Force Key Comparison EUROMET.M.F-K2

(50 kN and 100 kN)

EURAMET Project No 518

Final Report

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Pilot: NPL, United Kingdom

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Contents

1 Foreword

This report describes EURAMET Key Comparison EUROMET.M.F-K2, for force values of 50 kN and 100 kN.

2 Background to the comparison

The CCM Force Working Group met in October 1998 in Sydney and made decisions about CIPM Key Comparisons for the unit of force. These were to cover four force ranges, with four different pilot laboratories:

i)	5 kN – 10 kN	Pilot: MIKES-Raute, Finland
ii)	50 kN – 100 kN	Pilot: NPL, United Kingdom
iii)	500 kN – 1 MN	Pilot: PTB, Germany
iv)	2 MN – 4 MN	Pilot: NIST, USA

Key Comparison ii), piloted by NPL, was officially designated CCM.F-K2.a (Scheme A) and CCM.F-K2.b (Scheme B) by CIPM. EURAMET Project No 518 was initiated as a comparison to tie other European laboratories (and INMETRO from Brazil) into the results of this CIPM Key Comparison – as such, it was also piloted by NPL using similar equipment and protocols. The other common participant in the CIPM and EURAMET Key Comparisons was PTB from Germany.

This report gives the results for the EURAMET Key Comparison, designated EUROMET.M.F-K2 by BIPM.

3 Participants in the comparison

There were 12 laboratories including the pilot – these are listed in Table 1.

Country	Institute	Number	Scheme	Month
Brazil	INMETRO	11	A	4 / 2009
Czech Republic	CMI	6	A	6 / 2008
Denmark	FORCE	10	A	1 / 2009
Germany	PTB	5	A	5 / 2008
Greece	EIM	9	A	12 / 2008
Hungary	MKEH	4	A	4 / 2008
Netherlands	VSL	2	A	11 / 2007
Poland	GUM	12	B	7 / 2009
Portugal	IPQ	7	A	9 / 2008
Romania	INM	8	A	10 / 2008
Switzerland	METAS	3	A	3 / 2008
United Kingdom	NPL	1	A / B	Pilot

Table 1. Participating countries and laboratories, including the code number used in the report

4 Principles of the comparison

The purpose of Key Comparisons is to compare the units of measurement as realised throughout the world. In the area of force, the way this is done is by the use of high quality force transducers subjected to similar loading profiles in national force standard machines, following a strict measurement protocol and using similar instrumentation.

The following loading schemes were proposed by the CCM Force Working Group and used in both the CIPM and EURAMET Key Comparisons:

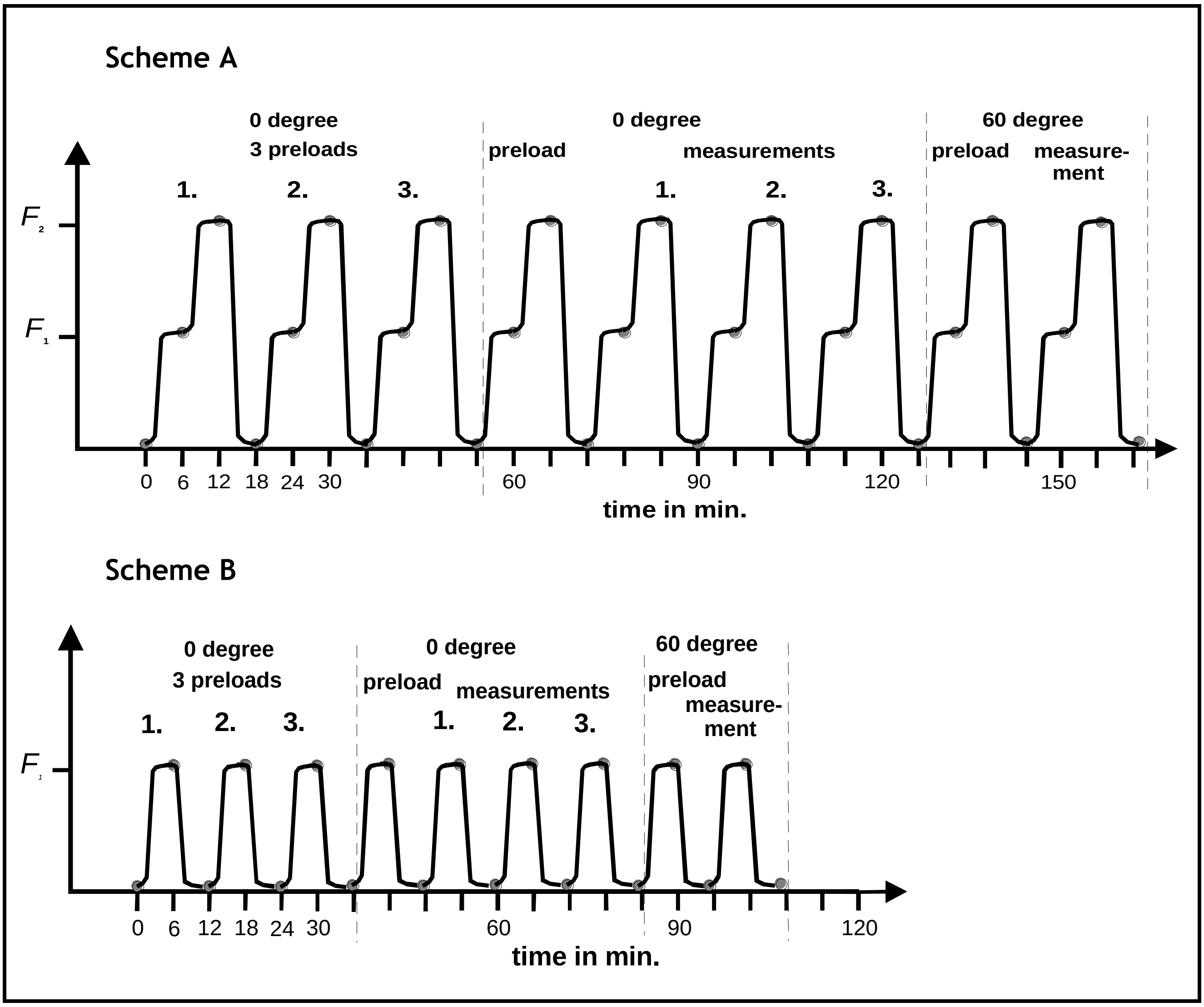

Figure 1. Loading scheme for both sets of transducers, at 50 kN and 100 kN (Scheme A) and at 50 kN (Scheme B).

The force transducer is rotated through a total of 720° in both schemes. One preload and one measurement (as at 60° in Figure 1) is carried out at 120°, 180°, 240°, 300°, 360°/0°, 60°, 120°, 180°, 240°, 300°, and 360°. The relatively long reading period of six minutes was selected to minimise the influence of creep.

The comparison was carried out using two transducers, both with nominal capacity 100 kN. The same transducers were used for the Scheme B work as for the Scheme A work because Scheme A had given much information about their performance at a force level of 50 kN. The transducers are detailed in Table 2.

Identification Code	Manufacturer	Serial Number	Capacity	Scheme
TrA	GTM	42793	100 kN	A/B
TrB	Sensy	19994730004	100 kN	A/B

Table 2. Transducers used in the comparison

At the conclusion of the comparison, the temperature sensitivities of the two transducers were determined, to enable corrections to be made for the effect of calibration temperatures differing from the nominal value of 20 °C. The results of these temperature tests are shown in Figures 2 and 3, and the assumption is made that the temperature sensitivity determined will be valid at any applied force.

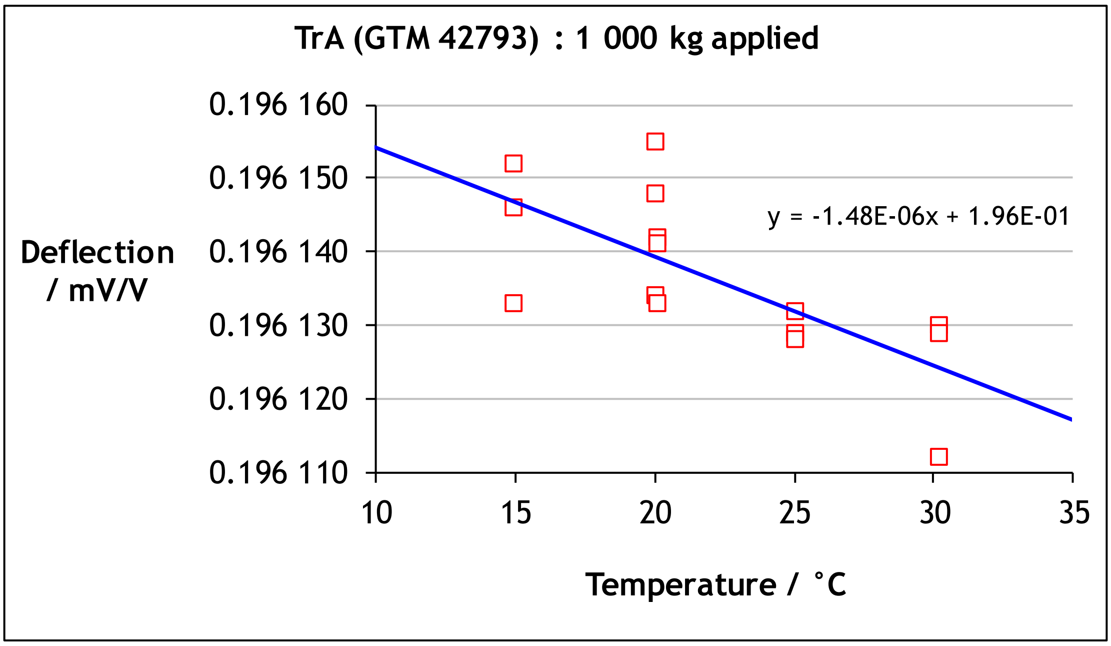

Figure 2. Temperature sensitivity results for TrA at an applied load of 1 000 kg

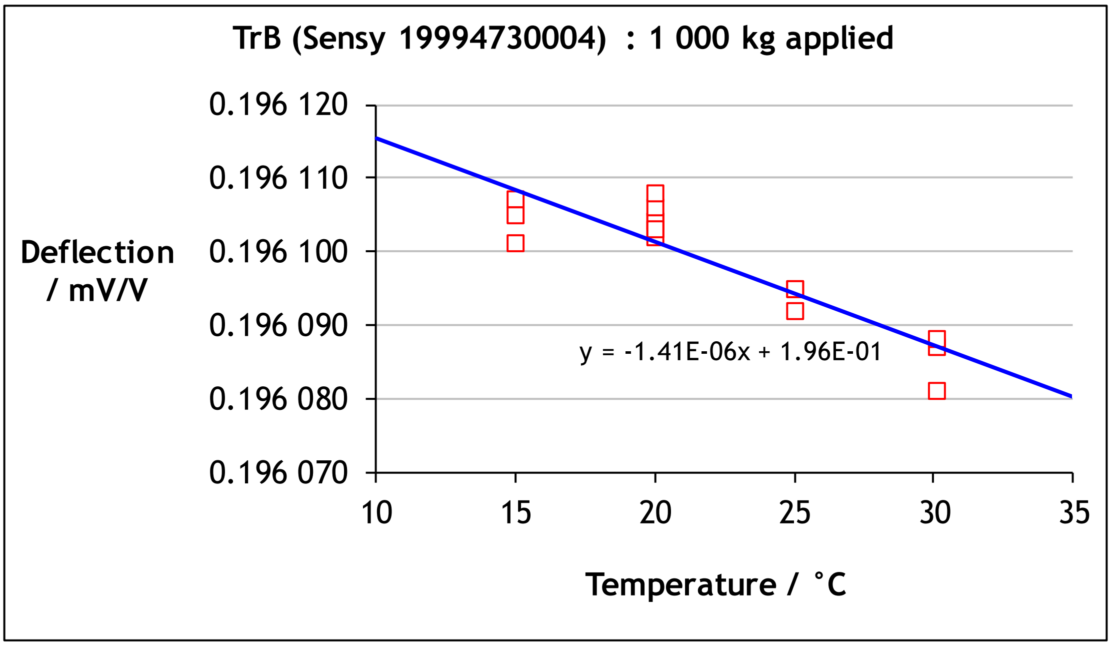

Figure 3. Temperature sensitivity results for TrB at an applied load of 1 000 kg

From these test results, relative temperature sensitivities for TrA and TrB of -7.56×10^{-6} K^{-1} and -7.17×10^{-6} K^{-1} respectively were determined. The relative uncertainty estimates associated with these values were determined, from analysis of the linear fit results, to be 4.10×10^{-6} K^{-1} and 1.99×10^{-6} K^{-1} respectively, at a 95 % (k = 2) level of confidence.

5 Format of the comparison

The comparison was made in a star format; the transducers came back to the pilot after each participating laboratory's measurements. One complete measurement cycle (pilot – participating laboratory – pilot) is called a loop. The pilot's first measurement is denoted the A-measurement and its second, after the participating laboratory, is called the B-measurement. The change at the pilot (B-measurement – A-measurement) is called the drift for that particular loop. The reference value for each loop is taken as the mean of the two pilot measurements – this is called the loop value.

6 Limitations of the comparison

Due to the fact that there is no real reference value to circulate (as the sensitivity of the force transducers varies over time), the following conditions apply:

- each measurement loop is independent of the others
- numerical values of different loops are not easily comparable
- only relative deviations can be compared
- there is no absolute numerical reference value

7 Instrumentation used in the comparison

In practice, it is not possible to calibrate the DMP40 measurement instruments used (one at each laboratory) against a single reference standard. The uniformity of the DMP40s used was confirmed by comparison against the same BN100 calibrator unit, circulated with the transducers. Each participating laboratory measured the indication of their DMP40 against the signal of the BN100 at a number of representative voltage ratios.

The deflections obtained at each laboratory, including the pilot, were adjusted using the assumption that the signal generated by the BN100 was absolutely correct. For example, if BN100 output settings of +0.0 mV/V and +2.0 mV/V resulted in DMP40 readings of +0.000 012 mV/V and +2.000 042 mV/V, giving a deflection of +2.000 030 mV/V instead of the nominal +2.000 000 mV/V, the assumption was made that the DMP40 was reading 1.5×10^{-5} too high and the measured deflection was reduced by this relative amount.

Figures 4 and 5 show the DMP40 readings at both the pilot and at each participating laboratory, at two different BN100 settings, corresponding to the different transducer deflection levels. The values obtained at the pilot vary over the period of the comparison by less than 1×10^{-5} – this indicates that the instability of the BN100 throughout the comparison is no greater than this value.

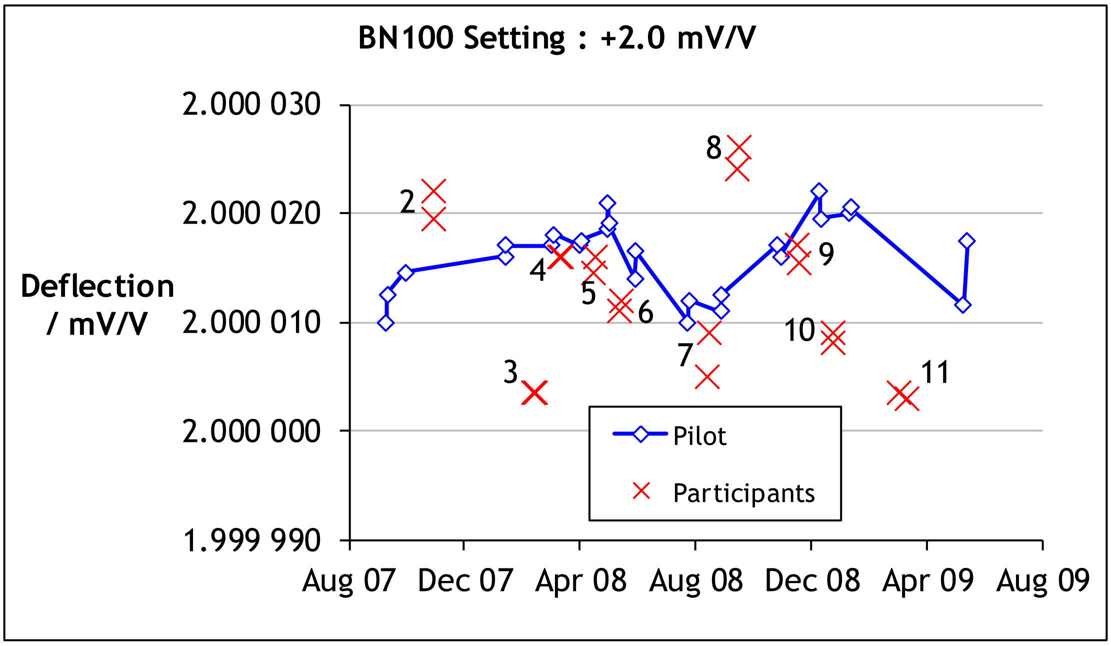

Figure 4. BN100 check results for a setting of +2.0 mV/V

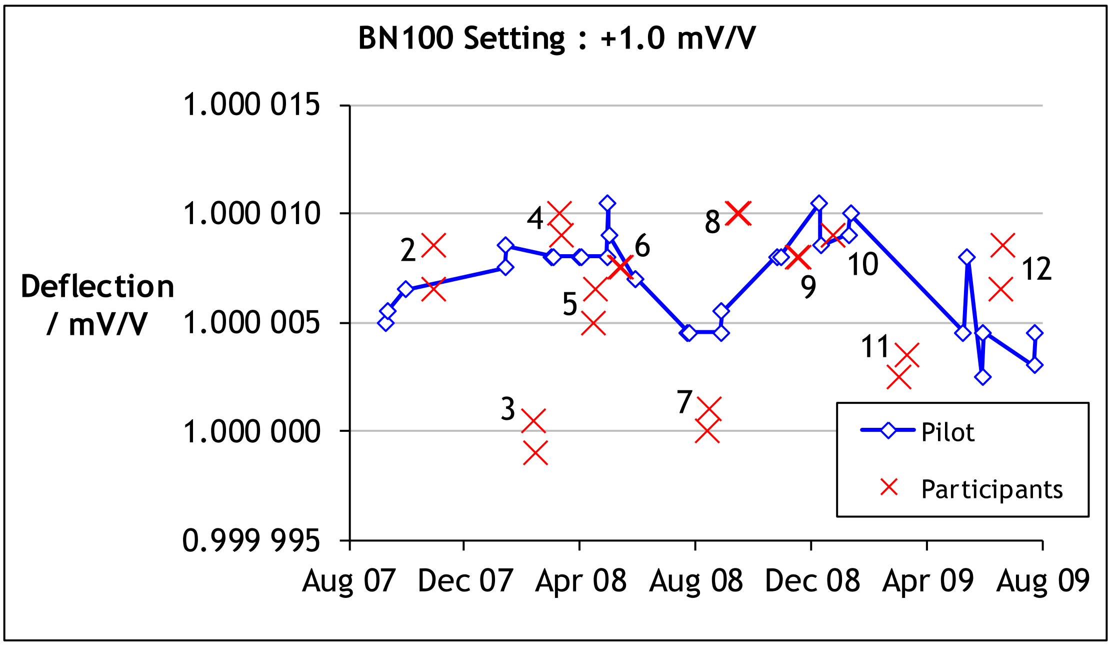

Figure 5. BN100 check results for a setting of +1.0 mV/V

8 Stability of transducer sensitivity

Because the quality of the comparison is dependent upon the three measurements made during each loop, the stability of each transducer's sensitivity is critical. Tables 3 to 6 detail the results obtained at the pilot and Figures 6 to 9 plot each transducer's mean deflection over the period of the comparison – these graphs also show the individual data points (at the twelve orientations) from which the mean deflection is calculated. For comparison purposes, in each graph, the y-axis gridline separation is approximately equal to a relative value of 5×10^{-5}. As TrA was also used during the CIPM Key Comparison, its results during this period are also plotted in the relevant figures to show its long-term stability.

The adjusted deflections take into account the results of the BN100 checks and also the difference in calibration temperature from the nominal 20 °C, using the temperature sensitivity values described earlier.

TrA (GTM 42793) – 50 kN			
Date	Deflection	Adjusted Deflection	Loop Value
	mV/V	mV/V	mV/V
9 Oct 2007	0.999 816	0.999 812	0.999 807 (P2)
11 Feb 2008	0.999 804	0.999 803	0.999 811 (P3)
31 Mar 2008	0.999 820	0.999 819	0.999 815 (P4)
29 Apr 2008	0.999 816	0.999 812	0.999 815 (P5)
28 May 2008	0.999 825	0.999 818	0.999 815 (P6)
26 Jun 2008	0.999 817	0.999 812	
20 Aug 2008	0.999 832	0.999 831	0.999 817 (P7)
24 Sep 2008	0.999 812	0.999 803	0.999 800 (P8)
21 Nov 2008	0.999 812	0.999 797	0.999 802 (P9)
5 Jan 2009	0.999 815	0.999 806	0.999 807 (P10)
4 Feb 2009	0.999 818	0.999 809	0.999 811 (P11)
9 Jun 2009	0.999 820	0.999 813	
25 Jun 2009	0.999 807	0.999 804	0.999 799 (P12)
18 Aug 2009	0.999 797	0.999 795	

Table 3. Results obtained from TrA (50 kN) at pilot laboratory

TrA (GTM 42793) - 100 kN			
Date	Deflection	Adjusted Deflection	Loop Value
	mV/V	mV/V	mV/V
9 Oct 2007	1.999 932	1.999 923	1.999 913 (P2)
11 Feb 2008	1.999 906	1.999 902	1.999 918 (P3)
31 Mar 2008	1.999 937	1.999 933	1.999 931 (P4)
29 Apr 2008	1.999 938	1.999 929	1.999 930 (P5)
28 May 2008	1.999 946	1.999 931	1.999 928 (P6)
26 Jun 2008	1.999 933	1.999 924	
20 Aug 2008	1.999 959	1.999 956	1.999 931 (P7)
24 Sep 2008	1.999 924	1.999 905	1.999 899 (P8)
21 Nov 2008	1.999 923	1.999 893	1.999 900 (P9)
5 Jan 2009	1.999 925	1.999 906	1.999 910 (P10)
4 Feb 2009	1.999 935	1.999 915	1.999 917 (P11)
9 Jun 2009	1.999 935	1.999 920	

Table 4. Results obtained from TrA (100 kN) at pilot laboratory

TrB (Sensy 19994730004) - 50 kN			
Date	**Deflection**	**Adjusted Deflection**	**Loop Value**
	mV/V	**mV/V**	**mV/V**
10 Oct 2007	0.999 671	0.999 665	
29 Oct 2007	0.999 699	0.999 692	0.999 700 (P2)
12 Feb 2008	0.999 709	0.999 707	0.999 730 (P3)
2 Apr 2008	0.999 755	0.999 753	0.999 808 (P4)
30 Apr 2008	0.999 865	0.999 862	0.999 876 (P5)
27 May 2008	0.999 897	0.999 890	
29 May 2008	0.999 932	0.999 926	0.999 923 (P6)
27 Jun 2008	0.999 925	0.999 921	
21 Aug 2008	0.999 949	0.999 947	0.999 945 (P7)
25 Sep 2008	0.999 949	0.999 942	0.999 942 (P8)
25 Nov 2008	0.999 955	0.999 941	0.999 945 (P9)
6 Jan 2009	0.999 954	0.999 948	0.999 959 (P10)
6 Feb 2009	0.999 980	0.999 970	0.999 944 (P11)
5 Jun 2009	0.999 920	0.999 917	
26 Jun 2009	0.999 933	0.999 929	0.999 932 (P12)
19 Aug 2009	0.999 938	0.999 935	

Table 5. Results obtained from TrB (50 kN) at pilot laboratory

TrB (Sensy 19994730004) - 100 kN			
Date	Deflection	Adjusted Deflection	Loop Value
	mV/V	mV/V	mV/V
10 Oct 2007	1.999 295	1.999 282	
29 Oct 2007	1.999 338	1.999 323	1.999 335 (P2)
12 Feb 2008	1.999 350	1.999 347	1.999 374 (P3)
2 Apr 2008	1.999 406	1.999 401	1.999 462 (P4)
30 Apr 2008	1.999 531	1.999 523	1.999 533 (P5)
27 May 2008	1.999 558	1.999 543	
29 May 2008	1.999 616	1.999 602	1.999 599 (P6)
27 Jun 2008	1.999 609	1.999 597	
21 Aug 2008	1.999 668	1.999 660	1.999 673 (P7)
25 Sep 2008	1.999 702	1.999 686	1.999 699 (P8)
25 Nov 2008	1.999 738	1.999 711	1.999 723 (P9)
6 Jan 2009	1.999 749	1.999 735	1.999 767 (P10)
6 Feb 2009	1.999 818	1.999 799	1.999 768 (P11)
5 Jun 2009	1.999 746	1.999 737	

Table 6. Results obtained from TrB (100 kN) at pilot laboratory

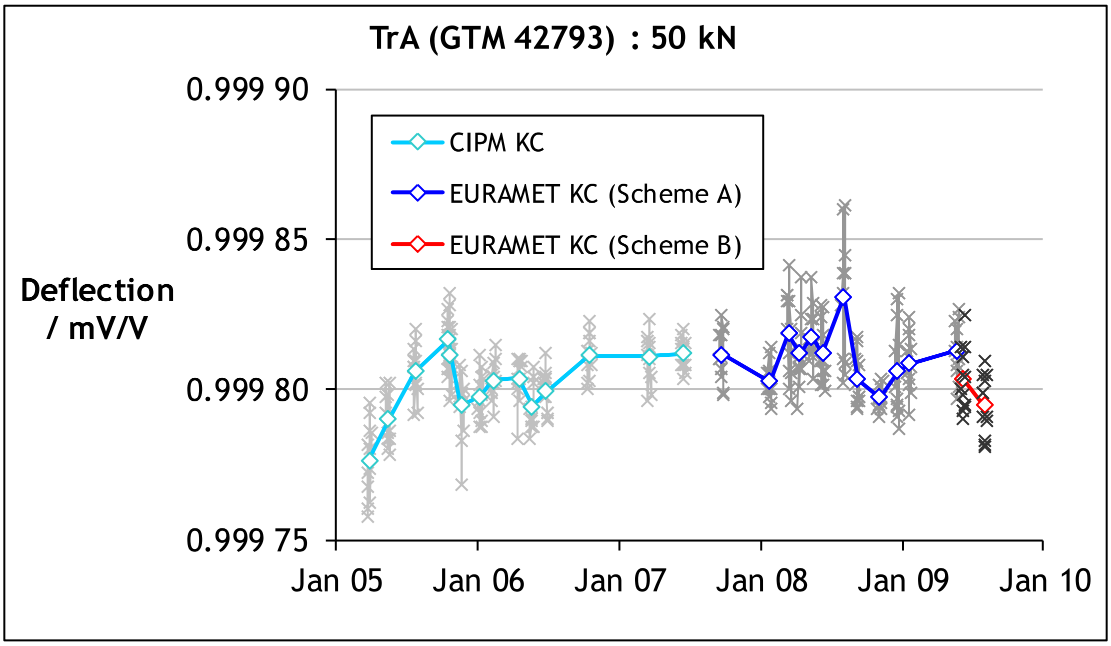

Figure 6. Stability of TrA at 50 kN throughout the comparison

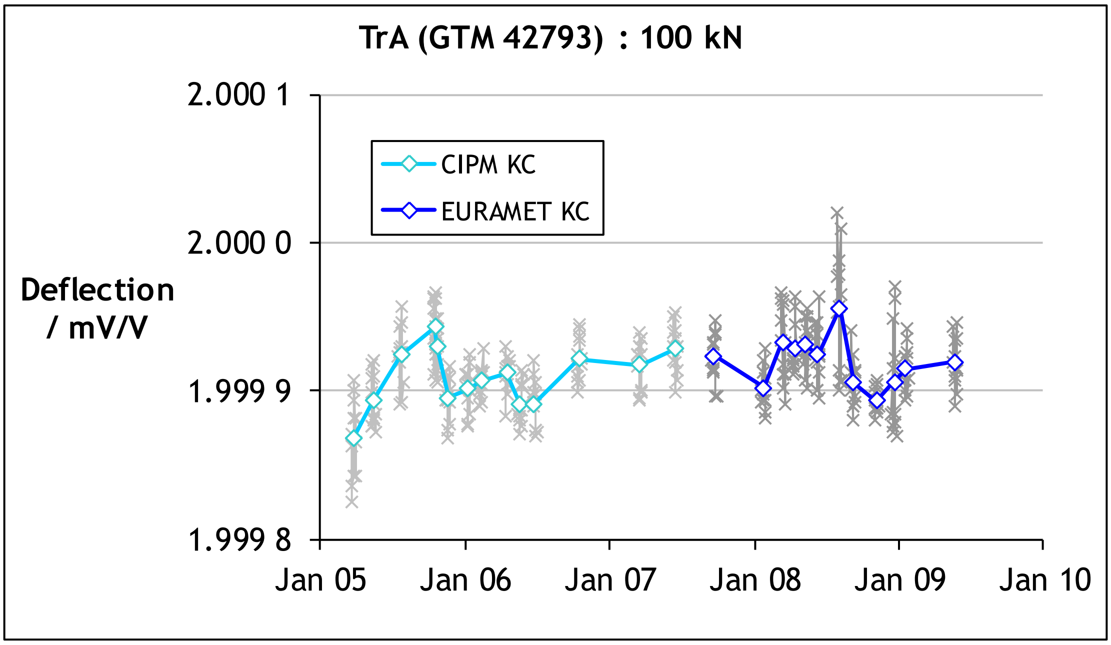

Figure 7. Stability of TrA at 100 kN throughout the comparison

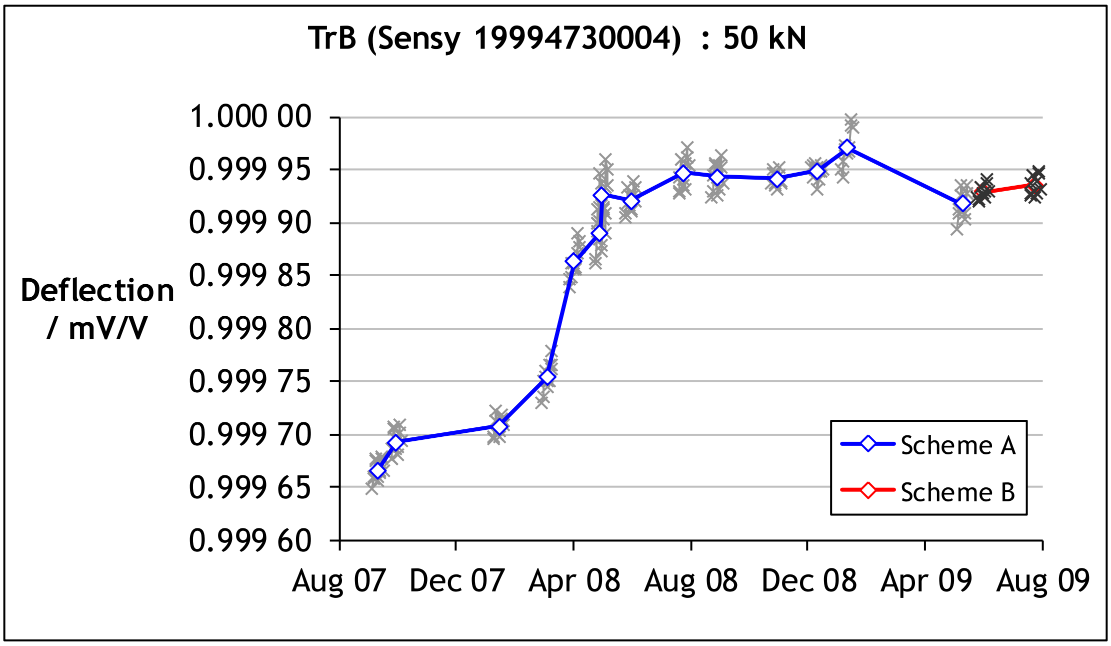

Figure 8. Stability of TrB at 50 kN throughout the comparison

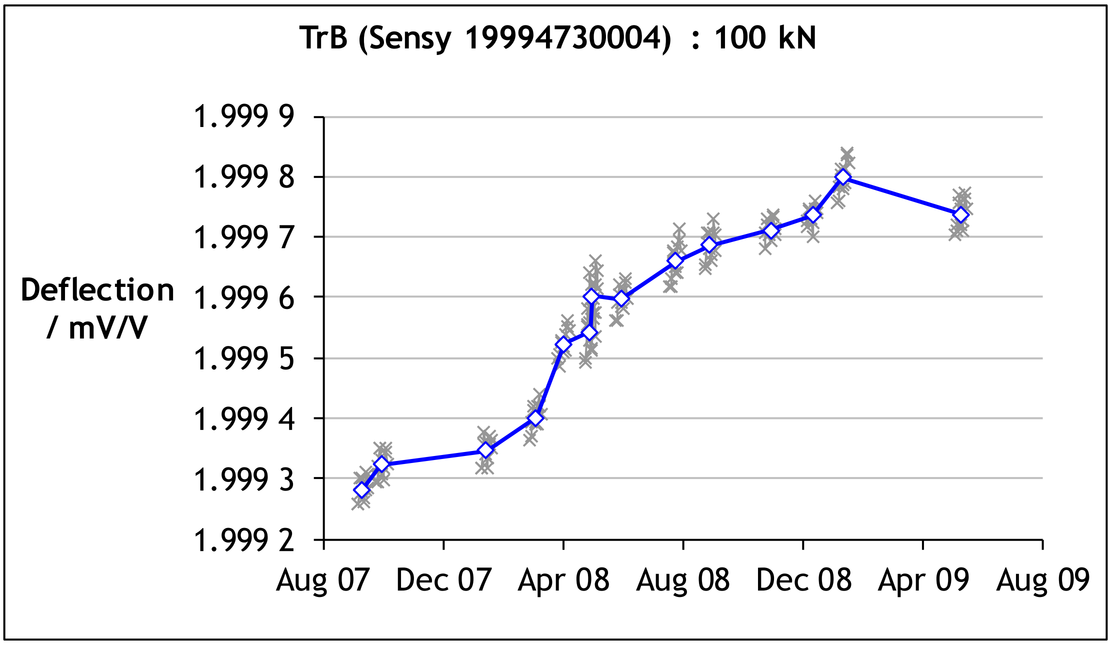

Figure 9. Stability of TrB at 100 kN throughout the comparison

9 Results obtained at participating laboratories

Tables 7 to 10 detail the results obtained at all participating laboratories and give the difference from each loop's reference value (given in Tables 3 to 6) in both absolute and relative terms. As with the calibrations at the pilot laboratory, the adjusted deflections compensate both for the BN100 values and for the calibrations not being performed at 20 °C.

TrA (GTM 42793) - 50 kN						
Code Number	Date	Deflection	Adjusted Deflection	Loop Value	Difference	
		mV/V	mV/V	mV/V	mV/V	Relative
2	28 Nov 2007	0.999 708	0.999 710	0.999 807	-0.000 098	-9.77E-05
3	12 Mar 2008	0.999 813	0.999 812	0.999 811	0.000 001	1.07E-06
4	8 Apr 2008	0.999 883	0.999 882	0.999 815	0.000 066	6.61E-05
5	15 May 2008	0.999 820	0.999 814	0.999 815	-0.000 001	-1.04E-06
6	10 Jun 2008	0.999 798	0.999 824	0.999 815	0.000 010	9.55E-06
7	10 Sep 2008	0.999 807	0.999 835	0.999 817	0.000 018	1.75E-05
8	10 Oct 2008	0.999 816	0.999 833	0.999 800	0.000 033	3.30E-05
9	12 Dec 2008	0.999 816	0.999 817	0.999 802	0.000 015	1.49E-05
10	19 Jan 2009	0.999 790	0.999 795	0.999 807	-0.000 013	-1.26E-05
11	6 Apr 2009	0.999 810	0.999 806	0.999 811	-0.000 005	-4.89E-06
12	14 Jul 2009	0.999 850	0.999 844	0.999 799	0.000 045	4.51E-05

Table 7. Results obtained from TrA (50 kN) at participating laboratories

TrA (GTM 42793) - 100 kN						
Code Number	Date	Deflection	Adjusted Deflection	Loop Value	Difference	
		mV/V	mV/V	mV/V	mV/V	Relative
2	28 Nov 2007	1.999 741	1.999 739	1.999 913	-0.000 174	-8.72E-05
3	12 Mar 2008	1.999 926	1.999 922	1.999 918	0.000 004	2.21E-06
4	8 Apr 2008	1.999 986	1.999 987	1.999 931	0.000 056	2.80E-05
5	15 May 2008	1.999 936	1.999 921	1.999 930	-0.000 009	-4.64E-06
6	10 Jun 2008	1.999 896	1.999 952	1.999 928	0.000 024	1.22E-05
7	10 Sep 2008	1.999 905	1.999 956	1.999 931	0.000 026	1.28E-05
8	10 Oct 2008	1.999 931	1.999 962	1.999 899	0.000 062	3.11E-05
9	12 Dec 2008	1.999 921	1.999 922	1.999 900	0.000 022	1.12E-05
10	19 Jan 2009	1.999 946	1.999 966	1.999 910	0.000 056	2.80E-05
11	6 Apr 2009	1.999 920	1.999 917	1.999 917	0.000 000	-7.15E-08

Table 8. Results obtained from TrA (100 kN) at participating laboratories

TrB (Sensy 19994730004) – 50 kN						
Code Number	Date	Deflection	Adjusted Deflection	Loop Value	Difference	
		mV/V	mV/V	mV/V	mV/V	Relative
2	27 Nov 2007	0.999 612	0.999 615	0.999 700	-0.000 084	-8.45E-05
3	13 Mar 2008	0.999 695	0.999 695	0.999 730	-0.000 035	-3.49E-05
4	10 Apr 2008	0.999 783	0.999 790	0.999 808	-0.000 018	-1.76E-05
5	14 May 2008	0.999 866	0.999 862	0.999 876	-0.000 015	-1.46E-05
6	12 Jun 2008	0.999 913	0.999 933	0.999 923	0.000 010	9.60E-06
7	12 Sep 2008	0.999 906	0.999 929	0.999 945	-0.000 015	-1.54E-05
8	13 Oct 2008	0.999 889	0.999 901	0.999 942	-0.000 041	-4.08E-05
9	15 Dec 2008	0.999 946	0.999 946	0.999 945	0.000 001	1.38E-06
10	20 Jan 2009	0.999 899	0.999 904	0.999 959	-0.000 055	-5.52E-05
11	30 Mar 2009	0.999 947	0.999 944	0.999 944	0.000 000	2.91E-08
12	16 Jul 2009	0.999 924	0.999 912	0.999 932	-0.000 020	-1.99E-05

Table 9. Results obtained from TrB (50 kN) at participating laboratories

TrB (Sensy 19994730004) – 100 kN						
Code Number	Date	Deflection	Adjusted Deflection	Loop Value	Difference	
		mV/V	mV/V	mV/V	mV/V	Relative
2	27 Nov 2007	1.999 164	1.999 164	1.999 335	-0.000 171	-8.56E-05
3	13 Mar 2008	1.999 364	1.999 359	1.999 374	-0.000 015	-7.40E-06
4	10 Apr 2008	1.999 437	1.999 454	1.999 462	-0.000 009	-4.34E-06
5	14 May 2008	1.999 542	1.999 528	1.999 533	-0.000 005	-2.47E-06
6	12 Jun 2008	1.999 608	1.999 652	1.999 599	0.000 052	2.62E-05
7	12 Sep 2008	1.999 612	1.999 652	1.999 673	-0.000 021	-1.04E-05
8	13 Oct 2008	1.999 615	1.999 632	1.999 699	-0.000 066	-3.32E-05
9	15 Dec 2008	1.999 725	1.999 726	1.999 723	0.000 002	1.24E-06
10	20 Jan 2009	1.999 754	1.999 774	1.999 767	0.000 007	3.61E-06
11	30 Mar 2009	1.999 760	1.999 755	1.999 768	-0.000 013	-6.64E-06

Table 10. Results obtained from TrB (100 kN) at participating laboratories

Figure 10 summarises all participants’ results, expressed as differences from the loop value and Figure 11 shows the unweighted mean relative difference from the loop value obtained by each participant at forces of 50 kN and, where applicable, 100 kN. Note that the weighted mean difference cannot be calculated until estimates of uncertainty have been made.

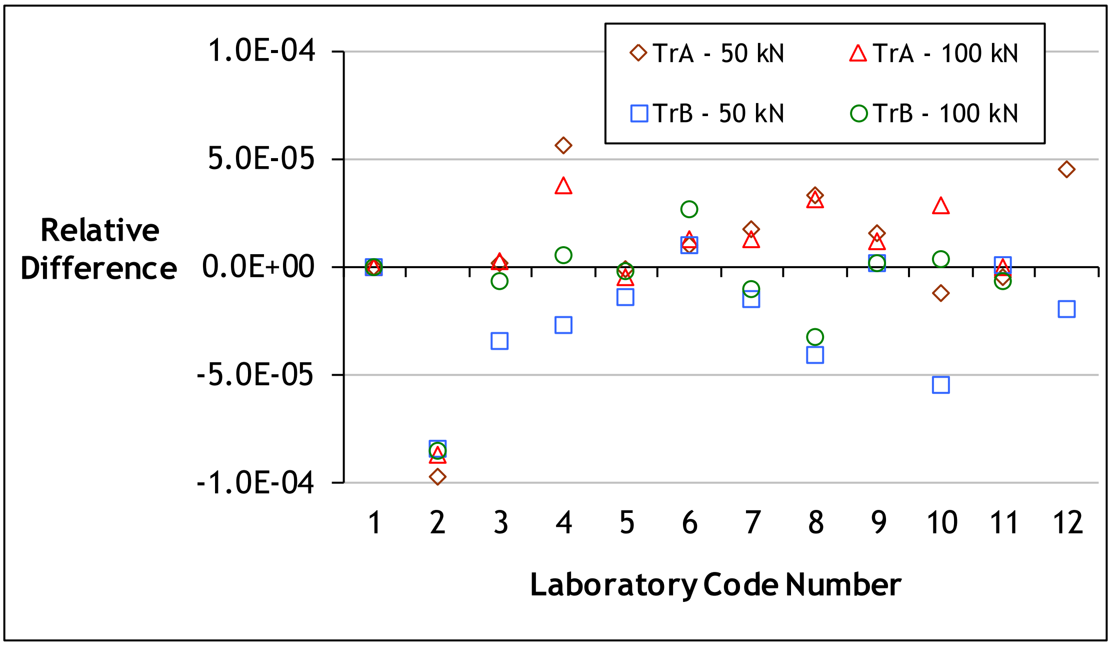

Figure 10. Summary of participants' results

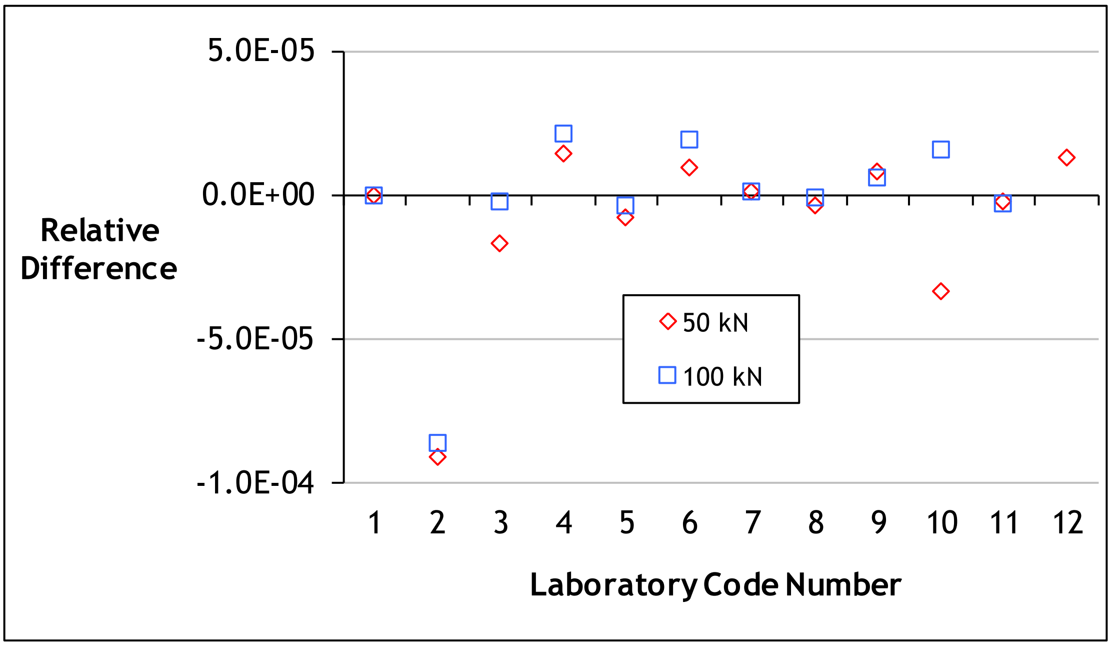

Figure 11. Mean differences obtained from both transducers

10 Uncertainty analysis

Table 11 calculates, for each mean deflection obtained in each participating laboratory, an expanded relative uncertainty value. Each uncertainty value is calculated in the same way, with contributions due to the applied force, the reproducibility of the readings (calculated as the standard deviation of the mean), and the resolution of the DMP40. No allowance is made yet for the BN100 checks or temperature corrections – these will be dealt with in conjunction with the effect of drift of the transducer at a later stage.

Code Number	**Transducer / Force**	**Relative Standard Uncertainty**			**Relative Expanded Uncertainty (*k*=2)**
		Force	**Reproducibility**	**Resolution**	
1	TrA / 50 kN	5.0E-06	3.1E-06	4.1E-07	1.2E-05
	TrA / 100 kN	5.0E-06	3.0E-06	2.0E-07	1.2E-05
	TrB / 50 kN	5.0E-06	3.7E-06	4.1E-07	1.2E-05
	TrB / 100 kN	5.0E-06	3.5E-06	2.0E-07	1.2E-05
2	TrA / 50 kN	5.0E-05	4.2E-06	4.1E-07	1.0E-04
	TrA / 100 kN	5.0E-05	3.8E-06	2.0E-07	1.0E-04
	TrB / 50 kN	5.0E-05	1.2E-04	4.1E-07	2.6E-04
	TrB / 100 kN	5.0E-05	8.3E-05	2.0E-07	1.9E-04
3	TrA / 50 kN	1.0E-05	3.9E-06	4.1E-07	2.1E-05
	TrA / 100 kN	1.0E-05	4.2E-06	2.0E-07	2.2E-05
	TrB / 50 kN	1.0E-05	7.4E-06	4.1E-07	2.5E-05
	TrB / 100 kN	1.0E-05	6.5E-06	2.0E-07	2.4E-05
4	TrA / 50 kN	1.0E-04	8.7E-06	4.1E-07	2.0E-04
	TrA / 100 kN	1.0E-04	8.1E-06	2.0E-07	2.0E-04
	TrB / 50 kN	1.0E-04	5.8E-06	4.1E-07	2.0E-04
	TrB / 100 kN	1.0E-04	9.4E-06	2.0E-07	2.0E-04
5	TrA / 50 kN	8.0E-06	6.0E-06	4.1E-07	2.0E-05
	TrA / 100 kN	8.0E-06	5.9E-06	2.0E-07	2.0E-05
	TrB / 50 kN	8.0E-06	5.4E-06	4.1E-07	1.9E-05
	TrB / 100 kN	8.0E-06	4.9E-06	2.0E-07	1.9E-05
6	TrA / 50 kN	1.0E-04	4.6E-06	4.1E-07	2.0E-04
	TrA / 100 kN	1.0E-04	4.8E-06	2.0E-07	2.0E-04
	TrB / 50 kN	1.0E-04	1.1E-05	4.1E-07	2.0E-04
	TrB / 100 kN	1.0E-04	1.1E-05	2.0E-07	2.0E-04
7	TrA / 50 kN	1.0E-05	3.0E-06	4.1E-07	2.1E-05
	TrA / 100 kN	1.0E-05	3.5E-06	2.0E-07	2.1E-05
	TrB / 50 kN	1.0E-05	9.6E-06	4.1E-07	2.8E-05
	TrB / 100 kN	1.0E-05	5.5E-06	2.0E-07	2.3E-05

8	TrA / 50 kN	2.5E-05	1.8E-06	4.1E-07	5.0E-05
	TrA / 100 kN	2.5E-05	1.2E-06	2.0E-07	5.0E-05
	TrB / 50 kN	2.5E-05	3.6E-06	4.1E-07	5.1E-05
	TrB / 100 kN	2.5E-05	1.6E-06	2.0E-07	5.0E-05
9	TrA / 50 kN	1.0E-05	2.9E-06	4.1E-07	2.1E-05
	TrA / 100 kN	1.0E-05	3.4E-06	2.0E-07	2.1E-05
	TrB / 50 kN	1.0E-05	5.3E-06	4.1E-07	2.3E-05
	TrB / 100 kN	1.0E-05	5.3E-06	2.0E-07	2.3E-05
10	TrA / 50 kN	5.0E-05	9.0E-06	4.1E-07	1.0E-04
	TrA / 100 kN	5.0E-05	9.9E-06	2.0E-07	1.0E-04
	TrB / 50 kN	5.0E-05	5.4E-06	4.1E-07	1.0E-04
	TrB / 100 kN	5.0E-05	8.2E-06	2.0E-07	1.0E-04
11	TrA / 50 kN	1.0E-05	7.0E-06	4.1E-07	2.4E-05
	TrA / 100 kN	1.0E-05	8.8E-06	2.0E-07	2.7E-05
	TrB / 50 kN	1.0E-05	1.6E-06	4.1E-07	2.0E-05
	TrB / 100 kN	1.0E-05	3.4E-06	2.0E-07	2.1E-05
12	TrA / 50 kN	5.0E-05	6.0E-06	4.1E-07	1.0E-04
	TrB / 50 kN	5.0E-05	7.8E-06	4.1E-07	1.0E-04

Table 11. Relative expanded uncertainty value for each mean deflection

For each deflection value obtained by a participating laboratory, the difference between it and the loop value is calculated. The uncertainty associated with this difference is a combination of the participant's uncertainty, the uncertainty of the loop value, and the uncertainty associated with the temperature corrections. The uncertainty of the loop value includes contributions due to the drift of the transducer, the effect of the BN100 corrections, and any change in the force applied at the pilot laboratory (note that this is smaller than the uncertainty of generated force, as the same masses are used for each pilot calibration – the main contribution will be a change in buoyancy force due to air pressure variation, and it is evident that this effect is negligible when compared to the magnitudes of the drift and BN100 effects).

Considering Figures 4 and 5, an estimate of a relative standard uncertainty associated with the BN100 corrections of 5×10^{-6} would not seem unreasonable, and the uncertainty associated with the temperature corrections is simply taken as the difference between calibration temperatures at the pilot and the participating laboratory multiplied by the standard uncertainty associated with the sensitivity value, so the remaining question is how to deal with the drift of the transducer - three alternative methods are possible:

1) Base the drift uncertainty contribution solely on the difference between the two pilot measurements, assuming a specific distribution. This has the disadvantage of basing the value on just two numbers – not a large sample. If the two pilot measurements are identical, this would lead to no contribution due to drift, even if the transducer displays significant values in other loops – it may just be chance that

the two measurements are the same for one particular loop, and it does not mean that the transducer sensitivity is not different during the participant's measurements.

2) Take a standard deviation or average value of drift throughout the complete exercise as a common drift component. This has the disadvantage of possibly underestimating the contribution for some loops and overestimating it for others – it is possible that the stability of the transducer will vary throughout the comparison, particularly if significant environmental factors are present.

3) A combination of the above two approaches – using a rectangular distribution for the two pilot values for a specific loop together with a proportion of the mean absolute drift added as a second rectangular distribution.

Using approach 3, with 50 % of the mean absolute drift used as the half-width of the second rectangular distribution, the expanded uncertainties due to drift for the loop values obtained for the two transducers at 100 kN are as shown in Figures 12 and 13 – these contributions do not appear unreasonable.

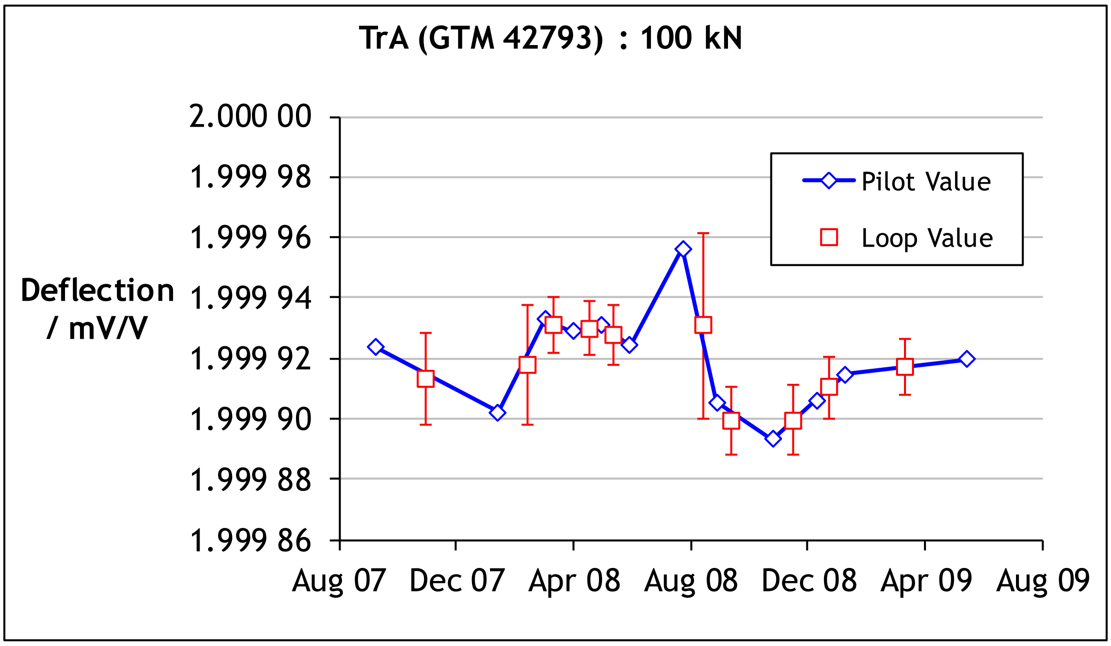

Figure 12. Pilot values and loop values with associated expanded drift uncertainties for TrA (100 kN)

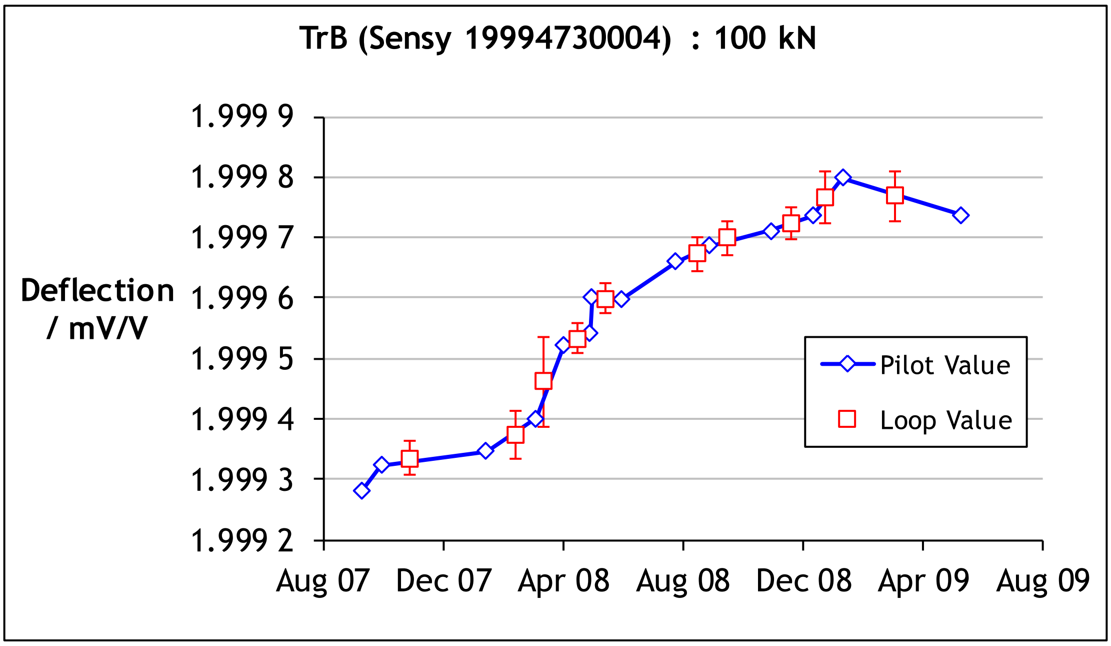

Figure 13. Pilot values and loop values with associated expanded drift uncertainties for TrB (100 kN)

When the drift, BN100, and temperature uncertainty contributions are incorporated with the uncertainty associated with the deflection at each laboratory, the results are as shown in Figures 14 to 17.

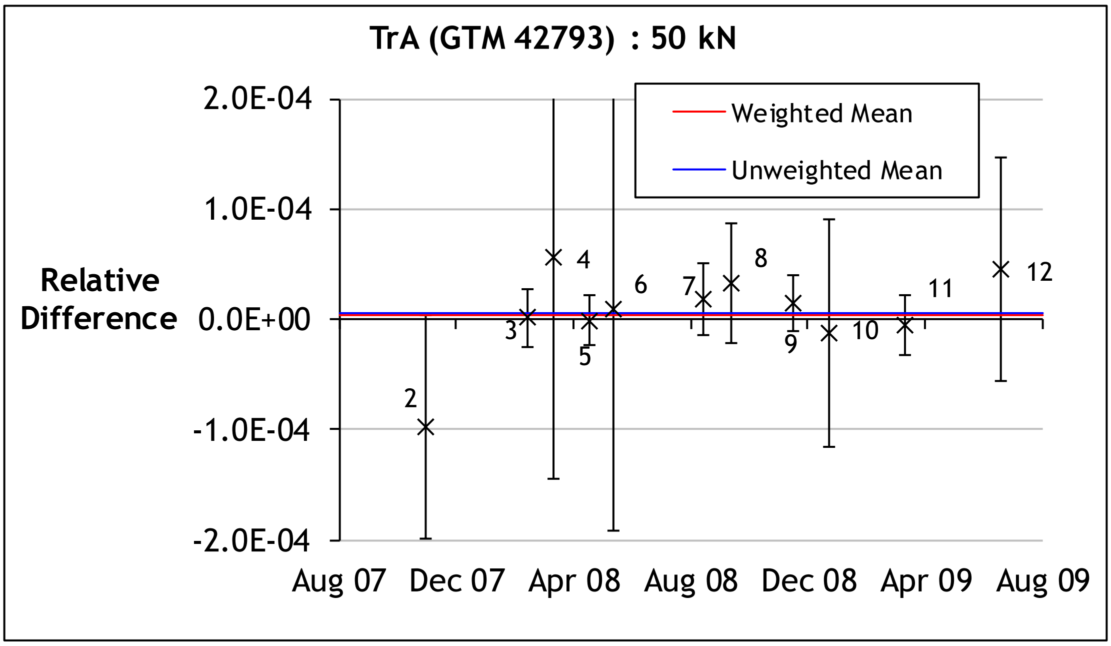

Figure 14. Differences from the loop value together with associated expanded uncertainties for TrA (50 kN)

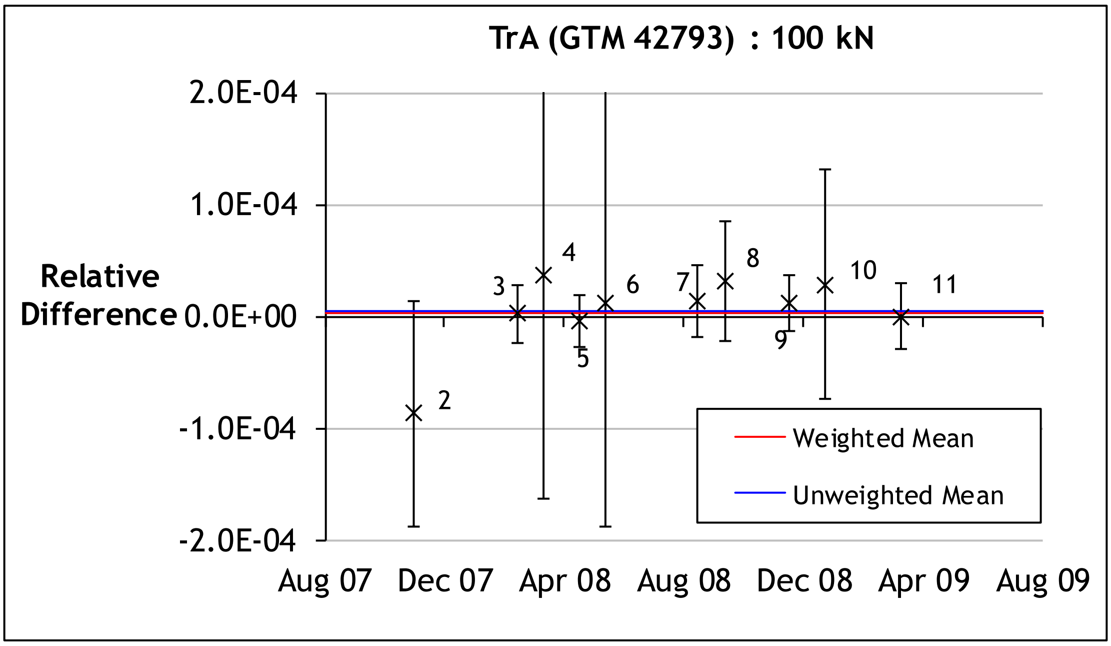

Figure 15. Differences from the loop value together with associated expanded uncertainties for TrA (100 kN)

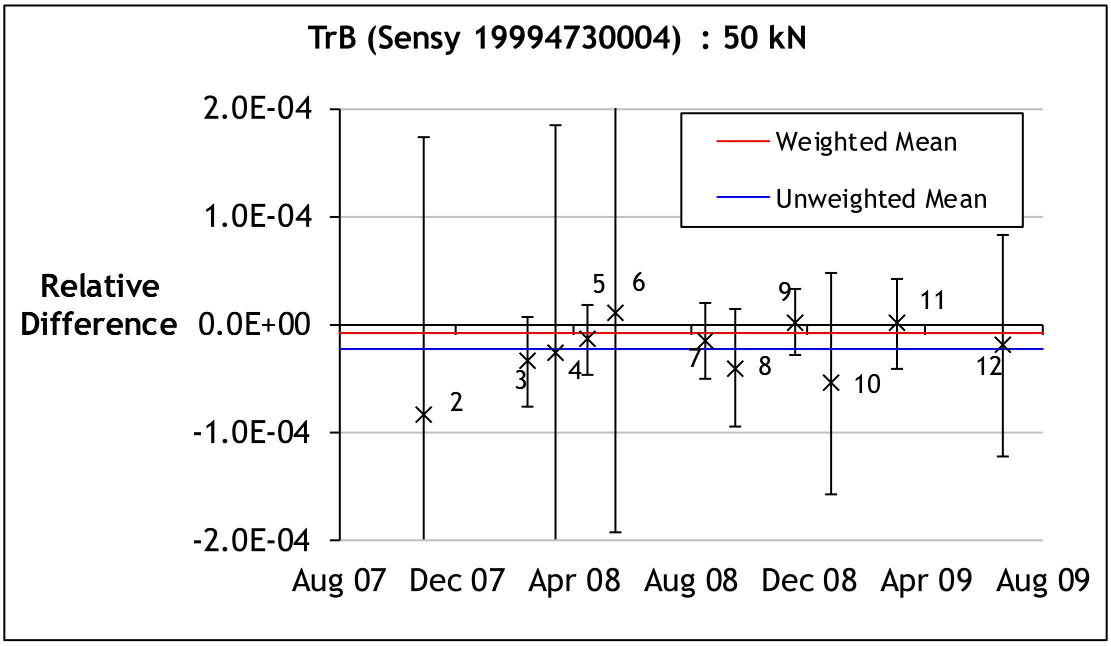

Figure 16. Differences from the loop value together with associated expanded uncertainties for TrB (50 kN)

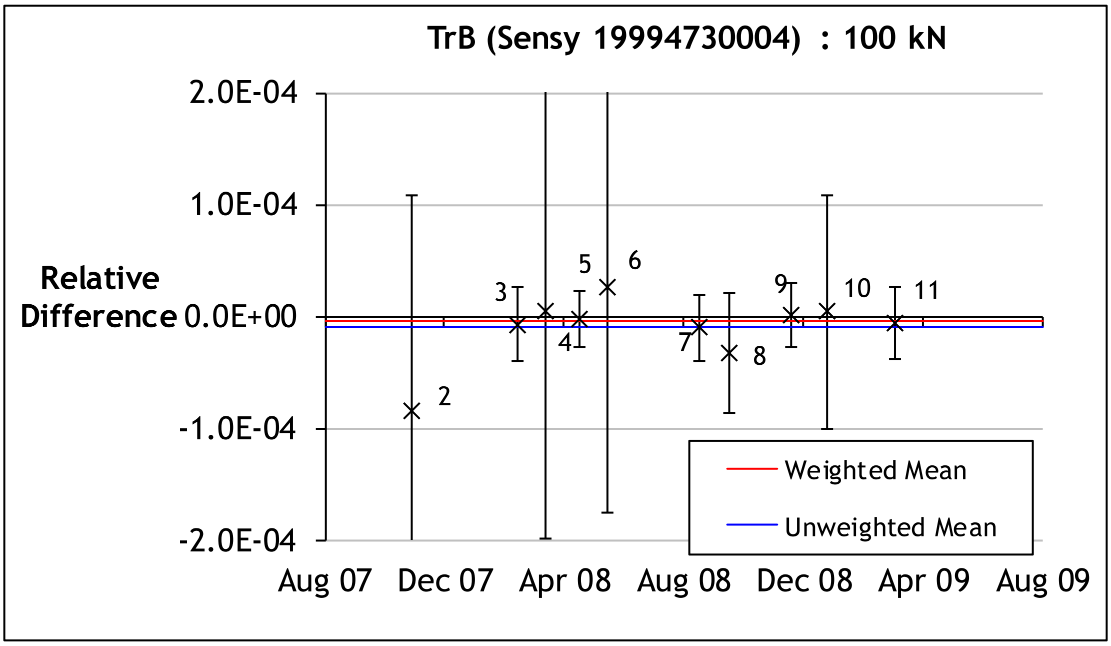

Figure 17. Differences from the loop value together with associated expanded uncertainties for TrB (100 kN)

Both the weighted and unweighted mean lines are based on the results from all laboratories including the pilot. In the calculation of each transducer's weighted mean, the uncertainty associated with the pilot laboratory value is taken as a combination of the contributions due to applied force, reproducibility, resolution, and BN100 corrections – the effects of drift and temperature corrections are incorporated within the uncertainties of the other participants and should not be counted twice.

The next step is to combine the results from each laboratory at each force and it seems reasonable that this should be done as a weighted mean of each laboratory's results, giving more weight to the values with lower associated uncertainties. The uncertainty associated with the pilot's weighted mean difference (of 0.00×10^{-5}) at each force is taken as the average of the uncertainties associated with the individual transducers used at the specific force levels – this is because using the calculated value associated with the weighted mean would result in an unrealistically low value, due to correlation between a number of the uncertainty contributions.

The resulting mean differences from the overall weighted mean values, and their associated expanded uncertainties, are shown in Figure 18 (for a force of 50 kN) and Figure 19 (for a force of 100 kN). The weighted mean deviations from the pilot of -0.08×10^{-5} (at 50 kN) and -0.05×10^{-5} (at 100 kN) have relative expanded uncertainties of 0.82×10^{-5} (at 50 kN) and 0.80×10^{-5} (at 100 kN).

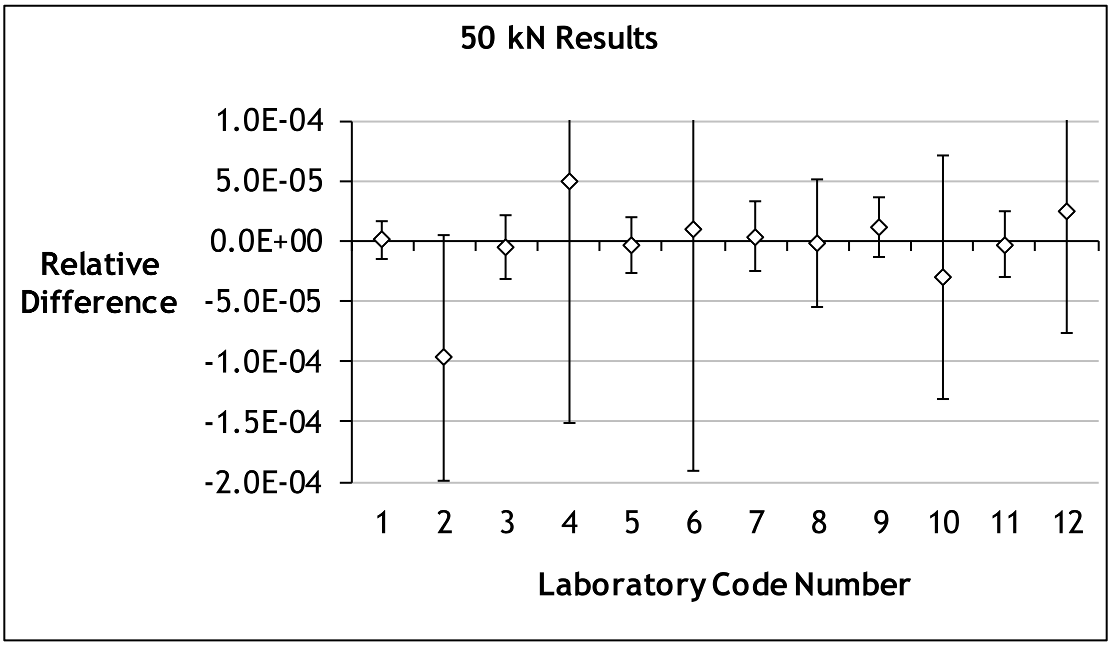

Figure 18. Differences from the weighted mean value together with associated expanded uncertainties at 50 kN

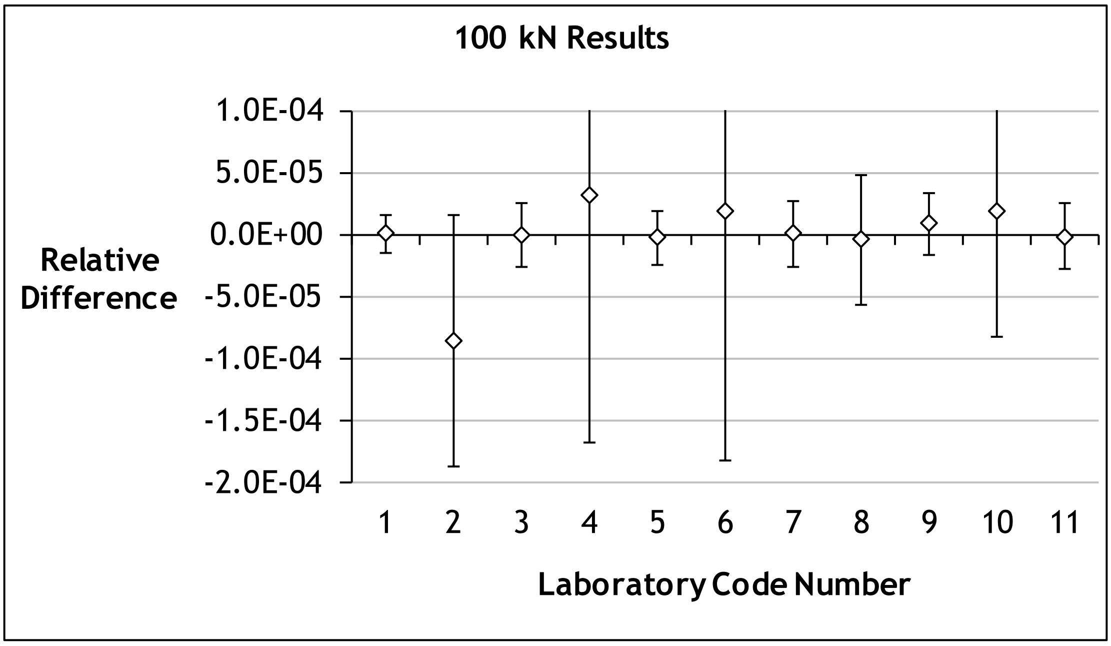

Figure 19. Differences from the weighted mean value together with associated expanded uncertainties at 100 kN

Appendix – Key Comparison Reference Values

If, for both the 50 kN and 100 kN force levels, the Reference Value for this comparison, denoted RV$_{\mathrm{EUR}}$, is taken as the weighted mean deviation from the pilot value, the degree of equivalence for each laboratory is expressed as (1) the deviation from RV$_{\mathrm{EUR}}$ and (2) the uncertainty of this deviation – this uncertainty is calculated as the sum in quadrature of the uncertainty associated with the deviation from the pilot value and the uncertainty of RV$_{\mathrm{EUR}}$, with the resulting values being given in Table 12. Figure 20 illustrates these results in a graphical format and demonstrates that there is no evidence that any laboratory's deviation from RV$_{\mathrm{EUR}}$ is statistically significant.

The degree of equivalence between a given pair of laboratories is expressed as the difference between their respective deviations from RV$_{\mathrm{EUR}}$ and the uncertainty of this difference – this uncertainty is calculated as the sum in quadrature of the uncertainties associated with these two deviations, and the resulting values are given in Tables 13 and 14 for force levels of 50 kN and 100 kN respectively. Results in which the magnitude of the difference exceeds its expanded uncertainty are shown in bold. These results suggest that, at 100 kN, there is no significant difference between any pair of laboratories. At a force of 50 kN, the only pair of laboratories whose results are significantly different are numbers 2 and 9 – when looking at Table 12 and Figure 20, it is apparent that it is laboratory number 2 whose uncertainty claims may need revising.

	50 kN		**100 kN**	
Laboratory	**Deviation from RV$_{\mathrm{EUR}}$**	**Expanded Uncertainty of Deviation**	**Deviation from RV$_{\mathrm{EUR}}$**	**Expanded Uncertainty of Deviation**
1	0.1	1.6	0.1	1.5
2	-9.7	10.2	-8.7	10.1
3	-0.5	2.7	-0.0	2.6
4	4.9	20.1	3.2	20.1
5	-0.4	2.3	-0.3	2.2
6	1.0	20.1	1.8	20.1
7	0.4	2.9	0.0	2.7
8	-0.2	5.3	-0.4	5.3
9	1.2	2.5	0.8	2.5
10	-3.0	10.2	1.9	10.2
11	-0.3	2.7	-0.2	2.7
12	2.5	10.1		

Table 12. Degrees of equivalence of individual laboratories, all relative figures × 10^{-5}

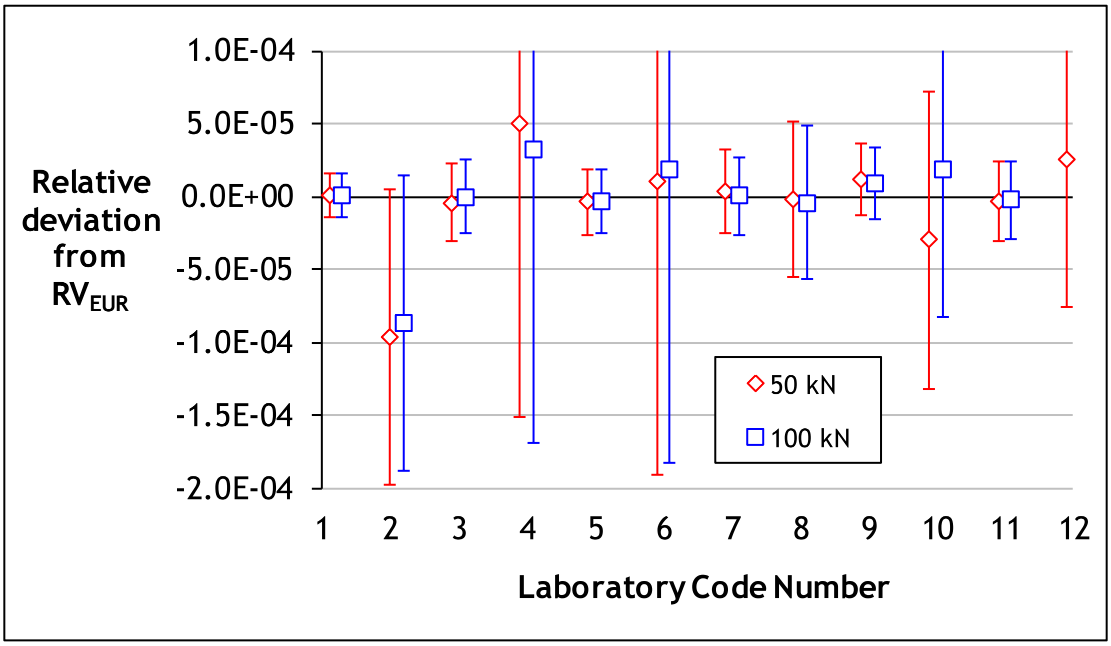

Figure 20. Deviations from RV_{EUR} with associated expanded uncertainties

		1	2	3	4	5	6	7	8	9	10	11	12
1	Δ		-9.8	-0.5	4.9	-0.5	1.0	0.3	-0.3	1.1	-3.1	-0.4	2.4
	U		10.3	3.1	20.2	2.8	20.2	3.3	5.5	2.9	10.3	3.1	10.3
2	Δ	9.8		9.2	14.6	9.3	10.7	10.0	9.5	**10.8**	6.7	9.4	12.2
	U	10.3		10.5	22.5	10.4	22.5	10.6	11.4	**10.5**	14.4	10.5	14.3
3	Δ	0.5	-9.2		5.4	0.1	1.5	0.8	0.2	1.6	-2.6	0.1	3.0
	U	3.1	10.5		20.3	3.5	20.3	3.9	5.9	3.6	10.5	3.8	10.5
4	Δ	-4.9	-14.6	-5.4		-5.3	-3.9	-4.6	-5.2	-3.8	-8.0	-5.3	-2.4
	U	20.2	22.5	20.3		20.3	28.4	20.3	20.8	20.3	22.5	20.3	22.5
5	Δ	0.5	-9.3	-0.1	5.3		1.4	0.8	0.2	1.6	-2.6	0.1	2.9
	U	2.8	10.4	3.5	20.3		20.2	3.7	5.8	3.4	10.4	3.6	10.4
6	Δ	-1.0	-10.7	-1.5	3.9	-1.4		-0.7	-1.3	0.1	-4.0	-1.3	1.5
	U	20.2	22.5	20.3	28.4	20.2		20.3	20.8	20.3	22.5	20.3	22.5
7	Δ	-0.3	-10.0	-0.8	4.6	-0.8	0.7		-0.6	0.8	-3.4	-0.7	2.2
	U	3.3	10.6	3.9	20.3	3.7	20.3		6.0	3.8	10.6	3.9	10.5
8	Δ	0.3	-9.5	-0.2	5.2	-0.2	1.3	0.6		1.4	-2.8	-0.1	2.7
	U	5.5	11.4	5.9	20.8	5.8	20.8	6.0		5.9	11.5	5.9	11.4
9	Δ	-1.1	**-10.8**	-1.6	3.8	-1.6	-0.1	-0.8	-1.4		-4.2	-1.5	1.4
	U	2.9	**10.5**	3.6	20.3	3.4	20.3	3.8	5.9		10.5	3.7	10.4
10	Δ	3.1	-6.7	2.6	8.0	2.6	4.0	3.4	2.8	4.2		2.7	5.5
	U	10.3	14.4	10.5	22.5	10.4	22.5	10.6	11.5	10.5		10.5	14.4
11	Δ	0.4	-9.4	-0.1	5.3	-0.1	1.3	0.7	0.1	1.5	-2.7		2.8
	U	3.1	10.5	3.8	20.3	3.6	20.3	3.9	5.9	3.7	10.5		10.5
12	Δ	-2.4	-12.2	-3.0	2.4	-2.9	-1.5	-2.2	-2.7	-1.4	-5.5	-2.8	
	U	10.3	14.3	10.5	22.5	10.4	22.5	10.5	11.4	10.4	14.4	10.5	

Table 13. Degrees of equivalence between laboratories at 50 kN (Δ = difference in deviations from RV_{EUR}, *U* = expanded uncertainty of difference), all relative figures × 10^{-5}

		1	2	3	4	5	6	7	8	9	10	11
1	*Δ*		-8.7	-0.1	3.2	-0.4	1.7	0.0	-0.5	0.8	1.8	-0.3
	U		10.3	3.0	20.2	2.7	20.2	3.1	5.5	2.9	10.3	3.1
2	*Δ*	8.7		8.6	11.9	8.3	10.5	8.7	8.2	9.5	10.5	8.4
	U	10.3		10.5	22.5	10.4	22.5	10.5	11.4	10.4	14.4	10.5
3	*Δ*	0.1	-8.6		3.2	-0.3	1.8	0.0	-0.4	0.8	1.9	-0.2
	U	3.0	10.5		20.3	3.4	20.3	3.7	5.8	3.6	10.5	3.7
4	*Δ*	-3.2	-11.9	-3.2		-3.5	-1.4	-3.2	-3.6	-2.4	-1.3	-3.4
	U	20.2	22.5	20.3		20.2	28.4	20.3	20.8	20.2	22.5	20.3
5	*Δ*	0.4	-8.3	0.3	3.5		2.1	0.3	-0.1	1.1	2.2	0.1
	U	2.7	10.4	3.4	20.2		20.2	3.5	5.7	3.3	10.4	3.4
6	*Δ*	-1.7	-10.5	-1.8	1.4	-2.1		-1.8	-2.2	-1.0	0.1	-2.0
	U	20.2	22.5	20.3	28.4	20.2		20.3	20.8	20.2	22.5	20.3
7	*Δ*	0.0	-8.7	0.0	3.2	-0.3	1.8		-0.4	0.8	1.9	-0.2
	U	3.1	10.5	3.7	20.3	3.5	20.3		5.9	3.7	10.6	3.8
8	*Δ*	0.5	-8.2	0.4	3.6	0.1	2.2	0.4		1.3	2.3	0.2
	U	5.5	11.4	5.8	20.8	5.7	20.8	5.9		5.8	11.5	5.9
9	*Δ*	-0.8	-9.5	-0.8	2.4	-1.1	1.0	-0.8	-1.3		1.0	-1.1
	U	2.9	10.4	3.6	20.2	3.3	20.2	3.7	5.8		10.5	3.6
10	*Δ*	-1.8	-10.5	-1.9	1.3	-2.2	-0.1	-1.9	-2.3	-1.0		-2.1
	U	10.3	14.4	10.5	22.5	10.4	22.5	10.6	11.5	10.5		10.5
11	*Δ*	0.3	-8.4	0.2	3.4	-0.1	2.0	0.2	-0.2	1.1	2.1	
	U	3.1	10.5	3.7	20.3	3.4	20.3	3.8	5.9	3.6	10.5	

Table 14. Degrees of equivalence between laboratories at 100 kN (*Δ* = difference in deviations from $\mathrm{RV}_{\mathrm{EUR}}$, *U* = expanded uncertainty of difference), all relative figures × 10^{-5}

In order to ensure that this comparison can be clearly linked to the CIPM one, it is necessary to compare the results obtained by the laboratories that participated in both exercises, i.e. NPL (laboratory number 1) and PTB (laboratory number 5). Table 15 details the degrees of equivalence for these two laboratories in the CIPM Key Comparison, together with a weighted mean deviation from the KCRV, with an associated uncertainty.

Force		NPL	PTB	Weighted Mean
50 kN	*Δ*	-0.5	-1.1	-0.7
	U	1.6	2.6	1.4
100 kN	*Δ*	-0.5	-1.4	-0.8
	U	1.7	2.6	1.4

Table 15. Degrees of equivalence for NPL and PTB in CIPM Key Comparison (*Δ* = deviation from KCRV, *U* = expanded uncertainty of deviation), all relative figures × 10^{-5}

This suggests that, at a force of 50 kN, the KCRV is 0.7 × 10^{-5} greater than the weighted mean of the two laboratories. At 100 kN, the KCRV is 0.8 × 10^{-5} greater than their weighted mean values. Table 16 calculates (from the figures in Table 12) the weighted mean of the two laboratories in the EURAMET Key Comparison.

Force		NPL	PTB	Weighted Mean
50 kN	Δ	0.1	-0.4	-0.1
	U	1.6	2.3	1.3
100 kN	Δ	0.1	-0.3	-0.1
	U	1.5	2.2	1.3

Table 16. Degrees of equivalence for NPL and PTB in EURAMET Key Comparison (Δ = deviation from RV_{EUR}, U = expanded uncertainty of deviation), all relative figures × 10^{-5}

Based on the assumption that the performance of the NPL and PTB machines has remained unchanged between the two comparisons, it can be concluded from the differences between these weighted means that, at a force of 50 kN, RV_{EUR} is 0.6×10^{-5} lower than the KCRV. At 100 kN, RV_{EUR} is 0.7×10^{-5} lower than the KCRV. In order to relate the results of the other laboratories to the KCRV, their Table 12 degrees of equivalence need to be adjusted by these differences, resulting in the values given in Table 17, with expanded uncertainties calculated as the quadrature sum of the Table 12 uncertainty and the uncertainty associated with the correction from RV_{EUR} to the KCRV. These results are then plotted in Figure 21.

	50 kN		100 kN	
Laboratory	**Deviation from KCRV**	**Expanded Uncertainty of Deviation**	**Deviation from KCRV**	**Expanded Uncertainty of Deviation**
1	-0.5	2.4	-0.7	2.4
2	-10.3	10.3	-9.4	10.3
3	-1.0	3.3	-0.7	3.2
4	4.4	20.2	2.5	20.2
5	-1.0	3.0	-1.0	2.9
6	0.4	20.2	1.1	20.2
7	-0.2	3.4	-0.7	3.3
8	-0.8	5.6	-1.1	5.6
9	0.6	3.1	0.1	3.1
10	-3.6	10.4	1.2	10.4
11	-0.9	3.3	-0.9	3.3
12	1.9	10.3		

Table 17. Degrees of equivalence of individual laboratories, all relative figures × 10^{-5}

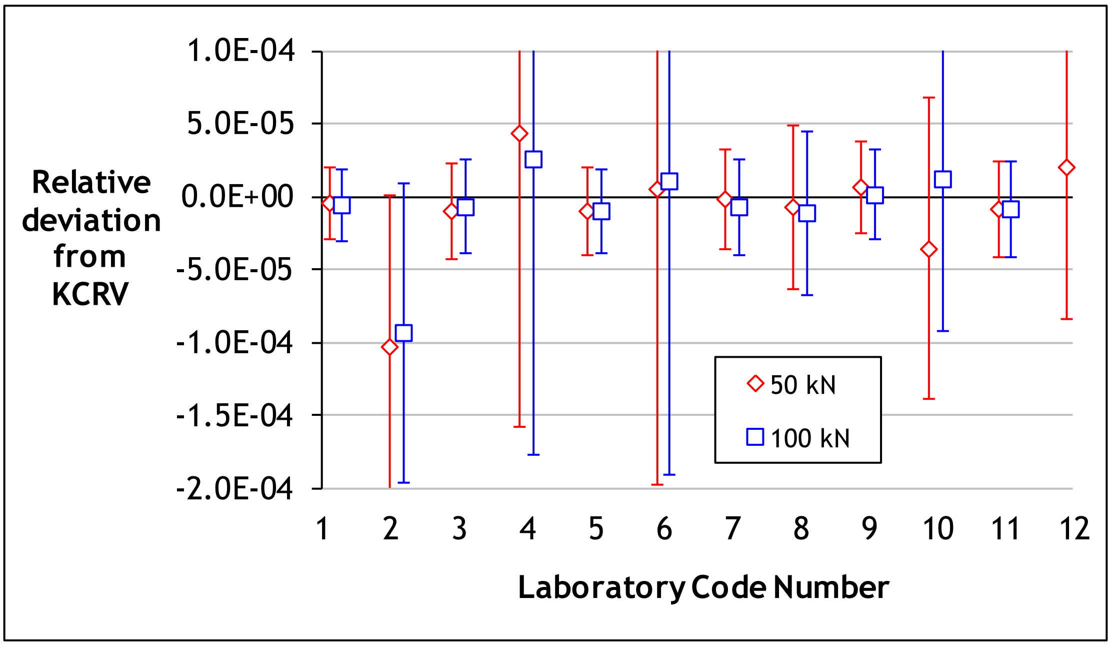

Figure 21. Deviations from the KCRV with associated expanded uncertainties