

**Final report on the key comparison,
CCM.M.P-K15.1 in the pressure range from
 1.0×10^{-4} Pa to 1.0 Pa**

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Abstract

At the CCM WG-LP meeting in Berlin in April 2011 the National Institute of Metrology of the people's republic of China asked for an opportunity to be linked to the CCM.M.P-K15 comparison which just ended. METAS, which had been active as pilot for the CCM.M.P-K15, agreed to make a link with NIM. The NIM introduced since short time a new static expansion system in his laboratory.

Two spinning rotor gauges and a control electronic are used as transfer standard. The circulation of the transfer standard was made in spring 2012. The drift of the accommodation coefficient of the two SRG was measured by the pilot before and after the comparison. The stability of the transfer standards was excellent and the equivalence between METAS and NIM has been demonstrated.

A link to the CCM.M.P-K15 has been established using the results from METAS. It has been possible to achieve a link with a small uncertainty due to the strong correlation of the measurements of METAS in the two comparisons.

Contents

1	Introduction	7
2	Participating laboratories.....	8
2.1	NIM.....	8
2.1.1	Description of the reference standard	8
2.1.2	Traceability of the system	10
2.2	METAS	10
2.2.1	Reference system.....	10
2.2.2	Measurements.....	10
2.3	Uncertainty of the participants on the reference pressure	11
3	Transfer standard	13
4	Method used for the measurements.....	14
4.1	Correction of the residual drag.....	14
4.2	Points of measurement.	14
4.3	Circulation of the transfer standard	14
4.4	Collection of the results.....	15
5	Results provided by the participants.....	16
5.1	Measurements of the pilot laboratory and stability.....	18
6	Method used for the calculation of the reference value.	20
6.1	Influence of the reference pressure in the transition regime.	20
6.2	Correction of the drift of the transfer standard	20
6.3	Pressure value for a participating laboratories	21
6.4	Reference value for a participating laboratory	21
6.5	Pressure value for the pilot laboratory.....	21
6.6	Reference value for the pilot laboratory.....	22
6.7	Reference value of the comparison.....	22
6.8	Normalization of the reference value.....	22
6.9	Relative deviation to the reference value.	23
6.10	Degree of equivalence	23
7	Method used for the determination of the uncertainty.....	24
7.1	Uncertainty on sigma measured by the participants	24
7.1.1	Equation of the SRG.....	24
7.1.2	Uncertainty on K.....	24

7.1.3	Uncertainty of the reference pressure	24
7.1.4	Contribution due to the uncertainty on the temperature	25
7.1.5	Contribution due to the uncertainty on the residual deceleration	25
7.1.6	Type A uncertainty for σ	25
7.2	Uncertainty on the value of sigma used to correct the drift.	25
7.3	Uncertainty on sigma measured by the pilot.....	26
7.4	Uncertainty on the reduced pressure for a participant.....	26
7.5	Uncertainty on the reduced pressure for the pilot, for one loop	26
7.6	Uncertainty on the reference value of a participant	27
7.7	Uncertainty on the reference value of the pilot	27
7.8	Uncertainty on the reference value of the comparison	27
7.9	Uncertainty on the relative deviation	27
8	Reduction to a weighted mean.....	29
8.1	Difference and uncertainty respective to the weighted mean.	30
9	Link to the key comparison CCM.P-K15.....	32
9.1	Mathematics used for linking to the key comparison.	32
9.2	Link to CCM.P-K15	33
10	Conclusion	37
11	References	38

Index of tables

Table 1: Characteristics of the definition of the pressure by the participants.	8
Table 2: Relative uncertainty on the definition of the reference pressure by the participants. The uncorrelated relative uncertainty of METAS is also given as it is used in the link with CCM.P-K15.1.	11
Table 3: Characteristics of the transfer standard.....	13
Table 4: Measurement method used for each pressure step.	14
Table 5: Schedule of the circulation of the transfer standard.....	15
Table 6: Accommodation coefficient measured by the laboratories for the transfer standard SRG1 and uncertainties associated calculated using the Eq. 26.....	16
Table 7: Accommodation coefficient measured by the laboratories for the transfer standard SRG2 and uncertainties associated calculated using the Eq. 26.....	17
Table 8: Normalized value of the participants as given by Eq. 16 and the associated uncertainty given by Eq. 37 and 38. The first column gives the normalized uncertainty of the comparison which by definition is equivalent to the nominal pressure and the associated uncertainty given by Eq. 39.....	29
Table 9: The relative difference, respective to the reference value of the comparison, as defined by Eq. 17, and relative uncertainty as defined by Eq. 40. The last line is the mean value of the relative deviation expressed in absolute number and depicts some kind of average agreement of the participant with the reference value.	30
Table 10: Determination of the link to CCM.P-K15 through the measurement of metas.....	33
Table 11: Offset and uncertainty (degree of equivalence) of NIM and METAS respective to the reference value of CCM.P-K15, expressed in Pa.	34
Table 12: Pair of offset and associated uncertainty (degree of equivalence) of NIM and METAS respective to the reference value of CCM.P-K15, expressed relatively to the reference value.	35

Index of figures

Fig. 1:	Diagram of the expansion system	8
Fig. 2:	Picture of the expansion system	9
Fig. 3:	Basic principle of the calibration method	10
Fig. 4:	Relative uncertainty on the definition of the reference pressure by the participants plotted versus the pressure in Pa.....	12
Fig. 5:	Accommodation coefficient measured on the SRG1 by the different participants.	17
Fig. 6:	Accommodation coefficient measured on SRG2 by the different participants.....	18
Fig. 7:	Accommodation coefficient relative to the initial value for the SRG1 (blue) and SRG2 (red). The numbers on the horizontal axis denote the successive measurements performed by the pilot before, during and after the time of the comparison. The transfer standards have been circulated between measurement 2 and 3.....	19
Fig. 8:	Offset and expanded uncertainty of the measurements of NIM, relative to the weighted mean of the results of CCM.P-K15.1.....	31
Fig. 9:	Offset and expanded uncertainty of the measurements of METAS, relative to the weighted mean of the results of CCM.P-K15.1.....	31
Fig. 10:	Offset and expanded uncertainty relative to the reference value of CCM.P-K15 for the measurements of NIM.....	36
Fig. 11:	Offset and expanded uncertainty relative to the reference value of CCM.P-K15 for the measurements of METAS.	36

1 Introduction

A bilateral comparison between the NIM (P. R. China) and METAS (Switzerland) in the range 0.1 mPa to 1 Pa was agreed at the CCM low pressure working group (WG-LP) in Berlin in April 2011. The comparison is registered as comparison CCM.P-K15.1 by the BIPM and will be linked to the comparison CCM.P-K15.

METAS is the pilot of this comparison and made all the processing of the information as well as the redaction of the report. The results from NIM were submitted to METAS.

2 Participating laboratories.

METAS which previously took part as pilot to the CCM.P-K15, and NIM took part to this comparison.

The following table gives a short form of the situation of the participants at the time of the comparison. METAS as pilot collected the result from NIM and did the calculations.

Table 1: Characteristics of the definition of the pressure by the participants.

Laboratory	Standard	Definition	Traceability	Role	CMC
NIM	Static expansion system	Primary	Independent	participant	NO
METAS	Static expansion system	Primary	Independent	pilot	NO

2.1 NIM

2.1.1 Description of the reference standard

Our standard is a two-stage expansion system, which covers the measuring range from 1.0×10^{-4} Pa to 1.0×10^3 Pa with the corresponding uncertainty from 0.4 % to 0.06 % ($k=2$). Fig. 1 shows a diagram of the system. The two transfer standards, as well as our reference standard SRG which is used to monitor whether the expansion system is working properly, are fixed on the equator of the calibration chamber and are calibrated at the same time. An extractor gauge, which is used to measure the limit pressure of the vacuum system, is fixed on the top of the calibration chamber and is switched off during calibration. Fig. 2 shows a picture of the system.

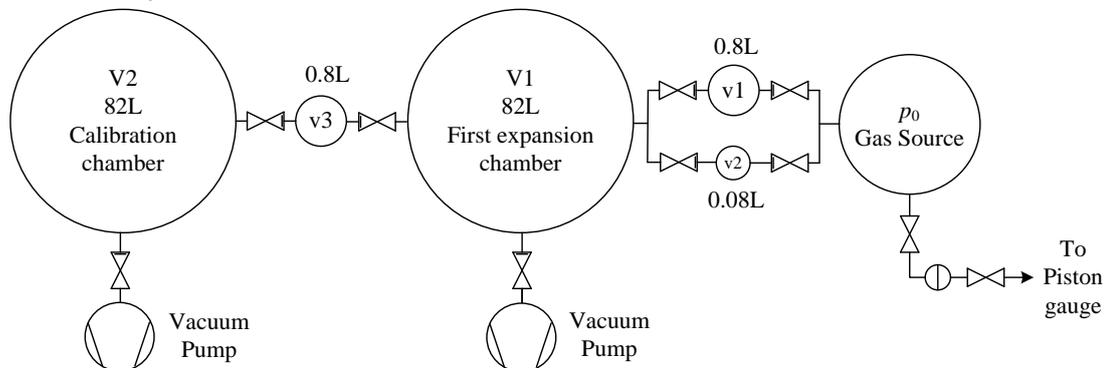


Fig. 1: Diagram of the expansion system



Fig. 2: Picture of the expansion system

The expansion ratio of the system for each stage is approximately 0.01, in order to obtain less expansion ratio, multiple expansion can be made in the first expansion chamber V_1 . In this comparison measurement, for the first three target points (on the order of 10^{-4} Pa) in each calibration cycle, two expansions are made successively in the chamber V_1 . Then, the mathematic model is:

$$p = p_0 f_1 \frac{T_1}{T_2} f_1 \frac{T_3}{T_4} f_2 \frac{T_5}{T_6} + p_b \quad (1)$$

For the six other points only one expansion is made in V_1 and the model is

$$p = p_0 f_1 \frac{T_1}{T_2} f_2 \frac{T_5}{T_6} + p_b \quad (2)$$

Where p_0 is the initial pressure, f_1 is the volume ratio of $\frac{v_1}{v_1 + V_1 + v_3}$, f_2 is the volume ratio of

$\frac{v_3}{v_3 + V_2}$, T_1 is the temperature of V_1 after the first expansion, T_2 is the temperature of v_1 before

the first expansion, T_3 is the temperature of V_1 after the second expansion, T_4 is the temperature of v_1 before the second expansion, T_5 is the temperature of V_2 after expansion, T_6 is the temperature of v_3 before expansion, p_b is the corrected residual pressure.

The initial pressure p_0 is measured by two capacitive diaphragm gauges (CDG) (1 Torr and 100 Torr) and a quartz resonator digital manometer. The initial pressures of all the target point except 1Pa are measured by CDGs, which are calibrated by piston gauge (model FRS5) immediately at the same pressure. Fig. 3 shows the basic principle of calibration method. CDG is mounted between the gas source and piston gauge respectively through two valves(K1 and K2). First, open K1 and close K2, then measure the pressure of gas source with CDG. Second, close K1 and open K2, then calibrate the CDG with piston gauge at the same pressure. Therefore, the initial pressures of the pressure lower than 1 Pa are meas-

ured with FRS5 piston gauge indirectly (the measuring upper limit is 11 kPa). The initial pressure of 1 Pa target point is approximately 12 kPa, it is measured by a quartz resonator digital manometer. The manometer is calibrated by a FPG8601 piston gauge (the measuring upper limit is 15 kPa.) right after the experiment.

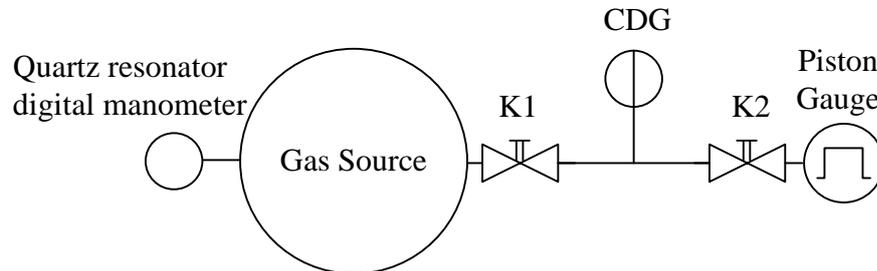


Fig. 3: Basic principle of the calibration method

2.1.2 Traceability of the system

Volume ratio f_1 was obtained by accumulation method in August 2011, and the relative standard uncertainty of f_1 was estimated to be 0.04%. The large volume in volume ratio f_2 , is measured by reference volume method, and the small volume of f_2 is measured by weighing method [1]. The uncertainty of corrected volume ratio (f_1 or f_2) is estimated to be 0.04%.

The temperature of T1~T6 are measured by eighteen PRTs(Pt100). In order to avoid correlation, temperature difference is actually used, i.e. the difference between the temperature of small volume before expansion and the temperature of large volume after expansion. In this measurement, the ultimate temperature difference between seven PRTs on the expansion chamber is (0.3~0.4) C, and the temperature difference between large volume and small volume is approximately (0.1~0.2) C. In addition, the mean temperature change of the large volume is below 0.05 C during each pressure point measurement.

Since the pressure will rise due to outgassing, for this reason, a slight correction is brought to the residual pressure during the SRG data acquisition. In this measurement, the pressure-rise rate of the expansion system is approximately 2.0×10^{-9} Pa/s, which was measured by a SRG. The corrected residual pressure is estimated according to the measurement time.

2.2 METAS

2.2.1 Reference system.

METAS realizes the pressure from 0.01 mPa to 100 Pa using a static expansion system with up to four expansion stages. Each stage has a typical expansion ratio of 100 except the last expansion stage that has two expand scheme with expansion ratio of either 50 or 200. The maximum pressure reduction ratio is then $2 \cdot 10^{-8}$. The initial pressure is regulated by a pressure generator PPC3 from DH-Instruments, the temperature of all the chambers is measured with an uncertainty of 0.1 K. The expansion process is automated and the closing time of the valves is optimized to avoid dynamic effects.

The expansion ratio of each stage has been characterized using the technique by addition as well as by depletion. [2].

The system is made of stainless steel chambers connected with conflate gaskets. The valves use polymer gaskets for the sealing with the chamber.

2.2.2 Measurements.

The measurements have been made on the two SRG used as transfer standards at the

same time. One of the SRG was connected to the MKS-SRG2 CE that was circulated and the other SRG was connected to an MKS-SRG2 that remained in METAS. On the second day of measurement the MKS-SRG2 units were swapped so that each SRG would be characterized using two different electronic units. The residual drag has been measured prior each measurement point. The value of the deceleration under pressure has been measured 5 times for pressure up to 30 mPa and 3 times for higher values. A linear regression is applied on the measurement points in order to compensate for possible outgassing at low pressure. The linear regression gives similar results to a mean value at high pressures. A spare SRG has been present on the last chamber during the time of the comparison to assess the stability of the system.

2.3 Uncertainty of the participants on the reference pressure

The uncertainties claimed by NIM and METAS are given in table 2.

In the case of METAS, the last column gives the uncorrelated part of the uncertainty which will be used in the determination of the uncertainty of the link. The uncorrelated part of the uncertainty comes from influence factors that are uncorrelated between CCM.P-K15 and CCM.P-K15.1 (temperature measurement, outgassing, starting pressure) while the correlated part of the uncertainty is given by the influence factors that remain similar in the two comparisons (error on the virial coefficient, ratio of the volume of the chambers).

Table 2: Relative uncertainty on the definition of the reference pressure by the participants. The uncorrelated relative uncertainty of METAS is also given as it is used in the link with CCM.P-K15.1.

	NIM	METAS	METAS
Pressure	relative uncertainty	relative uncertainty	uncorrelated relative uncertainty
Pa			
1.0×10^{-4}	0.0017	0.0100	0.0095
3.0×10^{-4}	0.0015	0.0050	0.0040
9.0×10^{-4}	0.0015	0.0050	0.0028
3.0×10^{-3}	0.0010	0.0030	0.0022
9.0×10^{-3}	0.0009	0.0030	0.0021
3.0×10^{-2}	0.0012	0.0030	0.0017
9.0×10^{-2}	0.0009	0.0030	0.0014
3.0×10^{-1}	0.0009	0.0020	0.0015
1.0	0.0010	0.0020	0.0014

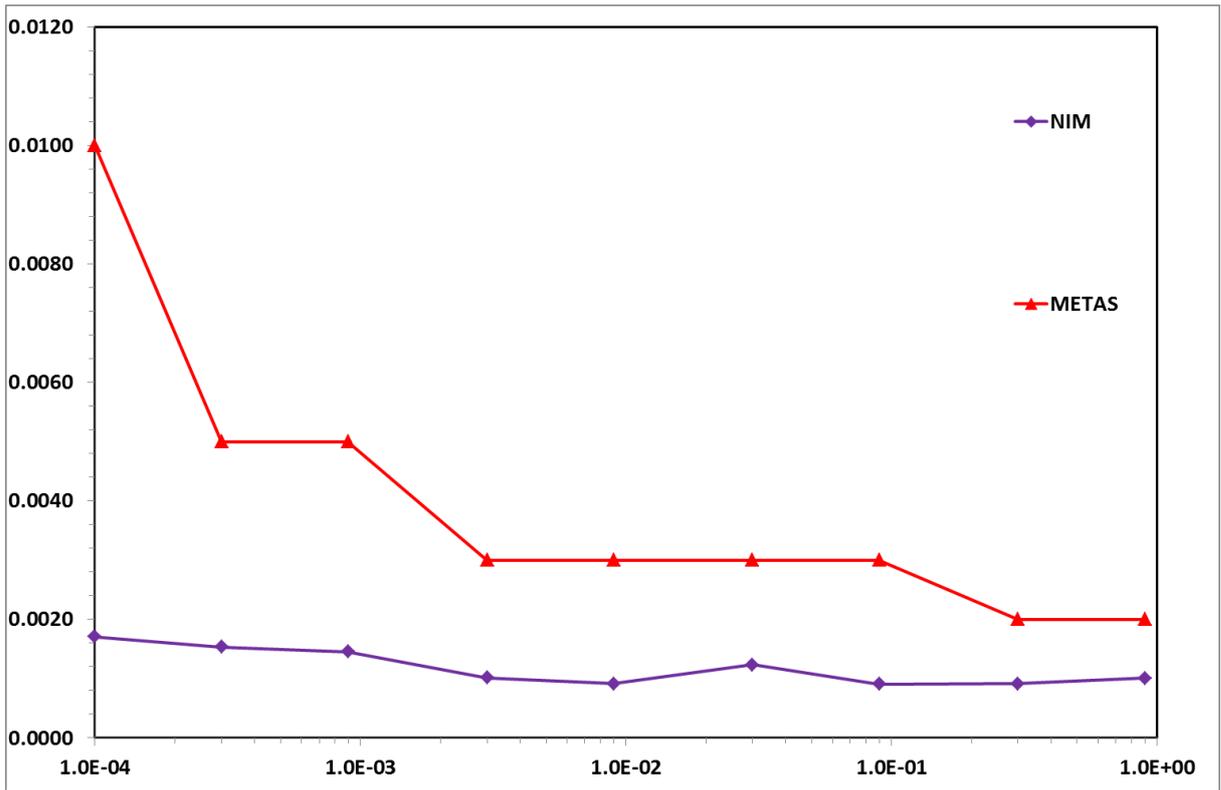


Fig. 4: Relative uncertainty on the definition of the reference pressure by the participants plotted versus the pressure in Pa.

3 Transfer standard

The transfer standards consist of a pair of SRG kept under vacuum using a Varian all metal valve. The specifications of the transfer standard are listed in the table 3. Some of the characteristics have not been measured but will be used as conventional values in order to determine the accommodation coefficient.

Table 3: Characteristics of the transfer standard.

Transfer Standard	SRG1	SRG2
Metas Number	006411	006412
SRG Part Number (MKS)	MKS SRG-BF	MKS SRG-BF
SRG Serial Number	G191872	G191943
Valve part number	Varian 9515027	Varian 9515027
Valve Serial Number	LVB90399	LVL70128
Dead volume, valve open	120 cm ³ (u=1 cm ³)	120 cm ³ (u=1 cm ³)
Ball diameter (nominal)	4.5 mm	4.5 mm
Ball density (nominal)	7700 kg/m ³	7700 kg/m ³
Rotation frequency	405 – 415 Hz	405 – 415 Hz

A stainless steel spring is mounted on the plate of the valve and is immobilising the ball once the valve is closed. In the open position, the spring is far enough from the ball and the measurement of the residual drag of the ball has shown no spurious drag due to an electromagnetic coupling between the ball and the spring via the magnetic field of the ball.

An electronic readout unit has been circulated in conjunction with the transfer standard. The participating laboratory had the choice either to use the readout unit provided or their own unit. The characteristics of the readout unit are as follow:

Part Number (MKS): SRG-2CE

Metas number: 005555

Serial number 500163G

The rotation frequency of the ball has been restricted to the range 405 Hz – 415 Hz. Preliminary measurements have shown that the frequency of rotation has a negligible effect on the residual drag measurement; however this range of frequency is covered by older electronic readout equipment.

A dynamometric wrench was also circulated and had to be used to close the all metal valve with the specified torque. This point has no influence on the metrological characteristics of the SRG but ensure a high number of cycles for the valve with a reproducible tightness.

4 Method used for the measurements

4.1 Correction of the residual drag

Two techniques have been proposed for the determination of the residual drag of the SRG used for the correction of the deceleration measured under vacuum. A first technique is the measurement of the residual drag prior to the series of measurement at the different pressure steps. The measurement has to be repeated for several values of rotation speed to determine the correction to apply due to the rotation speed. A second technique is the measurement of the residual drag before each measurement point. This second technique is well adapted to the measurements in a static expansion system as the SRG is under vacuum before each measurement.

4.2 Points of measurement.

The measurements have been performed at 9 pressure steps obtained using nitrogen.

The temperature of the system had to be monitored and to stay within 20 °C and 24 °C.

The deviation of the effective pressure from the nominal pressure had to be less than 10% from the nominal value for points lower than 4.0×10^{-2} Pa and less than 5% of the nominal value at higher pressure.

Each laboratory had to repeat each measurement point at least three times in each calibration sequence. The calibration sequence will be repeated at least twice making a total of at least 54 measurement points.

Depending on the pressure step, the duration of the measurement and the number of measurements recorded for the calculation of the average value was selected as shown in table 4.

Table 4: Measurement method used for each pressure step.

Number of pressure step (index <i>j</i>)	Pressure	Time (sec)	Measurements	Repetitions
1	$1.0 \cdot 10^{-4}$ Pa	30	10	3
2	$3.0 \cdot 10^{-4}$ Pa	30	10	3
3	$9.0 \cdot 10^{-4}$ Pa	30	10	3
4	$3.0 \cdot 10^{-3}$ Pa	30	10	3
5	$9.0 \cdot 10^{-3}$ Pa	30	5	3
6	$3.0 \cdot 10^{-2}$ Pa	30	5	3
7	$9.0 \cdot 10^{-2}$ Pa	30	5	3
8	0.3 Pa	10	5	3
9	1.0 Pa	10	5	3

4.3 Circulation of the transfer standard

The circulation of the transfer standard was made between METAS and NIM with the shipment to NIM beginning of March 2012 and the shipment back to METAS en of April 2012.

Table 5: Schedule of the circulation of the transfer standard.

Date	Laboratory	Index <i>l</i>
2012.01	METAS	1
2012.03	NIM	2
2012.07	METAS	3

4.4 Collection of the results

The results of NIM have been communicated directly to METAS after completion of the measurements. The electronic worksheet used for the collection of the results is similar to the one used previously in CCM.M.P-K15.

The participants had to provide the effective value of the pressure generated the deceleration of the SRG while exposed to the pressure and under vacuum, the uncertainty on the deceleration under vacuum (residual drag), the frequency of rotation, the temperature and the uncertainty on the temperature.

5 Results provided by the participants.

The results of NIM and METAS are presented in table 6 and 7. The accommodation coefficient versus pressure is displayed in Fig 5 and Fig 6 for respectively the SRG 1 and 2 in circulation. The value is the mean of the measurements made by the participants and the uncertainty is given by equation 29 for all participants.

The viscous effect above 0.03 Pa is responsible for a decrease of the value of the accommodation coefficient at pressure higher than 0.03 Pa.

Table 6: Accommodation coefficient measured by the laboratories for the transfer standard SRG1 and uncertainties associated calculated using the Eq. 26.

P_j (Pa)		METAS0	NIM	METAS1
1.0×10^{-4}	σ_{21}	1.0304	1.0347	1.0381
	$u(\sigma_{21})$	0.0155	0.0070	0.0155
3.0×10^{-4}	σ_{22}	1.0304	1.0347	1.0381
	$u(\sigma_{22})$	0.0062	0.0034	0.0062
9.0×10^{-4}	σ_{23}	1.0304	1.0353	1.0381
	$u(\sigma_{23})$	0.0054	0.0020	0.0054
3.0×10^{-3}	σ_{24}	1.0304	1.0350	1.0381
	$u(\sigma_{24})$	0.0033	0.0011	0.0033
9.0×10^{-3}	σ_{25}	1.0304	1.0347	1.0381
	$u(\sigma_{25})$	0.0033	0.0011	0.0033
3.0×10^{-2}	σ_{26}	1.0304	1.0348	1.0381
	$u(\sigma_{26})$	0.0033	0.0014	0.0033
9.0×10^{-2}	σ_{27}	1.0289	1.0331	1.0365
	$u(\sigma_{27})$	0.0033	0.0010	0.0033
3.0×10^{-1}	σ_{28}	1.0266	1.0293	1.0344
	$u(\sigma_{28})$	0.0048	0.0010	0.0048
1.0	σ_{29}	1.0123	1.0178	1.0202
	$u(\sigma_{29})$	0.0025	0.0010	0.0025

Table 7: Accommodation coefficient measured by the laboratories for the transfer standard SRG2 and uncertainties associated calculated using the Eq. 26.

P_j (Pa)		METAS0	NIM	METAS1
1.0×10^{-4}	σ_{11}	0.9799	0.9783	0.9784
	$u(\sigma_{11})$	0.0140	0.0056	0.0140
3.0×10^{-4}	σ_{12}	0.9799	0.9807	0.9784
	$u(\sigma_{12})$	0.0054	0.0035	0.0054
9.0×10^{-4}	σ_{13}	0.9799	0.9808	0.9784
	$u(\sigma_{13})$	0.0050	0.0020	0.0050
3.0×10^{-3}	σ_{14}	0.9799	0.9807	0.9784
	$u(\sigma_{14})$	0.0031	0.0011	0.0031
9.0×10^{-3}	σ_{15}	0.9799	0.9805	0.9784
	$u(\sigma_{15})$	0.0030	0.0009	0.0030
3.0×10^{-2}	σ_{16}	0.9799	0.9805	0.9784
	$u(\sigma_{16})$	0.0030	0.0014	0.0029
9.0×10^{-2}	σ_{17}	0.9785	0.9790	0.9772
	$u(\sigma_{17})$	0.0030	0.0009	0.0030
3.0×10^{-1}	σ_{18}	0.9766	0.9755	0.9754
	$u(\sigma_{18})$	0.0020	0.0009	0.0020
1.0	σ_{19}	0.9639	0.9655	0.9629
	$u(\sigma_{19})$	0.0018	0.0009	0.0018

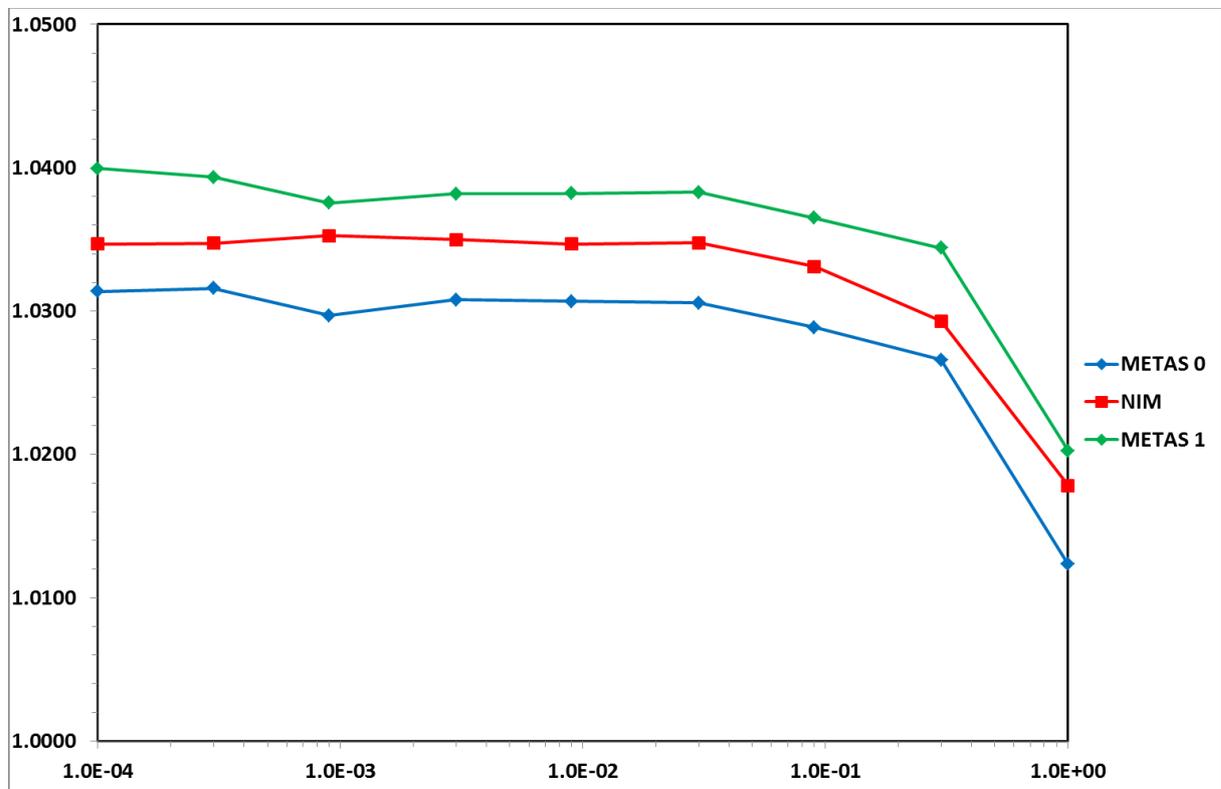


Fig. 5: Accommodation coefficient measured on the SRG1 by the different participants.

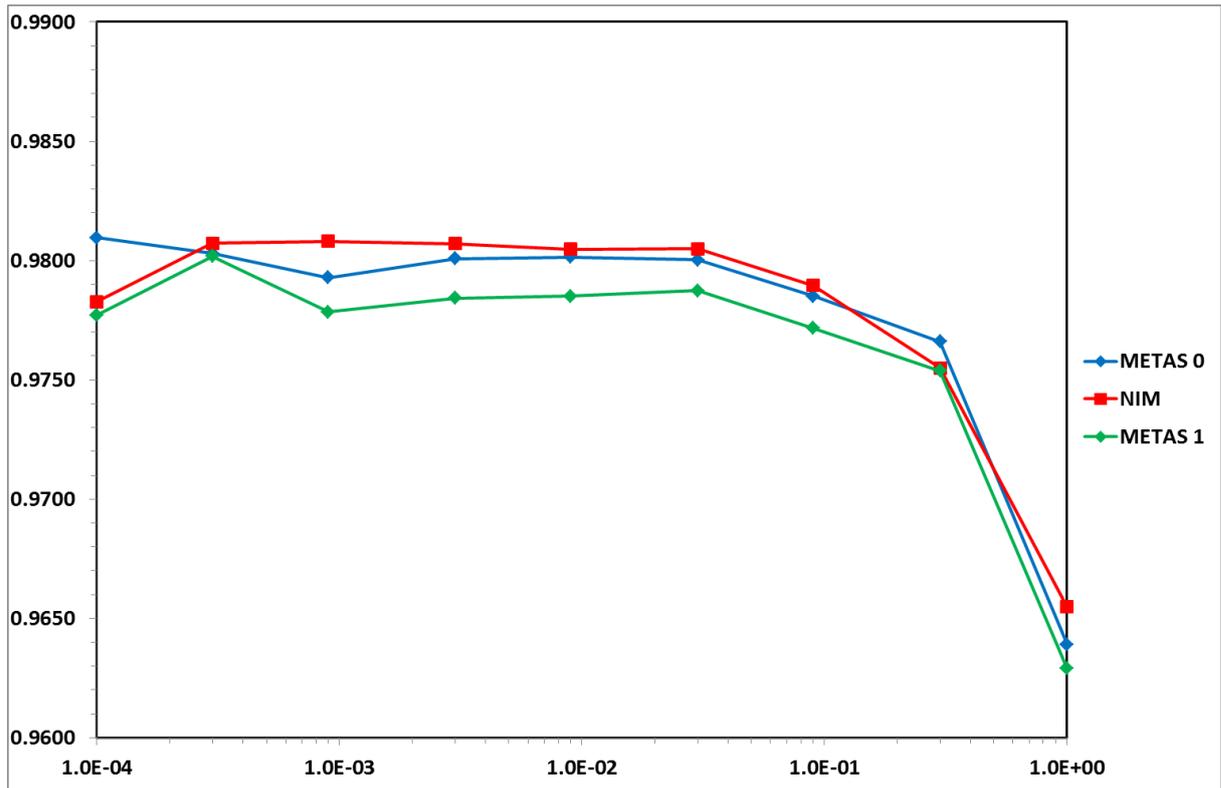


Fig. 6: Accommodation coefficient measured on SRG2 by the different participants.

5.1 Measurements of the pilot laboratory and stability.

The transfer standard has been measured three times by the pilot laboratory before the travel to China and two times after its return.

The two transfer standards have shown good stability over the five sets of measurement made up to five months before and after the circulation. The SRG 1 had a change of 0.5% after the circulation and it is important to mention that this increase of accommodation factor has been measured by NIM as mentioned under point 2.1.3. The SRG 2 has exhibited excellent stability leading to a great confidence in the results.

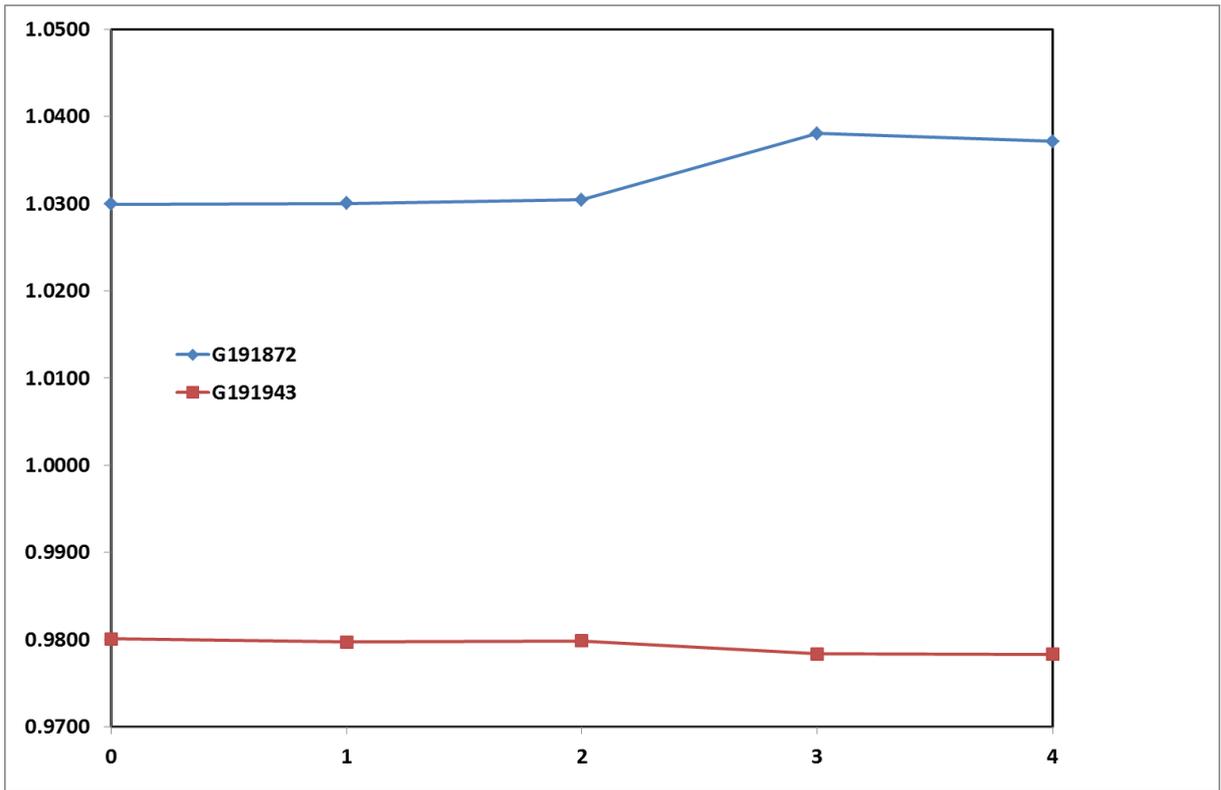


Fig. 7: Accommodation coefficient relative to the initial value for the SRG1 (blue) and SRG2 (red). The numbers on the horizontal axis denote the successive measurements performed by the pilot before, during and after the time of the comparison. The transfer standards have been circulated between measurement 2 and 3.

6 Method used for the calculation of the reference value.

The mesurand of the comparison is the accommodation factor of two SRG sensor determined at different values of pressure.

The values of the accommodation factor, for each participating laboratory, are given by:

$$\sigma_{ijkl}$$

Where

i is the number of the spinning rotating gage

j is the number of the nominal pressure step as defined in table 4

k is the number of the measurement in the series for a specific nominal value

l is the number of the measurement within the comparison, as defined in table 5, and is also used to designate the NMI that did the measurement.

6.1 Influence of the reference pressure in the transition regime.

Since the value of accommodation coefficient is dependent of the pressure in the transition regime, the measured accommodation factors have to be corrected for the nominal value of pressure for measurement at nominal pressure $9.0 \cdot 10^{-2}$ Pa and above. A linear regression is then made on the measurement and the slope is used to correct the effective measurement to what would have been obtained at the nominal pressure.

$$\sigma_{ijk} = \hat{\sigma}_{ijk} + \left(\tilde{P}_j - \hat{p}_{ijk} \right) m_i \quad (3)$$

Where:

\hat{p}_{ijk} is the effective pressure

\tilde{P}_j is the nominal pressure j

$\hat{\sigma}_{ijk}$ is the accommodation factor measured at the effective pressure \hat{p}_{ijk}

m_i is the slope of the linear regression over the set of measurement $[\hat{p}_{ijk}, \hat{\sigma}_{ijk}]$ for $j=1$ to 6 and k for the values where $p_k > 3.0 \times 10^{-2}$ Pa

The contribution to the uncertainty introduced by this correction will be neglected in the rest of the discussion as the \hat{p}_{ijk} are very closed from their respective p_k .

6.2 Correction of the drift of the transfer standard

As the transfer standard was not perfectly stable over the entire time of the comparison, it is necessary to make a correction of the drift by using the measurements made by the pilot laboratory at the beginning and at the end of the circulation.

Since the value of the accommodation coefficient is not a function of the pressure in the molecular flow regime, it was decided to calculate an average value $\bar{\sigma}_{ijl}(METAS)$ for all measurements made at pressure values $3.0 \cdot 10^{-2}$ Pa or lower. This average value is calculated for measurements performed at nominal pressures between 9.0×10^{-4} Pa and 3.0×10^{-2} Pa

$$\bar{\sigma}_{ijl}(METAS) = \frac{1}{24} \sum_{k=1}^{k=6} \sum_{j=3}^{j=6} \sigma_{ijkl}(METAS) \quad j=1..6, l=1, 3 \quad (4)$$

The value of the accommodation factor at pressure above 3.0×10^{-2} Pa is calculated for each pressure point.

$$\bar{\sigma}_{ijl}(METAS) = \frac{1}{6} \sum_{k=1}^{k=6} \sigma_{ijkl}(METAS) \quad j=7..9, l=1, 3 \quad (5)$$

The value of the accommodation value given by the pilot laboratory has been determined by a weighted mean of the values at the beginning and at the end of the loop:

$$\bar{\sigma}_{ijl}(METAS) = \frac{1}{2} (\bar{\sigma}_{ij(l-1)}(METAS) + \bar{\sigma}_{ij(l+1)}(METAS)) \quad l=2 \quad (6)$$

This way of calculating the reference value is optimal to correct a linear drift of the transfer standard during the loop.

6.3 Pressure value for a participating laboratories

The pressure realized in the participating laboratories has been calculated based on the average response of the sensor measured before and after the participating NMI by the pilot laboratory. In a first step, the average response of the SRG is calculated:

$$\sigma_{ijl} = \frac{1}{6} \sum_{k=1}^{k=6} \sigma_{ijkl} \quad l=2 \quad (7)$$

Then the measured pressure is:

$$p_{ij} = \tilde{P}_j \frac{\sigma_{ijl}}{\bar{\sigma}_{ijl}(METAS)} \quad l=2 \quad (8)$$

6.4 Reference value for a participating laboratory

The reference value for a participating laboratory is the weighted mean value of the reference value for each SRG. It is weighted by the combination of the type A uncertainties. This way of doing has the advantage that if an SRG had a large drift and the other was stable during the transport, the result of the participating laboratory is less affected by the unstable SRG.

$$p_j = \frac{\sum_{i=1}^{i=2} \frac{p_{ij}}{u_A^2(p_{ij})}}{\sum_{i=1}^{i=2} \frac{1}{u_A^2(p_{ij})}} \quad (9)$$

6.5 Pressure value for the pilot laboratory

The pressure determined by the pilot laboratory is taken as an average of the measurement at the beginning and at the end of each loop.

$$\sigma_{ij}(METAS, l, l+2) = \frac{1}{12} \sum_{k=1}^{k=6} \sigma_{ijkl}(METAS) + \sigma_{ijkl+2}(METAS) \quad l=1 \quad (10)$$

Then the pressure measured by the pilot laboratory for each loop of the comparison is given by:

$$p_{ij}(METAS, l, l+2) = P_j \frac{\sigma_{ij}(METAS, l, l+2)}{\sigma_{ij}(METAS, l, l+2)} \quad l=1 \quad (11)$$

It should be mentioned that for nominal pressure above 3.0×10^{-2} Pa the value of the pressure of the pilot laboratory is always the nominal pressure.

Finally the pressure value of the pilot laboratory is given by the weighted mean value of the reference pressure for all the loops. The weight coefficient is the combination of all the type A uncertainties.

$$p_{ij}(METAS) = \sum_{l=1} \frac{p_{ij}(METAS, l, l+2)}{u_A^2(p_{ij}(METAS, l, l+2))} \quad (12)$$

6.6 Reference value for the pilot laboratory

The reference value of the pilot laboratory is obtained like for the participating NMI's, through a weighted mean value in which the weight coefficient is the combination of the type A uncertainties:

$$p_j(METAS) = \frac{\sum_{i=1}^{i=2} \frac{p_{ij}(METAS)}{u_A^2(p_{ij}(METAS))}}{\sum_{i=1}^{i=2} \frac{1}{u_A^2(p_{ij}(METAS))}} \quad (13)$$

6.7 Reference value of the comparison

The reference value of the comparison is obtained as a weighted mean value on all the participants that have a primary definition of the pressure.

$$p_j(CCM.P - K15.1) = \frac{\sum_l \frac{p(l)}{u^2(p(l))}}{\sum_l \frac{1}{u^2(p(l))}} \quad (14)$$

Where l designate the NMI's selected to provide the reference value according to the number of the petal where the NMI did its measurement.

6.8 Normalization of the reference value

The reference value given in equation 14 is slightly biased because in equation 7 we take only the accommodation coefficient defined by the pilot. It is allowed to multiply by the same ratio all the reference pressure for all the participants without affecting the uncertainty of the comparison. This way it is possible to have as reference pressure of the comparison the nominal pressure. The coefficient of normalization is given by the weighted mean value among the laboratories who take part to the definition of the reference value.

$$P_{jc} = \frac{P_j}{p_j(CCM.P - K15.1)} \quad (15)$$

And this way, the new reference value is equivalent to the nominal value:

$$p_j(\text{CCM.P-K15.1}) \equiv P_j \quad (16)$$

6.9 Relative deviation to the reference value.

Due to the large span of value of pressure in this comparison, it is more convenient to express the deviation relative to the nominal value rather than in absolute number. This deviation is given by the following expression:

$$d_j = \frac{p_{jc}}{P_j} - 1 \quad (17)$$

6.10 Degree of equivalence

Finally the degree of equivalence gives the ratio between the deviation and the uncertainty of the deviation. The degree of equivalence is given by:

$$E_j(l) = \frac{p_j(l)}{U(d_j(l))} \quad (18)$$

7 Method used for the determination of the uncertainty.

7.1 Uncertainty on sigma measured by the participants

In the following discussion the measured quantity has, in some places, been replaced by the presumed value of this quantity (for example the effective pressure seen by the sensor has been replaced by the nominal value of the pressure). This is for the simplification of the calculation and has only a negligible effect on the uncertainty calculation as both values are very close.

7.1.1 Equation of the SRG

The value of sigma is determined by using the relation between the deceleration and the pressure including the influence factors (temperature, residual drag)

$$\sigma_{ijk} = (DCR_{ijk} - RD_{ijk}(\omega)) \frac{\pi a \rho}{10 P_{ijk}} \sqrt{\frac{2RT_{ijk}}{\pi m}} \quad (19)$$

For clarity we will rewrite it by putting all the constants together:

$$\sigma_{ijk} = (DCR_{ijk} - RD_{ijk}(\omega)) \frac{K}{P_{ijk}} \sqrt{T_{ijk}} \quad (20)$$

And the constant K is then given by:

$$K = \frac{\pi a \rho}{10} \sqrt{\frac{2R}{\pi m}} \quad (21)$$

The constant is the same for both SRGs as we have determined it based on the nominal value of mass, density and diameter. This way of doing has no influence on the uncertainty or final value as explained under 7.1.2.

The uncertainty of the sigma measured by the participants is estimated by taking into account the uncertainty of the generated pressure as provided by the participants, the uncertainty on the residual drag, the uncertainty on the temperature of the SRG as well as the standard deviation of the set of sigma measured.

7.1.2 Uncertainty on K

The uncertainty on the constant K has an influence on the calculation of the accommodation factor sigma. The factor K is however used by all the participants and this way this uncertainty is correlated over all the participants. It is then cancelled in the calculation of the pressure measured by the participants.

7.1.3 Uncertainty of the reference pressure

The uncertainty on the generated pressure has been provided by the participants for each measurement point. As the value of the relative uncertainty is not sensitive to small changes of pressure and as the values provided by the participants are similar from one cycle of measurement to another, only one value of uncertainty for each target pressure needs to be calculated. This uncertainty is a type B uncertainty and the sensitivity coefficient evaluated at the nominal pressure P_j is given by:

$$\frac{\partial \sigma_{ij}}{\partial P_j} = (DCR_{ij} - RD_{ij}(\omega)) \frac{-K}{P_j^2} \sqrt{T_j} \quad (22)$$

7.1.4 Contribution due to the uncertainty on the temperature

The collision rate of gas on the rotor depends on the temperature, therefore the SRG temperature uncertainty needs to be included in the combined uncertainty of the accommodation factor. The sensitivity coefficient of the temperature is given by:

$$\frac{\partial \sigma_{ij}}{\partial T_j} = (DCR_{ij} - RD_{ij}(\omega)) \frac{K}{2P_j} \sqrt{\frac{1}{T_j}} \quad (23)$$

7.1.5 Contribution due to the uncertainty on the residual deceleration

The residual deceleration at zero pressure is determined and used to correct the deceleration measured when the SRG is exposed to the gas. The sensitivity coefficient of the residual drag is given by:

$$\frac{\partial \sigma_{ij}}{\partial RD_i(\omega)} = -\frac{K}{P_j} \sqrt{T_j} \quad (24)$$

7.1.6 Type A uncertainty for σ

The standard deviation of a set of measured accommodation coefficients for a given SRG and a given nominal pressure is a type A uncertainty. It is generated by the repeatability of the measurement of the DCR due to non-systematic errors. This standard deviation has been corrected as explained by Kacker and Jones [3] to obtain the contribution to the uncertainty of sigma:

$$u(\text{repeatability}) = \sqrt{\frac{n-1}{n-3}} s(\sigma_{ij}) \quad (25)$$

Where:

n : is the number of measurements

s : is the standard deviation

Finally the uncertainty on the accommodation factor is given by:

$$u^2(\sigma_{ij}) = \frac{n-1}{n-3} s^2(\sigma_{ij}) + \left(\frac{\partial \sigma_{ij}}{\partial P_j} \right)^2 u^2(P_j) + \left(\frac{\partial \sigma_{ij}}{\partial T_j} \right)^2 u^2(T_j) + \left(\frac{\partial \sigma_{ij}}{\partial RD_i(\omega)} \right)^2 u^2(RD_i(\omega)) \quad (26)$$

It is useful for the calculation of the weighted mean value to determine the combination of the uncorrelated uncertainty contributions (mostly type A) to the accommodation coefficient.

$$u_A^2(\sigma_{ij}) = \frac{n-1}{n-3} s^2(\sigma_{ij}) + \left(\frac{\partial \sigma_{ij}}{\partial T_j} \right)^2 u^2(T_j) + \left(\frac{\partial \sigma_{ij}}{\partial RD_i(\omega)} \right)^2 u^2(RD_i(\omega)) \quad (27)$$

7.2 Uncertainty on the value of sigma used to correct the drift.

The uncertainty on the reference value of sigma used to compensate the drift of the SRG

(the $\bar{\sigma}_{ijl}(METAS)$ as defined in equation 4 or 5) is given by the stability of the transfer standard. The uncertainty due to the stability of the SRG is defined the following way:

$$u(\bar{\sigma}_{ijl}(METAS)) = 0.5 \left| \bar{\sigma}_{ijl}(METAS) - \bar{\sigma}_{ij(l+2)}(METAS) \right| \quad l=1 \quad (28)$$

The minimal value for the uncertainty has been set to 0.0015 as a same value before and after a given NMI could also involve some canceling of the drift.

7.3 Uncertainty on sigma measured by the pilot

The uncertainty of the accommodation coefficient determined by the pilot laboratory is slightly different from the uncertainty on the coefficient of the participating NMI's as the value of the pilot is an average value of two measurements. We make the assumption that the value of the uncertainty is the same for each cycle of measurement of the accommodation factor. The terms that correspond to type B uncertainty are unchanged while the terms of type A are slightly reduced due to the larger number of measurement:

$$u^2(\sigma_{ij}(METAS, l, l+2)) = \left(\frac{\partial \sigma}{\partial P_j} \right)^2 u^2(P_j) + \frac{1}{4} \sum_{L=l, l+2} \left(\frac{n-1}{n-3} s^2(\sigma_{ijL}) + \left(\frac{\partial \sigma}{\partial T_j} \right)^2 u^2(T_j) + \left(\frac{\partial \sigma}{\partial RD_i(\omega)} \right)^2 u^2(RD_{iL}(\omega)) \right) \quad l=1 \quad (29)$$

The combination of the uncorrelated uncertainties is given by:

$$u_A^2(\sigma_{ij}(METAS, l, l+2)) = \frac{1}{4} \sum_{L=l, l+2} \left(\frac{n-1}{n-3} s^2(\sigma_{ijL}) + \left(\frac{\partial \sigma}{\partial T_j} \right)^2 u^2(T_j) + \left(\frac{\partial \sigma}{\partial RD_i(\omega)} \right)^2 u^2(RD_{iL}(\omega)) \right) \quad l=1 \quad (30)$$

7.4 Uncertainty on the reduced pressure for a participant

The uncertainty on the reduced pressure can be easily treated as an incoherent addition of the relative uncertainty on the accommodation factor measured by the participating NMI and given by the pilot laboratory:

$$u(p_{ijl}) = \tilde{P}_j \sqrt{\left(\frac{u(\sigma_{ijl})}{\sigma_{ijl}} \right)^2 + \left(\frac{u(\bar{\sigma}_{ijl}(METAS))}{\bar{\sigma}_{ijl}(METAS)} \right)^2} \quad l=1 \quad (31)$$

Once again, the calculation of the combination of the uncorrelated uncertainties used for the weighted mean is given by:

$$u_A(p_{ijl}) = \tilde{P}_j \sqrt{\left(\frac{u_A(\sigma_{ijl})}{\sigma_{ijl}} \right)^2 + \left(\frac{u(\bar{\sigma}_{ijl}(METAS))}{\bar{\sigma}_{ijl}(METAS)} \right)^2} \quad l=1 \quad (32)$$

7.5 Uncertainty on the reduced pressure for the pilot, for one loop

The calculation of the uncertainty of the pilot on one loop is similar to the calculation for a participant; the only difference is the definition of the accommodation factor determined by the pilot which is the average value of two measurements.

$$u(p_{ij}(METAS, l, l+2)) = \tilde{P}_j \sqrt{\left(\frac{u(\sigma_{ij}(METAS, l, l+2))}{\sigma_{ij}(METAS, l, l+2)}\right)^2 + \left(\frac{u(\bar{\sigma}_{ij}(METAS, l, l+2))}{\sigma_{ij}(METAS, l, l+2)}\right)^2} \quad (33)$$

The combination of the type A uncertainties is given by:

$$u_A(p_{ij}(METAS, l, l+2)) = \tilde{P}_j \sqrt{\left(\frac{u_A(\sigma_{ij}(METAS, l, l+2))}{\sigma_{ij}(METAS, l, l+2)}\right)^2 + \left(\frac{u_A(\bar{\sigma}_{ij}(METAS, l, l+2))}{\sigma_{ij}(METAS, l, l+2)}\right)^2} \quad (34)$$

Finally, the uncertainty on the reference value of pressure, for a given SRG, obtained by the weighted mean value of the measurements of the 3 loops

$$u^2(p_{ij}(METAS)) = u^2(P_j) + u_A^2(p_{ij}(METAS, l, l+2)) \quad (35)$$

Where the combination of type A uncertainties is:

$$u_A^2(p_{ij}(METAS)) = u_A^2(p_{ij}(METAS, l, l+2)) \quad (36)$$

7.6 Uncertainty on the reference value of a participant

The uncertainty of the reference pressure of a given NMI for a given step of the comparison is given by:

$$u^2(p_j) = u^2(P_j) + \frac{1}{\sum_{i=1}^{i=2} \frac{1}{u_A^2(p_{ij})}} \quad (37)$$

7.7 Uncertainty on the reference value of the pilot

The uncertainty on the weighted mean value of the reference pressure obtained with the two SRG is given by:

$$u^2(p_j(METAS)) = u^2(P_j) + \frac{1}{\sum_{i=1}^{i=2} \frac{1}{u_A^2(p_{ij}(METAS))}} \quad (38)$$

7.8 Uncertainty on the reference value of the comparison

The uncertainty of the reference value of the comparison obtained with Eq.14 is given by Cox [4]:

$$u^2(p_j(CCM.P - K15.1)) = \frac{1}{\sum_l \frac{1}{u^2(p(l))}} \quad (39)$$

The uncertainty on the normalized reference values given by Eq. 16 is similar to the value obtained with Eq. 14 as the coefficient given by Eq. 15 is close to 1.

7.9 Uncertainty on the relative deviation

The uncertainty on the relative deviation calculated by Eq. 17 is given by Cox [4] and is as

follow for the laboratories participating to the definition of the reference value:

$$U(d_j(l)) = 2 \sqrt{\left(\frac{u(p_{jc})}{p_{jc}} \right)^2 - \left(\frac{u(p_j(CCM.P - K15.1))}{p_j(CCM.P - K15.1)} \right)^2} \quad (40)$$

8 Reduction to a weighted mean

The determination of a weighted mean has been made on the base of all the laboratories who took part to the comparison.

The consistency check defined by Cox [4] using the chi squared function has been applied to the two laboratories contributing to the reference value and it has been fulfilled with success.

The normalised reference pressure as given by Eq. 16 for each participant and the associated uncertainties as given by Eq. 37 and 38 are presented on table 8.

Table 8: Normalized value of the participants as given by Eq. 16 and the associated uncertainty given by Eq. 37 and 38. The first column gives the normalized uncertainty of the comparison which by definition is equivalent to the nominal pressure and the associated uncertainty given by Eq. 39.

P_j (Pa)		Reference	NIM	METAS
1.0×10^{-4}	p_1	1.0000×10^{-4}	9.9983×10^{-5}	1.0009×10^{-4}
	$u(p_1)$	4.46×10^{-7}	4.84×10^{-7}	1.15×10^{-6}
3.0×10^{-4}	p_2	3.0000×10^{-4}	3.0001×10^{-4}	2.9999×10^{-4}
	$u(p_2)$	8.32×10^{-7}	9.69×10^{-7}	1.62×10^{-6}
9.0×10^{-4}	p_3	9.0000×10^{-4}	9.0032×10^{-4}	8.9841×10^{-4}
	$u(p_3)$	1.93×10^{-6}	2.12×10^{-6}	4.70×10^{-6}
3.0×10^{-3}	p_4	3.0000×10^{-3}	3.0009×10^{-3}	2.9969×10^{-3}
	$u(p_4)$	4.71×10^{-6}	5.32×10^{-6}	1.01×10^{-5}
9.0×10^{-3}	p_5	9.0000×10^{-3}	9.0019×10^{-3}	8.9926×10^{-3}
	$u(p_5)$	1.36×10^{-5}	1.53×10^{-5}	3.00×10^{-5}
3.0×10^{-2}	p_6	3.0000×10^{-2}	3.0008×10^{-2}	2.9978×10^{-2}
	$u(p_6)$	5.08×10^{-5}	5.90×10^{-5}	9.97×10^{-5}
9.0×10^{-2}	p_7	9.0000×10^{-2}	9.0019×10^{-2}	8.9926×10^{-2}
	$u(p_7)$	1.36×10^{-4}	1.53×10^{-4}	2.99×10^{-4}
3.0×10^{-1}	p_8	3.0000×10^{-1}	2.9994×10^{-1}	3.0012×10^{-1}
	$u(p_8)$	4.23×10^{-4}	5.13×10^{-4}	7.48×10^{-4}
1.0	p_9	1.0000	1.0001	0.9998
	$u(p_9)$	1.38×10^{-3}	1.72×10^{-3}	2.33×10^{-3}

Table 9: The relative difference, respective to the reference value of the comparison, as defined by Eq. 17, and relative uncertainty as defined by Eq. 40. The last line is the mean value of the relative deviation expressed in absolute number and depicts some kind of average agreement of the participant with the reference value.

P_j (Pa)		NIM	METAS
1.0×10^{-4}	d_1	-0.0002	0.0009
	$u(d_1)$	0.0038	0.0211
3.0×10^{-4}	d_2	0.0000	0.0000
	$u(d_2)$	0.0033	0.0092
9.0×10^{-4}	d_3	0.0004	-0.0018
	$u(d_3)$	0.0019	0.0095
3.0×10^{-3}	d_4	0.0003	-0.0010
	$u(d_4)$	0.0016	0.0060
9.0×10^{-3}	d_5	0.0002	-0.0008
	$u(d_5)$	0.0015	0.0059
3.0×10^{-2}	d_6	0.0003	-0.0007
	$u(d_6)$	0.0020	0.0057
9.0×10^{-2}	d_7	0.0002	-0.0008
	$u(d_7)$	0.0015	0.0059
3.0×10^{-1}	d_8	-0.0002	0.0004
	$u(d_8)$	0.0019	0.0041
1.0	d_9	0.0001	-0.0002
	$u(d_9)$	0.0020	0.0037
$\Sigma(d_i)/N$		0.0002	0.0008

8.1 Difference and uncertainty respective to the weighted mean.

The relative differences to the weighted mean as given by Eq. 17 and the associated uncertainty are summarised in table 9. These results expressed as plots are displayed in Fig 8 and Fig. 9.

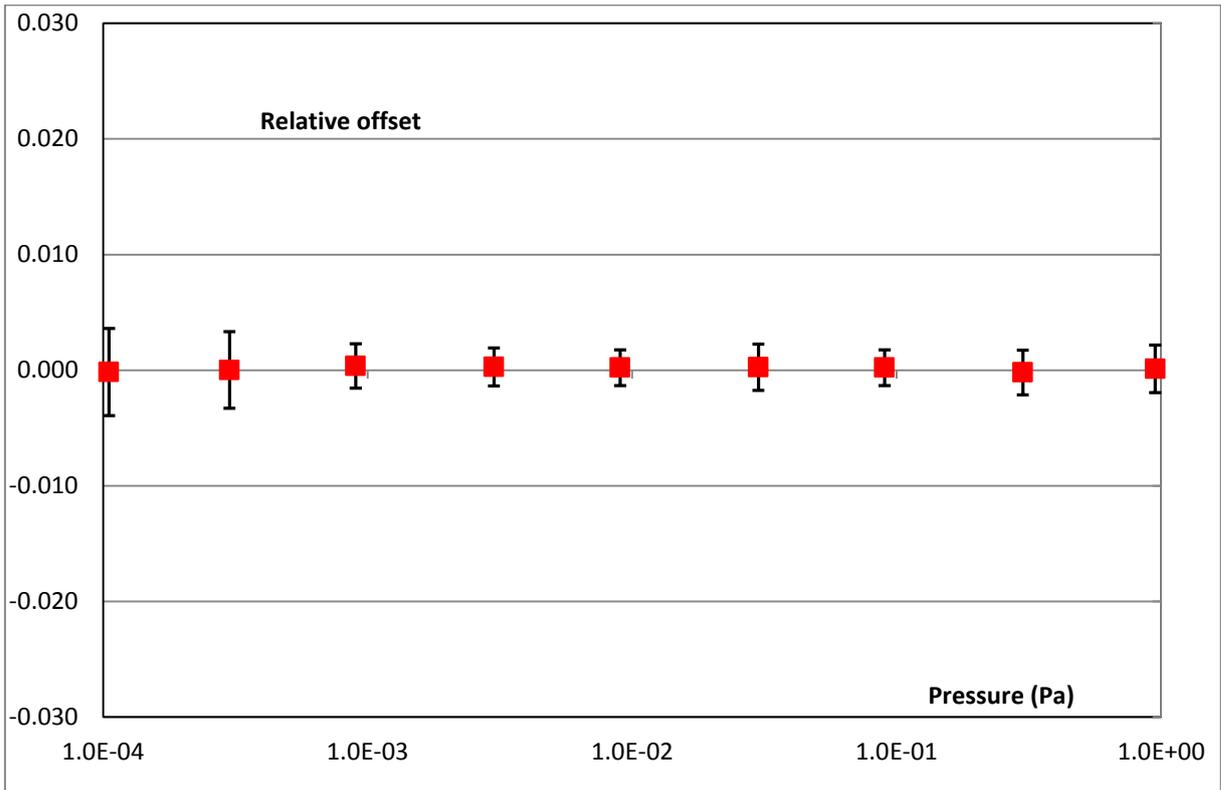


Fig. 8: Offset and expanded uncertainty of the measurements of NIM, relative to the weighted mean of the results of CCM.P-K15.1.

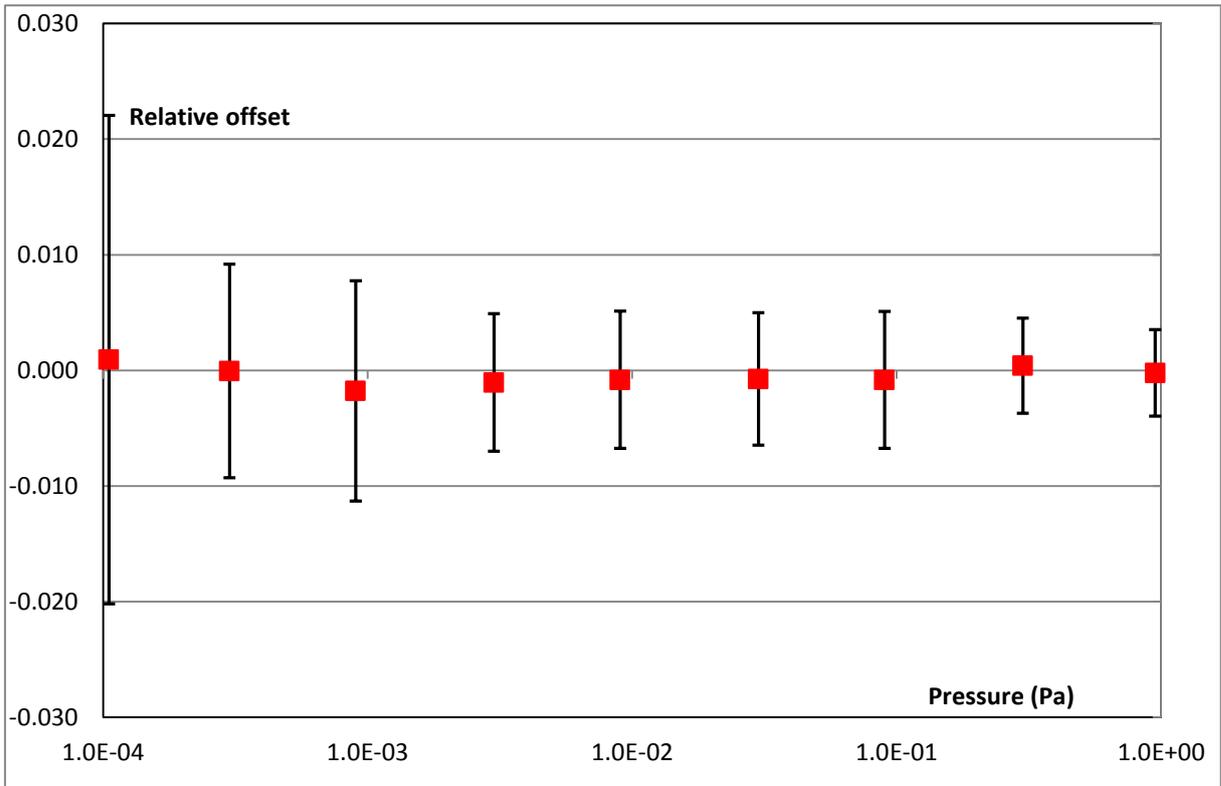


Fig. 9: Offset and expanded uncertainty of the measurements of METAS, relative to the weighted mean of the results of CCM.P-K15.1.

9 Link to the key comparison CCM.P-K15.

According to the document CIPM MRA-D-05 [5] the comparison CCM.P-K15.1 shall be linked to CCM.P-K15 [8] like it would be for a comparison organized by a RMO.

In this case the laboratory used for the link is METAS which was pilot on CCM.P-K15 which ended less than two years before this work.

9.1 Mathematics used for linking to the key comparison.

We will use a technique similar to [7] with the special situation that there is only one linking laboratory and that the mesurand (pressure) which is used for the link is the same in the two comparisons.

In a first time we calculated the offset of the linking laboratory respective to the reference value of the CCM.P-K15 comparison:

$$X_i = x_j(p_i) \quad (41)$$

where :

i designates the index of the pressure step

j designates the index of the participating laboratories and is 1 in our case

X_i is the offset of the weighted mean of the laboratories used for the link, respective to the reference value of the CCM comparison, for target pressure i

$x_j(p_i)$ is the offset respective to the CCM comparison reference value for laboratory j at target pressure i

$U(x_j(p_i))$ is the expanded uncertainty ($k=2$) associated to the deviation

and the expanded uncertainty ($k=2$) of the offset is given by:

$$U(X_i) = U(x_j(p_i)) \quad (42)$$

We calculate then a similar way the offset of the linking laboratory respective to the reference value for the CCM.P-K15.1 comparison:

$$Y_i = y_j(p_i) \quad (43)$$

Y_i is the offset of the weighted mean of the laboratories used for the link, respective to the reference value of the EURAMET comparison, for target pressure i

$y_j(p_i)$ is the offset respective to the EURAMET comparison reference value for laboratory j at target pressure i

$U(y_j(p_i))$ is the expanded uncertainty ($k=2$) associated to the deviation

and the expanded uncertainty ($k=2$) of the offset is given by:

$$U(Y_i) = U(y_j(p_i)) \quad (44)$$

The reference values of both comparisons are expressed in terms of pressure and are in fact similar mesurand that can be directly compared without conversion factor. This is also true for the respective uncertainties which can be combined without the use of a scaling factor.

The mean value of pressure obtained as reference for CCM.P-K15.1 is not equivalent to the reference value of CCM.P-K15 and the offset of the participants ($d_{i,j}$) have to be corrected the following way in order to obtain the offset relative to the reference value of CCM.P-K15:

$$D_{i,j} = d_{i,j} + X_i - Y_i \quad (45)$$

where $D_{i,j}$ represents the offset respective to the reference value of CCM.P-K15.

The uncertainty of the difference $X_i - Y_i$ is given by the combination of the uncorrelated uncer-

tainties of the pressure defined by METAS as given in the last column of table 2. We make the assumption that the uncorrelated uncertainty is the same in the two comparisons as the equipment is similar:

$$U(X_i - Y_i) = \sqrt{2}U_{uncorr}(p_{i,1}) \quad (46)$$

The uncertainty of the $D_{i,j}$ is then given by:

$$U(D_{i,j}) = \sqrt{\left(U(d_{i,j})\right)^2 + \left(U(P_i(CCM.PK15))\right)^2 + \left(U(X_i - Y_i)\right)^2} \quad (47)$$

9.2 Link to CCM.P-K15

The link to the comparison CCM.P-K15 is made through METAS which took part to both comparisons within a relatively short time. This allows taking into account the correlation between the uncertainties of METAS in both comparisons. This correlation comes from the calibration values of the system that are similar in both comparisons (expansion ratio of the chambers, influence of the valve,...) and to some constant included in our calculation (virial coefficient of nitrogen). This correlation allows eliminating a part of the uncertainty of METAS when performing the link for NIM.

The table 10 provides the values and the associated uncertainties used for establishing the link to CCM.P-K15. The uncertainty of the link is smaller than the uncertainty of the offset of METAS because we take into account only the uncorrelated uncertainty.

Table 10: Determination of the link to CCM.P-K15 through the measurement of metas.

CCM.P.K-15		CCM.P.K-15		CCM.P.K-15.1		Link	
Reference value		METAS		METAS		K-15.1 to K-15	
x_{ref}	$U(x_{ref})$	X_i	$U(X_i)$	Y_i	$U(Y_i)$	$X_i - Y_i$	$U(X_i - Y_i)$
Pa	Pa	Pa	Pa	Pa	Pa	Pa	Pa
1.00×10^{-4}	5.9×10^{-7}	1.99×10^{-7}	1.7×10^{-6}	9.32×10^{-8}	2.1×10^{-6}	1.06×10^{-7}	1.3×10^{-6}
3.00×10^{-4}	8.1×10^{-7}	3.36×10^{-7}	2.7×10^{-6}	-1.47×10^{-8}	2.8×10^{-6}	3.51×10^{-7}	1.7×10^{-6}
9.00×10^{-4}	1.6×10^{-6}	-8.52×10^{-7}	8.7×10^{-6}	-1.59×10^{-6}	8.6×10^{-6}	7.39×10^{-7}	3.6×10^{-6}
3.00×10^{-3}	4.8×10^{-6}	-9.27×10^{-7}	1.7×10^{-5}	-3.13×10^{-6}	1.8×10^{-5}	2.20×10^{-6}	9.3×10^{-6}
9.00×10^{-3}	1.2×10^{-5}	-4.14×10^{-6}	5.3×10^{-5}	-7.37×10^{-6}	5.3×10^{-5}	3.23×10^{-6}	2.7×10^{-5}
3.00×10^{-2}	4.0×10^{-5}	1.01×10^{-5}	1.7×10^{-4}	-2.21×10^{-5}	1.7×10^{-4}	3.22×10^{-5}	7.2×10^{-5}
9.00×10^{-2}	1.2×10^{-4}	2.82×10^{-5}	5.4×10^{-4}	-7.38×10^{-5}	5.3×10^{-4}	1.02×10^{-4}	1.8×10^{-4}
3.00×10^{-1}	3.6×10^{-4}	2.81×10^{-4}	1.2×10^{-3}	1.23×10^{-4}	1.2×10^{-3}	1.58×10^{-4}	6.4×10^{-4}
1.00	1.1×10^{-3}	1.02×10^{-4}	3.8×10^{-3}	-2.09×10^{-4}	3.7×10^{-3}	3.11×10^{-4}	2.0×10^{-3}

Once the link has been applied we obtain the degree of equivalence for the participating laboratories respective to the reference value of CCM.P-K15 that can be expressed in Pascal as shown in table 11.

Table 11: Offset and uncertainty (degree of equivalence) of NIM and METAS respective to the reference value of CCM.P-K15, expressed in Pa.

P_j (Pa)		Reference	NIM	METAS
1.0×10^{-4}	p_1	1.0000×10^{-4}	1.0009×10^{-4}	1.0020×10^{-4}
	$U(p_1)$	5.95×10^{-7}	1.52×10^{-6}	2.57×10^{-6}
3.0×10^{-4}	p_2	3.0000×10^{-4}	3.0036×10^{-4}	3.0034×10^{-4}
	$U(p_2)$	8.08×10^{-7}	2.13×10^{-6}	3.35×10^{-6}
9.0×10^{-4}	p_3	9.0000×10^{-4}	9.0106×10^{-4}	8.9915×10^{-4}
	$U(p_3)$	1.60×10^{-6}	4.27×10^{-6}	9.43×10^{-6}
3.0×10^{-3}	p_4	3.0000×10^{-3}	3.0031×10^{-3}	2.9991×10^{-3}
	$U(p_4)$	4.78×10^{-6}	1.16×10^{-5}	2.07×10^{-5}
9.0×10^{-3}	p_5	9.0000×10^{-3}	9.0051×10^{-3}	8.9959×10^{-3}
	$U(p_5)$	1.24×10^{-5}	3.26×10^{-5}	6.10×10^{-5}
3.0×10^{-2}	p_6	3.0000×10^{-2}	3.0040×10^{-2}	3.0010×10^{-2}
	$U(p_6)$	4.05×10^{-5}	1.02×10^{-4}	1.91×10^{-4}
9.0×10^{-2}	p_7	9.0000×10^{-2}	9.0121×10^{-2}	9.0028×10^{-2}
	$U(p_7)$	1.16×10^{-4}	2.54×10^{-4}	5.74×10^{-4}
3.0×10^{-1}	p_8	3.0000×10^{-1}	3.0010×10^{-1}	3.0028×10^{-1}
	$U(p_8)$	3.59×10^{-4}	9.33×10^{-4}	1.43×10^{-3}
1.0	p_9	1.0000	1.0004	1.0001
	$U(p_9)$	1.07×10^{-3}	3.04×10^{-3}	4.37×10^{-3}

In the table 12 we provide the degree of equivalence respective to the reference value of CCM.P-K15 in relative value as it is more convenient for displaying in a plot.

Table 12: Pair of offset and associated uncertainty (degree of equivalence) of NIM and METAS respective to the reference value of CCM.P-K15, expressed relatively to the reference value.

P_j (Pa)		NIM	METAS
1.0×10^{-4}	D_1	0.0009	0.0020
	$U(D_1)$	0.0152	0.0257
3.0×10^{-4}	D_2	0.0012	0.0011
	$U(D_2)$	0.0071	0.0112
9.0×10^{-4}	D_3	0.0012	-0.0009
	$U(D_3)$	0.0047	0.0105
3.0×10^{-3}	D_4	0.0010	-0.0003
	$U(D_4)$	0.0039	0.0069
9.0×10^{-3}	D_5	0.0006	-0.0005
	$U(D_5)$	0.0036	0.0068
3.0×10^{-2}	D_6	0.0013	0.0003
	$U(D_6)$	0.0034	0.0064
9.0×10^{-2}	D_7	0.0013	0.0003
	$U(D_7)$	0.0028	0.0064
3.0×10^{-1}	D_8	0.0003	0.0009
	$U(D_8)$	0.0031	0.0048
1.0	D_9	0.0004	0.0001
	$U(D_9)$	0.0030	0.0044
$\Sigma(D_i)/N$		0.0009	0.0007

The relative offset respective to the reference value and the associated uncertainty, are given in table 12. They are plotted, as degree of equivalence, in the Fig. 10 and 11, for NIM and METAS respectively. The values of the offset for METAS are in fact exactly the same as the values obtained at the time of CCM.P-K15, but the uncertainties are larger. The uncertainties obtained on the offset of NIM are in the same order of magnitude as the uncertainties obtained by the participants to CCM.P-K15.

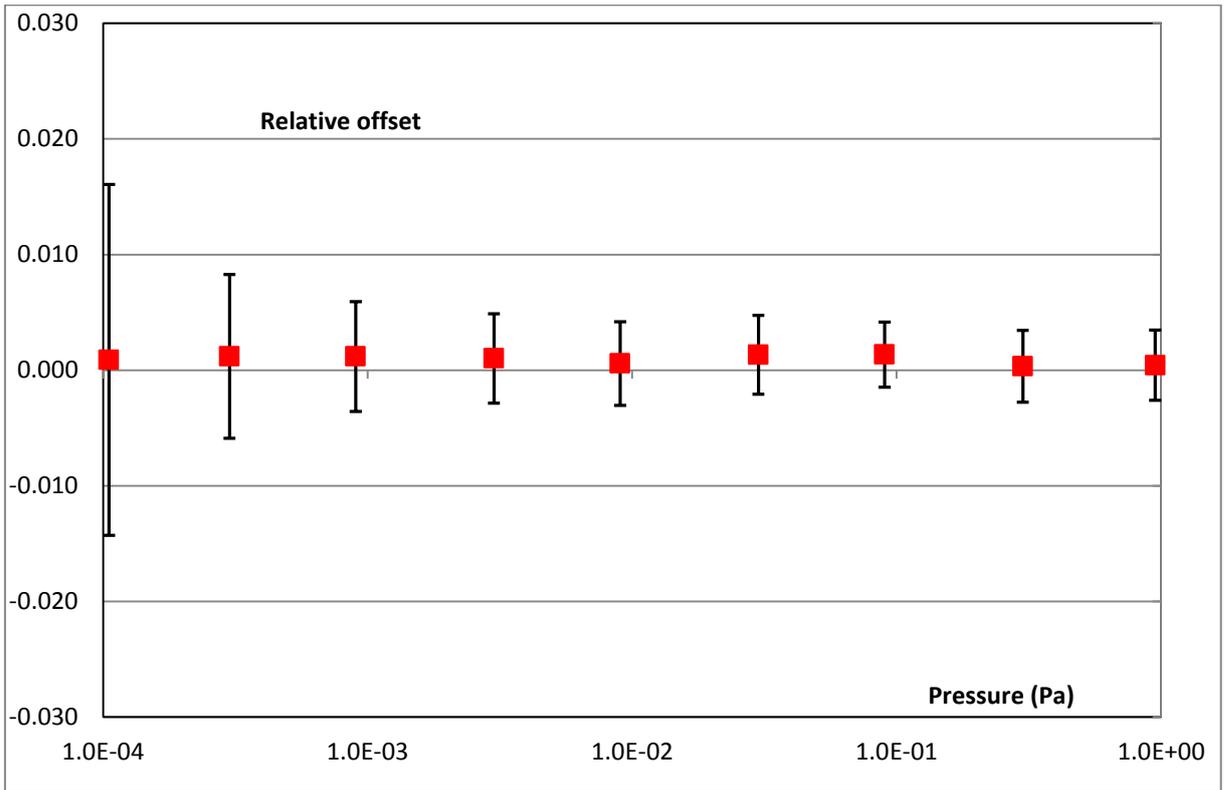


Fig. 10: Offset and expanded uncertainty relative to the reference value of CCM.P-K15 for the measurements of NIM.

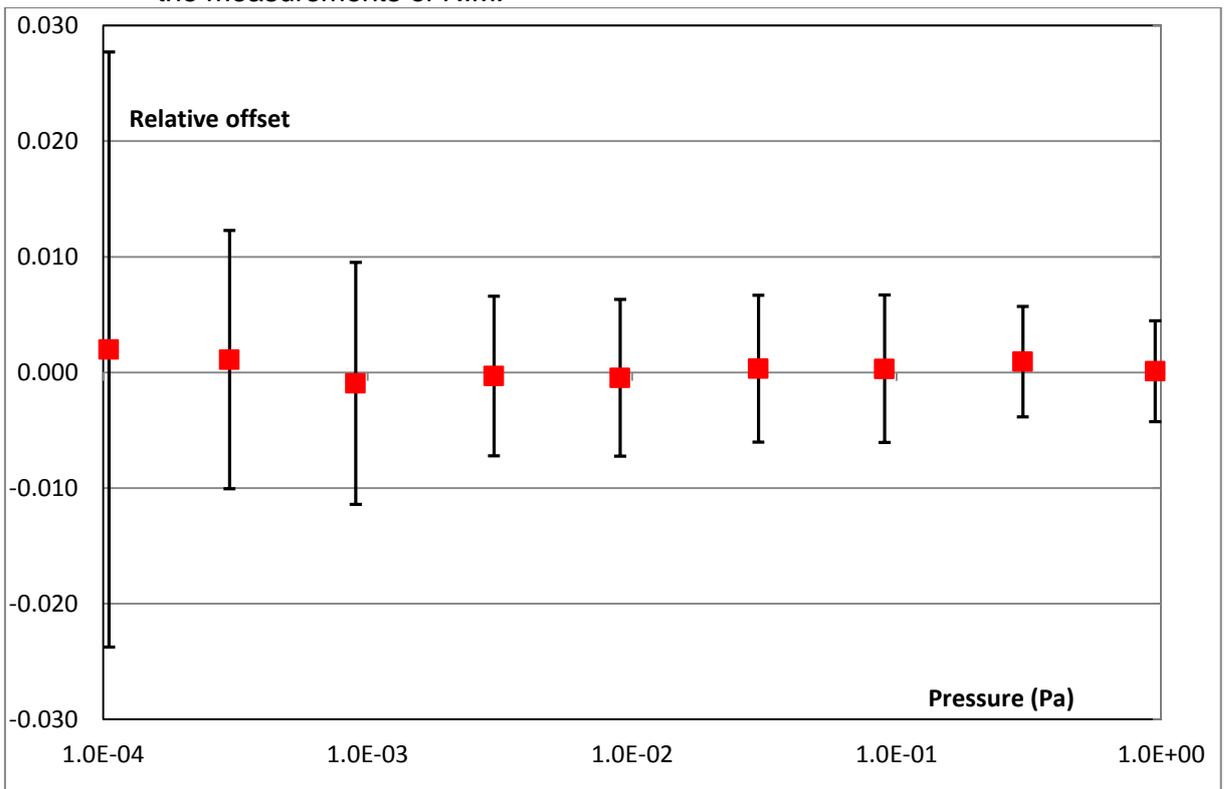


Fig. 11: Offset and expanded uncertainty relative to the reference value of CCM.P-K15 for the measurements of METAS.

10 Conclusion

This comparison gave the opportunity to the Chinese national metrology institute NIM to be linked to the CCM.P-K15 with the support of METAS. The transfer standards have shown an excellent stability and it was easy to demonstrate the equivalence between the two participants.

The link with CCM.P-K15 was optimal with the possibility to subtract the correlated uncertainties of METAS in order to obtain a small uncertainty in the link. The Equivalence of NIM with the reference value of CCM.P-K15 has been established.

The link of NIM to CCM.P-K15 is an important contribution for the APMP. The stability of the transfer standard was not optimal when it was circulated between the APMP participants at the time of CCM.P-K15 [8].

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