the following interval of duration nT, but are equal for all dates included in a given interval of duration nT. The clock behaviour observed during the whole interval of computation is taken into account. The resulting algorithm is dynamic and adaptive ***a posteriori*** at intervals of durationn nT.
  The advantage is the possibility of taking into account any abnormal behaviour of clocks occurring during this period. The disadvantage is the access to the time scale in deferred-time for the (n + 1) dates included in the interval of computation.
  This is the case for TA(F), for which T = 1 day and nT = 30 days (n = 30). The TAI is computed by the same process with T = 10 days and nT = 60 days (n = 6).

Another algorithm, at the NIST, uses both of these processes. This is TA2 which relies on the algorithm AT2 (AT1 plus abnormal behaviour detection), operating with T = 2 hours and $nT \approx$ 10 days, run forward and backward over a duration of one month [Weiss and Weissert 1994]. It follows that there is an iterative re-processing of the data throughout the month. The NIST thus has at its disposal two time scales, the real-time AT1 and the deferred-time TA2, the computation of the previous TA(NIST), based on a Kalman filter [Barnes 1982], being discontinued since mid-1993.

Most of the algorithms used in national laboratories adopt the first choice, updating TA(k) in real time or in near real-time without post-processing. In addition, some algorithms, as those for TAI at the BIPM, A.1(MEAN) at the USNO, or TA2 at the NIST, choose an iterative procedure to evaluate weights and frequency predictions: This takes the form of successive recomputations of TA for the same interval, with detection of outliers at each step, until results converge [Tavella and Thomas 1991].

Weighting procedure

Since time scale algorithms are designed to optimize frequency stability, each clock should be weighted according to its own frequency stability. The weight attributed to a given clock is thus basically chosen to be inversely proportional to its frequency variance σ_i^2.

$$\omega_i = \frac{1/\sigma_i^2}{\sum_{k=1}^{N} 1/\sigma_k^2}, \; i = 1, \ldots, N. \tag{21}$$

The reason for this is that, if the contributing clocks are independent and if weights are not artificially limited, the frequency variance of the resulting time scale may be written as:

$$\frac{1}{\sigma_{\mathrm{TA}}^2} = \sum_{i=1}^{N} \frac{1}{\sigma_i^2}, \tag{22}$$

which means that the time scale is, in principle, more stable than any contributing element. The choice of the variance type (classical, filtered, or Allan) depends on the purposes for which the time scale is generated, and can thus differ according to the algorithm which is considered. However, there are two limiting factors:

* The frequencies of clock H_i, used for the computation of its frequency variance, are estimated over an interval of duration τ. According to (22), the stability of the resulting time scale is optimized for averaging times close to τ. It is thus of pre-eminent importance to determine for which τ values the contributing clocks present their best stabilities, and to define what objective of stability the time scale should fulfil. In other words, the optimization of both short-term and long-term stability could call for contributions from different types of clocks, treated according to different procedures in the algorithm. This is the case for the software UTC(USNO) computed at the USNO [Breakiron 1991], and also for the KAS algorithms [Stein 1992].
* The frequencies of clock H_i, used for the computation of its frequency variance, are estimated by comparison with a reference. Very often this reference is the time scale itself, because its stability is supposed to be better than that of the contributing clocks. It follows that the computed variance is inherently biased [Yoshimura 1980] and ceases to represent the true quality of the clock. This is the so-called "clock-ensemble correlation" effect. An approach to the derivation of this effect has been published [Tavella *et al.* 1991], and gives:

$$\sigma_{i,\mathrm{bias}}^2 = \sigma_{i,\mathrm{true}}^2 (1 - \omega_i), \tag{23}$$

where $\sigma_{i,\mathrm{bias}}^2$ and $\sigma_{i,\mathrm{true}}^2$ are the "biased" and "true" frequency variances of clock H_i.

The effect of clock-ensemble correlation is proportional to the relative contribution of the clock within the ensemble. If not taken into account, a very stable clock is progressively more heavily weighted, which threatens the reliability of the time scale. The correction factor of (23) appears in most algorithms used in national laboratories, sometimes with a multiplication factor close to 1 [Tavella and Thomas 1991a]. However, it does not intervene in the TAI algorithm because the number of contributing clocks and the implementation of an upper limit of weight lead to a maximum contribution, ω_i, of a given clock, which has been smaller than 1% since beginning of 1993, and is thus negligible with respect to 1.

In addition to the fundamental aspects that have just been discussed, the weighting procedure must obey some other rules. The most important is the implementation of an upper limit of weight, necessary practically to make the time scale rely on the best clocks and yet avoid giving a predominant role to any one of them. Another is an objective criterion to safeguard the time scale against the possible abnormal behaviour of some clocks. It is important to stress that the existence of an upper limit of weight safeguards reliability but invalidates (22). It can thus lead to a time scale TA that is no better than the best single contributing clock.

To illustrate, we take the examples of the algorithms AT1(NIST) and ALGOS(BIPM) for which a complete comparison is available [Tavella and Thomas 1991a].

Weighting procedure in AT1(NIST)

In AT1(NIST), the weights used for the computation of AT1 at date t are deduced from the results of the computation of AT1 at date t_0 (t - t_0 = T). The weight $\omega_i(t)$ of clock H_i is obtained from (21) where $\sigma_i^2(t)$ results from an exponential filter written as:

$$\sigma_i^2(t) = \frac{1}{A+1}\left[\delta_i^2 + A.\sigma_i^2(t_0)\right], \tag{24}$$

with

$$\delta_i = \left|y_i(t_0) - y_{ip}(t_0)\right| + K_i/T. \tag{25}$$

The exponential filter is used to deweight the past behaviour of the clock. Its time constant A is usually set to 20 to 30 days. The term δ_i contains the shift between the actual frequency of clock H_i and its predicted value, thus giving an estimation of the predictability of the clock over T. The term K_i, added in (25), takes into account the correlation between the ensemble time and clock H_i. This is absolutely necessary in the AT1(NIST) algorithm, which is designed for the treatment of a small number of clocks ($\approx$ 10) and where the maximum contribution of a given clock can reach 20%. Since very recently, the term K_i has been chosen according to (23) in both the AT1 and TA2 algorithms [Weiss and Weissert 1994].

The AT1(NIST) weight determination keeps no memory of the absolute values of past frequencies, but rather relies on frequency variations. This is similar to the difference between an Allan variance and a classical variance. Although the clock frequency instability is tested, it should be noted that some information about long-term systematic variations could be lost.

The use of an exponential filter for the weight determination is efficient because it deweights the past: if a clock has a frequency "accident", and thus is intentionally deweighted, its deweighting is progressively removed over an interval of several integration times. In AT2(NIST) and TA2(NIST) a frequency-step detection is explicitly introduced [Weiss and Weissert 1994]: the basic idea is to detect a frequency difference larger than 4 times the level of frequency noise observed for that clock. In addition, an upper limit of weight is introduced in AT1(NIST) for the sake of reliability.

Weighting procedure in ALGOS(BIPM)

As already noted, ALGOS(BIPM) operates in post-processing, treating as a whole measurements taken over a basic period $nT = 60$ days. Measurements are available every $T =$ 10 days, on the MJD ending with 9. The time scale is updated for each of the six dates t included in the two-month period under consideration: $t = t_0 + mT$, with $m = 1, 2, 3, 4, 5, 6$. The date t_0 is the last date of the previous two-month interval, for which the time scale is maintained and not updated. The separation between updates is thus 10 days, but the gap between calculations is 60 days.

In ALGOS(BIPM), the weight $\omega_i(t)$ of clock H_i is constant over the two-month interval I of computation: it is thus valid for the seven dates $t = t_0 + mT$, with $m = 0, 1, 2, 3, 4, 5, 6$, continuity at t_0 being ensured by the clock frequency prediction. It can be written $\omega_i(I)$ and obeys (21), where $\sigma_i^2(I)$ are individual classical variances computed from six consecutive two-month frequencies of the clock H_i. These are the frequencies computed over interval I and the five previous two-month intervals. As the frequency over the interval I is not yet known, an iterative process is used [Tavella and Thomas 1991a] which begins with the weights obtained at the previous two-month computation, ending at date t_0; this gives an indication of the behaviour of each clock during the interval I and thus makes it possible to refine weights in the following iterations.

The ALGOS(BIPM) weight determination uses clock measurements covering a full year, so annual frequency variations and long-term drifts can lead to deweighting. This has helped to reduce the seasonal variation of TAI observed during the seventies and the eighties. In addition, the choice of 60 days, initially chosen to smooth the Loran-C data, corresponds to a good averaging time for the detection of frequency anomalies. Sampling over 60 days thus allows optimization of TAI stability in the long-term. With the increased use of GPS common-view links and of the newly designed HP clocks, nT could be reduced to 30 days. The weight could then be determined with 12 one-month samples.

In ALGOS(BIPM), the term for the clock-ensemble correlation of (23) is negligible and is thus not introduced [Tavella *et al.* 1991]. There is an upper limit of weights which corresponds to a minimum variance $\sigma_i^2(I)$ of $3{,}66 \times 10^{-14}$, which could be changed, if called for by improvements in clock performance. An algorithm to detect abnormal behaviour is also implemented: this tests frequency changes [Tavella and Thomas 1991a].

To conclude, the weights used in AT1(NIST) and ALGOS(BIPM) obey the same rules, in particular: optimization of the stability, detection of abnormal behaviour, minimization of the clock-ensemble correlation. The specific choices which have been made, match the available timing data and fulfil the fundamental requirement of access to a time scale in real time or deferred time.

Frequency prediction

The way the frequency of clock H_i is predicted depends on its statistical characteristics and on the duration for which the prediction should be valid. There are several pure cases:

* The predominant noise is white frequency noise: this is the case for commercial caesium clocks for averaging times τ ranging from 1 day to 10 days. The most probable frequency, estimated over an interval of duration τ, for the following τ interval is then given by the mean of the frequency values observed over a number of previous intervals of duration τ.
* The predominant noise is random walk frequency modulation: this is the case for commercial caesium clocks for averaging times τ ranging from 20 days to 70 days. The most probable frequency value for the following τ interval is then the last frequency value estimated over the previous interval of duration τ.
* The predominant frequency deviation is a linear drift: this is the case for some hydrogen masers for averaging times τ longer than several days. The most probable frequency for the following τ interval is then the last frequency calculated over the previous interval of duration τ corrected by a term deduced from the estimated frequency drift.

To optimize an ensemble it is thus necessary to have a good knowledge of the behaviour of the contributing clocks, and to be astute in selecting suitable modes of frequency prediction for the different clock types.

To illustrate, we consider the algorithms AT1(NIST) and ALGOS(BIPM).

Frequency prediction in AT1(NIST)

For AT1(NIST), the predicted frequency $y_{ip}(t)$ of clock H_i, for computation of AT1 at date t, is deduced from the results of the computation of AT1 at date t_0, with $t - t_0 = T$. This is obtained from an exponential filter written as:

$$y_{ip}(t) = \frac{1}{B_i + 1}\left[y_i(t_0) + B_i . y_{ip}(t_0)\right]. \tag{26}$$

The predicted frequency of clock H_i is an average of the frequencies of clock H_i over past periods with an exponential weighting. The time constant B_i of the exponential filter depends on the statistical properties of clock H_i and may thus differ from one clock to another. It allows an optimal estimation of the long-term behaviour of the clock, since it corresponds to the averaging time for which the clock reaches its flicker floor or for which a good estimation of the random walk component is possible.

Frequency prediction in ALGOS(BIPM)

As already noted, ALGOS(BIPM) operates in post-processing, treating as a whole measurements taken over a basic period nT = 60 days. As with its weight, the predicted frequency of clock H_i is constant over the two-month interval I of computation; it is thus valid for the seven dates $t = t_0 + mT$, with m = 0, 1, 2, 3, 4, 5, 6, and can be written $y_{ip}(I)$.

In ALGOS(BIPM), the predicted frequency used for the present two-month interval is equal to the frequency obtained over the previous two-month interval as a one-step linear prediction. This is the optimal prediction for averaging times of two months, for which the predominant noise is random walk frequency modulation. All clocks contributing to TAI are subjected to the same mode of frequency prediction; however, changes in the procedure are under discussion, in particular the introduction of a frequency drift estimation to predict the frequencies of hydrogen masers.

To conclude, the modes of frequency prediction in AT1(NIST) and ALGOS(BIPM) differ because each is adapted to the length of its own basic interval of computation and so to the statistical properties of the clocks over such averaging times.

2.4. Accuracy of the scale interval of a time scale

Improvement of the accuracy of a time scale is generally carried out outside the main algorithm, which deals only with optimizing stability.

For TAI, it is carried out by frequency steering the free-running time scale derived from the stability algorithm ALGOS(BIPM). The frequency corrections are smaller than the frequency fluctuations of the time scale to avoid a degradation of its stability. They are decided after comparison of the frequency of the computed time scale with a combination of the frequencies of primary frequency standards, continuously operating or occasionally evaluated, all around the world [Azoubib *et al.* 1977]. In this exercise, the effect of the gravitational red shift on the primary standard frequencies is taken into account. Only one frequency steering correction was applied in 1993: it amounted to $0{,}5 \times 10^{-15}$. The accuracy of TAI is expressed in terms of the mean duration of its scale unit, computed for two-month intervals, in SI seconds on the rotating geoid. It is published in the successive volumes of the *Annual Report of the BIPM Time Section*. For example, the mean duration of the TAI scale unit was equal to $(1+0{;}2 \times 10^{-14})$ SI second on the rotating geoid for the interval May-June 1993, with an uncertainty ($1\ \sigma$) equal to $1{,}3 \times 10^{-14}$.

For the NIST atomic time scales, accuracy is ensured by comparisons with the primary frequency standards NBS6 and NIST7.

2.5. Examples

This section exemplifies recent data, none older than January 1993.

* Figures 5, 6, 7, and 8 show the curves of the Allan standard deviation with respect to the averaging time, for the time comparisons between TAI and, respectively, TA(F), AT1, TA(PTB), and A.1(MEAN). These values are taken from data used for TAI computation in 1993 [Annual Report of the BIPM Time Section 1993].

The TA(F) is computed from 23 caesium clocks in laboratories distributed all over France, with an algorithm similar to ALGOS(BIPM). The minimal value of the Allan standard deviation is:

$$\sigma_y(\tau \approx 40 \text{ days}) \approx 8 \times 10^{-15}. \tag{27}$$

The AT1 is computed from about 10 caesium clocks maintained on a single site, using the AT1(NIST) algorithm. The minimal value of the Allan standard deviation is:

$$\sigma_y(20 \text{ days} \leq \tau \leq 40 \text{ days}) \approx 5 \times 10^{-15}. \tag{28}$$

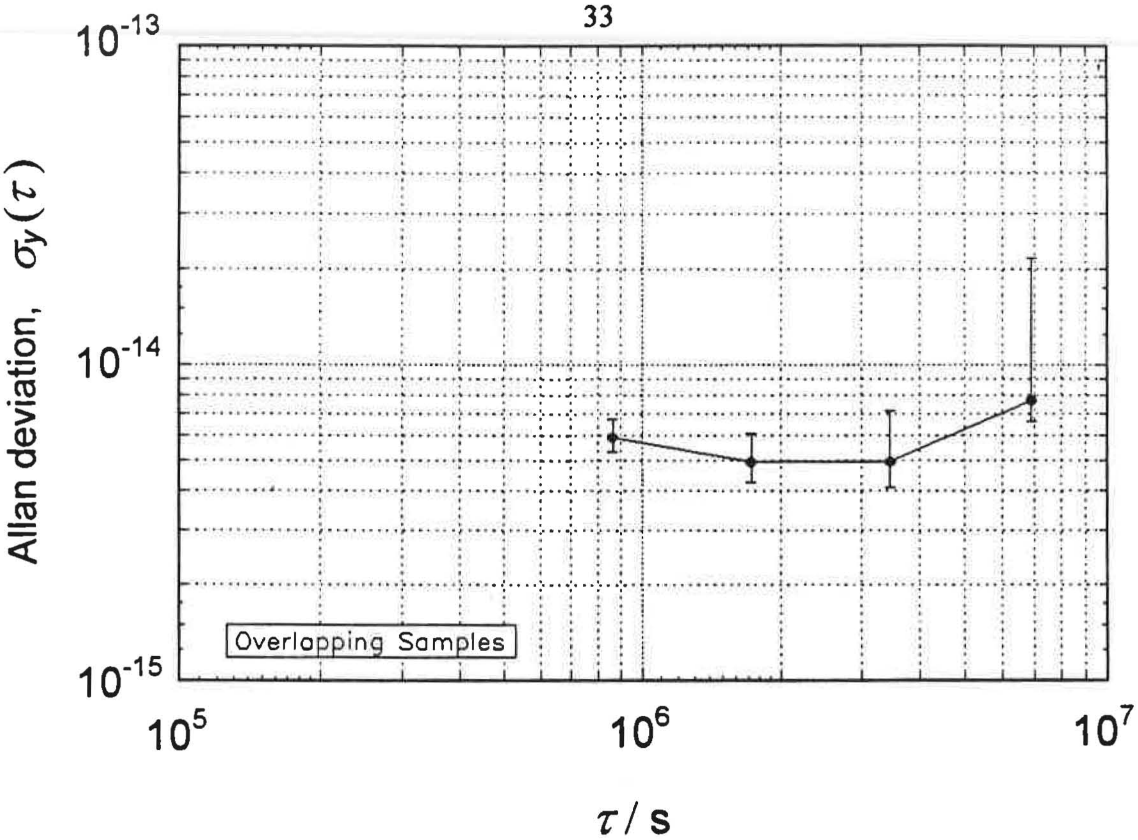


Figure 5. Allan standard deviation of timing data obtained from the comparison between TAI and TA(F) for 1993. The TA(F) is computed by the LPTF, Paris, France

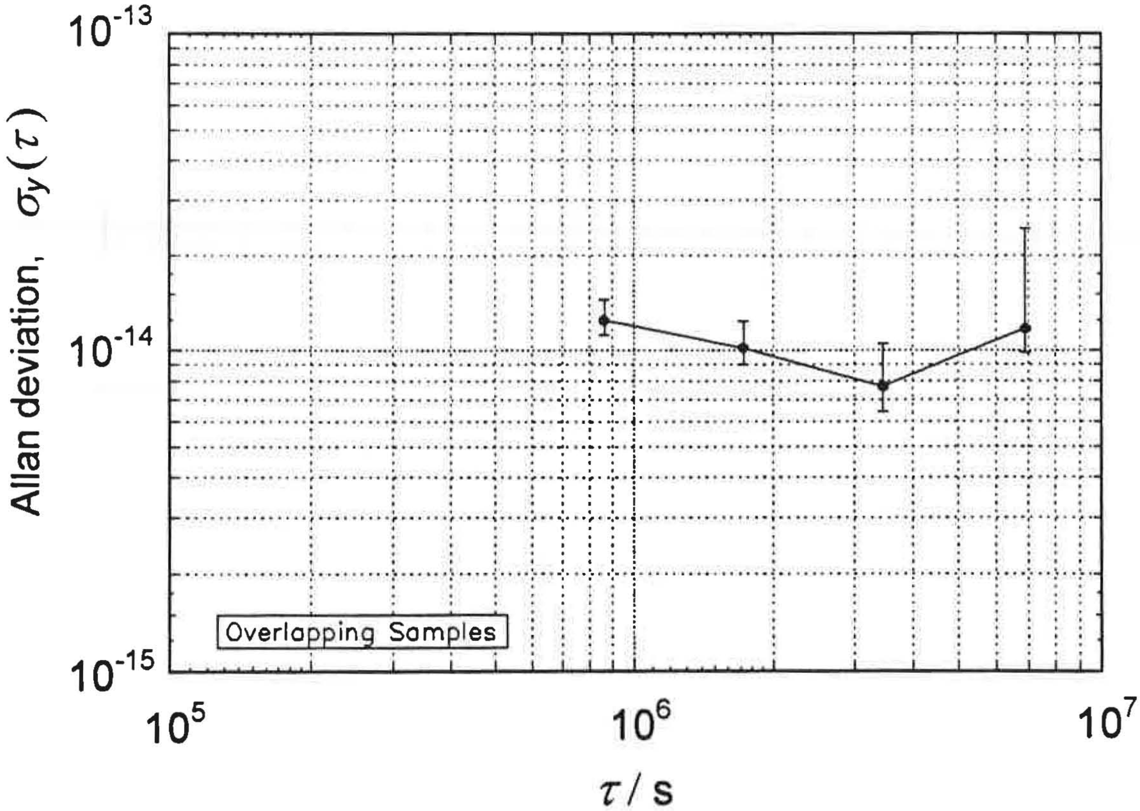


Figure 6. Allan standard deviation of timing data obtained from the comparison between TAI and AT1 for 1993. The AT1 is computed by the NIST, Boulder, CO, USA.

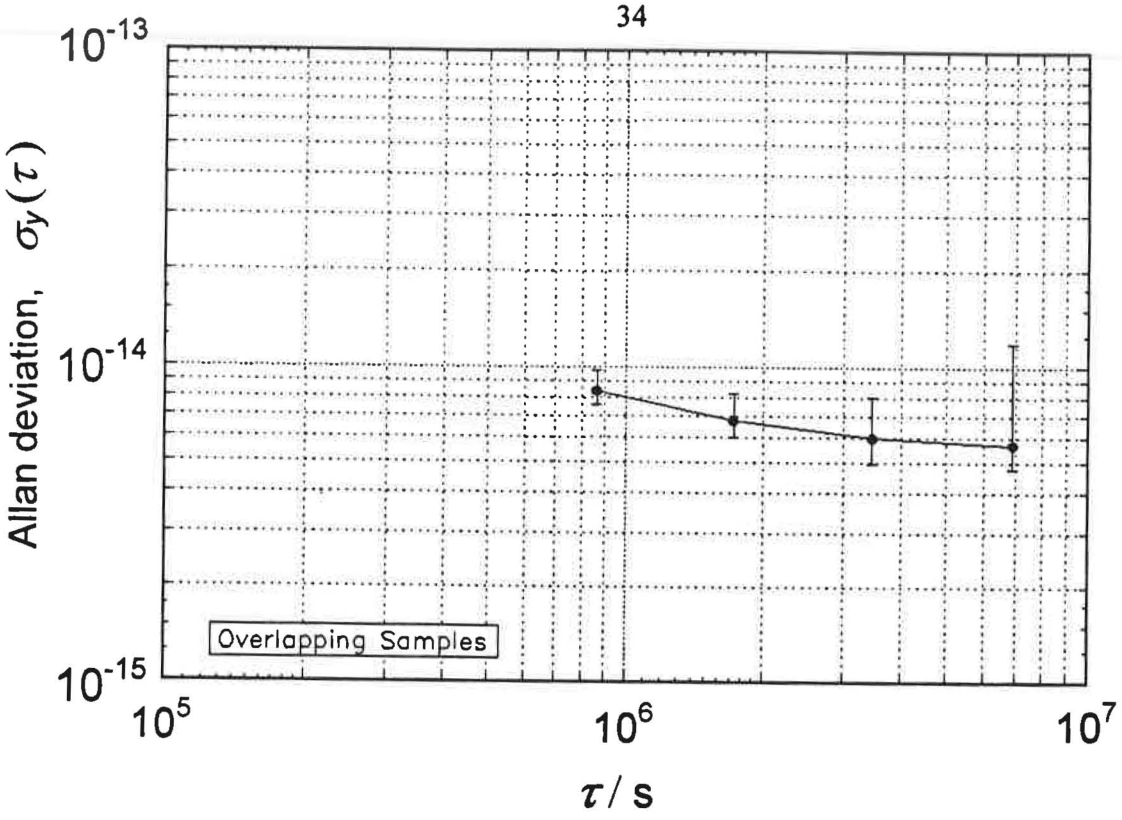


Figure 7. Allan standard deviation of timing data obtained from the comparison between TAI and TA(PTB) for 1993. The TA(PTB) is directly link to the output of the primary frequency standard PTB CS2, operating as a clock at the PTB, Braunschweig, Germany.

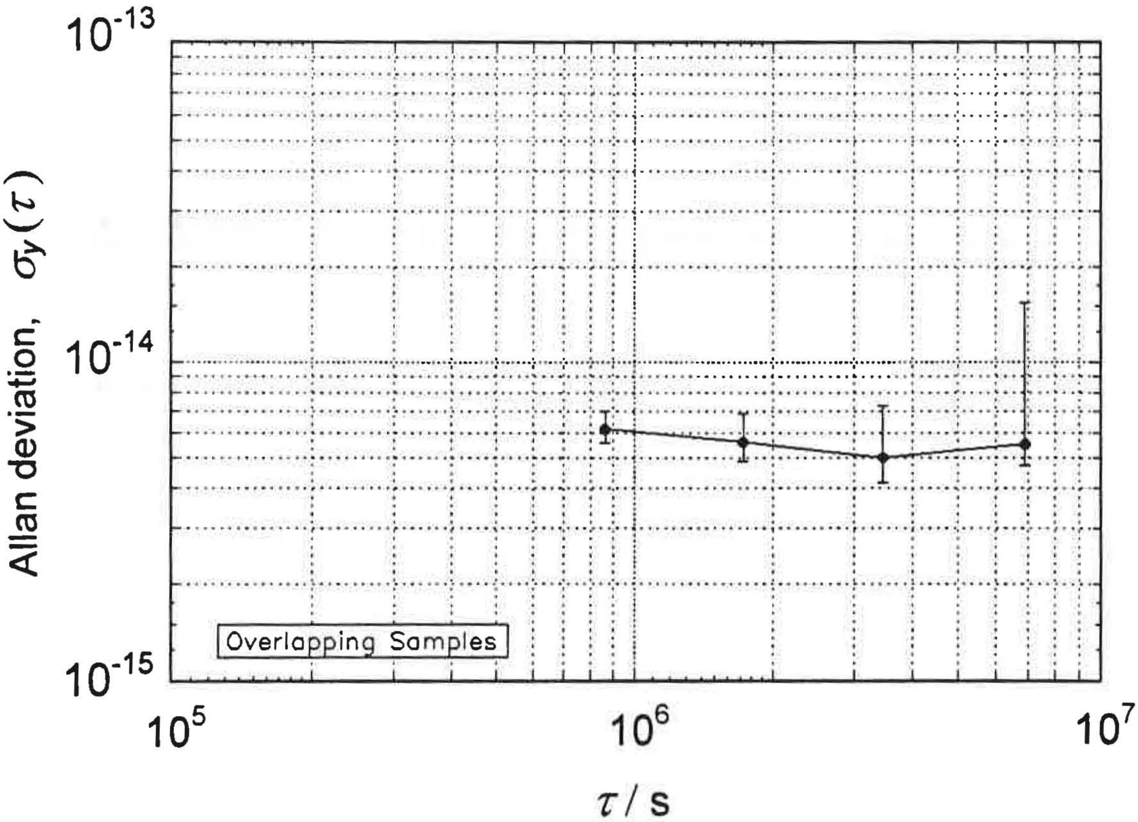


Figure 8. Allan standard deviation of timing data obtained from the comparison between TAI and A.1(MEAN) for 1993. The A.1(MEAN) is computed by the USNO, Washington, D.C., USA.

The TA(PTB) is not derived from a time scale algorithm. It is simply the output of the primary frequency standard PTB CS2, which operates continuously as a clock. The minimal value of the Allan standard deviation is:

$$\sigma_y(80 \text{ days} \leq \tau) \approx 6\text{x}10^{-15}. \qquad (29)$$

The TA(USNO) is the time scale A.1(MEAN) computed from about 50 caesium clocks (36 of them are HP 5071A units) and 14 hydrogen masers kept on site, with an algorithm which uses a double weighting procedure for optimization of both short-term and long-term stability. The minimal value of the Allan standard deviation is:

$$\sigma_y(\tau \approx 80 \text{ days}) \approx 5\text{x}10^{-15}. \qquad (30)$$

Since the values of Allan standard deviations given here describe the time differences between TAI and the independent time scales, the part of instability coming from TAI is not separated from that coming from the individual TAs. Application of the N-cornered-hat technique allows this separation provided that the time scales entering in the computation are statistically independent.

* Figure 9 shows the Allan standard deviation values for TAI obtained with a 4-cornered-hat technique, using data from comparisons between TAI and AT1, TAI and TA(SU) and TAI and TA(PTB) for the period January 1993 - April 1994. The values obtained are of order several parts in 10^{15}.

It is important to note that the reported values from the Allan deviation have decreased considerably for most of the independent time scales in the past few years. For TAI, since the introduction of the new HP 5071A clocks and the use of active auto-tuned hydrogen masers, the values have also fallen substantially.

* Figures 10 and 11 show two examples of time variations of comparisons between UTC and UTC(k) over one year ending in April 1994.

The UTC(OP), in Paris, is a hardware UTC derived from one single physical clock steered through a micro-phase-stepper. The change of the master clock in 1993 from an old-design HP unit to a HP 5071A unit is readily observed: stability is immediately improved. A frequency steering command was given in 1993 to bring UTC(OP) closer to UTC.

The UTC(NIST) kept at the NIST is a software UTC derived from an ensemble of physical clocks, and steered to UTC by software. This local UTC has several physical representations, obtained from hardware clocks, each steered every 6 minutes, through a microphase stepper. The UTC(NIST) makes slow and regular oscillations around UTC.

Another example of implementation of a time-scale algorithm is given in Cordara *et al*, 1993. This exemplifies the practical requirements and problems arising from the automatic computation of the Italian atomic time scale, TA(IEN), which is generated from a small ensemble of clocks.

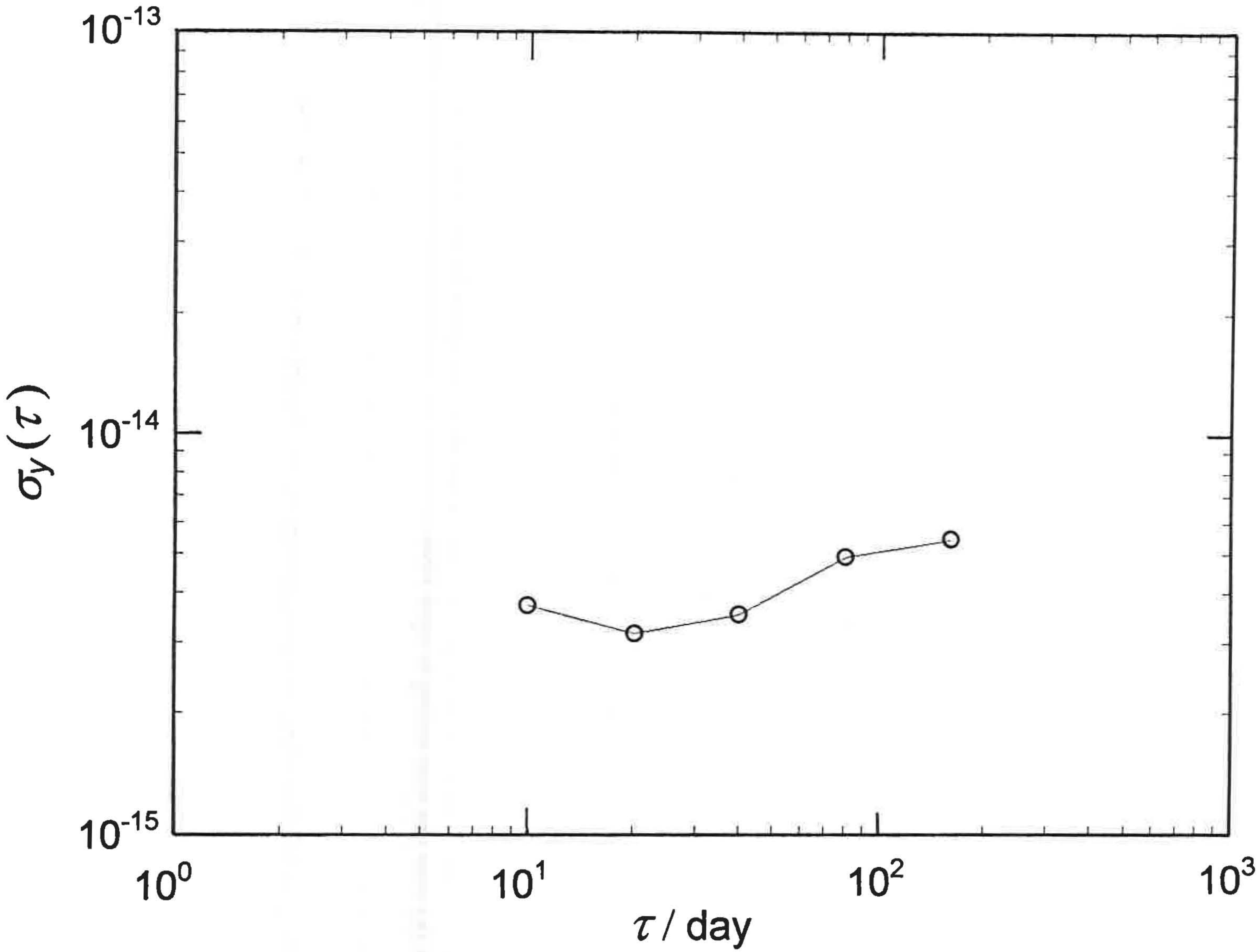


Figure 9. Allan standard deviation of TAI obtained from the 4-cornered-hat technique applied to timing data obtained from the comparison between TAI and TA, TAI and TA(PTB), and TAI and AT1 for the period January 1993 - April 1994.

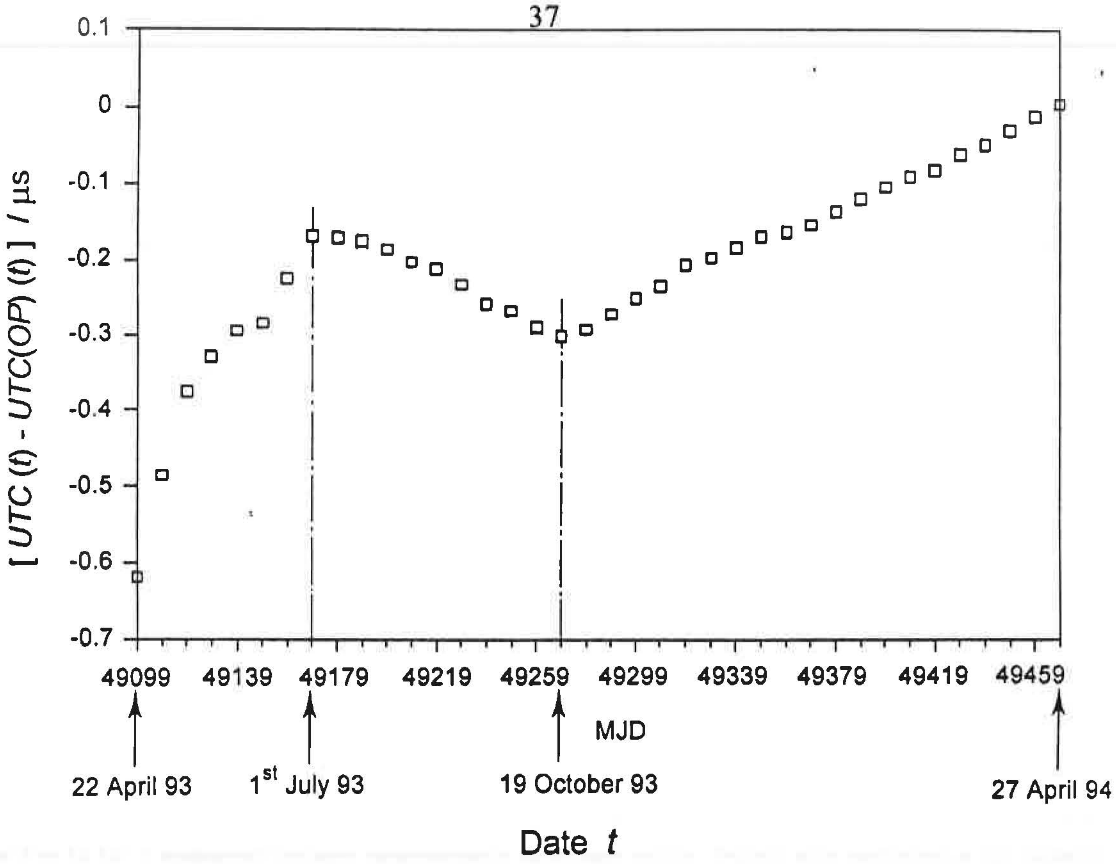


Figure 10. Timing data obtained from the comparison between UTC and UTC(OP) for one year ending in May 94.

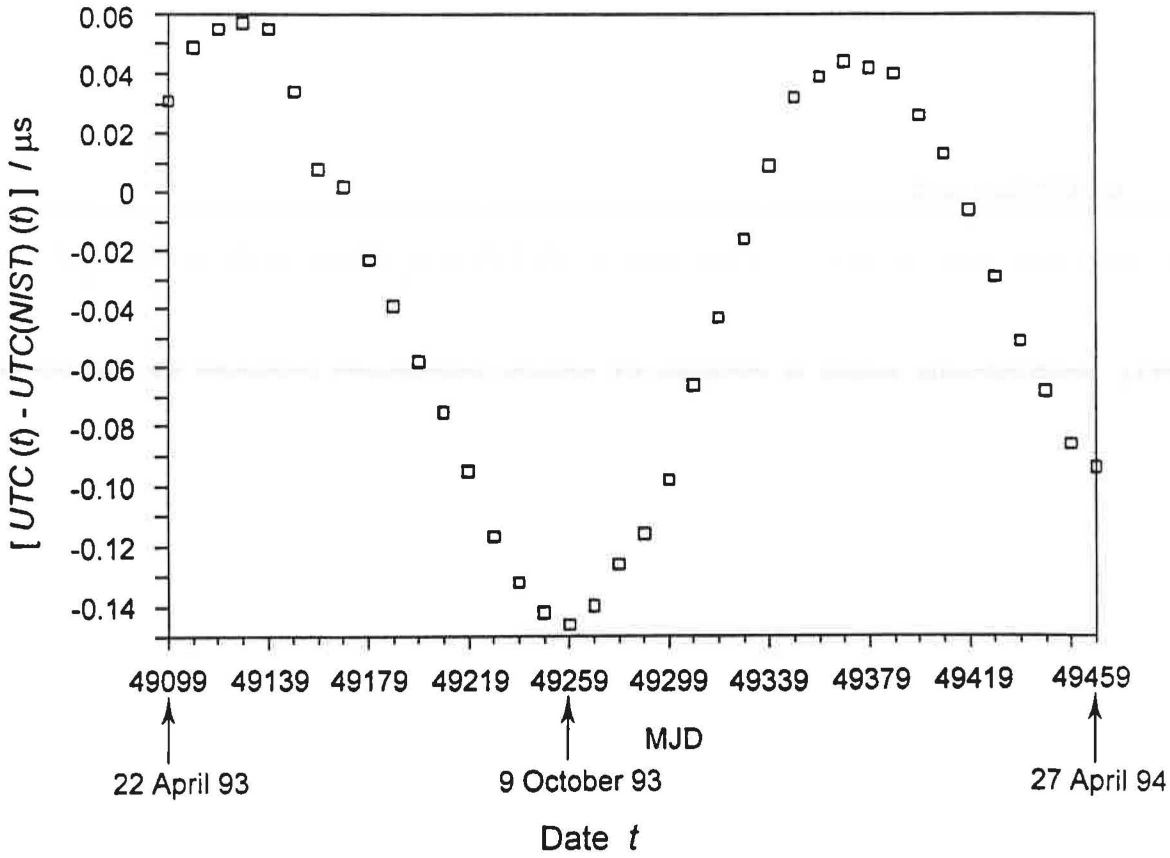


Figure 11. Timing data obtained from the comparison between UTC and UTC(NIST) for one year ending in May 94.

3. Dissemination of time scales

As already described above, a time scale can be derived only from a knowledge of the time difference between this time scale and another one, or from a physical clock, at a given date. Access to time scales is thus effected by the publication of time differences. The uncertainty of these values is generally better than 10 ns (1 σ).

Before considering particular examples, it is useful to note that an ensemble time scale can be disseminated by making a comparison with any other operating clock, even if this clock does not participate in the generation of the time scale, for it is sufficient to have a time link. It is thus important to distinguish between the generation and the dissemination of a time scale. In the extreme case TAI could be chosen to be the mean of a few ultra-stable clocks kept in a small number of laboratories, but the work carried out for its dissemination, *i.e.* the establishment of an international GPS network, would be exactly the same.

The dissemination of most time scales is performed by the publication of official documents, usually on paper sheets, but also via electronic mail:

* Figure 12 reproduces the first page of one issue of the monthly *Bulletin H* produced by the LPTF, the French primary laboratory for time and frequency . It contains several tables, in particular one which gives the time comparison values between UTC(OP) and GPS time, and between UTC(OP) and three European Loran-C chains. It covers a one-month period.
* Figure 13 reproduces the two first pages of one issue of the weekly *IERS Bulletin-A*. It contains tables of values of comparison between UT1 and UTC and information about the polar motion.
* Figure 14 reproduces the three first sections of one issue of the monthly *Circular T* produced by the BIPM. It contains tables of the values of comparisons between UTC and UTC(k) for the 45 local representations of UTC, and between TAI and TA(k) for the 17 independent atomic time scales computed throughout the world. The BIPM also gives a daily estimation of comparisons between UTC and GPS time, and UTC and GLONASS time.

For other time scales, such as GPS time and GLONASS time, dissemination is realized in real time through observations of the satellites which transmit it. It may be necessary to filter the measurements to remove observational noise and intentional degradation [Thomas 1992, 1993]. Deferred-time access is obtained by specific publications produced by the USNO [Series 4] (see Fig. 15), the BIPM [*Circular T*], and also by the NIST time and frequency services.

OBSERVATOIRE DE PARIS

Bulletin H 317

LABORATOIRE PRIMAIRE DU TEMPS ET DES FREQUENCES

Laboratoire primaire désigné par le Bureau National de Métrologie

TABLEAU 1 - MESURES DE TEMPS RAPPORTEES A UTC(OP)

		MESURES DE PHASE DES CHAINES DE LORAN-C UTC(OP) - SIGNAL à 10h UT			MESURES DU TEMPS GPS à 14h UT
Date 1994 Mai	Date MJD	SYLT 7970-W µs	ESTARTIT 7990-Z µs	LESSAY 8940-M µs	UTC(OP)-GPS -9 s + µs
1	49473	3.35	1.73	-0.27	0.070
2	49474	3.38	1.76	-0.28	0.074
3	49475	3.46	1.82	-0.29	0.072
4	49476	3.42	1.72	-0.25	0.077
5	49477	3.44	1.72	-0.27	0.062
6	49478	3.52	1.78	-0.21	0.084
7	49479	3.51	1.83	-0.23	0.088
8	49480	3.47	1.78	-0.23	0.092
9	49481	3.48	1.74	-0.21	0.114
10	49482	3.49 (1)	1.74	-0.16	0.111
11	49483	3.34	1.81	-0.17	0.090
12	49484	3.33	1.81	-0.17	0.087
13	49485	3.33	1.77	-0.17	0.087
14	49486	3.38	1.93	-0.17	0.099
15	49487	3.34	1.87	-0.11	0.096
16	49488	3.41	1.95	-0.18	0.099
17	49489	3.33	1.87	-0.11	0.074
18	49490	3.33	1.79	-0.11	0.087
19	49491	3.35	1.82	-0.10	0.072
20	49492	3.35	1.92	-0.06	0.082
21	49493	3.40	1.88	-0.07	0.084
22	49494	3.43	1.91	-0.11	0.078
23	49495	3.44	1.88	-0.10	0.072
24	49496	3.51	1.84	-0.09	0.061
25	49497	3.46	1.81	-0.05	0.063
26	49498	3.52	1.82	-0.07	0.067
27	49499	3.50	1.81	-0.08	0.073
28	49500	3.52	1.79	-0.06	0.074
29	49501	3.50	1.76	-0.03	0.087
30	49502	3.52	1.77	-0.05	0.070
31	49503	3.52	1.80	0.01	0.075

L P T F, Observatoire de PARIS, 61 Avenue de l'Observatoire, 75014 PARIS, FRANCE
Téléphone : (33-1) 40-51-22-21 - Télex : 270 776 - Téléfax : (33-1) 43-25-55-42
Internet: lptfop@opdaf1.obspm.fr - Span : opdaf1::lptfop

Figure 12. First page of *Bulletin H* (issue No 317), produced on a monthly basis at the LPTF, Paris, France.

IERS Bulletin - A

International Earth Rotation Service

NEOS Earth Orientation Bulletin

26 May 1994 Incorporating U.S.N.O. Series 7 Vol. VII No. 21

CONTENTS OF THIS ISSUE: PAGE

GENERAL INFORMATION:

MJD = Julian Date - 2 400 000.5 days

UT2-UT1 = 0.022 sin(2*pi*T) - 0.012 cos(2*pi*T)
- 0.006 sin(4*pi*T) + 0.007 cos(4*pi*T)

where pi = 3.14159265... and T is the date in Besselian years.

TDT = TAI + 32.184 seconds = A.1 + 32.150 seconds

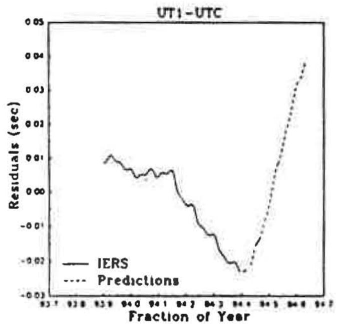
DUT1= (UT1-UTC) transmitted with time signals
= -0.2 seconds beginning 9 June 1994 at 0000 UTC

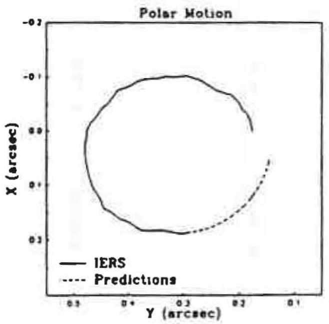
Beginning 1 July 1993:
TAI-UTC(BIPM) = 28.000 000 seconds
A.1(USNO)-UTC(USNO) = 28.034 381 7 seconds

A positive leap second will be introduced in UTC on 30 June 1994.

U.S.N.O. DATA AND PREDICTIONS OF UT1-UTC AND POLAR MOTION

T(PLOT) = UT1-UTC - .2811 + (MJD-49315) * .00226





OBSERVATIONS:

NEOS VLBI Intensives

MJD	UT1-UTC (s)	s.e. (s)
49483.7962	-.123264	.000035
49485.7863	-.127547	.000030
49486.7837	-.129754	.000030

P designates preliminary values based on preliminary pole positions

NEOS VLBI

MJD	x (")	s.e. (")	y (")	s.e. (")	UT1-UTC (s)	s.e. (s)
49490.253	.18678	.00010	.29495	.00007	-.137987	.000004

VLBI from Jet Propulsion Laboratory

MJD	UT0-UTC (sec)	s.d. (sec)	Baseline
49490.010	-.14455	.00025	1463
49493.850	-.14673	.00005	1463

DELFT UNIVERSITY OF TECHNOLOGY SLR

MJD	x (")	m.e (")	y (")	m.e. (")
49488.00	.18917	.00015	.30282	.00013
49491.00	.18758	.00018	.29647	.00014

GLOBAL POSITIONING SYSTEM COMBINED SOLUTION

MJD	x (")	m.e (")	y (")	m.e. (")
49489.0	.18837	.00040	.29903	.00038
49490.0	.18802	.00035	.29698	.00041
49491.0	.18771	.00033	.29494	.00037
49492.0	.18741	.00023	.29289	.00036
49493.0	.18710	.00024	.29086	.00036
49494.0	.18677	.00027	.28882	.00038
49495.0	.18646	.00029	.28678	.00038

COMBINED EARTH ORIENTATION PARAMETERS:

IERS Rapid Service

Date	MJD	x (")	error (")	y (")	error (")	UT1-UTC (s)	error (s)
1994 May 17	49489	.18847	.00030	.29906	.00016	-.134786	.000076
18	49490	.18816	.00032	.29700	.00018	-.137299	.000157
19	49491	.18783	.00031	.29495	.00018	-.139949	.000165
20	49492	.18748	.00031	.29290	.00017	-.142710	.000165
21	49493	.18712	.00026	.29086	.00017	-.145522	.000167
22	49494	.18677	.00024	.28882	.00018	-.148303	.000363
23	49495	.18642	.00024	.28678	.00011	-.150971	.000440

PREDICTIONS:

The following formulas will not reproduce the predictions given below, but may be used to extend the predictions beyond the end of this table.

x = .0270 - .0279 cos A + .0756 sin A + .1938 cos C - .1141 sin C
y = .3224 + .0721 cos A + .0337 sin A - .1141 cos C - .1938 sin C
UT1-UTC = -.1024 - .00231 (MJD - 49495) - (UT2-UT1)

Figure 13. First two pages of *IERS Bulletin-A* (issue of 26 May 1994), produced on a weekly basis at the IERS, Paris, France.

BIPM ISSN 1143-1393

BUREAU INTERNATIONAL DES POIDS ET MESURES

Circular T 76 (1994 May 25)

1 - Coordinated Universal Time UTC. Computed values of UTC-UTC(k) (1).

(From 1993 July 1, 0h UTC, to 1994 July 1, 0h UTC, TAI-UTC = 28 s)
(From 1994 July 1, 0h UTC, until further notice, TAI-UTC = 29 s)

Date 1994 0h UTC		Mar 28	Apr 7	Apr 17	Apr 27
MJD		49439	49449	49459	49469
Laboratory k		UTC-UTC(k) (Unit = 1 microsecond)			
AOS (Borowiec)		-1.153	-1.377	-1.520	-1.701
APL (Laurel)		1.249	1.181	1.126	1.061
AUS (Canberra)		0.498	0.476	0.447	0.408
BEV (Wien)		-	-	-	-
CAO (Cagliari)		-6.872	-7.066	-7.233	-7.483
CH (Bern)		1.929	1.921	1.826	1.693
CRL (Tokyo)		2.053	2.024	2.010	2.025
CSAO (Lintong)		-0.477	-0.452	-0.407	-0.402
CSIR (Pretoria)		-3.061	-2.924	-2.865	-2.826
FTZ (Darmstadt)		0.040	0.092	0.161	0.228
IEN (Torino)		0.065	0.103	0.131	0.177
IFAG (Wettzell)		-0.598	-0.575	-0.569	-0.522
IGMA (Buenos Aires)		-3.14	-3.14	-3.13	-3.15
INPL (Jerusalem)		-1.135	-1.280	-1.398	-1.474
JATC (Lintong)		-1.891	-1.283	-0.397	0.365
KRIS (Taejon)		-0.276	-0.289	-0.264	-0.251
LDS (Leeds)		-0.282	-0.281	-0.323	-0.356
MSL (Lower Hutt)		-0.434	-0.431	-0.388	-0.346
NAOM (Mizusawa)		-1.436	-1.477	-1.513	-1.539
NAOT (Tokyo)		-0.661	-0.876	-1.035	-1.262
NIM (Beijing)		7.78	7.81	7.78	7.80
NIST (Boulder)		-0.051	-0.068	-0.086	-0.094
NMC (Sofiya)		-	-	-	-
NPL (Teddington)		0.119	0.116	0.114	0.113
NPLI (New-Delhi)	(2)	-3.12	-3.22	-	-3.18
NRC (Ottawa)		5.265	5.367	5.468	5.567
NRLM (Tsukuba)		-9.641	-9.937	-10.233	-10.521
OMH (Budapest)		6.489	6.510	6.502	6.559
ONBA (Buenos Aires)		5.65	5.57	5.70	5.48
ONRJ (Rio de Janeiro)		-13.877	-	-	-
OP (Paris)		-0.047	-0.029	-0.010	0.005
ORB (Bruxelles)		-1.673	-1.712	-1.666	-1.755
PKNM (Warszawa)		0.454	0.373	0.241	0.212
PTB (Braunschweig)		2.748	2.753	2.754	2.772
RC (Habana)	(3)	-2.36	-3.00	-3.08	-2.80
ROA (San Fernando)		2.615	2.610	2.632	2.637
SCL (Hong Kong)		0.034	0.107	0.177	0.424
SNT (Stockholm)		0.065	0.085	0.086	0.067
SO (Shanghai)		2.14	-	2.16	1.78
SU (Moskva)		-3.375	-3.461	-3.548	-3.624
TL (Chung-Li)		-3.106	-3.049	-2.985	-2.914
TP (Praha)		-1.147	-1.135	-1.098	-1.069
TUG (Graz)		4.481	4.564	4.643	4.739
USNO (Washington DC)(USNO MC)		0.045	0.051	0.051	0.057
VSL (Delft)		0.094	0.132	0.166	0.174

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BIPM , T 76 (2)

2 - International Atomic Time TAI and local atomic time scales TA(k).

The following table gives the computed values of TAI-TA(k) (1).

Date 1994 0h UTC		Mar 28	Apr 7	Apr 17	Apr 27
MJD		49439	49449	49459	49469
Laboratory k		TAI-TA(k) (Unit = 1 microsecond)			
APL (Laurel)		2.712	2.644	2.589	2.524
AUS (Canberra)		-50.849	-51.020	-51.106	-51.273
CH (Bern)		-75.231	-75.059	-74.894	-74.767
CRL (Tokyo)		36.656	37.065	37.496	37.948
CSAO (Lintong)		14.992	14.887	14.803	14.678
F (Paris)		127.851	128.227	128.604	128.987
INPL (Jerusalem)		-	-196.410	-198.459	-200.492
JATC (Lintong)		9.415	10.044	10.708	11.488
KRIS (Taejon)		-3.486	-3.279	-3.054	-2.811
NIM (Beijing)		-8.73	-8.68	-8.70	-8.66
NISA (Boulder)	(4)	-45111.238	-45111.631	-45112.029	-45112.417
NRC (Ottawa)		21.334	21.436	21.537	21.636
PTB (Braunschweig)		-360.652	-360.647	-360.646	-360.628
RC (Habana)	(3)(5)	-325.85	-326.53	-326.66	-326.42
SO (Shanghai)		-45.43	-	-45.40	-45.81
SU (Moskva)	(6)	27246.625	27246.539	27246.452	27246.376
USNO (Washington DC)	(7)	-34695.858	-34696.529	-34697.211	-34697.880

3 - Notes on sections 1 and 2.

(1) Values UTC-UTC(k) and TAI-TA(k) are published within 1 ns except for laboratories which are not linked through GPS common views.

(2) NPLI.

MJD	UTC-UTC(NPLI)
49419	-3.29
49429	-3.03

(3) RC .

MJD	UTC-UTC(RC)	TAI-TA(RC) - 18 s
49419	-2.78	-326.18
49429	-2.54	-325.99

(4) NIST. TA(NISA) designates the scale AT1 of NIST.

(5) RC . Listed values are TAI-TA(RC) - 18 seconds.

(6) SU . Listed values are TAI-TA(SU) - 2.80 seconds.

(7) USNO. TA(USNO) designates the scale A1(MEAN) of USNO.

Figure 14. First two pages of *Circular T* (issue of 25 May 1994), produced on a monthly basis at the BIPM, Sèvres, France.

DAILY TIME DIFFERENCES, SERIES 4, NO. 1426 (CONTINUED)

GLOBAL POSITIONING SYSTEM (GPS)
BLOCK I AND BLOCK II SATELLITES

VALUES PRESENTED BELOW FOR NAVSTAR GPS SATELLITES ARE THE RESULT OF A LINEAR FIT THROUGH APPROXIMATELY 130 DATA POINTS REFERRED TO THE BEGINNING OF THE TRACKING PERIOD. TRACKING PERIODS START ON THE MINUTE AND RANGE FROM TWO TO THIRTEEN MINUTES.

GPS TIME IS AHEAD OF UTC BY NINE SECONDS. UNIT = ONE NANOSECOND

			NAVSTAR 10 PRN12		NAVSTAR 13 PRN02		NAVSTAR 14 PRN14		NAVSTAR 15 PRN15		NAVSTAR 16 PRN16	
		MJD	MC-GPS	GPS TIME	MC-GPS	GPS TIME	MC-GPS	GPS TIME	MC-GPS	GPS TIME	MC-GPS	GPS TIME
MAY	22	49494	0	(2102)	-96	(2005)	-40	(2135)	-7	(1901)	41	(0254)
	23	49495	-13	(2058)	-17	(2001)	-87	(2135)	-16	(1857)	-53	(0251)
	24	49496	-6	(2054)	-52	(1957)	67	(2123)	-9	(1853)	-34	(0247)
	25	49497	-5	(2050)	-8	(1953)	-49	(2119)	-15	(1849)	61	(0243)
	26	49498	2	(2046)	-137	(1949)	-29	(2115)	-20	(1845)	17	(0239)
	27	49499	-1	(2042)	-34	(1945)	-27	(2111)	-8	(1841)	-71	(0235)
	28	49500	9	(2038)	-7	(1941)	-42	(2107)	-5	(1837)	27	(0231)
	29	49501	-3	(2034)	38	(1937)	-79	(2103)	-5	(1833)	88	(0227)
	30	49502	0	(2030)	249	(1933)	-12	(2059)	-2	(1829)	10	(0223)
	31	49503	8	(2026)	0	(1930)	148	(2055)	4	(1825)	-109	(0219)

			NAVSTAR 17 PRN17		NAVSTAR 18 PRN18		NAVSTAR 19 PRN19		NAVSTAR 20 PRN20		NAVSTAR 21 PRN21	
		MJD	MC-GPS	GPS TIME	MC-GPS	GPS TIME	MC-GPS	GPS TIME	MC-GPS	GPS TIME	MC-GPS	GPS TIME
MAY	22	49494	73	(0607)	-9	(2327)	67	(1717)	18	(0954)	37	(0818)
	23	49495	-6	(0603)	-11	(2323)	29	(1713)	-10	(2351)	44	(0814)
	24	49496	-28	(0559)	119	(2319)	81	(1709)	-17	(2347)	4	(0810)
	25	49497	-2	(0555)	141	(2315)	97	(1705)	22	(2356)	6	(0806)
	26	49498	-51	(0551)	1	(2311)	-55	(1701)	-5	(2339)	176	(0802)
	27	49499	20	(0547)	216	(2307)	70	(1657)	0	(2335)	-85	(0758)
	28	49500	12	(0543)	-5	(2303)	11	(1653)	16	(2331)	32	(0754)
	29	49501	26	(0540)	-71	(2259)	-61	(1649)	13	(2327)	69	(0750)
	30	49502	-59	(0540)	190	(2358)	-25	(1645)	14	(2323)	91	(0746)
	31	49503	138	(0541)	-30	(2354)	101	(1641)	29	(2332)	62	(0742)

Figure 15. Second page of *USNO Series 4* (issue No 1426), produced on a weekly basis at the USA, Washington, D.C., USA.

Conclusions

In 1991, the International Astronomical Union clearly specified in terms of general relativity the framework within which time scales should be defined. A realization of Terrestrial Time, as explicitly mentioned in the IAU Resolution, is International Atomic Time, TAI, which is obtained from a combination of the readings of atomic clocks kept on the Earth.

While TAI is the international reference for timing, many other time scales are regularly computed and used for scientific purposes. Besides keeping local representations of UTC, the laboratories which compute these scales have to design algorithms for the generation of free-running and independent time scales based on data collected on site. Development of algorithms inevitably leads to a need to write an equation of definition, in the form of a weighted average, and to settle procedures for the determination of clock weight and the prediction of clock frequency. Many sophistications are possible, but the actual choices are guided by the purposes the time scale is intended to serve and by the noise affecting timing data.

In 1993, world-wide most stable time scales reached stabilities better than 1 part in 10^{14} for averaging times of order several weeks. Achieved accuracies are limited by the accuracy of the best primary frequency standards, and are, at present, characterized by an uncertainty (1 σ) of order 1 part in 10^{14}. Improvements in performance are rapid: it is probable that accuracies of order some parts in 10^{16}, for realization of the SI second, and several hundreds of picoseconds, for time comparisons, will be available in the year 2000.

Though the second is defined in atomic terms and time scales are generated from atomic clocks, time retains its close relation with astronomy: the international reference time scale is the purely atomic TAI, but coherence with the Earth rotation has been maintained by the production of UTC. The 21th century may see the relation with astronomy again reinforced through the use of millisecond pulsars for monitoring the long-term stability of TAI [Petit *et al.* 1992].

Note

Laboratory acronyms and locations can be found in Table 3, page 20 and 21 of the Annual Report of the BIPM Time Section, Volume 6, available on request from the BIPM, Pavillon de Breteuil, 92312 Sèvres Cedex, France.

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ANNEX I

Random frequency instability of an oscillator, in the time domain, may be estimated by several sample variances. The recommended measure [Recommendation 538-2, CCIR 1992] is the two-sample standard deviation, which is the square root of the two-sample zero dead-time variance $\sigma_y^2(\tau)$, also designated as Allan variance, defined as:

$$\sigma_y^2(\tau) = \frac{1}{2}\langle(\bar{y}_{k+1} - \bar{y}_k)^2\rangle, \tag{AI.1}$$

where $\langle\ldots\rangle$ denotes an infinite time average and where $\bar{y}_k$ can be estimated by:

$$\bar{y}_k = \frac{x_{k+1} - x_k}{\tau}. \tag{AI.2}$$

The terms x_k and x_{k+1} are the time measurements, obtained from the comparison between two oscillators, at date t_k and $t_{k+1} = t_k + \tau$, with $k = 1, 2, 3, \ldots$ The term $\bar{y}_k$ is the average normalized frequency departure of one of the oscillator against the other one, estimated over the averaging time τ.

The fixed sampling rate is $1/\tau$, it gives zero dead time between frequency measurements. In addition, it is supposed that the known systematic effects have been removed from the time measurements x_k.

Call $1/\tau_0$ the initial sampling rate. One obtain a more efficient estimate of $\sigma_y(\tau)$ using what is called the "overlapping estimate". This is obtained through the following equation (for $\tau \geq \tau_0$):

$$\sigma_y^2(\tau) = \frac{1}{2(N-2n)\tau^2} \sum_{i=1}^{N-2n} (x_{i+2n} - 2x_{i+n} + x_i)^2, \tag{AI.3}$$

where N is the number of original time departure measurements spaced by τ_0 and $\tau = n\tau_0$.

The original $\bar{y}_i(\tau_0)$ can be combined to estimate a set of $\bar{y}_k(\tau)$ averaged over the time τ:

$$\bar{y}_k = \frac{1}{n} \sum_{i=k}^{k+n-1} \bar{y}_i. \tag{AI.4}$$

The "overlapping estimate" of $\sigma_y(\tau)$, given in (AI.3), can also be obtained from:

$$\sigma_y^2(\tau) = \frac{1}{2(M-2n+1)} \sum_{k=1}^{M-2n+1} (\bar{y}_{k+n} - \bar{y}_k)^2, \tag{AI.5}$$

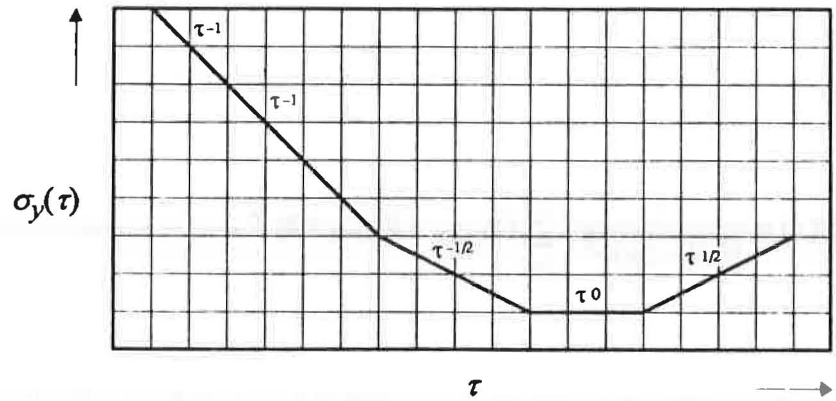
where M is the number of original frequency measurements of sample time τ_0 ($M = N$-1)

<u>*Note*</u>

If dead time exists between the frequency departure measurements and this is ignored in computing (AI.1), it has been shown that the resulting stability values, which are no longer Allan variances, will be biased, except for the white frequency noise. This bias has been studied and some tables for its correction published.

An interesting feature of $\sigma_y(\tau)$ is that its dependence with τ gives an indication of the type of random noise affecting the clock signal. A plot of $\sigma_y(\tau)$ versus τ for a frequency standard typically shows a behaviour consisting of elements as shown in Fig. AI.1.

* The first part, with $\sigma_y(\tau) \approx \tau^{-1}$, corresponds to a range of averaging times for which the white or flicker phase noise is predominant. Alternative techniques are necessary (see below) to decide whether the oscillator is effectively perturbed by white phase noise or by flicker phase noise.
* A second part, with $\sigma_y(\tau) \approx \tau^{-1/2}$, corresponds to a range of averaging times for which the white frequency noise is predominant.
* These τ^{-1} and/or $\tau^{-1/2}$ laws continue with increasing averaging time until the so-called flicker "floor" is reached, where $\sigma_y(\tau)$ is independent of the averaging time τ. This behaviour is found in almost all frequency standards; its level depends on the particular frequency standard and is not fully understood in its physical basis.
* Finally, the curve shows a deterioration of the stability with increasing averaging time, a predominant random walk of frequency leading to $\sigma_y(\tau) \approx \tau^{+1/2}$.



<u>Figure AI.1</u>: Characteristic plot of the Allan standard deviation $\sigma_y(\tau)$, versus the averaging time τ, for random frequency fluctuations of an oscillator.

The "modified Allan variance", $Mod.\sigma_y(\tau)$, has been developed, which has the property of yielding different dependencies on τ for white phase noise and flicker phase noise: these are respectively $\tau^{-3/2}$ and τ^{-1}. Using time departures x_i, the modified Allan variance is estimated using the following equation:

$$Mod.\sigma_y^2(\tau) = \frac{1}{2\tau^2 n^2 (N-3n+1)} \sum_{j=1}^{N-3n+1} \left[\sum_{i=j}^{n+j-1} (x_{i+2n} - 2x_{i+n} + x_i) \right]^2 , \qquad \text{(AI.6)}$$

where N is the original number of time measurements spaced by τ_0, and $\tau = n\tau_0$ the sampling time of choice.

The time instability in the time-domain for the above five power-law spectra may be measured using the second difference of adjacent time averages. This time variance is also relayed to *Mod.* $\sigma_y(\tau)$ through the following equation:

$$\sigma_x^2(\tau) = \tau^2 \frac{Mod.\sigma_y^2(\tau)}{3}. \qquad \text{(AI.7)}$$

Two useful properties of the time variance are as follows:

* $\sigma_x(\tau)$ is equal to the classical standard deviation of the time difference measurements for $\tau = \tau_0$, in case of white phase noise.
* $\sigma_x(\tau)$ is equal to the standard deviation of the mean of the time difference measurements for $\tau = N\tau_0$ (the data length), in case of white phase noise.

Table AI.1 gives the functional characteristics of five independent noise processes for frequency instability of oscillators.

More information about the characterization of random frequency instability, and about the calculation of the level of uncertainty affecting the estimates of the Allan variances can be found in the *NIST Technical Note 1337*, edited in 1990 by D.B. Sullivan, D.W. Allan, D.A. Howe and F.L. Walls, and in particular, in the paper "Time and Frequency (Time-Domain) Characterization, Estimation, and Prediction of Precision Clocks and Oscillators", by D.W. Allan, *IEEE Trans. Ultras. Ferro. Freq. Cont.*, **UFFC-34**, 1987, pp 647-654, also reproduced in the *NIST Technical Note 1337* on pages TN-121 to TN-128.

Noise process	Slope characteristics of log-log plot		
	Time domain		
	$\sigma_y^2(\tau)$	$Mod.\sigma_y^2(\tau)$	$\sigma_x^2(\tau)$
	μ	ν	η
White phase	-2	-3	-1
Flicker phase	-2	-2	0
White frequency	-1	-1	+1
Flicker frequency	0	0	+2
Random walk frequency	+1	+1	+3

Table AI.1: Characterization of noise processes for frequency instability of oscillators. The coefficients μ, ν, and η correspond to dependencies of the Allan variance, the modified Allan variance, and the time variance according to the following expressions:

$$\sigma_y^2(\tau) \approx \tau^{\mu},\ Mod.\sigma_y^2(\tau) \approx \tau^{\nu},\ \sigma_x^2(\tau) \approx \tau^{\eta}.$$