Consultative Committee on Mass and Related Quantities (CCM)
Working Group on Gravimetry (WGG)

Regional Comparison of Absolute Gravimeters SIM.M.G-K1 Key Comparison

Table Mountain Geophysical Observatory (TMGO)
Boulder, Colorado

Pilot laboratory: NIST-Gaithersburg

1. Introduction

The Regional Key Comparison of Absolute Gravimeters, SIM.M.G-K1, and a Pilot Study took place at the Table Mountain Geophysical Observatory (TMGO) near Boulder, Colorado between October 4 and October 16, 2016.

The procedures and schedule for the measurements were spelled out in the Technical Protocol, approved by all participants and the CCM-WGG. However, due to logistical constraints, a few teams were not able to attend, and the schedule was optimized accordingly.

Dr. David Newell of NIST, Gaithersburg, Maryland served as the Pilot Laboratory representative, while Dr. Derek van Westrum of NOAA-NGS, Boulder, Colorado served as the site host and coordinator. The comparison, SIM.M.G-K1, was organized in accordance with CIPM MRA-D-05 of the Consultative Committee on Mass and Related Quantities (CCM), and linked to the CCM.G-K2 [1] through EURAMET.M.G-K2 [2] via FG5X-216.

The list of participants and a description of the site (including vertical gravity gradients, observed tidal parameters, and superconducting gravimeter signal observed during the comparison) are presented, followed by a discussion of the analysis. The results of the data analysis, the degrees of equivalence (DoE) of the gravimeters, and the key comparison reference values (KCRVs) are also presented. For the final and official solution of the KCRVs, the contributions of absolute gravity data inconsistent at the 95% confidence level are investigated. Overall, the results and uncertainties indicate an excellent agreement among the gravimeters, with a standard deviation of the gravimeters' DoEs better than 1.3 µGal.

The Pilot Study is a complete solution where all participating gravimeters were treated as equivalent in terms of their contribution to the definition of the reference values (RVs). The results of the Pilot Study are given in Appendix A.

Table 1. Participants in the comparison.

#	Country	Institution	Gravimeter	NMI or DI	Operator(s)
1	USA	NIST-Gaithersburg	FG5-204	YES	David Newell
2	USA	NOAA-NGS	FG5X-102	NO	Derek van Westrum Jeff Kanney
3	Canada	National Research Council, Canada	FG5-105	YES	Jacques Liard
4	Luxembourg	University of Luxembourg	FG5X-216	YES	Olivier Francis
5	USA	Micro-g LaCoste	FG5X-302	NO	Brice Lucero Brian Ellis
6	Italy	INGV	FG5-238	NO	Filippo Greco Antonio Pistorio
7	Netherlands	TU Delft	FG5-234	NO	Rene Reudink
8	Mexico	CENAM & National University of Mexico	FG5X-252	YES	Alfredo Esparza Ramirez
9	Italy	Agenzia Spaziale Italiana	FG5-218	NO	Domenico Iacovone Francesco Baccaro
10	Canada	Natural Resources Canada	FG5-236	NO	Jason Silliker
11	USA	National Geospatial Intelligence Agency	FG5-107	NO	Robert D Wheeler
12	Germany	Federal Agency for Cartography and Geodesy	FG5-301	NO	Reinhard Falk Axel Ruelke

2. List of participants

Table 1 lists the participants in the comparison. All gravimeters were of the FG5(X) type with two bulk interferometer systems (FG5-107 and FG5-301). Four institutions were a National Metrology Institute (NMI) or a Designated Institute (DI). These are shaded in pink.

3. Site description and relative gravity measurements

A schematic of the TMGO facility, renovated in 2010, is shown in Figure 1. Six of the 10 available piers (AK is permanently reserved for the SG) were employed during SGIM.M.G-K1: AG, AH, AJ, AQ, AS, and AT. Each pier is constructed of approximately 1m^3 of concrete, isolated from the building's foundation. The building itself is located on top of a remote mesa, far from any cultural noise sources.

In the summer of 2014, LaCoste & Romberg gMeters D43 and G6 were used to measure relative gravity values on a fixed tripod at three heights above each pier at TMGO. Three heights allow the determination of any nonlinearity in the gradient. A given measurement consists of six or more "laps" of each gravimeter up the three-tiered tripod. Each tier occupation consists of approximately twenty 10 second gravity samples. After first rejecting any sample outliers (>3σ, k=1), the instrument drift and any significant tares are removed. A weighted least-squares analysis provides a quadratic fit to all measured gravity differences (both instruments) simultaneously, resulting in the following function of gravity with height:

$$g(z) = az^2 + bz + c. \qquad (1)$$

Note that when determining the difference in gravity value at two different heights, the constant term, *c*, cancels. Uncertainties of the parameters *a* and *b* are derived from the variance-covariance matrix. Because the two instruments are analyzed simultaneously, any systematic difference between them appears as an increase in *a* and *b*. Uncertainties in the height measurements (tier locations) are considered negligible. The parameters are listed (for all piers for completeness) in Table 2.

Finally, note that the gradient value is not expected to change at TMGO over the time-frame discussed. TMGO is located on a broad, flat, undisturbed expanse of land, and even unmodeled environmental factors like water table fluctuations will not cause a change in the value of the gradient.

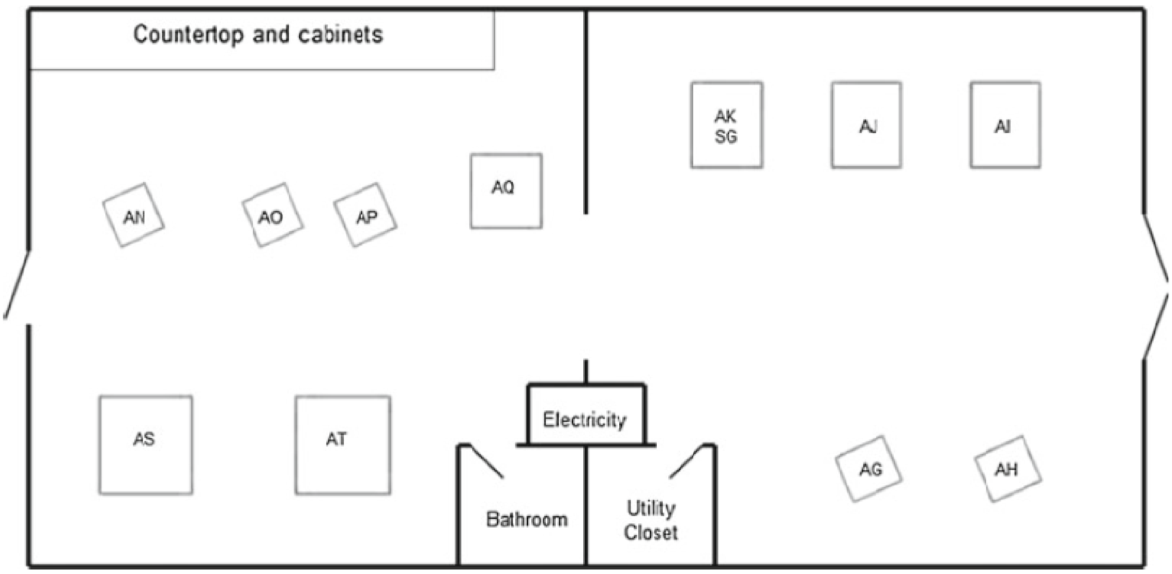

Figure 1. A schematic of the TMGO gravity piers. AK is permanently occupied by the SG, and only piers AG, AH, AJ, AQ, AS, and AT were used in the comparison.

Table 2. Vertical gravity gradients at the TMGO piers (only AG, AH, AJ, AQ, AS, and AT were used for the comparison).

Site	a /µGal m^{-2}	σ_a /µGal m^{-2}	b /µGal m^{-1}	σ_b /µGal m^{-1}	σ_{ab} /µGal^2 m^{-3}
AG	4.6	1.3	-322.3	1.7	2.2
AH	4.9	1.0	-318.4	1.3	1.3
AI	3.6	1.7	-310.4	2.1	3.6
AJ	1.1	1.1	-316.1	1.5	1.7
AN	3.6	1.3	-316.0	1.7	2.2
AO	3.3	1.2	-319.4	1.6	1.9
AP	0.6	1.8	-317.0	1.5	2.7
AQ	9.2	1.1	-332.5	1.4	1.8
AS	8.5	3.0	-330.0	4.0	12.0
AT	6.8	1.4	-329.2	1.9	2.7

The gravity difference between height z_1 and z_2 is given by:

$$\Delta g(z_1 - z_2) = g(z_2) - g(z_1) = a \times (z_2^2 - z_1^2) + b \times (z_2 - z_1) \qquad (2)$$

and the associated uncertainty

$$\sigma_{\Delta g}^2 = (z_2^2 - z_1^2)^2 \times \sigma_a^2 + (z_2 - z_1)^2 \times \sigma_b^2 + 2 \times (z_2^2 - z_1^2) \times (z_2 - z_1) \times \sigma_{ab} \qquad (3)$$

The participants reported gravity results at the "effective measurement height", h_{eff}, of their respective gravimeters where the gravity value is least sensitive to the effect of the gradient [3] [4]. The gradient formula for that respective pier was then used to transfer the value to a final, common height of 125 cm (an approximate average of the effective measurement height for FG5 and FG5-X gravimeters).

The observed tidal parameters – used by all participants to correct for earth tides and ocean loading simultaneously – were provided by Olivier Francis, based on an analysis of SG CT-024. They are listed in Table 3 [5].

4. Superconducting gravity meter measurements

A GWR superconducting gravity meter, CT-024, was operated continuously throughout the comparison on pier AK, situated near the center of the absolute piers. Figure 2 displays the results after the earth tide, ocean loading, and atmospheric pressure effects have been removed. The resulting change in gravity, ~1 µGal peak-to-peak, throughout the comparison was presumably observed by all gravity meters. This small, slow change in gravity is well within the systematic uncertainty of the absolute gravimeters, and its presence is thus neglected in the analysis. The mean times of absolute measurements are indicated by red circles.

Table 3. Observed tidal parameters for TMGO from SG CT-024 via Olivier Francis.

Wave	Start Frequency (cpd)	Stop Frequency (cpd)	Amplitude Factor	Phase Lag (degrees)
DC	0.000000	0.000001	1.00000	0.0000
long	0.000002	0.249951	1.16000	0.0000
Q1	0.721500	0.906315	1.16052	1.1570
O1	0.921941	0.940487	1.16468	1.1775
NO1	0.958085	0.974188	1.15951	1.0326
P1	0.989049	0.998028	1.16539	1.1041
S1	0.999853	1.000147	1.49457	15.9599
K1	1.001825	1.003651	1.15452	1.1761
PSI1	1.005329	1.005623	1.30377	1.3908
PHI1	1.007595	1.011099	1.20411	0.6319
J1	1.013689	1.044800	1.18028	1.1094
OO1	1.064841	1.216397	1.18279	0.3491
2N2	1.719381	1.872142	1.16806	-0.4567
N2	1.888387	1.906462	1.15681	-0.2398
M2	1.923766	1.942754	1.15945	0.1973
L2	1.958233	1.976926	1.16297	0.3812
S2	1.991787	2.002885	1.17172	-0.5305
K2	2.003032	2.182843	1.17348	-0.4844
M3	2.753244	3.081253	1.07285	-0.2409
M4	3.381379	4.347615	1.03900	0.0000

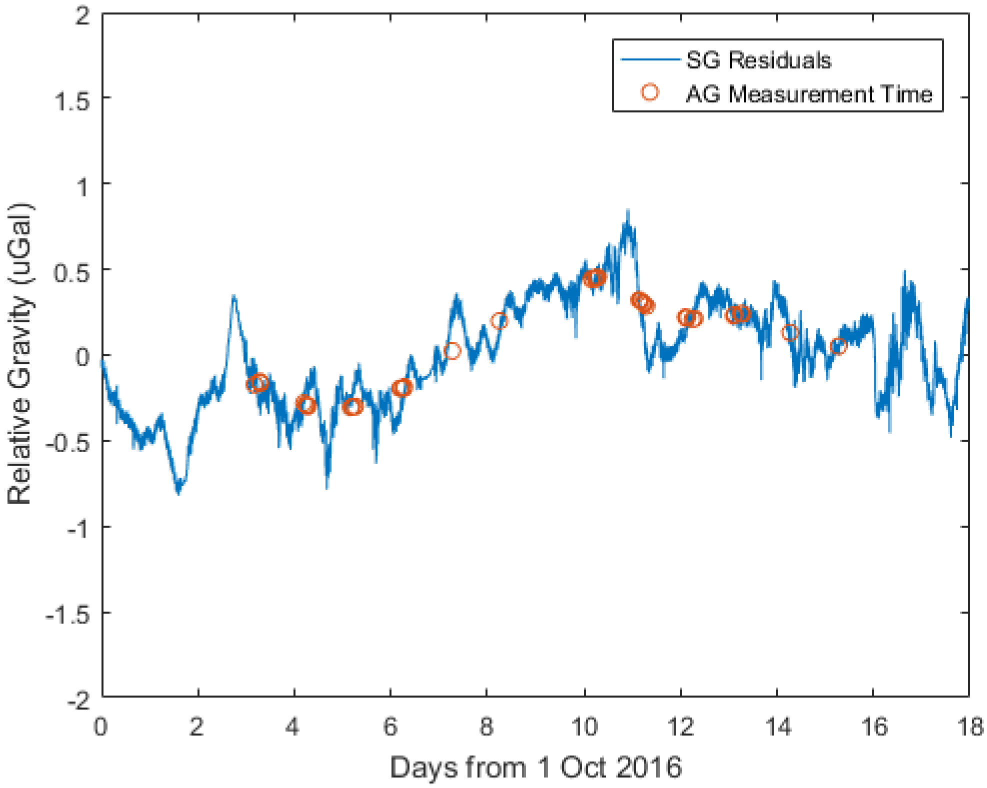

Figure 2. The output of the superconducting gravity meter, CT-024 with earth tides and barometric pressure effects removed (blue line). A maximum peak-to-peak signal of about 1.0 µGal is observed during the entire duration of the comparison. The red circles indicate the average times of each absolute gravity measurement.

5. Absolute gravity measurements

Each participant was free to measure gravity over a duration of their choosing within a window of approximately 24 hours (before being required to move to the next scheduled pier). Table 4 lists the measurement schedule (NMI/DI gravimeters are shaded in pink). The schedule was designed to maximize the overlap of gravimeters, minimize the number of occupied piers, and prohibit reoccupations of the same pier by a single gravimeter [6]. Every gravimeter overlapped with every other gravimeter at least two times and no more than three times.

Table 4. Nominal pier occupation schedule for the comparison. Days 1-4 were Monday through Thursday nights (3-6 October, 2016), and days 5-8 were Monday through Thursday nights the following week (10-13, October, 2016). Due to real-time logistical constraints, not all instruments could observe on their scheduled date. The actual occupation times are listed in Table 5. (All instruments did observe at their scheduled location.)

Pier\Day	1	2	3	4	5	6	7	8
AG	FG5X-252	FG5-301	FG5-218	FG5-234	FG5-107	FG5-204	FG5X-102	FG5-236
AH	FG5X-216	FG5X-252	FG5-301	FG5-218	FG5-105	FG5-107	FG5-204	FG5X-302
AJ	FG5-238	FG5X-216	FG5X-252	FG5-301	FG5-236	FG5-105	FG5-107	FG5X-102
AQ	FG5-234	FG5-238	FG5X-216	FG5X-252	FG5X-302	FG5-236	FG5-105	FG5-204
AS	FG5-218	FG5-234	FG5-238	FG5X-216	FG5X-102	FG5X-302	FG5-236	FG5-107
AT	FG5-301	FG5-218	FG5-234	FG5-238	FG5-204	FG5X-102	FG5X-302	FG5-105

Gravity was determined above each pier benchmark at h_{eff} unique to each gravimeter (and in principle, each gravimeter set up). Each raw gravity value is corrected for:

- Earth tides and ocean loading via a common set of observed tidal parameters [5]
- Barometric pressure changes from the nominal value of 826.74 mBar using a common admittance factor of -0.3 µGal/mbar [7]
- Polar Motion using common values obtained from the Earth Rotation and Reference Systems Service (IERS) [8]
- Self attraction of the gravimeter itself [9] [10]
- Diffraction correction due to the finite width of the laser beam [11]

These raw gravity values were then transferred to a common height of 125 cm using the quadratic gradient parameters for each pier listed in Table 2. The gravimeter results are listed in Table 5.

Table 5. Absolute gravity observations g_{ij}. u_{ij} is the standard measurement uncertainty (k=1). See text for discussion. NMI/DI gravimeters are shaded in pink, and a common value of 979 622 000 µGal has been subtracted from each value. The days are UTC dates in October 2016.

Gravimeter	Pier	Gravity @ h_{eff} (µGal)	h_{eff} (m)	diffC (µGal)	SAC (µGal)	Start Date	Start Time	Stop Date	Stop Time	Gravity g_{ij} (µGal)	Uncert u_{ij} (µGal)
FG5X-252	AG	737.8	1.311	1	-1.2	8	21:35	9	14:35	756.7	2.2
FG5X-252	AH	739.3	1.311	1	-1.2	4	21:26	5	15:26	758	2.2
FG5X-252	AJ	746.7	1.313	1	-1.2	5	20:40	6	15:40	766.4	2.2
FG5X-252	AQ	741.5	1.309	1	-1.2	6	19:03	7	15:03	759.7	2.2
FG5X-216	AH	749.1	1.27	1.2	-1.4	3	17:00	4	15:15	755.2	2.38
FG5X-216	AJ	759.2	1.27	1.2	-1.4	4	18:00	5	15:30	765.5	2.38
FG5X-216	AQ	750.9	1.27	1.2	-1.4	5	17:00	6	16:15	757.1	2.38
FG5X-216	AS	745.8	1.27	1.2	-1.4	6	18:00	7	15:10	752	2.38
FG5-238	AJ	777.8	1.217	1.05	-1.2	3	22:26	4	15:26	767.4	1.85
FG5-238	AQ	771.3	1.218	1.05	-1.2	4	21:42	5	13:41	761.3	1.87
FG5-238	AS	766.6	1.216	1.05	-1.2	5	20:22	6	13:22	755.9	2.06
FG5-238	AT	766.7	1.214	1.05	-1.2	6	21:36	7	15:37	755.5	1.9
FG5-234	AG	768.9	1.215	1	-1.5	7	0:06	7	12:19	758	1.99
FG5-234	AQ	769.7	1.214	1	-1.5	4	1:06	4	12:19	758.6	1.97
FG5-234	AS	766.6	1.213	1	-1.5	5	0:06	5	12:19	755.2	2.4
FG5-234	AT	768.3	1.211	1	-1.5	6	0:06	6	12:19	756.2	2.01
FG5-218	AG	773.6	1.205	1.20	-1.36	6	1:05	5	12:05	759.6	1.87
FG5-218	AH	769.8	1.21	1.20	-1.36	7	1:05	6	7:05	757.3	1.84
FG5-218	AS	772.2	1.205	1.20	-1.36	8	1:05	7	12:05	758.2	2.14
FG5-218	AT	776.9	1.201	1.20	-1.36	5	1:05	4	12:05	761.5	2.03
FG5-301	AG	764.4	1.22	2	-1.43	4	21:23	5	15:39	755.1	2.33
FG5-301	AH	767.1	1.22	2	-1.43	5	20:33	6	15:49	758	2.32
FG5-301	AJ	776.3	1.22	2	-1.43	6	21:16	7	15:33	766.9	2.33
FG5-301	AT	765.1	1.22	2	-1.43	7	21:57	8	15:14	755.8	2.34
FG5X-102	AG	748.1	1.286	1.05	-1.2	12	15:01	13	14:01	759.2	1.84
FG5X-102	AJ	754.9	1.285	1.05	-1.2	13	14:23	14	14:23	765.8	1.85
FG5X-102	AS	743.8	1.282	1.05	-1.2	10	20:41	11	14:41	753.7	1.87

FG5X-102	AT	745	1.281	1.05	-1.2	11	20:50	12	13:50	754.7	1.85
FG5-204	AG	764.1	1.231	1.98	-1.23	12	0:01	12	11:46	758.2	1.83
FG5-204	AH	763	1.232	1.98	-1.23	13	0:01	13	11:46	757.5	1.82
FG5-204	AQ	763.1	1.23	1.98	-1.23	14	0:06	14	11:51	756.9	1.83
FG5-204	AT	765.5	1.227	1.98	-1.23	11	0:11	11	11:56	758.3	2.13
FG5-107	AG	764.3	1.225	1.05	-1.2	10	21:01	11	14:01	756.5	1.88
FG5-107	AH	763	1.228	1.05	-1.2	11	16:23	12	14:23	756.2	1.85
FG5-107	AJ	775.8	1.227	1.05	-1.2	12	16:23	13	14:23	768.5	1.87
FG5-107	AS	763.8	1.226	1.05	-1.2	13	14:23	14	14:23	756.3	2.04
FG5-105	AH	756.6	1.242	0.9	-1.1	10	18:32	11	12:33	754.2	1.71
FG5-105	AJ	770	1.237	0.9	-1.1	11	17:48	12	13:48	765.9	1.72
FG5-105	AQ	758	1.237	0.9	-1.1	12	17:00	13	13:00	754	2.51
FG5-105	AT	758.8	1.235	0.9	-1.1	13	17:00	14	13:00	754.1	1.74
FG5-236	AG	761.4	1.226	0.82	-1.2	14	1:04	14	12:49	754	1.99
FG5-236	AJ	773.3	1.221	0.82	-1.2	11	1:08	11	12:53	764.2	2.02
FG5-236	AQ	764.4	1.223	0.82	-1.2	12	1:07	12	12:52	756.1	1.98
FG5-236	AS	762.4	1.217	0.82	-1.2	13	1:08	13	12:53	752.3	2.17
FG5X-302	AH	744.9	1.288	1.05	-1.2	14	21:16	15	15:47	756.4	1.81
FG5X-302	AQ	746	1.293	1.05	-1.2	10	18:37	11	13:37	759.1	1.81
FG5X-302	AS	742.2	1.285	1.05	-1.2	11	18:14	12	14:13	753	1.82
FG5X-302	AT	742.5	1.285	1.05	-1.2	15	19:04	16	18:04	753.4	1.81

6. Comparison Analysis Methods

Following Koo and Clare [12], the gravity values for each gravimeter on each pier are expressed as

$$g_{ij} = g_j + \delta_i + \varepsilon_{ij} \tag{4}$$

where g_{ij} is the gravity value measured on pier j by gravimeter i as given in Table 5, g_j is the true (unknown) gravity value at pier j, δ_i is the true (unknown) bias of gravimeter i, and ε_{ij} is the measurement error. A variance weighted least squares analysis (LSA) is performed to give the best estimates of the pier gravity Reference Values g_j and the gravimeter biases δ_i. Since we do not consider correlations among the gravity measurements, the input (or measurement) covariance matrix $\boldsymbol{V}$ is diagonal with diagonal elements u_{ij}^2 where u_{ij} is given in Table 5. Thus the corresponding weight matrix $\boldsymbol{W} = \boldsymbol{V}^{-1}$ is also diagonal with diagonal elements $w_{ij} = 1/u_{ij}^2$. The set of equations is ill defined with an infinite number of solutions, so an additional constraint on the weighted sum of the biases is imposed:

$$\sum_{i=1}^{n} \overline{w}_i \ \delta_i = d \tag{5}$$

where d is the linking converter for the KCRVs. The normalized variance weights $\overline{w}_i$ have the condition that

$$\sum_{i=1}^{n} \overline{w}_i \ = 1 \tag{6}$$

and are defined as

$$\overline{w}_i = \frac{w_i}{\sum_{i=1}^{n} w_i} \quad , \tag{7}$$

where $w_i = 1/u_i^2$ and u_i is the root mean square of u_{ij} given in Table 5 for gravimeter i. Initially d is taken to be zero. By definition, only NMI and DI gravimeters can contribute to the KCRVs. Therefore the only nonzero weights are 0.224 for FG5X-252, 0.192 for FG5X-216, 0.299 for FG5-204, and 0.286 for FG5-105. The link of the KCRVs to CCM.G-K2 is then provided by the DoE of FG5X-216 from CCM.G-K2 [1] through EURAMET.M.G-K2 [2].

Results for the Pilot Study where all participating gravimeters are treated as equivalent with nonzero normalized weights are given in Appendix A.

7. Results

7.1 Initial results

For the initial solution, all measurements presented by the operators as given in Table 5 were included in the LSA. For 31 degrees of freedom (49 input data, 18 unknowns), the chi square statistic is $\chi^2 = 16.3$ with probability $p(16.3|31) = 0.99$ and a Birge ratio of $R_B = \sqrt{16.3/31} = 0.73$, indicating a high level of consistency for the input data. The gravimeter biases (δ) and pier Reference Values (g) are presented in Table 6 and Figure 3.

Table 6. Initial biases (δ) of NMI/DIs (pink) and non-NMI/DIs and pier Reference Values (g) at a height of 125 cm of the comparison using all the reported absolute measurements. The constant value 979 622 000.0 µGal is subtracted from the Reference Values, U is the expanded standard uncertainty at 95% confidence (k=2).

Gravimeter	δ/µGal	U/µGal
FG5X-252	1.10	1.95
FG5X-216	-1.10	2.18
FG5-238	1.63	2.24
FG5-234	1.12	2.39
FG5-218	3.64	2.28
FG5-301	0.52	2.58
FG5X-102	0.37	2.22
FG5-204	1.29	1.62
FG5-107	1.18	2.23
FG5-105	-1.47	1.63
FG5-236	-0.69	2.34
FG5X-302	-0.29	2.14

Pier	g/µGal	U/µGal
AG	756.29	1.75
AH	755.86	1.57
AJ	766.36	1.70
AQ	757.90	1.66
AS	754.08	1.90
AT	755.23	1.73

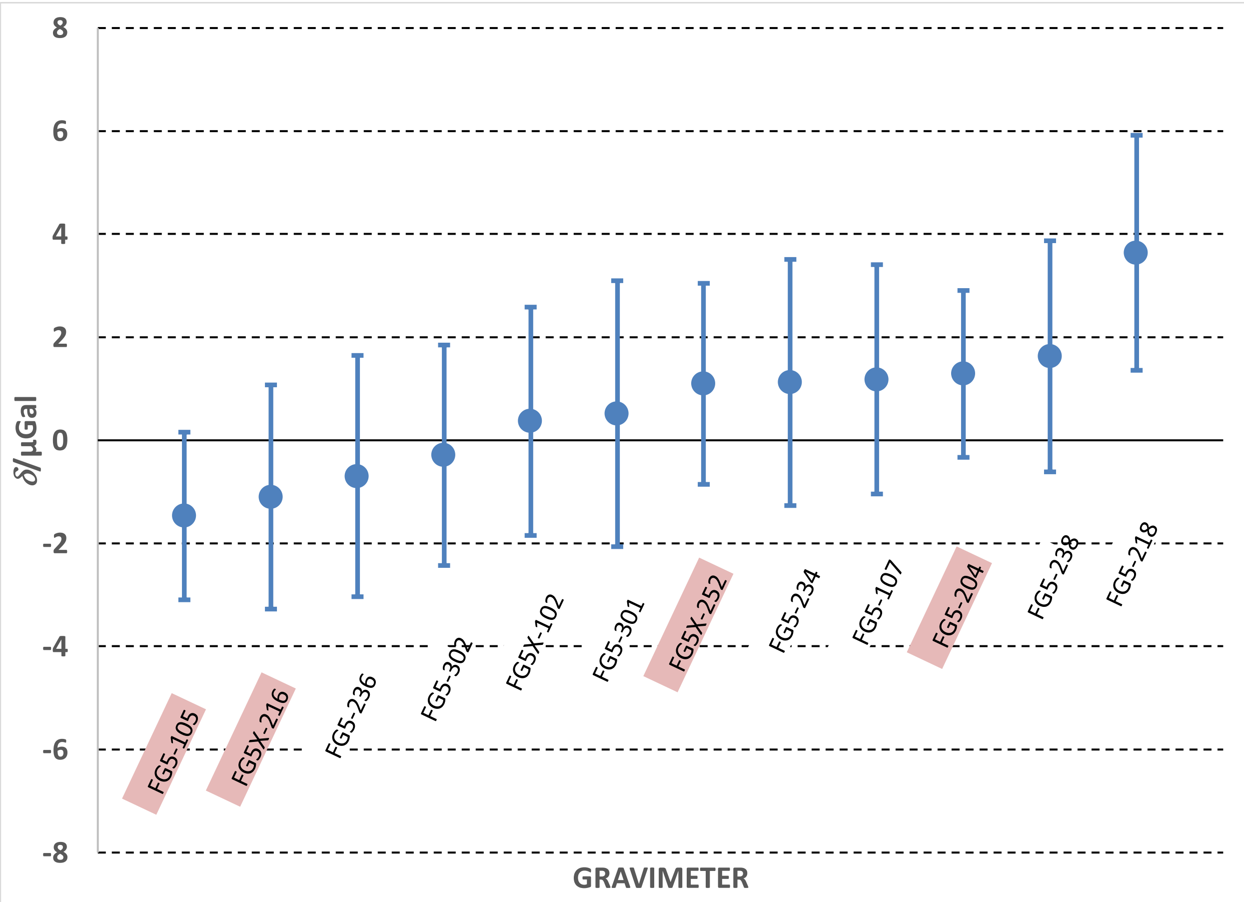

Figure 3. Initial biases δ of the gravimeters from the initial weighted LSA. Gravimeters of NMI/DIs are highlighted in pink. The error bars are the expanded standard uncertainties (k=2) from the weighted LSA and are listed in Table 6.

7.2 Consistency of input data

The consistency of the input data is investigated by calculating the normalized residuals R_{ij} from the LSA,

$$R_{ij} = \frac{(g_{ij} - g_j)}{U_{ij}} \quad , \tag{8}$$

where U_{ij} is the expanded uncertainty (k=2) of the measured value in Table 5. A less stringent test is to calculate the ratio of the difference between measured values of gravity and the reference value to the uncertainty of the difference, called the compatibility index E_{ij},

$$E_{ij} = \frac{(g_{ij} - g_j)}{\sqrt{U_{ij}^2 + U_j^2}} \quad , \tag{9}$$

where U_j is the expanded standard uncertainty (k=2) of the reference value g_j from the weighted LSA in Table 6. An absolute value of R_{ij} or E_{ij} larger than 1 indicates inconsistency at the 95% confidence level for the given test. Values of R_{ij} and E_{ij} are given in Table 7. Only one of the input data has an R_{ij} or E_{ij} value larger than 1, with FG5-218 on pier AT at R_{ij} = 1.54 and E_{ij} = 1.42. Since FG5-218 does not represent an NMI or DI, this value was omitted in the final analysis without further consideration.

Table 7. Consistency of input data: Comparison of measured gravity values g_{ij} (along with expanded standard uncertainties U_{ij}) with reference values g_j (along with expanded standard uncertainties U_j) by means of R_{ij} and E_{ij}. NMI/DI gravimeters are in pink. The constant value 979 622 000.0 µGal has been subtracted from the gravity measurements. R_{ij} and E_{ij} values larger than 1 are in yellow.

Gravimeter	Pier	g_{ij} /µGal	U_{ij} /µGal	g_j /µGal	U_j /µGal	g_{ij}-g_j /µGal	R_{ij}	E_{ij}
FG5X-252	AG	756.70	4.40	756.29	1.75	0.41	0.09	0.09
FG5X-252	AH	758.00	4.40	755.86	1.57	2.14	0.49	0.46
FG5X-252	AJ	766.40	4.40	766.36	1.70	0.04	0.01	0.01
FG5X-252	AQ	759.70	4.40	757.90	1.66	1.80	0.41	0.38
FG5X-216	AH	755.20	4.76	755.86	1.57	-0.66	-0.14	-0.13
FG5X-216	AJ	765.50	4.76	766.36	1.70	-0.86	-0.18	-0.17
FG5X-216	AQ	757.10	4.76	757.90	1.66	-0.80	-0.17	-0.16
FG5X-216	AS	752.00	4.76	754.08	1.90	-2.08	-0.44	-0.41
FG5-238	AJ	767.40	3.70	766.36	1.70	1.04	0.28	0.25
FG5-238	AQ	761.30	3.74	757.90	1.66	3.40	0.91	0.83
FG5-238	AS	755.90	4.12	754.08	1.90	1.82	0.44	0.40
FG5-238	AT	755.50	3.80	755.23	1.73	0.27	0.07	0.06
FG5-234	AG	758.00	3.98	756.29	1.75	1.71	0.43	0.39
FG5-234	AQ	758.60	3.94	757.90	1.66	0.70	0.18	0.16
FG5-234	AS	755.20	4.80	754.08	1.90	1.12	0.23	0.22
FG5-234	AT	756.20	4.02	755.23	1.73	0.97	0.24	0.22
FG5-218	AG	759.60	3.74	756.29	1.75	3.31	0.88	0.80
FG5-218	AH	757.30	3.68	755.86	1.57	1.44	0.39	0.36

FG5-218	AS	758.20	4.28	754.08	1.90	4.12	0.96	0.88
FG5-218	AT	761.50	4.06	755.23	1.73	6.27	1.54	1.42
FG5-301	AG	755.10	4.66	756.29	1.75	-1.19	-0.26	-0.24
FG5-301	AH	758.00	4.64	755.86	1.57	2.14	0.46	0.44
FG5-301	AJ	766.90	4.66	766.36	1.70	0.54	0.12	0.11
FG5-301	AT	755.80	4.68	755.23	1.73	0.57	0.12	0.11
FG5X-102	AG	759.20	3.68	756.29	1.75	2.91	0.79	0.71
FG5X-102	AJ	765.80	3.70	766.36	1.70	-0.56	-0.15	-0.14
FG5X-102	AS	753.70	3.74	754.08	1.90	-0.38	-0.10	-0.09
FG5X-102	AT	754.70	3.70	755.23	1.73	-0.53	-0.14	-0.13
FG5-204	AG	758.20	3.66	756.29	1.75	1.91	0.52	0.47
FG5-204	AH	757.50	3.64	755.86	1.57	1.64	0.45	0.41
FG5-204	AQ	756.90	3.66	757.90	1.66	-1.00	-0.27	-0.25
FG5-204	AT	758.30	4.26	755.23	1.73	3.07	0.72	0.67
FG5-107	AG	756.50	3.76	756.29	1.75	0.21	0.05	0.05
FG5-107	AH	756.20	3.70	755.86	1.57	0.34	0.09	0.09
FG5-107	AJ	768.50	3.74	766.36	1.70	2.14	0.57	0.52
FG5-107	AS	756.30	4.08	754.08	1.90	2.22	0.54	0.49
FG5-105	AH	754.20	3.42	755.86	1.57	-1.66	-0.48	-0.44
FG5-105	AJ	765.90	3.44	766.36	1.70	-0.46	-0.13	-0.12
FG5-105	AQ	754.00	5.02	757.90	1.66	-3.90	-0.78	-0.74
FG5-105	AT	754.10	3.48	755.23	1.73	-1.13	-0.33	-0.29
FG5-236	AG	754.60	3.98	756.29	1.75	-1.69	-0.43	-0.39
FG5-236	AJ	765.70	4.02	766.36	1.70	-0.66	-0.17	-0.15
FG5-236	AQ	757.30	3.96	757.90	1.66	-0.60	-0.15	-0.14
FG5-236	AS	754.40	4.28	754.08	1.90	0.32	0.08	0.07
FG5X-302	AH	756.40	3.62	755.86	1.57	0.54	0.15	0.14
FG5X-302	AQ	759.10	3.62	757.90	1.66	1.20	0.33	0.30
FG5X-302	AS	753.00	3.64	754.08	1.90	-1.08	-0.30	-0.26
FG5X-302	AT	753.40	3.62	755.23	1.73	-1.83	-0.51	-0.46

7.3 Linking converter

The linking converter of SIM.M.G-K1 to CCM.G-K2 is through the DoE of -0.4 µGal ± 2.65 µGal (k=1) determined for the DI gravimeter FG5X-216 during CCM.G-K2 [1] through EURAMET.M.G-K2 [2]. The final result of FG5X-216 for SIM.M.G.K1 with *d* set to zero and the datum of FG5-218 on pier AT removed is -1.15 µGal ± 1.09 µGal. Therefore the linking converter is *d* = 0.75 µGal ± 2.86 µGal (k=1) (±5.73 µGal k=2).

Since the linking converter of SIM.M.G-K1 to CCM.G-K2 relies on a single DI, a subsequent verification is performed by calculating a linking converter of SIM.M.G-K1 to CCM.G-K1 through the DoE of NMI gravimeter FG5-105 of -1.0 µGal ± 2.7 µGal (k=1) for CCM.G-K1. The final result of FG5-105 for SIM.M.G-K1 with *d* set to zero and the datum of FG5-218 on pier AT removed is -1.39 µGal ± 0.81 µGal. Therefore the linking converter for SIM.M.G-K1 to CCM.G-K1 would be *d* = 0.39 µGal ± 2.82 µGal (k=1) (±5.64 µGal k=2). The difference of 0.36 µGal between the two links are well within the uncertainties of the links, adding confidence in the stability of the link.

7.4 Final results

A final LSA was performed excluding the measurement of the FG5-218 at pier AT and with d = 0.75 µGal to obtain the best estimates for the KCRVs, given in Table 8. The expanded standard uncertainty U_j includes the expanded standard uncertainty of 5.73 µGal from the linking converter. For 30 degrees of freedom (48 input data, 18 unknowns), the chi square statistic is χ^2 = 13.8 with probability $p(13.8|30)$ = 0.99 and a Birge ratio of $R_B = \sqrt{13.8/30}$ = 0.68, indicating a very high level of consistency for the input data. Following Jiang et al. [13], the DoEs are the weighted average difference between the measurements of a gravimeter i and the KCRVs at given site j,

$$D_i = \left[\sum w_{ij}\left(g_{ij} - g_j\right)\right] \;/\; \sum w_{ij} \quad , \tag{10}$$

where the weights $w_{ij} = 1/U_{Dij}^2$ and U_{Dij} is the expanded uncertainty of the difference g_{ij} - g_j. The differences, expanded uncertainties, and DoEs are shown in Table 9. U_{Di} is the expanded uncertainty of the weighted differences. The official DoE results for comparison SIM.M.G-K1 are given in Table 10 and Figure 4. All the gravimeters are in excellent equivalence.

Table 8. Key Comparison Reference Values (KCRVs) linked to the CCM.G.K-2 using linking converter of (0.72 ± 5.73) µGal (k=2) related to DI gravimeter FG5X-216. The constant value 979 622 000.0 µGal is subtracted from the KCRVs. U is the expanded standard uncertainty at 95% confidence computed as the root mean square of the expanded standard uncertainty from the LSA and the expanded uncertainty of the linking converter.

Official Key Comparison Results		
Pier	KCRV/µGal	U/µGal
AG	755.68	5.99
AH	755.24	5.94
AJ	765.61	5.98
AQ	757.14	5.97
AS	753.43	6.04
AT	754.09	6.01

Table 9. DoEs of NMI/DIs (pink) and non-NMI/DIs. g_{ij} are the measured gravity values transferred to 125 cm with expanded uncertainty U_{ij}. g_j are the KCRVs with associated expanded (k=2) uncertainties U_j given in Table 8. U_{Dij} is the expanded uncertainty of differences g_{ij}-g_j. D_i is the final DoE computed according to Eq.10 along with the expanded uncertainty U_{Di}. The constant value 979 622 000.0 µGal was subtracted from the gravity measurements.

Gravimeter	Pier	g_{ij} /µGal	U_{ij} /µGal	g_j /µGal	U_j /µGal	g_{ij}-g_j /µGal	U_{Dij} /µGal	D_i /µGal	U_{Di} /µGal
FG5X-252	AG	756.70	4.40	755.68	5.99	1.02	7.44	1.79	3.71
FG5X-252	AH	758.00	4.40	755.24	5.94	2.76	7.39		
FG5X-252	AJ	766.40	4.40	765.61	5.98	0.79	7.42		
FG5X-252	AQ	759.70	4.40	757.14	5.97	2.56	7.41		
FG5X-216	AH	755.20	4.76	755.24	5.94	-0.04	7.61	-0.40	3.82
FG5X-216	AJ	765.50	4.76	765.61	5.98	-0.11	7.64		
FG5X-216	AQ	757.10	4.76	757.14	5.97	-0.04	7.63		
FG5X-216	AS	752.00	4.76	753.43	6.04	-1.43	7.69		
FG5-238	AJ	767.40	3.70	765.61	5.98	1.79	7.03	2.46	3.56

FG5-238	AQ	761.30	3.74	757.14	5.97	4.16	7.04		
FG5-238	AS	755.90	4.12	753.43	6.04	2.47	7.31		
FG5-238	AT	755.50	3.80	754.09	6.01	1.41	7.11		
FG5-234	AG	758.00	3.98	755.68	5.99	2.32	7.20	1.92	3.66
FG5-234	AQ	758.60	3.94	757.14	5.97	1.46	7.15		
FG5-234	AS	755.20	4.80	753.43	6.04	1.77	7.71		
FG5-234	AT	756.20	4.02	754.09	6.01	2.11	7.23		
FG5-218	AG	759.60	3.74	755.68	5.99	3.92	7.07	3.54	4.13
FG5-218	AH	757.30	3.68	755.24	5.94	2.06	6.99		
FG5-218	AS	758.20	4.28	753.43	6.04	4.77	7.40		
FG5-301	AG	755.10	4.66	755.68	5.99	-0.58	7.59	1.30	3.79
FG5-301	AH	758.00	4.64	755.24	5.94	2.76	7.54		
FG5-301	AJ	766.90	4.66	765.61	5.98	1.29	7.58		
FG5-301	AT	755.80	4.68	754.09	6.01	1.71	7.61		
FG5X-102	AG	759.20	3.68	755.68	5.99	3.52	7.03	1.15	3.53
FG5X-102	AJ	765.80	3.70	765.61	5.98	0.19	7.03		
FG5X-102	AS	753.70	3.74	753.43	6.04	0.27	7.10		
FG5X-102	AT	754.70	3.70	754.09	6.01	0.61	7.05		
FG5-204	AG	758.20	3.66	755.68	5.99	2.52	7.02	2.14	3.54
FG5-204	AH	757.50	3.64	755.24	5.94	2.26	6.97		
FG5-204	AQ	756.90	3.66	757.14	5.97	-0.24	7.00		
FG5-204	AT	758.30	4.26	754.09	6.01	4.21	7.36		
FG5-107	AG	756.50	3.76	755.68	5.99	0.82	7.08	1.87	3.55
FG5-107	AH	756.20	3.70	755.24	5.94	0.96	7.00		
FG5-107	AJ	768.50	3.74	765.61	5.98	2.89	7.05		
FG5-107	AS	756.30	4.08	753.43	6.04	2.87	7.29		
FG5-105	AH	754.20	3.42	755.24	5.94	-1.04	6.86	-0.85	3.55
FG5-105	AJ	765.90	3.44	765.61	5.98	0.29	6.90		
FG5-105	AQ	754.00	5.02	757.14	5.97	-3.14	7.80		
FG5-105	AT	754.10	3.48	754.09	6.01	0.01	6.94		
FG5-236	AG	754.60	3.98	755.68	5.99	-1.08	7.20	0.02	3.62
FG5-236	AJ	765.70	4.02	765.61	5.98	0.09	7.20		
FG5-236	AQ	757.30	3.96	757.14	5.97	0.16	7.16		
FG5-236	AS	754.40	4.28	753.43	6.04	0.97	7.40		
FG5X-302	AH	756.40	3.62	755.24	5.94	1.16	6.96	0.51	3.50
FG5X-302	AQ	759.10	3.62	757.14	5.97	1.96	6.98		
FG5X-302	AS	753.00	3.64	753.43	6.04	-0.43	7.05		
FG5X-302	AT	753.40	3.62	754.09	6.01	-0.69	7.01		

Table 10. Official DoE results for comparison SIM.M.G-K1 (according to Eq. 10). The uncertainty U_{DoE} is the expanded uncertainty at 95% confidence of the weighted differences.

Official Key Comparison Results		
Gravimeter	DoE/µGal	U_{DoE}/µGal
FG5X-252	1.79	3.71
FG5X-216	-0.40	3.82
FG5-204	2.14	3.54
FG5-105	-0.85	3.55

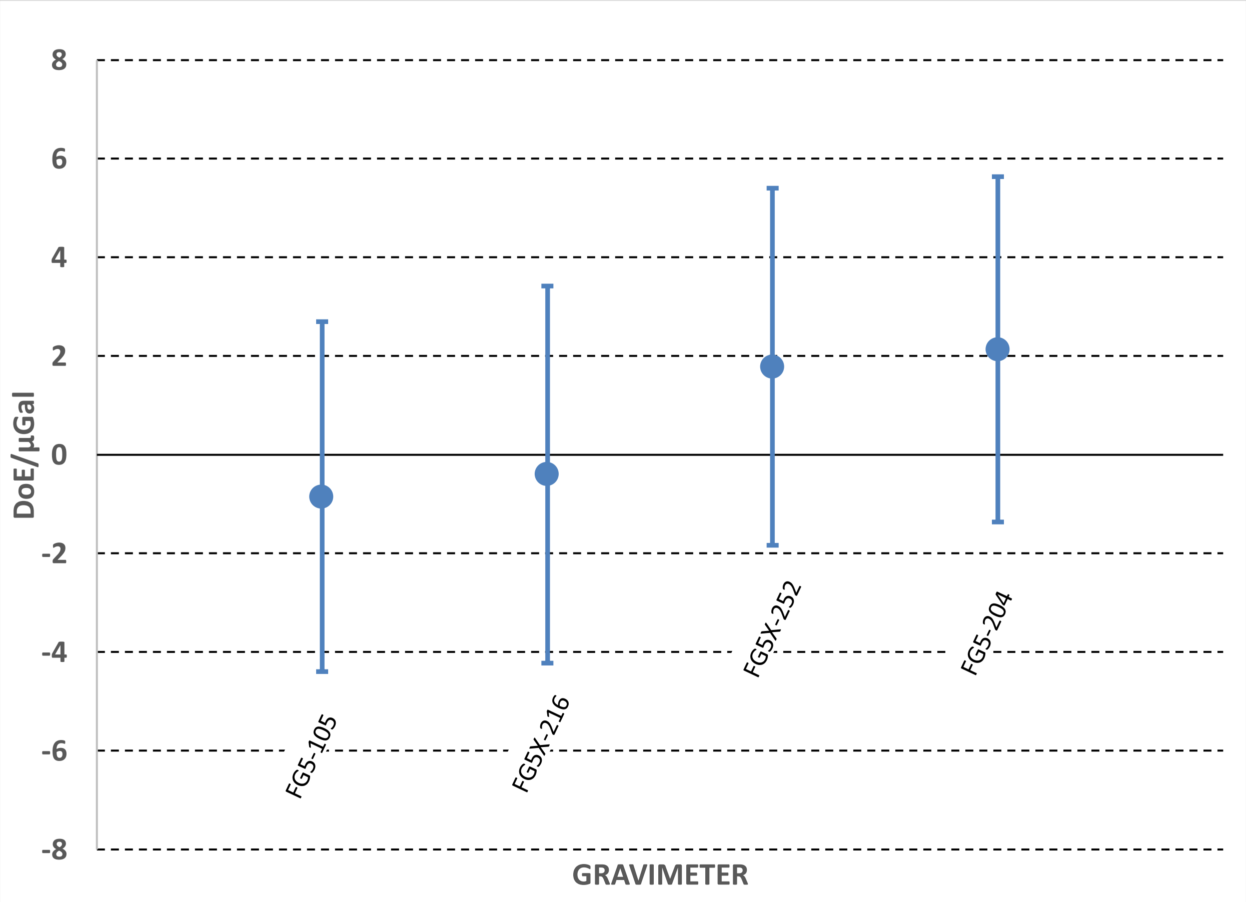

Figure 4. Official DoE results of the NMI/DI gravimeters participating in SIM.M.G-K1. The error bars represent the expanded uncertainties (U_{DoE}) of the DoE at 95% confidence.

8. Conclusions

Twelve absolute gravimeters were compared during the regional Key Comparison SIM.M.G-K1 of absolute gravimeters. Four gravimeters were from different NMIs and DIs. The comparison was linked to the CCM.G-K2 through EURAMET.M.G-K2 via the DI gravimeter FG5X-216. A subsequent verification of the link was provided by calculating the linking converter of SIM.M.G-K1 to CCM.G-K1 through the DoE of NMI gravimeter FG5-105 during CCM.G-K1. The two links differ by 0.36 µGal, well within the uncertainties of the links, adding confidence in the stability of the two links separated by four years. Non-NMI/DI gravimeters participating did not contributed to the determination of the KCRVs. Their *g*-values were used to determine relative gravity ties for a better estimation of gravity differences between the 6 sites used during the comparison. One measurement from a non NMI/DIs was found to be inconsistent at the 95% confidence level and was excluded in the final estimate of the KCRVs. Overall, the results and uncertainties indicate an excellent agreement among the gravimeters, with a standard deviation of the gravimeters' DoEs better than 1.3 µGal.

Appendix A: Pilot Study

The Pilot Study results presented below are from a comparison where all participating gravimeters are treated as equivalent in terms of their contribution to the definition of the reference values (RVs). No link is considered to CCM.G.K2. The analysis follows the same procedure as in Section 6. The set of equations are given in Eqs. 4 and 5 with $d = 0$ and the normalized weights calculated from Eq. 7. The normalized weights $\overline{w}_i$, the initial biases δ, and pier Reference Values g from the variance weighted LSA are given in Table A1.

Table A1. Normalized weights (w_i), initial biases (δ), and pier Reference Values (g) at a height of 125 cm for the Pilot Study using all the reported absolute measurements. The constant value 979 622 000.0 µGal is subtracted from the Reference Values. U is the expanded standard uncertainty at 95% confidence (k=2).

Gravimeter	$\overline{w}_i$	δ/µGal	U/µGal
FG5X-252	0.070	0.46	2.16
FG5X-216	0.059	-1.74	2.35
FG5-238	0.091	0.99	1.88
FG5-234	0.076	0.48	2.04
FG5-218	0.086	3.00	1.92
FG5-301	0.062	-0.12	2.29
FG5X-102	0.098	-0.27	1.81
FG5-204	0.093	0.65	1.85
FG5-107	0.092	0.54	1.87
FG5-105	0.089	-2.11	1.83
FG5-236	0.082	-1.33	1.99
FG5X-302	0.102	-0.93	1.77

Pier	g/µGal	U/µGal
AG	756.93	1.46
AH	756.50	1.45
AJ	767.00	1.48
AQ	758.54	1.50
AS	754.72	1.53
AT	755.87	1.44

The high level of consistency is the same as the initial results in section 7.1, with 31 degrees of freedom, chi square statistic $\chi^2 = 16.3$, probability $p(16.3|31) = 0.99$ and Birge ratio of $R_B = \sqrt{16.3/31} = 0.73$. As before, following the consistency investigation of Section 7.2, the input data of FG5-218 on pier AT has $R_{ij} = 1.39$ and $E_{ij} = 1.31$. This datum is omitted in the final analysis.

Following the procedure outlined in Section 7.4 and using Eq. 10, Table A2 and Figure A1 give the DoEs and their expanded standard uncertainties for the Pilot Study calculated from the differences between gravimeter measurements and the Reference Values in Table A1. All gravimeters are in equivalence. Note that the uncertainties U_j of the Reference Values do not include an uncertainty from a linking converter. As with the Key Comparison SIM.M.G-K1, the overall results and uncertainties indicate good agreement among the gravimeters, with a standard deviation of the gravimeters' DoEs better than 1.3 µGal

Table A2. DoEs from the Pilot Study (according to Eq. 10). The uncertainty U_{DoE} is the expanded uncertainty at 95% confidence of the weighted differences.

Gravimeter	DoE/µGal	U_{DoE}/µGal
FG5X-252	0.46	2.32
FG5X-216	-1.72	2.49
FG5-238	1.14	2.06
FG5-234	0.61	2.21
FG5-218	2.16	2.40
FG5-301	-0.02	2.45
FG5X-102	-0.16	2.00
FG5-204	0.74	2.03
FG5-107	0.52	2.04
FG5-105	-1.99	2.00
FG5-236	-1.32	2.16
FG5X-302	-0.81	1.96

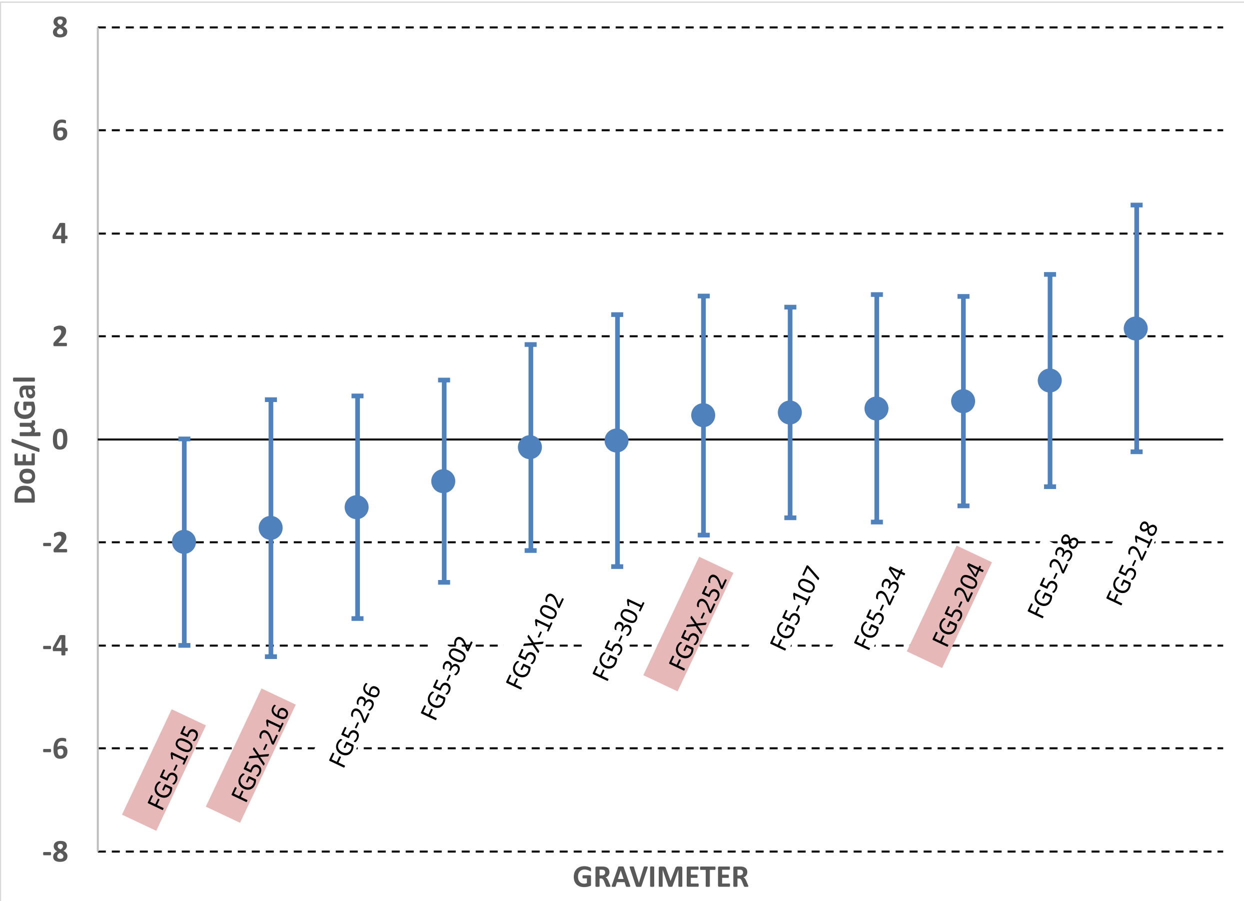

Figure A1. Degrees of Equivalence (DoE) from the Pilot Study calculated from the difference between the gravimeter measurements and the RVs. The error bars represent the expanded uncertainties (U_{DoE}) of the DoE at 95% confidence. Gravimeters of NMI/DIs are highlighted in pink.

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