8. Platinum Resistance Thermometers

8.1 General Remarks

The platinum resistance thermometer (PRT) is very widely used below 500 °C as a thermometric sensor. There is a wide range of quality of PRT available, from the standard instrument (SPRT) of the ITS-90 to some industrial types (IPRT) that are accurate only to within a few tenths of a kelvin or, perhaps, even a kelvin or more. The major difference of the industrial type of fabrication from the standard type is not just the purity of platinum, but also the less strain-free mounting of the film or wire which is embedded (partially or totally) in a cement (glass or refractory). Furthermore, in most cases, the thermometer body is not hermetically sealed.

The ITS restricts the quality of the thermometer that may be used as a standard instrument. For the ITS-90 an acceptable PRT must be made from pure, strain-free platinum, and it must satisfy at least one of the following two relations:

\[ W(29.7646 \, ^{\circ}C) \geq 1.118 \, 07 \]
\[ W(-38.8344 \, ^{\circ}C) \leq 0.844 \, 235 , \]

where \( W(t_{90}) = R(t_{90})/R(0.01 \, ^{\circ}C)^* \). If the PRT is to be used to the freezing point of silver, it must also satisfy the relation

\[ W(961.78 \, ^{\circ}C) \geq 4.2844 . \]

These relations are closely equivalent to the restriction that

\[ (dW(t_{90})/dt_{90})_{0.01 \, ^{\circ}C} \geq 3.986 \times 10^{-3} \, K^{-1} . \]

These relations, in essence, assure that the platinum in an SPRT is highly pure. They are equivalent to the requirement that \( W(H_2O \, b.p.) > 1.392 \, 44 \), whereas it is estimated that

* Note that this definition of \( W(t_{90}) \) is different from the equivalent one for \( W_{68}(t_{68}) \). The latter uses \( R(0 \, ^{\circ}C) \) as the reference in the ratio. It follows that \( W_{68}(t_{68}) = 1.000 \, 040 \, W(t_{90}) \) to better than 1 part in \( 10^6 \) for any SPRT.
for ideally pure platinum $W(H_2O \text{ b.p.}) \sim 1.39274$. No such restrictions apply, in principle, to a PRT used to approximate the ITS-90. Various national and international bodies place restrictions on the quality of PRT that is suitable to meet their requirements for accuracy, the restriction taking the form of a specification on $W(H_2O \text{ b.p.})$. In addition, in conjunction with the use of standard tables of resistance, the value of $R(0 \degree C)$ is specified. Also, the resistance ratio $W(4.2 K)$ is sometimes used as a quality indicator. The most accurate thermometers (corresponding to $W(H_2O \text{ b.p.}) > 1.3925$ have $W(4.2 K) < 4 \times 10^{-4}$, whereas industrial thermometers (corresponding to $W(H_2O \text{ b.p.}) \sim 1.385$) have $W(4.2 K)$ nearer $2 \times 10^{-2}$. It must also be remembered that for industrial purposes the platinum sensing element is usually mounted within a protective sheath which can modify some of the characteristics of the sensor itself. It is the latter that has been most often studied in the references to follow.

In Section 8.2 we outline a few simple methods for approximating the ITS-90* based upon fixed-point calibrations and simple interpolation formulae to be used with SPRTs, with indications of the accuracies that may be achieved. Discussion of IPRTs is given in Chapter 16 in Part 2.

8.2 Interpolation Equations for Standard Platinum Resistance Thermometers*

The approximations described here are concerned with SPRTs calibrated with simplified methods for medium to high accuracy. Several such simplified procedures have been suggested but only some of the more promising of them are described here. Some of the others appear either to be too complicated or to have been insufficiently tested (e.g. too few thermometers) to be suitable for recommendation.

(a) Perhaps the simplest secondary realization is an extrapolation below 273 K of the following defining equations of the IPTS-68 from above 273 K:

$$t_{68} = t' + 0.045 \left( \frac{t'}{100 \degree C} - 1 \right) \left( \frac{t'}{419.58 \degree C} - 1 \right) \left( \frac{t'}{630.74 \degree C} - 1 \right)$$

$$W_{68}(t') = 1 + At' + Bt'^2$$

* All of these methods were devised as approximations to the IPTS-68 but they are also applicable to the ITS-90.
For a group of 45 thermometers having $W_{68}(H_2O \text{ b.p.}) > 1.3925$, Bedford (1972b) showed that if the thermometers were calibrated only at the triple and boiling points of water and if a common value of $B$ were assumed for every thermometer, then the extrapolated temperatures ($T_e$) diverge smoothly from $T_{68}$ (or $T_{90}$) by up to 1 K at 100 K, 1.7 K at 70 K, and -12 K near 20 K, and variations in these temperatures are small (Fig. 6.1). Using a tabulated or functional correction, one can measure $T_{68}$ (and, consequently, $T_{90}$ using the tables of Appendix A) to within ±3 mK down to 150 K, ±15 mK at 77 K, and ±30 mK at 30 K. We note that:

(i) The method is simple. The thermometers are calibrated only at the triple and boiling points of water. If $B$ were measured by a tin-point or zinc-point calibration, the extrapolated scale uncertainty might be marginally smaller. This would be feasible for a long-stem thermometer but not for many capsule thermometers.

(ii) The procedure is equally valid for capsule or long-stem PRTs.

(iii) There is no definitive test of the extrapolation; there is a high probability, but no certainty, that any particular PRT will fall within the uncertainties quoted.

(b) Another simple secondary realization for the range 90 K to 273 K uses only two fixed points and obtains a third from an experimentally-observed linear correlation between $W(O_2 \text{ b.p.})$ and $W(H_2O \text{ b.p.})$. This correlation was first pointed out by Brodskii (1968), who found the following relation for thermometers with $W_{68}(H_2O \text{ b.p.}) > 1.3920$:

$$W(O_2 \text{ b.p.}) = a_1 + b_1 (W(H_2O \text{ b.p.}) -1) . \quad (8.3)$$

Berry [(1963), (1972)] examined similar correlations in detail. For platinum of the highest available purity, he determined [Berry (1972)] a linear relation of the form of Eq. (8.3) corresponding to various equilibrium concentrations of quenched-in vacancies. This is shown as curve V in Fig. 8.2, which Berry suggested would be an upper bound for all real PRTs. Similarly, his curve L [Berry (1963)] approximates the mean behaviour for his large group of thermometers. From Fig. 9 of Berry (1963) one can deduce a lower bound (curve M) below which no SPRT is likely to lie.
Fig. 8.1: Differences from $T_{90}$ of temperatures $T_e$ extrapolated below 0 °C using the IPTS-68 defining equations for above 0 °C. The hatched area shows the maximum observed deviations $\Delta (T_{90} - T_e)$ in $(T_{90} - T_e)$ for a group of 45 thermometers [Bedford (1986)].

Fig. 8.2: Linear relationships between $W(O_2$ b.p.) and $W(H_2O$ b.p.) for platinum: B, after Brodskii (1968); V, ideally pure platinum [Berry (1972)]; L, mean of many thermometers [Berry (1963)]; M, lower bound for SPRT [Berry (1963)]; S, result when argon replaces oxygen, converted to oxygen [Seifert (1980), (1984)]; [Bedford (1986)].
More recently, Seifert [(1980), (1984)] considered a relation similar to Eq. (8.3) relating $W_{68}(\text{Ar t.p})$ to $W_{68}(H_20 \ b.p.)$:

$$W_{68}(H_20 \ b.p.) = -0.52529 \ W_{68}(\text{Ar t.p.}) + 1.506108 \quad (8.4)$$

Using the $W_{CCT-68}(T_{68})$ ratio of 1.128402 for $W_{68}(O_2 \ b.p.)/W_{68}(\text{Ar t.p.})$, Seifert's equation can be converted to the form of Eq. (8.3); the result is shown as curve S in Fig. 8.2. Curve S has a more negative slope than the others and rises above V at lower values of $W(H_20 \ b.p.)$. The coefficients $a_1$ and $b_1$ of Eq. (8.3) are given in Table 8.1 for all of the curves in Fig. (8.2).

Table 8.1: Coefficients $a_1$, $b_1$ of Eq. (8.3) for the Curves in Fig. (8.1).

<table>
<thead>
<tr>
<th>Curve</th>
<th>$a_1$</th>
<th>$b_1$</th>
<th>reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0.97786</td>
<td>-1.8700</td>
<td>Brodskii (1963)</td>
</tr>
<tr>
<td>V</td>
<td>0.982826</td>
<td>-1.882572</td>
<td>Berry (1972)</td>
</tr>
<tr>
<td>L</td>
<td>0.903485</td>
<td>-1.680508</td>
<td>Berry (1963)</td>
</tr>
<tr>
<td>M</td>
<td>0.779641</td>
<td>-1.365254</td>
<td>Berry (1963)</td>
</tr>
</tbody>
</table>

Equation (8.3) allows a secondary realization from 90 K to 273 K with calibration at only the triple point of water and the boiling point of either oxygen or water, the other being estimated from Eq. (8.3). The coefficients $a_1$, $b_1$ for curve L are probably the most representative. Used with the IPTS-68 defining equation for the range 90.188 K to 273.15 K, this secondary realization is probably accurate to within ±30 mK when $W(H_20 \ b.p.) \sim 1.3920$ and ±10 mK when $W(H_20 \ b.p.) > 1.3925$. In Eq. (8.3), $W(O_2 \ b.p.)$ could be replaced with $W(\text{Ar t.p.})$ and $W(H_20 \ b.p.)$ by $W(ln \ f.p.)$ (with suitable adjustment of the coefficients $a_1$ and $b_1$), and an equivalent accuracy would result. According to the finding of Bedford (1972b), extrapolation to about 70 K will produce little further degradation in accuracy. Although temperatures between 125 K and 273 K are less sensitive to an error in $W(O_2 \ b.p.)$ than in $W(H_20 \ b.p.)$, it is probably better and usually more convenient to calibrate at...
the oxygen or argon point and deduce the resistance ratio at the steam point. Moreover, few laboratories now maintain a steam point, and most capsule thermometers cannot safely be heated as high as the tin freezing point, another reason to prefer calibration at the oxygen point and deduction of \( W(H_2O \text{ b.p.}) \). As indicated by Seifert, a relation similar to Eq. (8.3) based upon the argon triple point is even better. It obviates the need for any boiling-point measurement, requiring only two simple triple-point determinations. Seifert (1984) has extended this method by using one- or two-point comparisons with a standard PRT (at, e.g., 160 K and 78 K) to replace the argon-triple-point measurement. The accuracy is little degraded thereby.

(c) Several possibilities for secondary realization based upon Cragoe Z-functions rather than simple resistance ratios have been proposed. Among early investigations were those of Cragoe (1948), Corruccini (1960), (1962), and Barber (1962). The Z-function is defined by

\[
Z(T) = \frac{R(T) - R(T_1)}{R(T_2) - R(T_1)}, \quad (8.5)
\]

where \( T_1 \) and \( T_2 \) are fixed-point calibration temperatures at or near the extremities of the range. The Z-function for a particular choice of calibration temperatures is tabulated and temperatures are calculated by interpolating deviations from the table in some specified way. The Z-function tabulation acts as a reference function equivalent to \( W(T_{90}) \). Besley and Kemp (1978) incorporate the Z-function into a reference function using \( T_1 = 4.2 \) K (boiling helium) and \( T_2 = 273.15 \) K (melting ice) in Eq. (8.5). They define a reference function of the form

\[
T^* = \sum_{n=0}^{16} A_n \left[ \frac{\ln(Z) + 3.54}{3.54} \right]^n \quad (8.6)
\]

using the mean value of \( Z \) of a group of 19 thermometers each with \( W(4.2 \) K) < 4 x 10^{-4}. Temperature \((T = T^* - \Delta T^*)\) is determined for any particular thermometer in terms of a deviation from Eq. (8.6) of the form

\[
\Delta T^* = a \left[ 1 - \left( \frac{T^*}{40 \text{ K}} \right)^{1/2} \right] \Delta T, \quad (8.7)
\]

where

\[
\Delta T = b + c \, W(4.2 \text{ K}) + d \, W(4.2 \text{ K})^2, \quad T^* \leq 40 \text{ K} \quad (8.8)
\]

and

\[
\Delta T = 0, \quad T^* > 40 \text{ K} \quad (8.9)
\]
For thermometers with $W(4.2 \text{ K}) < 4 \times 10^{-4}$ this scheme allows calibration of thermometers to within $\pm 0.02 \text{ K}$ from 14 K to 273 K.

Using the same method for a group of 31 thermometers having $4 \times 10^{-4} < W(4.2 \text{ K}) < 7 \times 10^{-4}$, but using a different reference function (in place of Eq. (8.6)) and correction polynomial (in place of Eq. (8.8)), Besley and Kemp (1978) obtained an accuracy within $\pm 0.02 \text{ K}$ above 40 K for 31 thermometers, and within $\pm 0.04 \text{ K}$ below 40 K for 29 thermometers.

In summary, the thermometer is calibrated only at 4.2 K and 273.15 K. A temperature T is obtained by measuring $R(T)$, calculating $Z(T)$ from Eq. (8.5), calculating $T^*$ from Eq. (8.6), and calculating $\Delta T^*$ from Eqs. (8.7) - (8.9). When $4 \times 10^{-4} < W(4.2 \text{ K}) < 7 \times 10^{-4}$, the procedure is similar, but uses different reference and correction polynomials. With this proposal:

(i) only a relatively simple 2-point calibration is required,
(ii) only SPRTs are likely to have $W(4.2 \text{ K}) < 4 \times 10^{-4}$,
(iii) it is not applicable to long-stem thermometers; to measure any temperature below 273 K, a calibration at 4.2 K is necessary,
(iv) there is no test for the accuracy of interpolation for any particular thermometer.

Several other useful approximations to the ITS have been described (see the original papers for details). Kirby et al. (1975) proposed a single deviation function from 14 K to 273 K determined by calibration against a previously-calibrated SPRT at four points using three boiling liquids (He, H₂, N₂) and the ice point; the resulting inaccuracy is about $\pm 10 \times 10^{-6}$ in W. Pavese et al. (1978) suggested using the same equations as in the IPTS-68 but a different set of fixed points that included only triple points. Tiggelman and Durieux (1972a) showed that a polynomial in the form

$$W(T) = \sum_{n=0}^{6} A_n T^n$$  \hspace{1cm} (8.10)

can fit 14 calibration points between 2 K and 15 K with a standard deviation of $\pm 2 \text{ mK}$ at 4.2 K decreasing to $\pm 0.3 \text{ mK}$ above 11 K for 10 high quality SPRTs ($W(\text{H}_2 \text{O b.p.}) > 1.3926$). In the range 4 - 15 K a mere five points allow an accuracy of $\pm 3 \text{ mK}$. 