Final Results of Bilateral Comparison between NIST and PTB for Flows of High Pressure Natural Gas

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ABSTRACT

In 2009 NIST developed a U.S. national flow standard to provide traceability for flow meters used for custody transfer of pipeline quality natural gas. NIST disseminates the SI unit of flow by calibrating a customer flow meter against a parallel array of turbine meter working standards, which in turn are traceable to a pressure-volume-temperature-time (PVTt) primary standard. The calibration flow range extends from 0.125 actual m³/s to 9 actual m³/s with an expanded uncertainty as low as 0.22 % at high flows, and increasing to almost 0.40 % at the lowest flows. Details regarding the traceability chain and uncertainty analysis are documented in prior publications. The current manuscript verifies NIST’s calibration uncertainty via a bilateral comparison with the German national metrology institute PTB. The results of the bilateral are linked to the 2006 key comparison results between three EURAMET national metrology institutes (i.e., PTB, VSL, and LNE). Linkage is accomplished in spite of using a different transfer standard in the bilateral versus the key comparison. A mathematical proof is included that demonstrates that the relative difference between a laboratory’s measured flow and the key comparison reference value is independent of the transfer package for most flow measurement applications. The bilateral results demonstrate that NIST’s natural gas flow measurements are within their specified uncertainty and are equivalent the EURAMET national metrology institutes.

1. INTRODUCTION

In 2009, NIST established a natural gas calibration facility to provide traceability to flow meters used for custody transfer of pipeline quality natural gas. Only three other national metrology institutes (NMIs) in the world disseminate the SI unit for pipeline scale flows of natural gas, including PTB¹ (Germany), VSL² (Netherlands), and LNE³ (France). These three EURAMET NMIs conducted the first key comparison of high pressure natural gas flow in 2006 [1] in order to verify their calibration and measurement capabilities (CMCs) and to determine their degree of equivalence. Likewise, NIST also needs to assess its CMCs and determine if its natural gas calibration service is metrologically equivalent to the EURAMET NMIs. Therefore NIST participated in a bilateral comparison with PTB after commissioning its calibration service in 2009. The results of the bilateral comparison are linked to the 2006 key comparison results (herein referred to as CCM.FF-K5a)⁴ between PTB, VSL, and LNE. Linkage to the previous key comparison is achieved by accounting for 1) PTB’s results in the previous key comparison, 2) the difference between the results of NIST and PTB in this subsequent bilateral comparison, and 3) the long term stability of PTB’s flow standards. The results of the comparison show that NIST is fully equivalent with the EURAMET NMIs within its specified uncertainty.

The approach used in this bilateral is unique among other key comparisons in flow measurement done between NMIs. First, the bilateral results are linked to a key comparison done three years prior (i.e., CCM.FF-K5a). Second, the linkage to the prior key comparison is established using a different transfer package than was used in 2006. Previously, some flow experts believed that it was essential for all participants in a key comparison to use the same transfer package. However, we present a mathematical proof that shows the essential result of a key comparison (i.e., the relative difference between the flow measurement made by participating laboratories and the key comparison reference

¹ Physikalisch-Technische Bundesanstalt
² Van Swinden Laboratorium (sometimes abbreviated NMi-VSL, or NMi).
³ Laboratoire National d’Essais
⁴ The acronym CCM.FF-K5a is the name used for key comparisons in the area of high pressure natural gas. Numeric extensions appended to this acronym such as CCM.FF-K5a.1 and CCM.FF-K5a.2 denote bilateral comparisons subsequent to the key comparison. For example, CCM.FF-K5a.1 is a bilateral comparison between PTB and the secondary laboratory TransCanda Calibrations Ltd. while CCM.FF-K5a.2 is the bilateral comparison between PTB and NIST described in this manuscript.
value) is almost always independent of the transfer package. That is, the uncertainty attributed to using a different transfer package is normally more than an order of magnitude less than the CMCs of the participants and therefore can be neglected. As a result, a key comparison done among a small group of NMIs using a given transfer package can be disseminated to additional NMIs (or secondary laboratories) using a different transfer package.5

The remainder of this manuscript briefly discusses the traceability chains of PTB and NIST, presents a mathematical proof that explains why this bilateral comparison between PTB and NIST can be linked to the CCM.FF-K5a key comparison done in 2006. Finally, we explain the measurement program (flow set points, transfer package geometry and reproducibility, reproducibility and shift of PTB facility, etc.), and present the comparison results.

2. TRACEABILITY CHAINS OF PTB AND NIST

The German national metrology institute PTB establishes traceability to the SI unit for flows of high pressure natural gas using a piston prover primary standard. The primary standard is housed and operated in the test facility *pigssar*TM. For flows up to 400 m³/h, customer calibrations are done using turbine meter working standards that are routinely calibrated against the primary standard. A scaling procedure is used to provide traceability at higher flows up to 6500 m³/h [2, 3]. The volumetric flow uncertainty claimed in the CMC database is 0.16 % with a coverage factor of two (i.e., \( k = 2 \) or at a 95 % confidence level). This uncertainty is supported by the 2006 key comparison results as well as by numerous unofficial comparisons with both VSL and LNE that date back more than a decade. Since the 2006 key comparison, ongoing comparisons between PTB, VSL, and LNE have been used to establish the European Harmonized Reference Value (EHRV) which is disseminated to the customers of all three NMIs.

The U.S. national standard for natural gas flow is traceable to a pressure-volume-temperature-time (PVTt) system. Natural gas calibrations are done using an array of turbine meter working standards that are traceable to the PVTt system through a complex scale-up procedure using parallel arrays of critical flow venturis [4, 5, 6]. The turbine meters are housed and maintained at the CEESI Iowa flow facility in Garner, Iowa, while the PVTt standard is located on NIST campus in Gaithersburg, Maryland. NIST ensures the quality and accuracy of offsite natural gas calibrations by maintaining metrological control over the measurement process as explained in previous publications [5, 6]. The calibration flow range extends from 450 m³/h to 32400 m³/h at a nominal pressure of 7500 kPa ±1500 kPa at ambient temperatures. The expanded uncertainty for volumetric flow ranges from 0.22 % at high flows to 0.4 % at the lowest flows.

3. BASIS FOR LINKING BILATERAL RESULTS TO PREVIOUS KEY COMPARISON

Various types of flow meters or meters under test (MUT) have been used as transfer standards to compare the primary flow standards of NMIs. Depending on the operating principle of the MUT the indicated flow quantity or measurand can be any of the following: 1) totalized volume, 2) totalized mass, 3) volumetric flow, or 4) mass flow. While primary flow standards directly measure the flow quantity, the transfer standards used to compare primary flow standards are flow meters whose performance are characterized by their calibration coefficient, and not by the flow quantity alone. For this reason, the flow meter calibration coefficient is often the parameter compared in inter-comparisons. A parameter related to the calibration coefficient is the *meter deviation*6

\[
 f_i = \frac{Q_{\text{MUT}_i}}{Q_{\text{Lab}_i}} - 1
\]

which quantifies the difference between the flow quantity indicated by the MUT \( Q_{\text{MUT}_i} \) and the reference quantity measured by the Lab \#i \( Q_{\text{Lab}_i} \). Once the meter deviation has been determined by all of the participating laboratories, the key comparison reference value (KCRV) of the meter deviation \( f_{\text{KCRV}} \) is calculated as a weighted mean (\( w_i \) is the weight for Lab \#i),

5 Subsequent comparisons must include at least one NMI that participated in the original key comparison to establish linkage.

6 The ratio \( Q_{\text{MUT}_i}/Q_{\text{Lab}_i} \) has a value close to unity and is directly (or inversely) proportional to the calibration coefficient or meter factor.
\[ f_{\text{KCRV}} = \sum w_i \cdot f_i = \frac{Q_{\text{MUT}}}{Q_{\text{KCRV}}} - 1. \]  

(2)

Equivalently, the KCRV for the meter deviation can also be expressed as a relative deviation of the meter indicated flow quantity to the KCRV flow quantity. The degree of equivalence, which is in wide use in comparisons, is defined by two parameters: 1) the difference between Lab \#i and the KCRV meter deviations

\[ d_i = f_i - f_{\text{KCRV}}, \]  

and 2) the corresponding expanded uncertainty of this difference, \( U(d_i) \).

Another widely used parameter in inter-comparisons is the relative deviation

\[ \frac{\Delta Q_{\#i, \text{rel}}}{Q_{\text{KCRV}}} = \frac{Q_{\text{Lab}\#i} - Q_{\text{KCRV}}}{Q_{\text{KCRV}}}, \]  

(4)

which compares the reference quantity provided by the laboratory \( Q_{\text{lab}} \) to the KCRV flow quantity \( Q_{\text{KCRV}} \). The common understanding among flow experts is that the magnitude of \( \Delta Q_{\#i, \text{rel}} \) is equal; however, the mathematical relationship was never expressed in detail in the previous CCM.FF-protocols. Here, we derive the conditions for which the magnitudes of the difference \( d_i \) and the relative deviation \( \Delta Q_{\#i, \text{rel}} \) are approximately equal.

The derivation begins by multiplying the first term on the right hand side of Eq. (4) by unity \( (i.e., \frac{Q_{\text{MUT}}}{Q_{\text{MUT}}}) \) and applying the definitions of the meter deviations in Eq. (1) and Eq. (2). The second term in Eq. (4) \( (i.e., \text{negative one}) \) is expressed as \( -(f_i + 1)/(f_i + 1) \) and added to the first term resulting in Eq. (5a). Next, the numerator is expressed as a geometric series as shown in Eq. (5b). Because \( f_i \) is much smaller than unity \( (i.e., f_i << 1) \) the geometric series can be approximated as unity without any significant loss in accuracy.7 The final outcome is that \( d_i \) equals the negative value of the original quantity of interest \( \Delta Q_{\#i, \text{rel}} \).

\[
\Delta Q_{\#i, \text{rel}} = \frac{Q_{\text{Lab}\#i} - Q_{\text{KCRV}}}{Q_{\text{KCRV}}} = \frac{f_{\text{KCRV}} + 1}{f_i + 1} - \frac{f_i + 1}{f_i + 1} = f_{\text{KCRV}} - f_i
\]  

(5a)

\[
= (f_{\text{KCRV}} - f_i)(1 - f_i^2 + f_i^4 + \cdots +)
\]  

(5b)

\[
\Delta Q_{\#i, \text{rel}} \approx (f_{\text{KCRV}} - f_i) = -d_i
\]  

(5c)

The expansion by \( Q_{\text{MUT}}/Q_{\text{MUT}} \) in Eq. (5a) is based on the fact that the meter deviation \( f_i \) only weakly depends on the flow quantity \( Q \). In most cases the dependence is small and can be neglected. This is especially true for small changes of the flow quantity.8 The important conclusion from Eq. (5a) is that the expansion is independent of any special value of \( Q_{\text{MUT}} \), and therefore also independent of the meter under test. Consequently, \( \Delta Q_{\#i, \text{rel}} \) is also independent of the MUT. Although flow experts have made use of this result in past CCM.FF-KCs,9 this fact has never been explicitly stated. We rely on this fact herein given that the MUT (or transfer package) used in the previous key comparison is different than the transfer package used in this bilateral comparison between PTB and NIST.

By twice applying Eq. (3) the difference between two laboratories Lab \#i and Lab \#j equals

\[ d_{ij} = d_i - d_j = (f_i - f_{\text{KCRV}}) - (f_j - f_{\text{KCRV}}) = f_i - f_j \]  

(6)

7 For example, if the meter deviations \( f_i \) are on the order of 0.5 \%, the maximum possible error attributed to the approximation used in Eq. (5c) is less than \( \pm 0.005 \% \). In the field of high pressure gas flow measurement this additional uncertainty is insignificant compared to the CMC uncertainties.

8 This assumption is of course not exactly satisfied and leads to additional uncertainty. However, if the level of uncertainty is not acceptable, curve fitting can be used to determine an appropriate mathematical function to account for the dependency of \( f \) on \( Q \) so that any additional uncertainty is minimized (as was done e.g. in CCM.FF-K6).

9 In CCM.FF-K’s 1 through 6 (except 4) the independence of \( \Delta Q_{\#i, \text{rel}} \) from the MUT was used when the results of two different transfer standards were combined into a single result. Moreover, in the case of CCM.FF-K6 the results of 4 different MUTs were combined.
the difference of the meter deviations of the two laboratories. For the bilateral comparison herein the
difference between NIST and PTB is
\[
\langle d_{\text{NIST,PTB}} \rangle_{\text{SC}} = \langle f_{\text{NIST}} \rangle_{\text{SC}} - \langle f_{\text{PTB}} \rangle_{\text{SC}} = \langle f_{\text{NIST}} \rangle_{\text{SC}} - \langle f_{\text{PTB}} \rangle_{\text{KC}} - \Delta_{\text{PTB}}
\]
where the brackets \( \langle \cdot \rangle \) with the indices “KC” and “SC” indicate values determined in the key comparison CCM.FF-KC5a [1] and the subsequent comparison between PTB and NIST detailed in this manuscript. In addition, \( \Delta_{\text{PTB}} = \langle f_{\text{PTB}} \rangle_{\text{SC}} - \langle f_{\text{PTB}} \rangle_{\text{KC}} \) accounts for any shift in PTB’s meter deviation that occurred since the key comparison.

The desired result needed to complete the linkage is the difference between NIST and the KCRV for the previous KC. Using the definition in Eq. (3) for the difference in conjunction with Eq. (7) we obtain
\[
d_{\text{NIST}} = \langle f_{\text{NIST}} \rangle_{\text{SC}} - f_{\text{KCRV}} = \langle d_{\text{PTB}} \rangle_{\text{KC}} + \Delta_{\text{PTB}} + \langle d_{\text{NIST,PTB}} \rangle_{\text{SC}} \approx -\left( \frac{Q_{\text{lab},\text{NIST}}}{Q_{\text{KCRV}}} - 1 \right)
\]
where \( \langle d_{\text{PTB}} \rangle_{\text{KC}} \) is documented in the protocol of the previous KC [1], and the approximation is based on Eq. (5c). Equations (7) and (8) are the basis for linking NIST results to the previous key comparison. Under normal circumstances the value \( \Delta_{\text{PTB}} \) would be taken to be zero with an uncertainty equal to the reproducibility of the Lab #PTB (i.e., stability versus time). However, in the special case of this subsequent comparison it is known that \( \Delta_{\text{PTB}} \) is different from zero due to the complete and new recalibration of the test facility pigsar [3] in 2007, and the new cycle within the European Harmonization Group that started in 2008. We estimate \( \Delta_{\text{PTB}} \) based on measurements done at pigsar in 2004, 2005 (i.e., the harmonization cycle 2004-2007), and in 2009 (i.e., the harmonization cycle 2008-2011).

4. THE MEASUREMENT PROGRAM

Description of the Transfer Package
The transfer package contains two flow meters in series, an ultrasonic flow meter followed by a turbine meter. The nominal diameter of the flow meters and connecting piping is \( D = 300 \text{ mm (12 inch)} \), and the total length is 32 \( D \) (i.e., 9.6 m). The transfer package geometry, the location of the two flow meters and flow conditioners, as well as the two temperature tap locations are shown in Fig. 1. Both flow conditioners are installed 10 \( D \) upstream of flow meter, and the piping immediately downstream of each flow meter is 3 \( D \) in length with the temperature tap located at the mid-section.

Test Protocol
The Reynolds number can be used to characterize the performance of both flow meters in the transfer package (i.e., the turbine meter and ultrasonic flow meter) as was done with a similar transfer package that was used in the comparison between PTB and TCC\(^{10}\) documented in CCM.FF.K5a.1 [7]. Similarly, the transfer package in this bilateral comparison is calibrated over the same Reynolds number range by both PTB and NIST. Measurements are made at 7 points equally spaced on a logarithmic scale between the minimum (5.7 \( \times 10^5 \)) and maximum (2.5 \( \times 10^7 \)) Reynolds numbers. NIST calibrated the transfer package in natural gas at 8.8 MPa over a flow range extending from 840 m\(^3\)/h to 3670 m\(^3\)/h (actual flow). PTB matched the Reynolds numbers by calibrating the transfer

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\(^{10}\) TransCanada Calibrations Ltd.
package at 5 MPa from 1470 m³/h to 6440 m³/h (actual flow). As shown in Table 1 PTB calibrated the transfer package twice, once at the start of the comparison in March 2009 and a second time at the end of the comparison in December 2009. On both occasions the transfer package was calibrated in natural gas at two pressures, 1.6 MPa and 5 MPa. An additional calibration performed 5 years prior in March 2004 was used in conjunction with the two recent calibrations to assess the stability of the transfer package and to link the results to the previous key comparison CCM.FF-K5a. However, only the PTB calibrations done at 5 MPa along with the NIST calibration at 8.8 MPa are used to determine the degree of equivalence between NIST and PTB-pigsar. PTB also recalibrated the transfer package used in CCM.FF-KC5a and CCM.FF-KC5a.1 to provide additional data to establish the link between results from this bilateral and the KCRV of the last key comparison.

Table 1: Transfer package test schedule and calibration conditions. (Only the calibrations done by PTB in natural gas at 5 MPa and by NIST at 8.8 MPa are used to determine the degree of equivalence between NIST and PTB-pigsar.)

<table>
<thead>
<tr>
<th>No.</th>
<th>NMI</th>
<th>Date</th>
<th>Pressure</th>
<th>Working Fluid</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PTB</td>
<td>March 2009</td>
<td>1.6 &amp; 5 MPa</td>
<td>Natural Gas</td>
<td>Initial calibration of transfer package</td>
</tr>
<tr>
<td>2</td>
<td>PTB</td>
<td>March 2009</td>
<td>1.6 &amp; 5 MPa</td>
<td>Natural Gas</td>
<td>Recalibration of transfer package used in CCM.FF-KC5a and CCM.FF-KC5a.1 =&gt; information for linking to KCRV ($\Delta_{PTB}$)</td>
</tr>
<tr>
<td>3</td>
<td>NIST</td>
<td>July 2009</td>
<td>8.8 Mpa</td>
<td>Natural Gas</td>
<td>calibration of transfer package</td>
</tr>
<tr>
<td>4</td>
<td>PTB</td>
<td>Dec. 2009</td>
<td>1.6 &amp; 5 MPa</td>
<td>Natural Gas</td>
<td>closing calibration of transfer package</td>
</tr>
</tbody>
</table>

Reproducibility of the transfer package and the PTB-pigsar facility

When two flow meters in series (e.g., the transfer package in Fig. 1) are calibrated by a common reference standard, correlation techniques can be used to separate the overall reproducibility of the measured meter deviations in components attributed to each of the flow meters and to the reference standard. In this manuscript we split the overall reproducibility into its respective components using the correlation technique of Pöschel [8]. This technique has been used successfully in two prior key comparisons, CCM.FF-K5a [1] and CCM.FF-K5b [9], and a full description of the correlation technique as it applies to comparisons is described in these references.

The scatter plot in Fig. 2 qualitatively portrays the level of correlation between simultaneous measurements of the two flow meters in series shown in Fig. 1. The measurements are made at PTB-pigsar in natural gas at 1.6 MPa and 5 MPa. The residuals (i.e., $f - f_{ave}$) from flow meter 1 (i.e., the turbine flow meter) and flow meter 2 (i.e., the ultrasonic flow meter) are plotted on the x-axis and y-axis, respectively. The residual is defined as the difference between measured values of the meter deviation ($f$) with the average value ($f_{ave}$). For each flow meter $f_{ave}$ is the least square fit (LSF) to all of its calibration data as a function of Reynolds. The standard deviation of the horizontal (or vertical) scatter in the residuals is the overall reproducibility of the measured meter deviations for flow meter 1 (or flow meter 2). After separating the reproducibility into its respective components, we find the reproducibility attributed to flow meter 1 is 0.015 % and 0.022 % for flow meter 2 both at the 68 % confidence level (i.e., $k = 1$). Both values are repeated in Table 2 for CCM.FF-K5a.2 at the 95 % confidence level. The table also shows the reproducibility of the PTB-pigsar facility for CCM.FF-K5a.2 to be 0.062 $^{+0.019}_{-0.012}$. This level of reproducibility is commensurate with the level of reproducibility determined in past comparisons (i.e., CCM.FF-K5a, CCM.FF-K5a.1, and EURAMET.M.FF-K5.a) which are also shown in the table.

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11 Note that $0.062^{+0.019}_{-0.012}$ should be interpreted as 0.062 % for the estimated value and 0.062 % - 0.012 % = 0.05 % for the lower confidence level as well as 0.062 %+ 0.019 % = 0.081 % for the upper confidence level ($k = 2$).
Figure 2: Correlation plot of the residuals of the meter deviation for the two flow meters used in CCM.FF-K5a.2. (Data taken at PTB-pigsar.)

Table 2: Tabulated results for reproducibility $U_{\text{reprod}} (k = 2)$ of the transfer meters and the pilot lab in all comparison loops related to CCM.FF-K5

<table>
<thead>
<tr>
<th>Flow meter Diameter</th>
<th>Reproducibility [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D = 150$ mm</td>
</tr>
<tr>
<td></td>
<td>Meter #1 83034949</td>
</tr>
<tr>
<td>Type of Comparison</td>
<td></td>
</tr>
<tr>
<td>CCM.FF-K5a [1]</td>
<td></td>
</tr>
<tr>
<td>Key Comparison</td>
<td>0.050$^{+0.009}_{-0.007}$</td>
</tr>
<tr>
<td>between PTB, VSL, &amp; LNE</td>
<td></td>
</tr>
<tr>
<td>CCM.FF-K5a.1 [7]</td>
<td></td>
</tr>
<tr>
<td>Bilateral Comparison</td>
<td>0.038$^{+0.007}_{-0.005}$</td>
</tr>
<tr>
<td>PTB &amp; TCC</td>
<td></td>
</tr>
<tr>
<td>CCM.FF-K5a.2</td>
<td></td>
</tr>
<tr>
<td>Bilateral Comparison</td>
<td>--</td>
</tr>
<tr>
<td>PTB &amp; NIST</td>
<td></td>
</tr>
<tr>
<td>EURAMET.M.FF-K5.a</td>
<td></td>
</tr>
<tr>
<td>European Flow Lab Comparison</td>
<td>0.064$^{+0.021}_{-0.013}$</td>
</tr>
</tbody>
</table>

Shift in the calibration value at PTB-pigsar and linkage to the KCRV of CCM.FF-KC5a

The complete recalibration of the test facility pigsar [3] in 2007 and the new cycle within the European Harmonization Group starting in 2008 (i.e., PTB, LNE, and VSL recalibrate their facilities) defined a new calibration value at the test facility PTB-pigsar. To link the NIST test results to the KCRV we account for any shift at PTB between 2004 and 2009, $\Delta_{\text{PTB}} = \langle f_{\text{PTB}} \rangle_{\text{SC}} - \langle f_{\text{PTB}} \rangle_{\text{KC}}$. The amount of shift is determined using the following data sets measured at PTB-pigsar before and after recalibration:
- both flow meters used in CCM.FF-K5a.2
- both flow meters used in EURAMET.M.FF-K5a
- all flow meters used inside harmonisation between PTB, VSL, and LNE
- all working standards of PTB-pigsar

The shifts for the different sets of flow meters are plotted in Fig. 3 below.
As shown in Fig. 3 the average shift attributed to recalibration is $\Delta_{PTB} = 0.082 \pm 0.075\%$ where the 0.075 % is the uncertainty at the 95 % confidence level.

**Difference between PTB and the KCRV in CCM.FF-K5a**

The values for $\langle d_{PTB}\rangle_{KC}$ in CCM.FF-K5a were determined in a range of mass flow rate between 1040 kg/h and 37600 kg/h as shown in Fig. 4. The expanded uncertainty of $\langle d_{PTB}\rangle_{KC}$ ranged from 0.08% to 0.10%.

**Figure 3:** Shift in the calibration values for meters under test at PTB-pigsar due to recalibration in 2007 for different sets of flow meters. The shift is plotted versus the mass flow rate $Q_m$ because the Reynolds number is not comparable due to different pipe sizes of the meters (100 mm to 400 mm)

**Figure 4:** Results for the difference of PTB $\langle d_{PTB}\rangle_{KC}$ to the KCRV in the CCM.FF-K5a.
5. COMPARISON RESULTS

Measurement results from NIST and PTB are shown for the turbine meter (TM 74174) in Fig. 5 and for the ultrasonic flow meter (USM 2740) in Fig. 6. In both figures the meter deviation is plotted against the Reynolds number on a logarithmic scale. The solid lines are least square fits (LSF) to the measured data for both PTB and NIST.

![Figure 5](image-url)  
**Figure 5:** Plot of meter deviation ($f$) versus Reynolds number ($Re$) for TM 74174.

The evaluation of the results for the ultrasonic meter (USM) was based directly on the four path velocities $v_{path#i}$ documented in the log files without any correction. For each path, a meter deviation is defined as follows: $f_{USM,path#i} = \left( \frac{v_{path#i}}{v_{bulk}} - 1 \right) \times 100$ where $v_{bulk}$ is calculated by dividing the actual flow measured by the laboratory by the cross sectional area of the USM ($v_{bulk} = Q_{LabRef}/A_{meter}$).

![Figure 6](image-url)  
**Figure 6:** Plot of meter deviation ($f_{USM}$) for each path of the ultrasonic meter (USM 2740) versus Reynolds number. The solid lines for each path are a LSF expressed as a linear function of log($Re$).
Differences between NIST versus PTB, and NIST versus the KCRV of CCM.FF-K5a

The differences between the measurement results of NIST and PTB (d_{NIST,PTB}) for this bilateral comparison CCM.FF-Ka.2 are calculated using Eq. (7). These differences are plotted in Fig. 7 for the turbine meter, the ultrasonic flow meter, and their average over the tested Reynolds number range (i.e., from $5.7 \times 10^6$ to $2.5 \times 10^7$). Similarly, the differences between the measurement results of NIST and the KCRV of CCM.FF-K5a (d_{NIST}) are calculated using Eq. (8) over the same range of Reynolds numbers, and the corresponding results are plotted in Fig. 8. In both figures the expanded uncertainty indicated by the dashed lines are given as an acceptance band.

![Figure 7: Plot of differences between NIST and PTB versus Reynolds number (Re) for both meters (turbine meter and USM) and their average value.](image1)

**Figure 7:** Plot of differences between NIST and PTB versus Reynolds number (Re) for both meters (turbine meter and USM) and their average value.

![Figure 8: Plot of differences between NIST and KCRV of last key comparison (CCM.FF-K5a) versus Reynolds number (Re). Results are shown for both meters (turbine meter and ultrasonic flow meter) and their average value.](image2)

**Figure 8:** Plot of differences between NIST and KCRV of last key comparison (CCM.FF-K5a) versus Reynolds number (Re). Results are shown for both meters (turbine meter and ultrasonic flow meter) and their average value.

Applying the propagation of uncertainty to the difference between NIST and the PTB (d_{NIST,PTB}) in Eq. (7) the resulting expanded uncertainty is
Similarly, the propagation of uncertainty is applied to Eq. (8) to determine the expanded uncertainty of the difference between NIST and the KCRV of the previous key comparison (CCM.FF-K5a)

\[ U(d_{\text{NIST, PTB}}) = \sqrt{U^2(d_{\text{NIST}}) + U^2(d_{\text{PTB}})}. \]  \hspace{1cm} (9)

where the last term is the covariance between the measurements of PTB during the CCM.FF-KC5a and this subsequent comparison. The covariance accounts for correlation between the two sets of PTB measurements and in general has a value between zero and unity. To obtain the most conservative estimate of \( U(d_{\text{NIST}}) \) the covariance is taken to be zero so that the expanded uncertainty is given by

\[ U(d_{\text{NIST}}) = \sqrt{U^2(d_{\text{PTB,KCRV}}) + U^2(d_{\text{NIST, PTB}}) + U^2(\Delta_{\text{PTB}}) - 2\text{cov}(f_{\text{PTB,SC}}, f_{\text{PTB,KC}})}. \]  \hspace{1cm} (10)

The differences for the meter deviations as given in Fig. 7 and Fig. 8 are compared with their expanded uncertainty by means of the standardized degree of equivalence defined as \(^{12}\n
\[ E_n = \frac{|d|}{U(d)}. \]  \hspace{1cm} (12)

Measurement results are considered fully equivalent for \( E_n < 1 \). Figure 9 plots \( E_n \) values (averaged for both flow meters) for CCM.FF-K5a2 (i.e., difference between NIST and PTB) and CCM.FF-K5a (i.e., difference between NIST and the KCRV of the previous key comparison). In both cases the \( E_n \) values are significantly less than unity indicating that the NIST results are fully equivalent with EURAMET NMIs. Tables 3 and 4 show numerical values of the data.

\[ \text{Figure 9: Plot of degree of equivalence (} E_n \text{) versus Reynolds number (} R_e \text{). (Results are the average of both meters.)} \]

\(^{12}\) The absolute value in the numerator in Eq. (12) is a carryover from previous Euramet key comparisons between PTB, VSL, and LNE. It was introduced to prevent calibration customers of these NMIs from taking advantage of biases between these laboratories.
7. SUMMARY

A bilateral comparison was done between NIST and PTB for flows of high pressure natural gas. The transfer package included two flow meters in series, a turbine meter and an ultrasonic flow meter. The meter deviation (or calibration coefficient) was compared for each flow meter over the same Reynolds numbers range from $5.7 \times 10^6$ to $2.5 \times 10^7$. For NIST these Reynolds numbers corresponded to a flow range from $837 \text{ m}^3/\text{h}$ to $3672 \text{ m}^3/\text{h}$ at 8.8 MPa. The NIST results were linked to a previous key comparison between PTB and two EURAMET NMIs. This key comparison was done in 2006 using a different transfer package. A mathematical proof is developed that shows that the essential result of a key comparison (i.e., the relative difference between a flow measurement made by a participating laboratory and the key comparison reference value) is independent of the transfer package for uncertainty levels realized in most flow comparisons. The stability of the PTB facility played a critical role in linking the bilateral results to the previous key comparison since PTB calibrated both transfer packages. For this reason the linkage accounted for a known shift of 0.08% at the PTB facility which occurred in the time period between the two comparisons and resulted from recalibrating and improving its flow standards. The results of the comparison demonstrate that NIST’s flow measurements of high pressure natural gas are within their uncertainty specifications and are metrologically equivalent with the EURAMET NMIs.
6. REFERENCES


