Report on on-going CCL Key Comparison for the year 2015
Comparison of optical frequency and wavelength standards

CCL-K11

Final

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1 Document control


2 Introduction

The metrological equivalence of national measurement standards and of calibration certificates issued by national metrology institutes is established by a set of key and supplementary comparisons chosen and organized by the Consultative Committees of the CIPM or by the regional metrology organizations in collaboration with the Consultative Committees.

The BIPM.L-K10 (K10) key comparison was initiated in 1993 to provide a basis for demonstrating equivalence of national realizations of wavelength-standards used for the realization of the definition of the metre according to the method (c) in what was called the Mise en Pratique (MeP, refers to the document “Practical realization of the definition of the metre”). Such a comparison seemed of particular importance since the whole field of dimensional metrology had to be traceable to such realizations of the metre. The K10 comparison took only the 633 nm He-Ne standards into consideration. The measurand of the comparison was the difference of the average frequency of the hyperfine components d, e, f, and g in the R(127) 11-5 line as obtained by matrix measurements. The frequency of the reference laser BIPM4 was used as the key comparison reference value, representing the value recommended in the MeP.

The situation for realization of the SI-metre has changed due to the introduction of new techniques for absolute frequency measurements. This has opened up the alternative method (b) in the MeP to realize a frequency/wavelength standard traceable to the SI-second. The practical consequences of this development are that at least two methods are at the moment being used to realize the metre, and that standards of different wavelengths, important for dimensional metrology applications, can now demonstrate traceability with relative ease. Considering these circumstances the 11th CCL meeting which was held in October 2003 at the BIPM decided to close the K10 comparison and initiate a new key comparison named BIPM.L-K11. First measurements in BIPM.L-K11 were made at the BIPM in May 2004. Results from BIPM.L-K10 and BIPM.L-K11 can be found at http://kcdb.bipm.org.

Subsequently, the CIPM has decided, that the comb-related work, which used to provide external services, should stop at the BIPM at the end of 2006. This decision had direct implications on the activity which supported the BIPM.L-K11 that consequently were closed down at the end of year 2006. A proposal for a new scheme for the comparison, based on a group of node-laboratories in the different RMOs and piloted by the Bundesamt für Eich- und Vermessungswesen (BEV, Austria) was therefore made. This proposal, which had been agreed on by the President of the CCL, was given support by the CIPM at its 95th meeting and was endorsed by the 13th meeting of CCL in September 2007. The technical protocol (available from the BIPM web page) defines the procedures to follow in this new comparison, now transferred to the CCL, and named CCL-K11.

This document constitutes the seventh final report for the ongoing key comparison CCL-K11.
3 Organization

3.1 Participants

Table 1. List of participant (and node) laboratories and their contacts.

<table>
<thead>
<tr>
<th>Laboratory Code</th>
<th>Contact person, Laboratory</th>
<th>Phone, Fax, email</th>
</tr>
</thead>
<tbody>
<tr>
<td>VSL</td>
<td>Steven van den Berg VSL – Dutch Metrology Institute Thijsseweg 11, 2629 JA Delft The Netherlands</td>
<td>Tel. +31 15 2691586 e-mail: <a href="mailto:svdberg@vsl.nl">svdberg@vsl.nl</a></td>
</tr>
<tr>
<td>GUM</td>
<td>Dariusz Czulek Central Office of Measures (GUM) 2Elektoralna St., 00-139 Warsaw Poland</td>
<td>Tel. +48 22 5819332 e-mail: <a href="mailto:length@gum.gov.pl">length@gum.gov.pl</a></td>
</tr>
<tr>
<td>VTT MIKES (node)</td>
<td>Jeremias Seppä VTT Technical Research Centre of Finland Ltd Centre for Metrology MIKES Tekniikantie 1, FI-02150 Espoo, Finland</td>
<td>Tel. +358 50 410 5503 e-mail: <a href="mailto:jeremias.seppa@vtt.fi">jeremias.seppa@vtt.fi</a></td>
</tr>
<tr>
<td>BEV (pilot, node)</td>
<td>Michael Matus Bundesamt für Eich- und Vermessungswesen BEV Arltgasse 35, 1160 Wien Austria</td>
<td>Tel. +43 1 21110 6540 Fax +43 1 21110 996000 e-mail: <a href="mailto:michael.matus@bev.gv.at">michael.matus@bev.gv.at</a></td>
</tr>
<tr>
<td>BIPM (observer)</td>
<td>Lennart Robertsson BIPM Pavillon de Breteuil, 92312 Sèvres France</td>
<td>Tel. +33 1 45 07 70 53 Fax +33 1 45 34 20 21 e-mail: <a href="mailto:lroberts@bipm.org">lroberts@bipm.org</a></td>
</tr>
</tbody>
</table>

3.2 Schedule

Table 2 lists the measurements in chronological order, specifying the participants, the places and the dates. It is a characteristic of this comparison to receive the data immediately after completing the measurements which are performed in the respective node or host laboratories.

Table 2. Schedule of the comparison.

<table>
<thead>
<tr>
<th>RMO</th>
<th>Laboratory (country code)</th>
<th>Date of measurement</th>
<th>Node laboratory (place of measurements)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>EURAMET</td>
<td>VSL (NL)</td>
<td>June 2015</td>
<td>VTT MIKES</td>
<td>–</td>
</tr>
<tr>
<td>EURAMET</td>
<td>GUM (PL)</td>
<td>December 2015</td>
<td>VTT MIKES</td>
<td>–</td>
</tr>
</tbody>
</table>

4 Artefacts

4.1 Description of artefacts

Artefacts in this campaign are iodine stabilized HeNe-lasers at \( \lambda \approx 633 \) nm. All are stabilized on the \( f \) component of the \( ^{127}I_2 \) \( R(127) \) 11-5 transition. The designations of the artefacts, as chosen by the owner, are summarized in table 3.
Table 3. Artefacts participating.

<table>
<thead>
<tr>
<th>Laboratory (country code)</th>
<th>Designation of standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>VSL (NL)</td>
<td>NMi-5</td>
<td>MeP, commercial, Winters Electro-Optics, Model 100, Nr. 171</td>
</tr>
<tr>
<td>GUM (PL)</td>
<td>GUM1</td>
<td>MeP, commercial, Thompson-CSF-Jaeger</td>
</tr>
</tbody>
</table>

5 Measuring instructions

5.1 Measurands

Measurements reported here were performed according to the so-called method m1 (Absolute frequency measurement traceable to the realisation of the SI second). Setups of the node laboratories are outlined in the appendices 1-2 of this report.

Initially to the actual measurements each participating laboratory had to state:

- The expected frequency of the standard, $f_e$. This should normally be the frequency used in their calibration service. It is either the recommended value or a value determined by some other means.
- The standard uncertainty $u_e$ of the expected value. This should be a value compatible with the uncertainty given in the CMC for this service.
- The operational parameters used to obtain the two values mentioned above (if applicable).
- Sensitivity coefficients with uncertainties for parameters appearing in the uncertainty budget for the standard (if applicable).

The stated frequency $f_e$ is the actual measurand in this type of key comparison. It is compared on a per lab basis with the measured frequency $f_m$ possibly corrected to the reference operational parameters as given below. One has to note, that the comparison is blind; the participant is not told the result of the measurement before stating his value for $f_e$.

The standard uncertainty of the determined frequency is composed of two parts, one from the frequency measurement, $u_0$, and one from the uncertainty in the settings of the working (and other) parameters, $u_p$. The latter, the uncertainties related to the standard itself are to be estimated by each operator in accordance with their quality system. The uncertainty stemming from the measurements, $u_0$, is estimated by the operator of the experiment alone, or together with personnel involved in the comparison, again in accordance with a quality procedure. These uncertainties are reported in sections D8 and D9 (of the Technical Protocol) and are given as standard uncertainties following GUM\textsuperscript{1} practice. The combined uncertainty of $u_0$ and $u_p$, $u_m$, reported in D10 are given as the root sum squares of $u_0$ and $u_p$.

Table 4 gives the values used for the most important working parameters for each laser. Additional information can be found in the appendices.

\textsuperscript{1} GUM: Guide to the Expression of Uncertainty in Measurement, not to be confused with the participant.
Table 4. Working parameter values for the standards with estimated standard uncertainties in parenthesis as given in the measurement reports included in the appendices.

<table>
<thead>
<tr>
<th>Standard</th>
<th>Power in µW</th>
<th>Modulation width (peak to peak) in MHz</th>
<th>I₂ cold-finger temperature in °C</th>
<th>Cell wall temperature in °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMi-5</td>
<td>102 (2)</td>
<td>6,0 (0,1)</td>
<td>14,96 (0,10)</td>
<td>– (–)</td>
</tr>
<tr>
<td>GUM1</td>
<td>82 (1,2)</td>
<td>6,0 (0,1)</td>
<td>15,0 (0,06)</td>
<td>24 (1,2)</td>
</tr>
</tbody>
</table>

6 Results

6.1 Results and standard uncertainties as reported by participants

The stated frequencies \( f_e \) and the measured frequencies \( f_0 \) (see section 7) and \( f_m \) are given in table 5. The allocated standard uncertainties \( u_e \), \( u_0 \) and \( u_m \), respectively, are included in parenthesis. Both participants estimate \( f_e \) and \( u_e \) by absolute laser frequency calibrations performed at their home labs. Both have respective CMC entries for comb-measurement services.

The data from table 5 are used to calculate the final results according to equations (5-7). The results are given in table 6 and figure 1, respectively.

Table 5. Expected frequencies \( f_e \), measured (uncorrected) frequencies \( f_0 \), and measured frequencies, corrected for influence of operational parameters \( f_m \), together with the respective standard uncertainties of the values.

<table>
<thead>
<tr>
<th>Standard</th>
<th>( f_e (u_e) / \text{kHz} )</th>
<th>( f_0 (u_0) / \text{kHz} )</th>
<th>( f_m (u_m) / \text{kHz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMi-5</td>
<td>599,5 (1,9)</td>
<td>598,53 (0,23)</td>
<td>598,5 (0,23)</td>
</tr>
<tr>
<td>GUM1</td>
<td>605,4 (12)</td>
<td>596,43 (0,32)</td>
<td>596,4 (0,32)</td>
</tr>
</tbody>
</table>

Table 6. Degree of equivalence and \( E_n \) values for the standards.

<table>
<thead>
<tr>
<th>Standard</th>
<th>( \Delta f_e )</th>
<th>( U_r = 2u_e )</th>
<th>( E_n = \Delta f_e / U_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMi-5</td>
<td>+2,1 \cdot 10^{-12}</td>
<td>8,1 \cdot 10^{-12}</td>
<td>+0,26</td>
</tr>
<tr>
<td>GUM1</td>
<td>+19 \cdot 10^{-12}</td>
<td>51 \cdot 10^{-12}</td>
<td>+0,38</td>
</tr>
</tbody>
</table>


7 Analysis

7.1 Calculation of the KCRV

It is a distinctive feature of this key comparison, that the KCRV is determined on a per participant basis. Thus each participant has its own KCRV which is used to test consistency.

Denote the measured (uncorrected) frequency $f_0$ with standard uncertainty $u_0$, and the measured frequency, corrected for influence of operational parameters $f_m$ with standard uncertainty $u_m$. Then the following holds:

$$f_m = f_0 - \delta$$

(1)

The symbol $\delta$ denotes the condensed information about the influence of the actual working parameters and other quantities on the laser frequency. A linear model is commonly used:

$$\delta = \sum_i s_i \cdot \Delta x_i + \sum_i \delta_i$$

(2)

Where the $s_i$ denote the sensitivity coefficients and $\Delta x_i$ the deviations of the respective working parameters from the nominal values (care must be taken choosing the correct signs for both quantities). All other influence quantities (e.g. electronic offsets, cavity alignments, etc.) are modelled with the quantities $\delta_i$. These have usually zero expectation values but non-zero uncertainties. The uncertainties are thus derived in a straightforward way as:
\[ u_p = \sqrt{\sum_i (u(s_i)^2 \cdot \Delta x_i)^2 + \sum_i (s_i \cdot u(\Delta x_i))^2 + \sum_i u(\delta_i)^2} \]  
\text{(3)}

and

\[ u_m = \sqrt{u_p^2 + u_0^2} \]  
\text{(4)}

Denote the expected frequency \( f_e \) with standard uncertainty \( u_e \), and the measured frequency, corrected for influence of operational parameters \( f_m \) with standard uncertainty \( u_m \). In the nomenclature of the CIPM-MRA \( f_m \) (together with its standard uncertainty \( u_m \)) denotes the KCRV and \( f_e \) (together with its standard uncertainty \( u_e \)) the measurand.

For a particular standard, \( i \), construct the dimensionless quantities

\[ \Delta f_i(i) = \frac{f_e(i) - f_m(i)}{f_m(i)} \]  
\text{(5)}

\[ u_r(i) = \frac{\sqrt{u_e^2(i) + u_m^2(i)}}{f_m(i)} \]  
\text{(6)}

It must be noted that \( f_e \) and \( f_m \) should be transferred to the same (usually nominal) working parameters for the standard, which would be expected to coincide with those for which \( f_e \) is valid if no other instructions are given by the participating laboratory.

### 7.2 Calculation of Degrees of Equivalence

To test consistency between the measured values and the expected ones, hypothesis testing at a confidence level of 95\% is to be performed. The result will serve as a basis for the review of the CMC and indicate the compatibility with the claimed capabilities. In this framework the “degree of equivalence” (DoE) can be obtained in the usual way. Thus the (relative) DoE is \( \Delta f_r \) (equ. 5) with its standard uncertainty \( u_r \) (equ. 6). The consistency can thus be checked by the following condition:

\[ -1 \leq E_r = \frac{\Delta f_r(i)}{U_r(i)} \leq 1 \text{ with } U_r(i) = 2 \cdot u_r(i) \]  
\text{(7)}

As discussed at the 14\textsuperscript{th} CCL meeting, June 2009, it is neither necessary nor useful to determine a pair-wise degree of equivalence. For all results reported the expanded uncertainty to a 95\% confidence level can be obtained by multiplying the standard uncertainties with \( k = 2 \).

### 7.3 Discussion of results

Frequency measurements have been carried out on two national wavelength standards. A good agreement between the stated and the measured frequency values was found.

Both participants have respective CMC for this kind of service. The uncertainty stated in this comparison is equal or smaller than the CMC uncertainty. The homogenized data is summarized in table 7.
Table 7. Relative expanded uncertainties stated in this comparison versus published uncertainties in Appendix C of the KCDB. All values are recalculated to multiples of $10^{-12}$ for ease of comparison.

<table>
<thead>
<tr>
<th>Laboratory (country code)</th>
<th>$U_e = 2u_e$</th>
<th>$U_{CMC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VSL (NL)</td>
<td>$8 \cdot 10^{-12}$</td>
<td>$8 \cdot 10^{-12}$</td>
</tr>
<tr>
<td>GUM (PL)</td>
<td>$51 \cdot 10^{-12}$</td>
<td>$51 \cdot 10^{-12}$</td>
</tr>
</tbody>
</table>

CCL-K11 is not intened to derive a better value for any of the frequencies from the list of recommended radiations for the realisation of the metre and other optical frequency standards (formally known as MeP). Therefore it is not mandatory that $f_e$ is a value out of this list, nor is it necessary to correct for the nominal working parameters. It is however necessary for each participant to follow his internal working procedures like for any calibration for the respective CMC entry.

7.4 Linking of result to other comparisons

Plotting the DoE of all participants in the same graph links the results of this on-going key comparison. This is possible even for different nominal frequencies since the DoE are defined as relative quantities.

Appendix A Equipment and measuring processes of the participants

Details on the individual equipment and standards can be found in the measurement reports collated in the appendices 1 to 2 of the supplementary data to this report. These files are electronic copies; the respective node laboratories keep the signed originals.