APMP.M.F-K4.b Key Comparison
for 2 MN Force

Final Report
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Pilot Institute: NMIJ/AIST

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1. Introduction

The APMP.M.F-K4.b key comparison (KC) in the very high force range is initiated based on a decision of the APMP TCM meeting in October 2004 in Beijing, China. This comparison is planned to demonstrate the degree of equivalence between the national standards of force and to provide evidence for the calibration and measurement capability (CMC) claimed by participating national metrology institutes (NMIs) in the Asia-Pacific region, especially for those who did not participate in the global CCM.F-K4 KC. The APMP TCM approved that NMIJ/AIST of Japan to serve as the pilot institute and KRISS of Korea would be an advisory institute.

This document describes the comparison scheme and reports the results of the comparison.

2. Organization

2.1 Participants

Seven institutes were participated in this KC. A few more institutes showed their interests of participation, but they called off during the process of arranging or even rearranging the circulation schedule. NIST, who piloted the CCM.F-K4 KC, was later invited to this KC for the purpose of tightening the link to the CCM.F-K4 KC. The participated institutes and used force standard machines (FSMs) are listed in Table 1.

Table 1: Participated institutes and force standard machines

| Country / Economy (alphabetical order) | Institute       | Force standard machine | Capacity / MN | Type                          | Relative standard uncertainty of applied force | Note                                                                 |
|--------------------------------------|-----------------|------------------------|---------------|-------------------------------|-----------------------------------------------|                                                                     |
| Australia                            | NMIA            |                        | 4.5           | Build-up                      | 20x10^{-5}                                   | Reference transducer traceable to NMIA itself                  |
| Chinese Taipei                       | CMS/ITRI        |                        | 2             | Comparator                    | 25x10^{-5}                                   | Reference transducer traceable to PTB, Germany                  |
| Hong Kong, China                     | SCL             |                        | 3             | Hydraulic amplification       | 25x10^{-5}                                   |                                                                      |
| Japan                                | NMIJ/AIST       |                        | 20            | Hydraulic amplification       | 4.4x10^{-5}                                  | Link lab                                                            |
| Korea                                | KRISS           |                        | 2             | Hydraulic amplification       | 5.0x10^{-5}                                  | Link lab                                                            |
| Singapore                            | A*STAR (ex SPRING) |                    | 2             | Lever amplification           | 5.0x10^{-5}                                  |                                                                      |
| USA                                  | NIST            |                        | 4.448222      | Deadweight                    | 0.50x10^{-5}                                  | Link lab                                                            |

2.2 Comparison scheme

The scheme of this APMP.M.F-K4.b KC basically adheres to that of the CCM force KCs decided by the CCM Force Working Group meeting in October 1998 in Sydney.

2.2.1 Force range

A provisional survey revealed that APMP NMIs did not have force standard machines (FSMs) of over 3 MN capacities with the exception of one institute and those who already participated in the CCM.F-K4 KC. Taking this situation into account, it was agreed that force range of the APMP.M.F-K4 KC was simplified to
only 2 MN range (corresponding to the CCM.F-K4.b KC) and 4 MN range (corresponding to the CCM.F-K4.a KC) comparison do not take place this time.

2.2.2 Formation

The comparison was carried out in the star formation due to the limitation of long-term stability of traveling standards, even though state-of-the-art force transducers were adopted for this purpose.

A typical schedule of one cycle of the circulation was as follows.

1) Measurements by the pilot: 1 week
2) Transport to a participant: 2 weeks
3) Measurements by the participant: 2 weeks
4) Transport back to the pilot: 2 weeks
5) Measurements by the pilot: 1 week

2.2.3 Environmental conditions

Considering the climate in the Asia-Pacific region, the reference ambient temperature of this KC was chosen to be 23 °C instead of 20 °C of the CCM KCs. Temperature fluctuation exceeding ±0.2 °C, which is the limit in the CCM KCs, could be accepted in the APMP.M.F-K4.b KC provided that the influence of temperature fluctuation on measurement results was properly taken into account. The reference ambient temperature and its allowance were the only departures from the conditions of the CCM KCs.

2.2.4 Measurement sequence

The measurement sequence of this KC is shown in Figure 1, which adhered to that of the CCM KCs. Note that creep behavior in force transducer’s output is exaggerated in this diagram.

1) A measurement series with orientation changes from 0 ° to 720 ° in 60 ° pitch in a day, requiring approximately 7 h for one series.

OPTION: Two measurement series with orientation changes from 0 ° to 360 ° in 60 ° pitch in each of two consecutive days, requiring approximately 4.5 h for each series.

2) In the transducer’s orientation of 0 °, three preloads (from zero to 2 MN), one preload (from zero to 2 MN) and three measurement cycles (from zero to 2 MN). In the following orientations from 60 ° to 720 °, one
preload (from zero to 2 MN) and one measurement cycle (from zero to 2 MN).

3) Reading at each force step in 6 min time interval.

4) Orientation changes (rotations) of the force transducer just after unloading.

5) Calibration of each participant’s HBM DMP40 measuring amplifier by referring to the traveling HBM BN100A bridge calibration unit just before and just after each measurement series.

2.3 Traveling artifacts

The following equipment was circulated in one package as the traveling artifacts.

1) Force transducer 1 (Tr1)
   Capacity: 2 MN in compressive force,
   Manufacturer: HBM GmbH, Type: C18/2MN, Serial number: 00281RX3

2) Force transducer 2 (Tr2)
   Capacity: 2 MN in compressive force,
   Manufacturer: A&D Orientec Co. Ltd., Type: UL-2MN-KE1, Serial number: 110030

3) Bridge calibration unit
   Manufacturer: HBM GmbH, Type: BN100A, Serial number: 08112

4) Temperature and humidity logger
   Manufacturer: Veriteq Instruments, Inc., Type: SP-2000,
   Resolution: 0.2 °C, 2 %RH

5) Shock recorder
   Manufacturer: SRIC Corporation, Type: G-MEN 3GT,
   Acceleration range: up to 98 m/s², Resolution: 0.98 m/s², Frequency band: 0 Hz to 500 Hz

3. Results of the comparison

3.1 Traveling conditions

During the transportation of the traveling artifacts, temperature and humidity were varied from 12.8 °C to 29.8 °C and from 38 %RH to 74 %RH, respectively, and peak acceleration up to 81 m/s² was recorded. However, no serious damage was found on the equipment.

3.2 Reported results and uncertainties

Table 2 shows chronological order of the measurements, reported results and uncertainties. Most of the participants reported the values with corrections for the DMP40 indications and for temperature difference from the reference temperature of 23.0 °C, and with extrapolation to exactly 2 MN in the case of NIST. On the results of two participants, these corrections were made at the pilot.

At A*STAR, its FSM had a mechanical trouble during the measurement on Tr1. A*STAR gave up continuation of the measurements in consultation with the pilot, since it was estimated to take more than another two weeks to repair the FSM. Therefore, A*STAR reported only the measurement results of Tr2.
3.3 Stability of the traveling artifacts

3.3.1 Stability of the BN100A bridge calibration unit

In each measurement at the pilot, output signals of the BN100A bridge calibration unit were monitored by the same DMP40 amplifier always kept in the laboratory environment. The signals at 2.0 mV/V, 2.4 mV/V and 2.5 mV/V were found to be stable within a relative fluctuation of $5 \times 10^{-6}$. Thus, the uniformity of different DMP40 amplifiers of the participating institutes was secured by referring to the same traveling BN100A bridge calibration unit.

3.3.2 Stability of the force transducers

Figures 2 and 3 show stability of the traveling force transducers over the period of this KC. The figures indicate relative deviations of measurement results at the pilot with respect to an arithmetical mean value of them. The uncertainty bars represent relative expanded uncertainties of the measurements with the coverage factor $k=2$. 

---

<table>
<thead>
<tr>
<th>Institute</th>
<th>Code</th>
<th>Date</th>
<th>Deflection X / mV/V</th>
<th>Relative expanded uncertainty $W(X) \times 10^{-4}$</th>
<th>Date</th>
<th>Deflection X / mV/V</th>
<th>Relative expanded uncertainty $W(X) \times 10^{-4}$</th>
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<td>Pilot</td>
<td>P1</td>
<td>2005-06-14</td>
<td>2.001 536</td>
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<td>2005-06-15</td>
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<td>2.001 628</td>
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<tr>
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<td>P3</td>
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<td>2.001 639</td>
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<td>2005-09-26</td>
<td>2.437 115</td>
<td>1.18</td>
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<tr>
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<td>2005-11-29</td>
<td>2.001 655</td>
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<td>2005-11-30</td>
<td>2.437 104</td>
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<tr>
<td>SCL</td>
<td>L3</td>
<td>2006-04-10</td>
<td>2.002 686</td>
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<td>5.05</td>
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<td>2006-06-27</td>
<td>2.001 727</td>
<td>1.96</td>
<td>2006-06-26</td>
<td>2.437 267</td>
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<td>P8</td>
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<td>2.001 717</td>
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<td>2.437 317</td>
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<td>2006-10-25</td>
<td>2.001 802</td>
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<td>2006-10-24</td>
<td>2.437 199</td>
<td>0.97</td>
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<tr>
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<td>2.001 846</td>
<td>1.62</td>
<td>2006-12-12</td>
<td>2.437 369</td>
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<td>A*STAR</td>
<td>L5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2007-02-09</td>
<td>2.437 573</td>
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<td>0.96</td>
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<td>1.78</td>
<td>2007-04-10</td>
<td>2.437 403</td>
<td>1.01</td>
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<td>0.27</td>
<td>2007-05-23</td>
<td>2.437 565</td>
<td>0.21</td>
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Neither of the force transducers exhibited sufficient stability which enables comparison of the measurement results of all loops at once. Therefore, the measurement results of each loop should be evaluated separately by referring to the measurements made by the pilot just beginning and ending of the loop. It would be
preferable if more stable force transducers than current ones could become available.

3.4 Corrections

3.4.1 Correction for the use of different DMP40 amplifier

Deviations of different DMP40s were within 18 nV/V except for CMS/ITRI. A DMP40 of CMS/ITRI exhibited exceptionally large deviations of 170 nV/V at about 2.000 mV/V and 214 nV/V at about 2.437 mV/V. However, the BN100A was found to be stable within a relative drift of 1.3x10⁻⁶ in the proceeding and succeeding measurements at the pilot for CMS/ITRI. Therefore, corrections for different DMP40s were made systematically by referring to the same traveling BN100A.

3.4.2 Correction for the different ambient temperature

Before circulating the traveling artifacts, the pilot had evaluated temperature coefficients of the two traveling force transducers by varying the laboratory temperature from 22 °C to 26 °C. The relative temperature coefficients were determined to be +2.2x10⁻⁵ K⁻¹ for Tr1 and +1.1x10⁻⁵ K⁻¹ for Tr2. However, associated uncertainties were estimated to be 2.6x10⁻⁵ for Tr1 and 1.3x10⁻⁵ for Tr2 at around 23 °C.

Temperature fluctuation during any measurement series did not exceed ±0.2 °C and average temperature did not deviate more than 0.5 °C from the reference temperature of 23.0 °C, except for the case of A*STAR. Therefore, it is not always necessary to make corrections for the temperature differences except for A*STAR, as the uncertainties associated with the corrections are larger than the corrections themselves.

Since it was quite difficult for A*STAR to change its laboratory temperature from ordinary temperature of 20 °C to the KC reference temperature due to the limitation of air conditioning facility, the temperature correction was applied to the results of A*STAR and associated uncertainty was taken into account.

3.4.3 Correction for the sensitivity drift

As described above, the traveling force transducers were not stable enough. Therefore, corrections were made for the sensitivity drifts of the force transducers by referring to the results of proceeding and succeeding measurements at the pilot for each of the loops.

3.4.4 Correction for the creep effect

It takes from 1 min to 3 min for the participated FSMs to increase force from zero up to 2 MN, while the relative magnitude of creep is estimated to be only 0.6x10⁻⁵ for Tr1 or 1.3x10⁻⁵ for the Tr2 in the period between 3 min and 5 min after loading. On the other hand, uncertainties associated with evaluation of the creep are estimated to be approximately 2x10⁻⁵ for both force transducers, which are also larger than the corrections themselves. Therefore, no correction against the creep effect was applied to the measurement results.

3.5 Relative deviation of the measured values from the mean value of the pilot

For each of the two force transducers, a measured result \( X_{Li} \) of participant \( i \) is an arithmetical mean of 12 values measured at orientations from 60 ° to 720 °. Relative deviation \( d_{Li,P} \) of the measured result \( X_{Li} \) of participant \( i \) from that of the pilot \( X_{Pm,i} \) is calculated by referring to the proceeding measurement result \( X_{Pa,i} \) and the succeeding measurement result \( X_{Pb,i} \) at the pilot for each loop. Uncertainty \( u(d_{Li,P}) \) is evaluated with
considering sensitivity drift of the force transducer and correlation between the proceeding and succeeding measurement results at the pilot. The correlation coefficient \( r(X_{Pa,i}, X_{Pb,i}) \) is not always zero because the two measurements were made by the same force standard machine of the pilot. The relative deviations \( d_{L,P} \) and their expanded uncertainties \( U(d_{L,P}) \) with the coverage factor \( k=2 \) are tabulated in Table 3 shown later in this section.

\[
d_{L,P} = \frac{X_{Li} - X_{Pm,i}}{X_{Pm,i}} \quad (1)
\]

\[
u(d_{L,P}) = \sqrt{\left(\frac{u(X_{Li})}{X_{Pm,i}}\right)^2 + \left(\frac{u(X_{Pm,i})}{X_{Pm,i}}\right)^2 + \frac{1}{12}\left(\frac{\Delta X_{P,i}}{X_{Pm,i}}\right)^2} \quad (2)
\]

where

\[
X_{Pm,i} = \frac{X_{Pa,i} + X_{Pb,i}}{2} \quad (3)
\]

\[
u(X_{Pm,i}) = \sqrt{\frac{1}{4}\left[\left(\frac{u(X_{Pa,i})}{X_{Pa,i}}\right)^2 + \left(\frac{u(X_{Pb,i})}{X_{Pb,i}}\right)^2 + 2r(X_{Pa,i}, X_{Pb,i})u(X_{Pa,i})u(X_{Pb,i})\right]} \quad (4)
\]

\[
\Delta X_{P,i} = |X_{Pa,i} - X_{Pb,i}| \quad (5)
\]

Mean value \( \bar{d}_{\text{Link-P}} \) of the relative deviations of the three link institutes from the pilot and its uncertainty \( u(\bar{d}_{\text{Link-P}}) \) are calculated as follows by taking a weighted mean. These values are shown in the bottom row of Table 3.

\[
\bar{d}_{\text{Link-P}} = \frac{d_{\text{KRISS-P}}/u(d_{\text{KRISS-P}})^2 + d_{\text{NIST-P}}/u(d_{\text{NIST-P}})^2 + d_{\text{NIMJ-P}}/u(d_{\text{NIMJ-P}})^2}{1/u(d_{\text{KRISS-P}})^2 + 1/u(d_{\text{NIST-P}})^2 + 1/u(d_{\text{NIMJ-P}})^2} \quad (6)
\]

\[
u(\bar{d}_{\text{Link-P}}) = \sqrt{\frac{1}{1/u(d_{\text{KRISS-P}})^2 + 1/u(d_{\text{NIST-P}})^2 + 1/u(d_{\text{NIMJ-P}})^2}} \quad (7)
\]

The chi-squared test is applied to the results of the three link institutes [1]. The observed chi-squared values \( \chi^2_{\text{obs}} \) are calculated as 3.34 for Tr1 and 0.91 for Tr2, while the critical value \( \chi^2_{0.05,2} \) is 5.99 for two degrees of freedom and at 5% point of the chi-squared distribution. Thus, the results of the three link institutes are confirmed to be consistent in the both cases of Tr1 and Tr2.

Figures 4 and 5 respectively show the relative deviations \( d_{L,P,i} \) of the participant’s results from the pilot’s results for Tr1 and Tr2. In these figures, bar lengths represent the expanded uncertainties \( U(d_{L,P,i}) \) with the coverage factor \( k=2 \). Broken line shows the mean value \( \bar{d}_{\text{Link-P}} \) of the relative deviations of the three link institutes, and width of the gray band corresponds to the associated expanded uncertainty \( U(\bar{d}_{\text{Link-P}}) \) with the coverage factor \( k=2 \). It is worth noting that all of the uncertainty bars cross the broken line in both cases of Tr1 and Tr2.
Figure 4: Relative deviation of the measured values on Tr1 from the mean value of the pilot

Figure 5: Relative deviation of the measured values on Tr2 from the mean value of the pilot
Table 3: Relative deviation of the measured values on Tr1 and Tr2 from the mean value of the pilot and relative overall deviation

Relative overall deviation $D_{L,P}$ of participant $i$ from the pilot is calculated by taking a weighted mean of relative deviations of measurements on Tr1 and Tr2 as follows. Its uncertainty $u(D_{L,P})$ is evaluated with considering correlation between the relative deviations $d_{L,P,Tr1}$ of Tr1 and $d_{L,P,Tr2}$ of Tr2, because the two measurements were made by the same force standard machine of each participant and that of the pilot. These values, along with the weighted mean value $D_{Link,P}$ of the relative overall deviations of the three link institutes, are also shown in the columns on the right side of Table 3.

$$D_{L,P} = \frac{d_{L,P,Tr1}u(d_{L,P,Tr1})^2 + d_{L,P,Tr2}u(d_{L,P,Tr2})^2}{1/u(d_{L,P,Tr1})^2 + 1/u(d_{L,P,Tr2})^2}$$

$$u(D_{L,P}) = \frac{u(d_{L,P,Tr1})u(d_{L,P,Tr2})}{u(d_{L,P,Tr1})^2 + u(d_{L,P,Tr2})^2} \times \sqrt{u(d_{L,P,Tr1})^2 + u(d_{L,P,Tr2})^2 + 2r(d_{L,P,Tr1}, d_{L,P,Tr2})u(d_{L,P,Tr1})u(d_{L,P,Tr2})}$$

3.6 Degree of equivalence to the KCRV

Table 4 shows the relative overall deviations $D_{Link,KCRV}$ of the three link institutes from the key comparison reference value (KCRV) and their expanded uncertainties $U(D_{Link,KCRV})$ at the 2 MN force step [2].

Table 4: Relative overall deviations of the link institutes from the KCRV and their uncertainties

<table>
<thead>
<tr>
<th>Link Institute</th>
<th>$D_{Link,KCRV}$ / x10^-4</th>
<th>$U(D_{Link,KCRV})$ / x10^-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>KRISS</td>
<td>-0.28</td>
<td>2.02</td>
</tr>
<tr>
<td>NIST</td>
<td>0.01</td>
<td>0.20</td>
</tr>
<tr>
<td>NMIJ/AIST</td>
<td>0.31</td>
<td>0.92</td>
</tr>
</tbody>
</table>
Taking a weighted mean of these values of the CCM.F-K4 KC, mean deviation $\bar{D}_{\text{Link-KCRV}}$ of the three link institutes from the KCRV and associated expanded uncertainty $U(\bar{D}_{\text{Link-KCRV}})$ are calculated as follows.

$$\bar{D}_{\text{Link-KCRV}} = 0.02 \times 10^{-4}$$ (10)

$$U(\bar{D}_{\text{Link-KCRV}}) = 0.19 \times 10^{-4}$$ (11)

NOTE: NMIJ, the pilot, had revised the amplification ratio of its hydraulic amplification FSM at a time between the measurements of the CCM.F-K4 KC and this APMP KC based on an intra-laboratory comparison with its deadweight FSM using a set of build-up type force transducers. Therefore, the results of NMIJ in the CCM.F-K4 KC were corrected by $+1.35 \times 10^{-4}$ when calculating the mean deviation.

For each of the participating institutes, the degree of equivalence $D_i$ to the KCRV and its expanded uncertainty $U(D_i)$ with the coverage factor $k=2$ are calculated in the following manner.

$$D_i = D_{LJ-P} - \bar{D}_{\text{Link-P}} + \bar{D}_{\text{Link-KCRV}}$$ (12)

$$U(D_i) = k \sqrt{u(D_{LJ-P})^2 + u(\bar{D}_{\text{Link-P}})^2 + u(\bar{D}_{\text{Link-KCRV}})^2}$$ (13)

In the cases of the link institutes, $u(\bar{D}_{\text{Link-P}})$ is not taken into account, since it is correlated to $u(D_{LJ-P})$ of each link institute as shown in Equation (7). These values in relative representation are given in Table 5. It is worth noting that the deviations $D_i$ of the link institutes from the KCRV, especially that of NIST, are relatively large in this KC compared with those $D_{\text{Link-KCRV}}$ in the CCM.F-K4 KC shown in Table 4. Difference between the results of two KCs is attributed to the uncertainties of measurements by the pilot institute using the hydraulic amplification type FSM. Another reason for this may be due to the fact that the traveling standards used in this KC were completely different from those in the CCM.F-K4 KC.

Table 5: Degree of equivalence to the KCRV and corresponding expanded uncertainty in relative representation

<table>
<thead>
<tr>
<th>Institute</th>
<th>$D_i$ / 10^{-4}</th>
<th>$U(D_i)$ / 10^{-4}</th>
</tr>
</thead>
<tbody>
<tr>
<td>KRISS</td>
<td>0.2</td>
<td>1.5</td>
</tr>
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<td>NMIA</td>
<td>0.8</td>
<td>4.3</td>
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<td>SCL</td>
<td>3.7</td>
<td>5.6</td>
</tr>
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<td>CMS/ITRI</td>
<td>1.6</td>
<td>5.2</td>
</tr>
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<td>A*STAR</td>
<td>0.5</td>
<td>1.9</td>
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<tr>
<td>NIST</td>
<td>0.4</td>
<td>1.0</td>
</tr>
<tr>
<td>NMIJ/AIST</td>
<td>-0.6</td>
<td>1.2</td>
</tr>
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</table>

In the field of force metrology, since no stable artifact is currently available for realizing and maintaining real KCRV over a long period of time, there exists no absolute numerical reference value. Thus the nominal force of exactly 2 MN is taken as the KCRV $F_{\text{KCRV}}$ for the purpose of reporting the results in the unit of force. The
degree of equivalence $D_{Fi}$ to the KCRV and its expanded uncertainty $U(D_{Fi})$ ($k=2$) in the unit of force are calculated as follows for each participant:

$$D_{Fi} = F_{KCRV} D_i$$  \hspace{1cm} (14)

$$U(D_{Fi}) = F_{KCRV} U(D_i)$$  \hspace{1cm} (15)

These values are shown in Table 6 and depicted in Figure 6. In Figure 6, lengths of the bars represent the corresponding expanded uncertainties $U(D_{Fi})$ with the coverage factor $k=2$ and solid circles stands for the link institutes.

Table 6: Degree of equivalence to the KCRV and corresponding expanded uncertainty

<table>
<thead>
<tr>
<th>Institute</th>
<th>$D_{Fi}$ / kN</th>
<th>$U(D_{Fi})$ / kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>KRISS</td>
<td>0.05</td>
<td>0.30</td>
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<td>NMIA</td>
<td>0.15</td>
<td>0.86</td>
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<td>SCL</td>
<td>0.75</td>
<td>1.11</td>
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<td>0.33</td>
<td>1.04</td>
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<td>A*STAR</td>
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<td>0.38</td>
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<td>NIST</td>
<td>0.07</td>
<td>0.20</td>
</tr>
<tr>
<td>NMIJ/AIST</td>
<td>-0.12</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Figure 6: Degree of equivalence to the KCRV
These table and figure indicate that the measurement results of the all participants coincide with the KCRV demonstrating the equivalences within their uncertainties.

### 3.7 Degree of equivalence between pairs of participating institutes

For each pair of participating institutes, the degree of equivalence $D_{F,i,j}$ between institute $i$ and institute $j$ and associated expanded uncertainty $U(D_{F,i,j})$ ($k=2$) in the unit of force are calculated as follows.

\[
D_{F,i,j} = F_{KCRV} (D_{L,i-P} - D_{L,j-P})
\]

\[
U(D_{F,i,j}) = F_{KCRV} k \sqrt{(u(D_{L,i-P}))^2 + (u(D_{L,j-P}))^2}
\]

As shown in Table 1, no pair of participants has common traceability sources, thus it is not necessary to take correlation between any pair into account.

Table 7 shows these values. It can be recognized that all pairs of the participating institutes are equivalent to each other within their uncertainties.

Table 7: Degree of equivalence $D_{F,i,j}$ between institute $i$ (left column) and institute $j$ (top row) and corresponding expanded uncertainty $U(D_{F,i,j})$

<table>
<thead>
<tr>
<th>Inst. $j$</th>
<th>KRISS</th>
<th>NMIA</th>
<th>SCL</th>
<th>CMS/ITRI</th>
<th>A*STAR</th>
<th>NIST</th>
<th>NMIJ/AIST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inst. $i$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KRISS</td>
<td>$D_{F,i,j}$</td>
<td>-0.10</td>
<td>-0.70</td>
<td>-0.28</td>
<td>-0.05</td>
<td>-0.03</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>$U(D_{F,i,j})$</td>
<td>0.90</td>
<td>1.14</td>
<td>1.07</td>
<td>0.47</td>
<td>0.36</td>
<td>0.38</td>
</tr>
<tr>
<td>NMIA</td>
<td>0.10</td>
<td></td>
<td>$D_{F,i,j}$</td>
<td>-0.60</td>
<td>-0.18</td>
<td>0.05</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>0.90</td>
<td></td>
<td>$U(D_{F,i,j})$</td>
<td>1.39</td>
<td>1.33</td>
<td>0.92</td>
<td>0.87</td>
</tr>
<tr>
<td>SCL</td>
<td>0.70</td>
<td>0.60</td>
<td></td>
<td>$D_{F,i,j}$</td>
<td>0.42</td>
<td>0.65</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>1.14</td>
<td>1.39</td>
<td></td>
<td>$U(D_{F,i,j})$</td>
<td>1.51</td>
<td>1.16</td>
<td>1.12</td>
</tr>
<tr>
<td>CMS/ITRI</td>
<td>0.28</td>
<td>0.18</td>
<td></td>
<td>-0.42</td>
<td>$D_{F,i,j}$</td>
<td>0.23</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>1.07</td>
<td>1.33</td>
<td></td>
<td>$U(D_{F,i,j})$</td>
<td>1.09</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>A*STAR</td>
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<td></td>
<td>-0.65</td>
<td>-0.23</td>
<td>$D_{F,i,j}$</td>
<td>0.02</td>
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<tr>
<td></td>
<td>0.47</td>
<td>0.92</td>
<td></td>
<td>1.16</td>
<td>1.09</td>
<td>$U(D_{F,i,j})$</td>
<td>0.41</td>
</tr>
<tr>
<td>NIST</td>
<td>0.03</td>
<td>-0.08</td>
<td></td>
<td>-0.67</td>
<td>-0.25</td>
<td>-0.02</td>
<td>$D_{F,i,j}$</td>
</tr>
<tr>
<td></td>
<td>0.36</td>
<td>0.87</td>
<td></td>
<td>1.12</td>
<td>1.05</td>
<td>$U(D_{F,i,j})$</td>
<td>0.41</td>
</tr>
<tr>
<td>NMIJ/AIST</td>
<td>-0.17</td>
<td>-0.27</td>
<td></td>
<td>-0.87</td>
<td>-0.45</td>
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</tr>
<tr>
<td></td>
<td>0.38</td>
<td>0.88</td>
<td></td>
<td>1.13</td>
<td>1.05</td>
<td>$U(D_{F,i,j})$</td>
<td>0.43</td>
</tr>
</tbody>
</table>

### 4. Summary

The APMP.M.F-K4.b key comparison in the 2 MN force range revealed that all results of the seven participants are equivalent to the KCRV within their uncertainties and that these participants are equivalent to each other.
5. References

[1] Cox M. G. 2007 The evaluation of key comparison data: determining the largest consistent subset
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