Final Report

EUROMET.EM.M-S1

Euromet Project 597

‘Intercomparison of magnetic flux by means of coil transfer standard’

1. Introduction

Magnetic flux is one of the basic quantities of a magnetic field. Magnetic flux standards are used for the calibration of fluxmeters and for DC magnetic material measurements. These standards are also used (together with coil standards of magnetic flux density) for the calibration of the area of search coils or of the magnetic moment per electrical current of coils. This comparison was proposed in order to determine the capabilities of the NMIs in Europe in the measurement of magnetic flux.

Euromet project no.597 titled “Intercomparison of magnetic flux by means of transfer standard” started in the beginning of 2001 with participants from five European countries including CMI (Czech Republic) as the pilot laboratory. Each participant was asked to use their established measurement method to transfer the unit of magnetic flux.

2. Symbols and units

The equivalent units weber per ampere (Wb/A) and henry (H) are used concurrently in this Final Report.

3. Participants

The participants and their affiliations, the five institutes involved, are listed in Table 1 in order of transfer standard circulation:

Table 1

J Kupec, V Kohout	Czech Metrology Institute (CMI), Prague, Czech Republic - acting as the pilot laboratory
J Belliss, S Bryant	National Physical Laboratory (NPL), Teddington, United Kingdom
F Fiorillo	Instituto Elettrotecnico Nazionale 'Galileo Ferraris' (IEN), Torino, Italy
V Shifrin, V Khorev	Mendeleyev Institute for Metrology (VNIIM), St.Petersburg, Russia
K Weyand, H Ahlers	Physikalisch-Technische Bundesanstalt (PTB), Braunschweig, Germany

3. Organization of the comparison

The coil transfer standard was transported in a robust aluminium case, prepared especially for the comparison (cubic, with sides of about 0.5 m). The first measurement was performed in the pilot laboratory of CMI, Prague in March, 2001. The standard was transported from Prague to NPL, Teddington (measurement in April), then from the United Kingdom to IEN, Torino (measurement in May 2001). As other custom formalities were needed for transportation to Russia the standard had to return first to Prague, before it was sent to VNIIM, St.Petersburg. In the meantime, a control measurement was performed at CMI, Prague, to check the stability of the value of the transfer standard (July 2001). No changes were identified. The comparison continued in VNIIM, St.Petersburg in September 2001. Upon its return to Prague, there was some time delay due to Czech customs. The last comparison outside CMI was performed in PTB, Braunschweig, Germany in December 2001. The final measurement was performed again at CMI, Prague in February 2002. No damage to the transfer coil standard had occurred thanks to the well-constructed transport case.

4. Transfer standard

The Czech Metrology Institute, acting as the pilot laboratory, selected a coil with two cylindrical windings on a ceramic cylindrical form (ser. no. 7704). The nominal value is 10 mWb/A (mutual inductance 10 mH). Primary windings were wound with a copper conductor with a diameter of 1 mm and separated secondary windings were wound with a copper conductor with a diameter of 0.8 mm. The resistance of the primary 450 windings is 4.4 Ω, the resistance of the secondary 1013 windings is 25 Ω. The capacitance between primary and secondary windings is 36 pF. As this capacitance is low and the self capacitances of the primary and secondary windings are of similar values, the value of magnetic flux per unit current in the secondary windings (mutual inductance of the primary and secondary windings) does not depend on frequency in the range from 0 Hz to 20 Hz with a supposed relative uncertainty of mutual inductance higher than 1.10^{-5}. The resonance frequency f_r of the primary and secondary windings connected in the series was tested in CMI and calculated at 30 kHz ± 5kHz. A very simplified equation for the mutual inductance of the coil M for frequency f is:

$$M(f) = M_0 + kf^2/f_r^2, \qquad (1)$$

where M_0 is the mutual inductance of the coil at zero frequency, the value of factor k is ± 1 according to the method used and f is the frequency. The influence of the self-capacitances of both windings and the influence of the capacitance between the windings on the results of the measurement of the mutual inductance (magnetic flux per unit current) depends on the frequency used and on the connection of the coil, thus on the method of the measurement. This influence is also indispensable for impulse methods of measurements (commuting or switching off the DC current), which are called DC methods with zero frequency used. Nevertheless, it is usually possible to compensate these small influences due to capacitance impurity effects. The correction factor of the constant of the coil (constant is defined in part 5) for a temperature around 23 °C was measured in CMI 0.0003 mH/ °C with an uncertainty (k=2) of 0.0001 mH/ °C. The higher the temperature is, the higher the coil constant (mutual inductance) is.

5. Quantity measured

The quantity that was measured is the value of the constant k_Φ of the inductor. This is either the value of the constant in Wb/A, which is the magnetic flux (Wb) in the secondary winding for an electric current of 1 A in the primary winding (when the DC method is used), or the value of the mutual inductance (H) of the inductor for a frequency lower than or equal to 20 Hz (when AC method is used).
These DC and AC values should differ less than 1.10^{-5} of the value and therefore either the DC method or the low frequency AC method is sufficient.
Participants were asked to report both the original measured value of the coil constant and the value corrected at 23 °C.

6. Methods of measuring the coil constant

CMI - The comparison was performed with elements of the group coil standard for magnetic flux. The value of coil constant No. 7704 was determined from the mean value of the group standard. All coils from the group standard and coil transfer standard were compared to one another. Commutation of the DC current through primary windings of both coils was performed. The difference between the induced electrical impulses in the secondary windings was measured with a sensitive fluxmeter with a fotoelectrical amplifier. Parasitic fields and parasitic capacitances were compensated by transposition methods (commutations of electrical connections and commutations of location of coils). The same DC current differential method was used for all three comparisons at CMI (at the beginning in March 2001, in the middle in July 2001 and at the end in February 2002). The temperature in the laboratory during the measurement was between 22.4 °C and 23.6 °C.

NPL – The value of the transfer standard was measured with the primary and secondary windings connected in series, in the straight and then opposing senses, consequently.
The resulting self-inductances were measured. The mutual inductance was calculated using the sum and difference method. A screened four-arm bridge with an inductive voltage divider was used. The measured inductance was in the first arm of the bridge, the standard inductance with a decade-switched variable resistance box in the second arm, the other arms were made with an inductive voltage divider. At NPL, the unit of inductance is derived from capacitance, which is traceable to the quantum Hall resistance standard and the consensus value of the von Klitzing constant. The temperature in the laboratory during the measurement was between 20.4 °C and 20.6 °C.

IEN – The value of the transfer standard was determined both under DC and AC excitation. For the DC comparison (direct measurement of magnetic flux with a calibrated electronic fluxmeter), the peak current value of 90 mA in the primary winding was measured at a temperature of 26.5 °C. The relative combined standard uncertainty of the constant of the magnetic flux of the transfer standard was determined as being 1.10^{-3}. The value of the transfer standard for 23 °C was 9.99740 mWb/A.
AC comparison – the sinusoidal current of peak amplitude < 0.5 A through primary winding was measured by a digital voltmeter, the voltage on the secondary winding was simultaneously measured by an identical voltmeter operated in the synchronously sub-sampled mode. The measurement was performed at two frequencies:
10 Hz with a relative combined standard uncertainty of $1.7\ .10^{-4}$ with transfer coil constant of 9.99768 mH for a temperature of 23 °C (measuring temperature: 26.5 °C),
20 Hz with the same standard uncertainty, with a transfer coil constant of 9.99705 mH for 23 °C (the same measuring temperature 26.5 °C). Only the measurement at 20 Hz was used in later tables and calculations in agreement with IEN.

VNIIM – The compensation induction method at low frequencies (from 14 Hz to 32 Hz) was used. The calculable mutual inductor 10 mWb/A of Campbell type together with an additional variable mutual inductor of 10 to 30 µWb/A was applied as a standard for the comparison. The differential signal was measured by means of a synchronous detector, the output of which is connected to two DC-voltmeters for indicating reactive and active (having the same phase as the current) components of the signal. The phase of the synchronous detector was adjusted so as to be in such a state, as for the second (active) digital voltmeter to be independent of variable mutual inductor variation. The reactive component was measured by the first voltmeter. This voltmeter indication was calibrated by variable mutual inductor changing. The method of transposition by reversing the connections was used. The temperature during measurement was between 20.2 °C and 21.4 °C.

PTB – The current through the primary winding as well as the induced voltage in the secondary winding were measured at a frequency of 16.667 Hz. A burster resistor 1 Ω was used for the primary current measurement, a digital oscilloscope (with filters and two 15-bit, 100kHz A/D converters) was used for the simultaneous measurement of the voltages.
The measured arrays of voltages were transmitted to a computer, where the Fourier fundamental wave analysis was performed and the corrections were calculated.
The temperature during the measurement was between 22.8 °C and 23.1 °C.

Table 2: Measurement methods and conditions

Institute	Transfer measurement method	Current through primary winding in A	Temperature in °C
CMI	Comparison with group coil standard of magnetic flux - DC impulse method. Compensation with sensitive fluxmeter as indicator of difference	0.50	22.4 – 23.6
NPL	Bridge method of measurement of inductance at 20 Hz	0.005	20.4 – 20.6
IEN	Measurement of induced voltage and primary current at 20 Hz	0.35	26.0
VNIIM	Comparison with calculable Campbell standard with AC current at frequencies from 14 Hz to 31 Hz	0.30	20.2 – 21.4
PTB	Measurement of induced voltage and primary current with digital oscilloscope at 16,667 Hz	0.10	22.8 – 23.1

7. Results of transfer coil standard constant determination

Transfer standard no. 7704, as one coil of the Czech group standard of magnetic flux, has not been used for international comparisons in recent years. The mean value of the constants of the inductors participating in this group standard was obtained from comparisons with VNIIM prior to 1980. It was declared as the Czechoslovak National Standard of Magnetic Flux. This value of the group standard was slightly corrected (-0.2 µWb/A) after comparison with VNIIM in 1988 and after calibrations of inductor no.7705 (participating in group standard) at NPL in 1996 and 1998. The group standard was declared as the Czech National Standard of Magnetic Flux with the declared fixed value equal the mean value of the constants of the participating inductors in February of 2000. The value of transfer standard no.7704 was determined at 9996.60 µWb/A (µH) as a part of the Czech group standard at that time (February, 2000).

All of the comparisons carried out between CMI and VNIIM were based on a Russian Campbell mutual inductor, the value of which was calculated from length measurement.

The methods of comparison at VNIIM in 1988 and before 1980 were DC methods.

The method used by VNIIM in the Euromet 597 comparison in 2001 was a completely different AC method based also on the Campbell mutual inductor. The uncertainty of the Campbell inductor is only a small part of the reported VNIIM uncertainty of the Euromet 597 comparison in 2001 (about 3 percent). In the comparisons from 1988 and before 1980 this part of the uncertainty is even smaller. The correlation between the CMI Euromet 597 measurement and the VNIIM Euromet 597 measurement is supposed to be negligible due to the very small influence of the common part of the uncertainty and to the long interval of time since the earlier comparisons. The possible correlation of the CMI results with the NPL results presents a similar situation. The values of the calibrations of inductor no. 7705 at NPL in 1996 were used only in smaller part for a slight correction of the Czech group standard

compared to the VNIIM results. The NPL value from 1998 was not used at all. The method of measurement of NPL was similar in 1996 and in Euromet 597 but the uncertainty of common basic standards is only a small part of the NPL Euromet 597 reported uncertainty. Negligible correlation of the results of CMI with the results of NPL is supposed due to above described circumstances and to the long time interval (5 years) between the NPL measurements and also due to opposite symmetry (the NPL value from 1996 is smaller than the CMI declared value of the coil no. 7704 and NPL value from 2001 is greater than the CMI declared value of the coil No. 7704 in the indirect comparison). No other possibilities of correlation between the participant measurements were identified.
The first measurement was the comparison at CMI (as the coordinating laboratory) with the value of the CMI group standard in March, 2001. The result was 9996,25 μWb/A, uncertainty 2σ = 2.4 μWb/A. In July 2001 measurements at CMI were performed with the same method and with the same result, 9996.25 μWb/A, and the same uncertainty, 2σ = 2.4 μWb/A. The last measurement at CMI was performed with the same method with a result of 9996.51 μWb/A, and an uncertainty of 2σ = 2.4 μWb/A in February 2002. As the differences are small, no drift in the value of the transfer standard had occurred in the period between March 2001 and February 2002 and no corrections were needed.
The mean value of the CMI measurements, 9996.34 μWb/A, and the uncertainty of the mean value, 2σ = 2.40 μWb/A, was reported for the results of the transfer coil constant determination.

Table 3 shows the coil constant values as reported: the reported values calculated for the temperature of 23 °C using the coil temperature coefficient, the number of measurements and expanded combined uncertainties as quoted by the various Institutes. The CMI performed the measurement three times: before, during and after circulating the transfer standard to determine whether the coil values had changed during the transportation. The mean value of these three measurements together with the values of other Institutes was used for the calculation of the mean value of the constant of the coil and the listed uncertainty for the mean of CMI measurements together with the uncertainties of other Institutes were used for uncertainty calculations. As the uncertainties differ by more than one order of magnitude, the reciprocal of the individual uncertainty was defined as the weighting factor $W_i = 1/\sigma_I$ in order to determine a weighted mean value, $K_{\Phi W}$, according to:

$$K_{\Phi W} = \frac{\sum K_{\Phi 23i} W_i}{\sum W_i} \tag{2}$$

To determine the standard deviation, the individual deviations from the weighted mean, listed in the last column of Table 3, were weighted in the same way as the individual coil constant values:

$$\sigma_W = \frac{\sum | K_{\Phi W} - K_{\Phi 23i} | \, W_i}{\sum W_i} \tag{3}$$

The final result is

$$\boldsymbol{K_{\Phi W} \pm k\sigma_W = (9996.48 \pm 0.66)\ \mu\mathrm{Wb/A}\ (\mu\mathrm{H}) = 9996.48\,(1 \pm 66.10^{-6})\ \mu\mathrm{Wb/A}\ (\mu\mathrm{H}),}$$

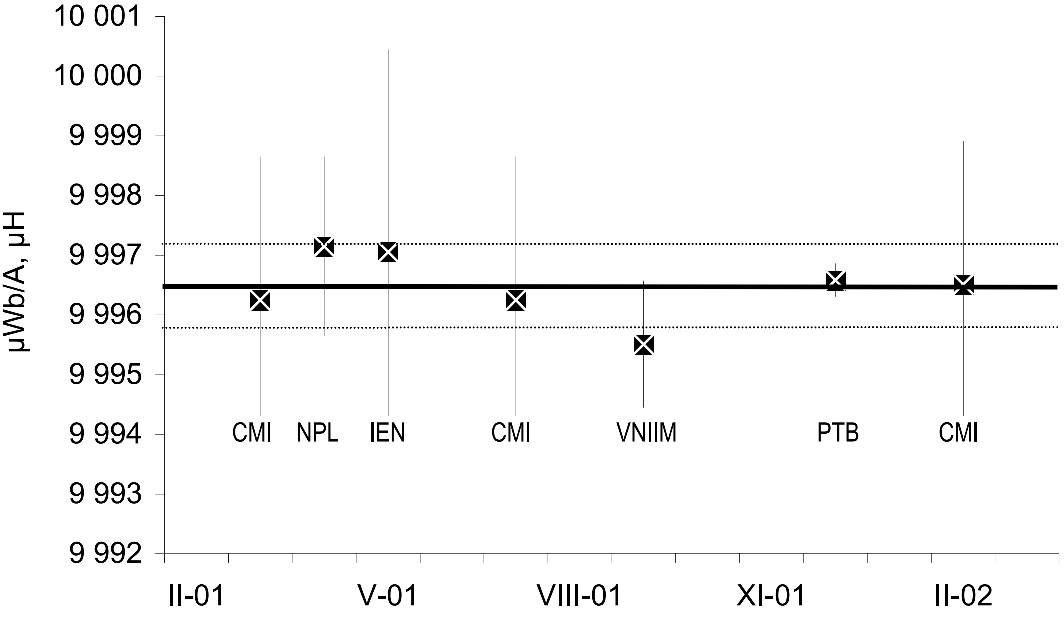
where $k\sigma_W$ is the value of the expanded weighted deviation, with σ_W determined from the equation (3) $\sigma_W = 0.33$ µWb/A (µH) and a coverage factor of $k = 2$. All participants agree with the weighted mean within the stated coverage factor ($k = 2$). The way of calculating and expressing the uncertainties is the same as in the first magnetic value Euromet comparison no.446 (1).

Table 3: Results of transfer coil standard of magnetic flux constant determination

Institute	**Date of measurements**	**Reported value $K_{\phi 23}$ (calculated for 23 °C, k= 2) µWb/A (µH)**	**Number of measurements**	**Reported 2σ µWb/A (µH)**	**$K_{\phi 23}$ - $K_{\phi W23}$ µWb/A (µH)**
NPL	April 2001	9997.15	4	1.50	+0.67
IEN	May 2001	9997.05	10	3.40	+0.57
CMI	March 2001 July 2001 Feb 2002	9996.34	8 8 8	2.40	-0.14
VNIIM	Sept 2001	9995.51	12	1.06	-0.97
PTB	Dec 2001	9996.58	6	0.28	+0.10
Weighted mean value		**9996.48**			**0.33**

Figure 1 shows the relative deviation of the coil constant determinations of the various Institutes, and their reported 2σ uncertainties in the measurement period.

Figure 1



8. Final remarks

Euromet Project 597 ‘Intercomparison of magnetic flux by means of coil transfer standard’ is the second Euromet comparison of magnetic quantity using an artefact as a transfer standard.
The first Euromet comparison of magnetic quantity was Project no.446, a comparison of magnetic flux density (1). It is important for magnetic laboratories to have these two quantities at their disposition for the possibility of calibrating other magnetic quantities (magnetic moment, windings area). The agreement in the results presented for magnetic flux provides a good foundation for establishing capability for other magnetic quantities.

With respect to the measurements of the transfer coil constant of magnetic flux, all participants’ results are in accord with the weighted mean within a declared 2σ uncertainty. The good accord of the comparisons is very valuable with respect to the different methods and measurement techniques that were used.

References

(1) Weyand, K. and others: International comparison of magnetic flux density by means of field coil standard – Final Report (Draft B) on Euromet Project No. 446, Metrologia, 2001, 38, 187-191.

Annexes:

Annex 1 – Measurement report NPL, United Kingdom
Annex 2 – Measurement report IEN, Italy
Annex 3 – Measurement report CMI, Czech republic
Annex 4 – Measurement report VNIIM, Russia
Annex 5 – Measurement report PTB, Germany

Annex A1

Report of the EUROMET Project 597
Comparison of magnetic flux by means of a coil transfer standard.

Janet H Belliss

Introduction

The purpose of the comparison is to compare the capabilities of NMIs in Europe in either the measurement of magnetic flux in Wb/A or the mutual inductance in mH of a transfer standard. This report details the measurements of mutual inductance of the transfer device at a frequency of 20 Hz.

Transfer Standard

On arrival the transfer standard S/N 7704 was unpacked and visually examined and the resistance of the primary and secondary windings were measured. The coil was found to be visually undamaged and functioning correctly. The device was then placed in a humidity chamber for a period to stabilize before measurements commenced.

Measurement chain and traceability

At NPL the unit of inductance, the henry, is derived from capacitance, the farad (1), which is traceable to the quantum Hall resistance standard and the consensus value of the von Klitzing constant R_{k-90}. By using a series of DC bridges the value of a 1000 Ω quadrifilar resistor, whose DC and AC values only differ by a few parts in 10^9, is determined. Then using a series of coaxial AC bridges the values of resistors and capacitors are determined culminating in the establishment of our primary 10 pF capacitance standard (2). To derive inductance from this traceable capacitance scale a high Q self-inductor is resonated with a variable capacitor in a parallel tuned circuit, at a measured frequency, so the impedance of the combination is almost purely resistive. Then at the same frequency the conductance and residual capacitance of the combination are measured. The capacitance and shunt conductance of the variable capacitor are also measured. The inductance of the high Q self-inductor is then calculated from these measured values. The inductance comparison bridge, shown in figure 1, relates the high Q inductor to our primary 3·8 mH inductance standard (3) and in turn other inductance standards to establish a traceable scale of inductance.

Inductance comparison bridge

The four-arm bridge shown in Figure 1 equates the ratio of the impedances of the two bridge arms containing the inductors L_1 and L_2 according to the balance equation (1).

$$\frac{L_1}{L_2} = \frac{R_1}{R_2 + R} = \frac{n}{(1-n)} \qquad (1)$$

where L_1, R_1 and L_2,(R_2+R) are the total values of the series inductance and resistance in arms 1 and 2. The ratio of the impedances of the two bridge arms containing the inductors is equated, on nulling the detector D, to the ratio n/(1-n) of the inductive voltage divider (IVD), A, where n is its dial reading. The quadrature component of the IVD ratio is negligible. Arm

2 of the bridge also includes the decade-switched variable resistance box R whose resistance and residual inductance at various combinations has been calibrated. The effects of capacitance currents within the bridge network are eliminated by completely screening all the bridge components and setting the potential of the screen to that of the output of IVD A by adjusting the output of IVD B to which it is connected.

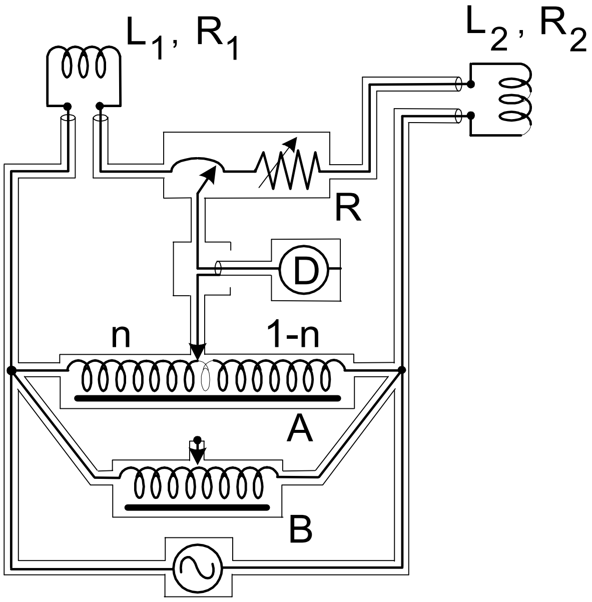


Figure 1

The mutual inductance of the transfer coil S/N 7704 was measured in terms of NPL standard self-inductors by a sum and difference method using bridge shown in figure 1 at a frequency of 20 Hz. The primary and secondary coils of the mutual inductor were connected in series in the same and then opposing senses and the resulting self-inductances L_A and L_B measured. The mutual inductance M was calculated from L_A and L_B using equation (2).

$$M = \frac{(L_A - L_B)}{4} \tag{2}$$

Measurement Conditions

The coil was kept at a relative humidity of 50 ± 5 % for a period to stabilize prior to measurement.
Measurement frequency : 20 Hz.
Mean measurement temperature: 20·53 ± 0·5 °C .

Measurement Results

The results of the four measurements are listed in the table below. The results at 23°C where obtained by calculation assuming a temperature coefficient of 0·0003 mH /°C.

Measurand: Coil constant k_Φ

Date of measurement	**T** **^{o}C**	**$I_{c*)}$** **mA**	**k_Φ** **mH**	**$k_{d\Phi}$** **{T=23^{o}C}** **mH** **by calculation**	comb. stand. uncert. **$u(k_{d\Phi})$** **mH**	eff. degree of freedom **ν_{eff}**
26 April 2001	20.5	5.5	9.996 82	9.997 57	0.000 75	*25*
27 April 2001	20.4	5.5	9.996 05	9.996 83	0.000 75	*25*
30 April 2001	20.6	5.5	9.996 25	9.996 97	0.000 75	*25*
2 May 2001	20.6	5.5	9.996 52	9.997 24	0.000 75	*25*
Mean: 9.996 41 mH at 20.53 ^{o}C					**0.000 75 mH**	**25**
Mean: 9.997 15 mH at 23 ^{o}C						

**)* Since the mutual inductance of coil has been measured the current was also measured , with a 1% accuracy.

Uncertainty budget for $k_{d\Phi}$

Transfer standard CMI 7704:

Quantity X_i	*estimate* x_i	*Standard uncertainty* $u(x_i)$	*Probability distribution / method of evaluation(A,*B)	*Sensitivity coefficient* c_i	*Uncertainty contribution* $u_i(k_{d\Phi})$ *ppm*	*Degree of freedom* ν_I
C	0·1 ppm	0.29 ppm	Normal/B	1·0	0.29	◊
H	1 ppm	10 ppm	Normal/B	1·0	10.0	◊
S	20 ppm	50 ppm	Normal/B	1·0	50.0	◊
T	20·53 °C	0·5 °C	Rectangular/B	30.0	8.7	◊
BR	1 ppm	35 ppm	Rectangular/B	1·0	20.2	◊
R	30 ppm	50 ppm	Normal/A	1·0	50.0	5
$k_{d\Phi}$					**$u(k_{d\Phi})$ 75 ppm**	ν_{eff} ***25***

Where Quantity, X_i :

C = Determination of our Primary 10 pF Capacitor from QHE.
H = Determination of our Primary 3·8 mH inductor from the 10 pF capacitor.
S = Build-up of the NPL standard inductors at 20 Hz.
T = Temperature of Transfer standard S/N 7704.
BR = Bridge resolution.
R = Repeatability of bridge measurements.

References

1. G H Rayner, B P Kibble, M J Swan. *On Obtaining the Henry from the Farad.* NPL Report DES 63. March 1980.

2. Janet H Belliss. *The NPL standard of Capacitance and its Dissemination.* NPL Memorandum DES 66. November 1992

3. G H Rayner, B P Kibble. *The NPL Inductance Comparison Bridge.* NPL Memorandum DES 52 August 1984. Revised by Janet H Belliss. January 1989.

Reference: L706.020

Date of issue: 27 July 2001
Revision Date: 25 October 2002

Participating Laboratory : National Physical Laboratory, UK.

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Annex A2

Transfer standard CMI 7704

Measurement report, part A

Participating laboratory: IEN Galileo Ferraris

The value of the transfer constant of the mutual inductor CMI 7704 has been determined at IEN Galileo Ferraris both under DC and AC excitation. In the latter case, the measurement was performed at two frequencies: 10 Hz and 20 Hz.

1) *DC measurement*

This measurement was carried out by means of a ballistic setup. The current in the primary winding was switched between symmetric values $i_p = \pm$ 90 mA and the flux linked with the secondary winding was correspondingly determined by means of a calibrated electronic fluxmeter. The peak current value in the primary winding was determined by the measurement of the voltage drop on a 10 Ω calibrated resistor (Tinsley 1659) through a digital voltmeter HP 3458A. The measurement was repeated, under identical conditions, six times. Measuring temperature: 26.5 °C.

1) *AC measurement*

A sinusoidal current of peak amplitude $i_p < 0.5$ A was fed into the primary winding by use of a universal source HP 3245A and a bipolar operational amplifier KEPCO BOP 72-5. Two tests were performed, at 10 Hz and 20 Hz respectively. The supply current was again measured by means of the 10 Ω calibrated resistor Tinsley 1659 and a digital voltmeter HP 3458A. The voltage on the secondary winding was simultaneously measured by an identical digital voltmeter HP 3458A. Both voltmeters operated in the synchronously sub-sampled mode. Ten repeated measurements were carried out, under identical conditions, for each frequency. Measuring temperature: 26.5 °C.

Date:...25 July 2001..

Signature..............

Measurement results
for transfer standard CMI 7704

Measurand: Coil constant k_{Φ}

DC measurement using the ballistic method and a calibrated fluxmeter

Date of measurement	**T** °C	I_c *) A	k_{Φ} **mWb/A or mH**	$k_{d\Phi}$ **{T=23°C} mWb/A or mH**	comb. stand. uncert. $u(k_{d\Phi})$ (relative)	eff. Degrees of freedom ν_{eff}
25 June 2001	26.5	0.090	9.99843	9.99740	$1 \cdot 10^{-3}$	∞
Mean:						

Humidity*: 60 %*

Laboratory: IEN Galileo Ferraris

Measurement results
for transfer standard CMI 7704

Measurand: Coil constant $\boldsymbol{k}_{\Phi}$

AC measurement at 10 Hz

Date of measurement	**T** °C	I_c *) **A**	k_{Φ} **mWb/A or mH**	$k_{d\Phi}$ **{T=23°C}** **mWb/A or mH**	comb. stand. uncert. $u(k_{d\Phi})$ (relative)	eff. Degrees of freedom ν_{eff}
26 June 2001	26	0.49	9.99858	9.99768	$1.7 \cdot 10^{-4}$	∞
Mean:						

Humidity*: 60 %*

Laboratory: IEN Galileo Ferraris

Measurement results
for transfer standard CMI 7704

Measurand: Coil constant $\boldsymbol{k}_{\Phi}$

AC measurement at 20 Hz

Date of measurement	**T** °C	I_c *) A	k_{Φ} **mWb/A or mH**	$k_{d\Phi}$ **{T=23°C} mWb/A or mH**	comb. stand. uncert. $u(k_{d\Phi})$ (relative)	eff. Degrees of freedom ν_{eff}
26 June 2001	26	0.35	9.99795	9.99705	$1.7 \cdot 10^{-4}$	∞
Mean:						

Humidity*: 60 %*

Laboratory: IEN Galileo Ferraris

Uncertainty budget for $K_{d\Phi}$

DC measurement using the ballistic method and a calibrated fluxmeter

Transfer standard CMI 7704:

Source of uncertainty X_i	*estimate* x_i	*Relative standard uncertainty* $u(x_i)$	*Probability distribution / method of evaluation(A,*B)	*Sensitivity coefficient* c_i	*Relative uncertainty contribution* $u_i(k_{d\Phi})$	*Degree of freedom* ν_i
Magnetic flux		$0.9\cdot10^{-3}$	*Rectangular/B*	*1*	$0.9\cdot10^{-3}$	∞
Voltage on the calibrated shunt		$1\cdot10^{-5}$	*Rectangular/B*	*1*	$1\cdot10^{-5}$	∞
Shunt resistance value		$6\cdot10^{-5}$	*Rectangular/B*	*1*	$6\cdot10^{-5}$	∞
Repeatability		$1.2\cdot10^{-4}$	*Normal/A*	*1*	$1.2\cdot10^{-4}$	*5*
$k_{d\Phi}$ **9.99740**					$u(k_{d\Phi})$ $1\cdot10^{-3}$	ν_{eff} ∞

Humidity*: 60 %*

Laboratory: IEN Galileo Ferraris

Uncertainty budget for $K_{d\Phi}$

AC measurement at 10 Hz

Transfer standard CMI 7704:

Source of uncertainty X_i	*estimate* x_i	*Relative standard uncertainty* $u(x_i)$	*Probability distribution / method of evaluation(A,B)*	*Sensitivity coefficient* c_i	*Relative uncertainty contribution* $u_i(k_{d\Phi})$	*Degree of freedom* ν_i
Secondary voltage		*$1.1\cdot10^{-4}$*	*Rectangular/B*	*1*	*$1.1\cdot10^{-4}$*	∞
Voltage on the calibrated shunt		*$1.1\cdot10^{-4}$*	*Rectangular/B*	*1*	*$1.1\cdot10^{-4}$*	∞
Shunt resistance value		*$6\cdot10^{-5}$*	*Rectangular/B*	*1*	*$6\cdot10^{-5}$*	∞
Repeatability		*$1.1\cdot10^{-5}$*	*Normal/A*	*1*	*$1.1\cdot10^{-5}$*	*9*
$k_{d\Phi}$ ***9.99768***					**$u(k_{d\Phi})$** **$1.7\cdot10^{-4}$**	**ν_{eff}** ∞

Humidity: *60 %*

Laboratory: IEN Galileo Ferraris

Uncertainty budget for $K_{d\Phi}$

AC measurement at 20 Hz

Transfer standard CMI 7704:

Source of uncertainty X_i	*estimate* x_i	*Relative standard uncertainty* $u(x_i)$	*Probability distribution / method of evaluation(A,B)*	*Sensitivity coefficient* c_i	*Relative uncertainty contribution* $u_i(k_{d\Phi})$	*Degree of freedom* ν_i
Secondary voltage		$1.1 \cdot 10^{-4}$	*Rectangular/B*	*1*	$1.1 \cdot 10^{-4}$	∞
Voltage on the calibrated shunt		$1.1 \cdot 10^{-4}$	*Rectangular/B*	*1*	$1.1 \cdot 10^{-4}$	∞
Shunt resistance value		$6 \cdot 10^{-5}$	*Rectangular/B*	*1*	$6 \cdot 10^{-5}$	∞
Repeatability		$2 \cdot 10^{-5}$	*Normal/A*	*1*	$2 \cdot 10^{-5}$	*9*
$k_{d\Phi}$ **9.99705**					$u(k_{d\Phi})$ $1.7 \cdot 10^{-4}$	ν_{eff} ∞

Humidity*: 60 %*

Laboratory: IEN Galileo Ferraris

Date: 25 July 2001

Annex A3
Euromet project 597

Measurement report

Participating laboratory: Czech Metrology Institute (CMI)

Part A

Comparison with elements of Czech group coil standard of magnetic flux was performed. The value of the coil transfer standard No.7704 constant was determined from the mean value of the group standard. Each of the coils of the group standard and coil transfer standard were compared mutual with each other. Direct comparison with commutation of DC current through primary windings of both coils and with measurement of difference of induced electrical impulses in secondary windings with sensitive fluxmeter (Etalon, F119) with fotoelectrical amplifier was used. DC electric current through primary windings was measured with Keithley voltmeter type 2001. Compensation of parasitic fields and parasitic capacitances with transposition methods was used (commutations of electrical connections, commutation of place of the coils). The same DC current differential method was used in the beginning of comparison in March 2001, in the middle of comparison in July 2001 and in the end in February 2002. Temperature during measurement was from 22.4 °C to 23.6 °C.

Date: 24. May 2002 Signature..............

Part B

Measurement results for transfer standard CMI 7704

Measurand: Coil constant $\boldsymbol{k}_{\Phi}$

Date of measurement	T °C	I_c *) A	K_{Φ} **mWb/A or mH**	$k_{d\Phi}$ **{T=23°C} mWb/A or mH**	comb. stand. uncert. $u(k_{d\Phi})$ (relative)	eff. Degrees of freedom ν_{eff}
26 March 2001	23	0.5	9.99625	9.99625	$1.2\cdot10^{-4}$	37
20 July 2001	23	0.5	9.99625	9.99625	$1.2\cdot10^{-4}$	37
21 February 2002	23	0.5	9.99651	9.99651	$1.2\cdot10^{-4}$	37
Mean			**9.99634**		$\mathbf{1.20\cdot10^{-4}}$	**56**

Humidity*: 55 %*

Czech Metrology Institute (CMI)

Uncertainty budget for K_{Φ}

DC measurement using the differential method and a sensitive fluxmeter

Transfer standard CMI 7704:

Source of uncertainty X_i	estimate x_i	Relative standard uncertainty $u(x_i)$	Probability distribution / method of evaluation(A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(k_{d\Phi})$	Degree of freedom ν_I
Magnetic flux Group standard		$1.1 \cdot 10^{-4}$	Rectangular/B	1	$1.1 \cdot 10^{-4}$	∞
Temperature of laboratory		$1 \cdot 10^{-5}$	Rectangular/B	1	$1 \cdot 10^{-5}$	∞
Electrical current in primary windings		$2 \cdot 10^{-5}$	Rectangular/B	1	$2 \cdot 10^{-5}$	∞
Temperature of windings due to electrical current		$3 \cdot 10^{-5}$	Rectangular/B	1	$3 \cdot 10^{-5}$	∞
External parasitic fields		$2 \cdot 10^{-5}$	Rectangular/B	1	$2 \cdot 10^{-5}$	∞
Repeatability		$3 \cdot 10^{-5}$	Normal/A	1	$3 \cdot 10^{-5}$	7
$k_{d\Phi}$ **9.99625** **9.99625** **9.99651**					**$u(k_{d\Phi})$** **$1.2 \cdot 10^{-4}$** **$1.2 \cdot 10^{-4}$** **$1.2 \cdot 10^{-4}$**	**ν_{eff}** **37** **37** **37**

Czech Metrology Institute (CMI)

Uncertainty budget for mean value of K_Φ

DC measurement using the differential method and a sensitive fluxmeter

Transfer standard CMI 7704:

Source of uncertainty X_i	estimate x_i	Relative standard uncertainty $u(x_i)$	Probability distribution / method of evaluation(A,B)	Sensitivity coefficient c_i	Relative uncertainty contribution $u_i(k_{d\Phi})$	Degree of freedom ν_I
Magnetic flux Group standard		$1.1\cdot10^{-4}$	Rectangular/B	1	$1.1\cdot10^{-4}$	∞
Temperature of laboratory		$1\cdot10^{-5}$	Rectangular/B	1	$1\cdot10^{-5}$	∞
Electrical current in primary windings		$2\cdot10^{-5}$	Rectangular/B	1	$2\cdot10^{-5}$	∞
Temperature of windings due to electrical current		$3\cdot10^{-5}$	Rectangular/B	1	$3\cdot10^{-5}$	∞
External parasitic fields		$2\cdot10^{-5}$	Rectangular/B	1	$2\cdot10^{-5}$	∞
Repeatability		$2\cdot10^{-5}$	Normal/A	1	$2\cdot10^{-5}$	23
$k_{d\Phi}$ **9.99634**					**$u(k_{d\Phi})$** **$1.20\cdot10^{-4}$**	**ν_{eff}** **56**

Czech Metrology Institute (CMI)

May 2002

Annex A4

Euromet project 597

Comparison of magnetic flux by means of coil transfer standard

Measurement report

(September 2001, St.Petersburg, Russia,)

Description of the measurement method

The compensation induction method on low frequency was used. The calculable mutual inductor of 10 mWb/A together with an additional variable mutual inductor (AVMI) of 10 to 30 μWb/A was applied as a standard for the comparison.

Differential signal was measured by means of a synchronous detector (SD) which output is connected to two DC-voltmeters (DVM) for indicating of reactive and active (having same phase with current) components of signal. Phase of SD was adjusted to have the situation, when the second (active) DVM is independent from the AVMI variation. The reactive component was measured by the first DVM.

The first DVM indication was calibrated by AVMI changing, so we didn't need measurement of current.

Result of each measurement is the mean of four connection situations: initial one; both windings of CSMI-7704 reversed; both primary windings reversed; both windings of the Russian standard reversed.

Relevant instruments

Calculable mutual inductor of Campbell type; additional variable mutual inductor of astatic type (without interaction with surrounding devices); generator; synchronous detector; two DC-voltmeters; AC-current indicator.

Participating laboratory:....**Mendeleyev Institute for Metrology, Russia**

Date:...September .24, 2001 Signature……............... Vladlen Ya. Shifrin

Measurement results for transfer standard CMI 7704

Measurement results for transfer standard CMI 7704

Measurand: Coil constant $\boldsymbol{k}_{\Phi}$

Date of measurement	**f, Hz**	**T, $^{\circ}$C**	**I, A**	**k_{Φ}, mWb/A**	**$k_{\mathrm{d}\Phi}$ {T=23°C}, mWb/A**
13.09.01	18.7	21.2	0.27	9.9949	9.9954
	18.9	21.4	0.33	9.9947	9.9952
14.09.01	14.3	21.0	0.30	9.9949	9.9956
	18.9	21.2	0.30	9.9949	9.9954
	32.0	21.1	0.30	9.9953	9.9959
15.09.01	14.3	21.2	0.30	9.9951	9.9956
	18.9	21.2	0.30	9.9952	9.9958
	32.0	21.1	0.30	9.9948	9.9954
16.09.01	14.3	20.3	0.30	9.9946	9.9954
	18.7	20.2	0.30	9.9949	9.9957
	31.3	20.3	0.30	9.9947	9.9955
17.09.01	18.9	21.2	0.30	9.9947	9.9952
Mean:					**9.99551 mWb/A**
Statistical standard deviation:					**0.0007 %**

Date:...September .24, 2001 Signature…................. Vladlen Ya. Shifrin

Uncertainty budget for $k_{d\Phi}$

Quantity X_i	*Value* x_i	*Uncertainty effecting the total result, %*			
		Estimate	*Evaluation*	*Distribution*	$u(x_i)$
1. Campbell-coil constant	*10016.5 μWb/A*	*0.002*	*B, max*	*Rectangular*	*0.0012*
2. Additional mutual inductance	*(10-30) μWb/A*	*1*	*B, max*	*Rectangular*	*0.6*
3. Zero of AVMI	*<0.1 μWb/A*	*100*	*B, max*	*Rectangular*	*58*
4. Measured reactive DC-voltage	*(1-3) V*	*0.2*	*B, max*	*Rectangular*	*0.12*
5. Variation of active DC-voltage by AVMI-changing	*<0.1 V*	*100*	*B, max*	*Rectangular*	*58*
6. Conversion factor of SD (linearity)	$\sim 10^4$	*1*	*B, max*	*Rectangular*	*1.2*
7. Mutual induction between coils	*<1 μWb/A*	*100*	*B, max*	*Rectangular*	*58*
8. Temperature correction	*<0.3 μWb/A*	*10*	*B, max*	*Rectangular*	*5.8*
9. Heating effect under current (<1°C)	*<0.3 μWb/A*	*50*	*B, max*	*Rectangular*	*29*
10. Effect of magnetic environment	*<0.7 μWb/A*	*100*	*B, max*	*Rectangular*	*58*
11. Standard deviation (n=12)	*0.07 μWb/A*	*100*	*A, std*	*Normal*	*100*
Total: **$k_{d\Phi}$ = 9.99551 mWb/A**					

Date:...September .24, 2001 Signature…................. Vladlen Ya. Shifrin

Annex A5 Measuring report, part A

A) description of the measurement method and relevant instruments

Principle of measurement

The primary winding of the mutual inductor to be calibrated and a 4-terminal precision resistor of low inductance are connected in series and an alternating current is passed through this circuit. The voltages at the secondary winding of the mutual inductor and at the terminals of the resistor are measured as well as the frequency. The secondary voltage of a mutual inductor is

$$U_2 = M \cdot \frac{\mathrm{d}I_1}{\mathrm{d}t} . \tag{1}$$

M mutual inductance, = k_Φ
I_1 primary current,
t time.

For an alternating current

$$\overline{|U_2|} = M \cdot 4f \cdot \hat{I}_1 , \tag{2}$$

is valid even in the nonsinusoidal case.

$\overline{|U_2|}$ mean value of the rectified secondary voltage,
f frequency,
$\hat{I}_1$ peak value of the primary current.

If the r.m.s values are measured:

$$\overline{|U_2|} = \frac{\tilde{U}_2}{F} \tag{3}$$

and

$$\hat{I}_1 = \tilde{I}_1 \cdot C \tag{4}$$

$\tilde{U}_2$ r.m.s. value of the secondary voltage,
F form factor of the secondary voltage,
$\tilde{I}_1$ r.m.s. value of the primary current,
C crest factor of the primary current.

Consequently

$$M = \frac{\tilde{U}_2}{4f \cdot \tilde{I}_1 \cdot C \cdot F} . \tag{5}$$

For a pure sinusoidal waveform

$$F = \frac{\pi}{2\sqrt{2}} \tag{6}$$

and

$$C = \sqrt{2} \tag{7}$$

is valid. In this case we get

$$M = \frac{\tilde{U}_2}{2\pi f \cdot \tilde{I}_1} = \frac{\tilde{U}_2 \cdot R}{2\pi f \cdot \tilde{U}_\mathrm{R}}. \tag{8}$$

R resistance of the precision resistor,

$\tilde{U}_\mathrm{R}$ r.m.s. value of the voltage at the terminals of the precision resistor.

In the case of deviations from the pure sinusoidal waveform there are harmonics, which were enhanced by the differentiation of the mutual inductor. Additional magnetic disturbing fields at power line frequency and harmonics and at high frequencies will superpose parasitic voltages to the secondary voltage of the mutual inductor. These effects can be suppressed by means of Fourier fundamental wave analysis. Equation (8) changes to

$$M = \frac{U_\mathrm{2F} \cdot R}{2\pi f \cdot U_\mathrm{RF}} \tag{9}$$

U_2F fundamental amplitude of the secondary voltage,

U_RF fundamental amplitude of the voltage at the terminals of the precision resistor.

If both voltages are approximately of the same amplitude, they can be measured in the same measuring range and a second measurement can be performed with interchanged inputs. The mean of both results is almost insensitive to the calibration of the voltage measuring devices. With a resistance of 1 Ω and a mutual inductance of 10 mH , $U_\mathrm{2F} = U_\mathrm{RF}$ is valid for 15,915 Hz. For the optimal elimination of the 50 Hz line frequency and its harmonics by fundamental Fourier component calculation, the best frequency is 16 2/3 Hz. This frequency differs by less than 5 % from 15,915 Hz, so thus the method of interchanging the channels will be still applicable.

Instrumentation

Digital oscilloscope:

For the simultaneous measurement of the voltages a 2-channel digital oscilloscope Nicolet 4094C with 4851 plug in was used, which uses two 15-bit,100 kHz A/D converters. For noise reduction we used the 1 kHz filters and sweep averaging. The time per point was 10 µs, so that 6000 pairs of points were sampled within one period of 16 2/3 Hz. The input resistance of each channel is 1 MΩ ± 2%, and the input capacitance is less than 60 pF.

Resistor:

Burster 1 Ω, planar type with heat sinks.

Value calibrated with 50 mA DC: 0,9998751 Ω, with a relative uncertainty of $5 \cdot 10^{-7}$.

Value at 0,5 A is 0,999870 Ω.

AC-current source:

Synthesizer with 16-bit D/A converter and 1024 points per period.

Filter amplifier, power amplifier with current output.

To stabilize the current amplifier, which tends to oscillate due to the inductive load, a capacitor of 92 µF was connected parallel to its output.

Frequency counter:

Toellner TOE 6723 was switched to period length mode.

Digital voltmeter for determining the r.m.s. value of the current through primary winding by measuring the voltage across the Burster resistor (measurement used only for monitoring the current, but not for determining the mutual inductance): Solartron 7071.

Computer:
The measured arrays of voltages were transmitted via IEEE-488 bus to an HP 300 computer, where the Fourier transform was performed and the corrections and ratios were calculated.

Thermometer:
Mercury-Glass 0,1°C uncertainty.

Ambient conditions

The measurements were performed in an air conditioned room at a temperature of (23,0 ± 1,0) °C and a relative humidity of (30 ± 10) %.
The travelling standard coil was aligned transverse to the main magnetic parasitic field, so that the influence was reduced to a minimum. The axis of the coil was close to the west-east direction. The line frequency portion (50 Hz and harmonics) induced a voltage with an r.m.s. value of about 82 µV, the noise and higher frequency influence can be characterised by an r.m.s. value of about 48 mV.

Sensitivity coefficient for parasitic field

For testing the parasitic field influence on the measurement result, a transformer was placed close to the travelling standard coil and connected to power line. The r.m.s. value of the disturbance signal was 29 mV, whereas the r.m.s. value of the signal without disturbance was 105 mV. The measured mutual inductivity without disturbance signal was 10,0393 mH, the value measured with superposed disturbance signal was 10,0461 mH. Certainly the value of 10,0393 mH is too high because of the disconnected transformer close to the travelling standard coil, but it can be seen that the relative deviation of both results was $6{,}8 \cdot 10^{-4}$, the sensitivity coefficient being $2{,}35 \cdot 10^{-4}$ H/V. The influence of noise and high frequency will be included in the statistic evaluation of the uncertainty (type A), the influence of the low frequency parasitic field will be less than 20 nH.

Measurements

The current through primary winding had an r.m.s. value of 0,1 A at the frequency of 16 2/3 Hz. Six series of ten measurements were performed in the time of 14. Jan. to 18. Jan., each of the measurements was performed twice with interchanged inputs. The results were averaged and corrected for the resistance of the secondary winding and the input resistance of the digital oscilloscope and for the temperature deviation. The measured resistance of the primary winding including connection cable was 4,49 Ω and that of the secondary winding was 25,26 Ω. Measurement of the resistances before and after the mutual inductivity measurements show only very small differences, from which it can be concluded, that the temperature increase during the series of measurements was less than 0,02°C.

Participating laboratory: Physikalisch-Technische Bundesanstalt

Date: 24. Jan. 2002

Signature

by order
H. Ahlers

Measurement results for transfer standard CMI 7704

Measurand: Coil constant k_Φ

Date of measurement	T °C	I_c (r.m.s) [1)] A	k_Φ mH	$k_{d\Phi}$ $\{T = 23°C\}$ mH	Comb. stand. uncert. $u(k_{d\Phi})$ mH	Eff. Degrees of freedom ν_{eff}
14. Jan. 02	23,6	0,09998	9,99728	9,99710	0,00027	11,8
16. Jan. 02	23,6	0,09999	9,99686	9,99668	0,00023	13,2
16. Jan. 02	23,5	0,09997	9,99700	9,99685	0,00032	10,8
16. Jan. 02	23,5	0,09997	9,99620	9,99605	0,00026	11,9
17. Jan. 02	23,4	0,09999	9,99644	9,99632	0,00023	12,9
18. Jan. 02	23,6	0,09999	9,99667	9,99649	0,00026	12,0
Mean:				9,99658	0,00014	199

[1)] Monitored value, used not for $k_{d\Phi}$.

Frequency: 16,6651 Hz

Uncertainty budget for $K_{\mathrm{d}\Phi}$

Model equation:

$$k_{\mathrm{d}\Phi} = M = \frac{\left(U_{\mathrm{2FM}} - \delta U_{\mathrm{2FM}}\right)\cdot k_1 \cdot \left(R_\mathrm{I} + R_2\right)\cdot R}{\left(U_{\mathrm{RFM}} - \delta U_{\mathrm{RFM}}\right)\cdot k_2 \cdot \left(R_\mathrm{I} + R\right)\cdot 2\pi f} - \frac{\partial M}{\partial T}\cdot \left(T - 23\right)$$

$k_{\mathrm{d}\Phi}$, M	mutual inductance, coil constant at 23°C,
U_{2FM}	fundamental amplitude of the secondary voltage, measured value,
δU_{2FM}	16 2/3 Hz portion influence of parasitic fields after filtering and sweep averaging and deviations due to nonlinearity of the oscilloscope input,
k_1	correction factor of the first oscilloscope channel,
U_{RFM}	fundamental amplitude of the voltage at the terminals of the precision resistor, measured value,
δU_{RFM}	deviations due to nonlinearity of the oscilloscope input,
k_2	correction factor of the second oscilloscope channel,
R_I	input resistance of the oscilloscope,
R_2	resistance of the secondary winding, including connecting cable resistance,
R	resistance of the precision resistor,
f	fundamental frequency,
$\frac{\partial M}{\partial T}$	temperature coefficient of the mutual inductance,
T	temperature of the coil.

Due to the interchanging of the channels the mean value is:

$$M = \frac{M_1 + M_2}{2} = \frac{1}{2}\cdot\left(\frac{\left(U_{\mathrm{2FM1}} - \delta U_{\mathrm{2FM1}}\right)\cdot k_1}{\left(U_{\mathrm{RFM2}} - \delta U_{\mathrm{RFM2}}\right)\cdot k_2} + \frac{\left(U_{\mathrm{2FM2}} - \delta U_{\mathrm{2FM2}}\right)\cdot k_2}{\left(U_{\mathrm{RFM1}} - \delta U_{\mathrm{RFM1}}\right)\cdot k_1}\right)\cdot\frac{\left(R_\mathrm{I} + R_2\right)\cdot R}{\left(R_\mathrm{I} + R\right)\cdot 2\pi f} - \frac{\partial M}{\partial T}\cdot\left(T - 23\right)$$

With $\delta = \frac{k_1}{k_2} - 1$,

$$\frac{\left(U_{\mathrm{2FM1}} - \delta U_{\mathrm{2FM1}}\right)}{\left(U_{\mathrm{RFM2}} - \delta U_{\mathrm{RFM2}}\right)} = X_1,$$

$$\frac{\left(U_{\mathrm{2FM2}} - \delta U_{\mathrm{2FM2}}\right)}{\left(U_{\mathrm{RFM1}} - \delta U_{\mathrm{RFM1}}\right)} = X_2,$$

and

$$X = \frac{X_1 + X_2}{2}$$

$$M = X\cdot\frac{\left(R_\mathrm{I} + R_2\right)\cdot R}{\left(R_\mathrm{I} + R\right)\cdot 2\pi f}\cdot\left(1 + \frac{1}{2}\delta^2\right) - \frac{\partial M}{\partial T}\cdot\left(T - 23\right)$$

is valid .

The measured mean of ratios is

$$X_{\mathrm{M}} = \frac{1}{2} \cdot \left(\frac{U_{\mathrm{2FM1}}}{U_{\mathrm{RFM2}}} + \frac{U_{\mathrm{2FM2}}}{U_{\mathrm{RFM1}}} \right).$$

$$M = X_{\mathrm{M}} \cdot \frac{(R_{\mathrm{I}} + R_2) \cdot R}{(R_{\mathrm{I}} + R) \cdot 2\pi f} \cdot \left(1 + \frac{1}{2} \delta^2 \right) \cdot \left(1 + \frac{\delta U_{\mathrm{RFM}}}{U_{\mathrm{RFM}}} - \frac{\delta U_{\mathrm{2FM}}}{U_{\mathrm{2FM}}} \right) - \frac{\partial M}{\partial T} \cdot (T - 23)$$

Transfer standard CMI 7704:

1. series

Quantity X_i	*estimate* x_i	*Standard uncertainty* $u(x_i)$	*Probability distribution / method of evaluation(A,*B)	*Sensitivity coefficient* c_i	*Uncertainty contribution* $u_i(k_{d\Phi})$	*Degree of freedom* ν_i
X_{M}	1,046899	$1 \cdot 10^{-6}$	Rect./B	9,55 mH	$9{,}55 \cdot 10^{-6}$ mH	∞
R_{I}	$1 \cdot 10^{6}$ Ω	$10 \cdot 10^{3}$ Ω	Rect./B	$-2{,}43 \cdot 10^{-10}$ mH/Ω	$-2{,}43 \cdot 10^{-6}$ mH	∞
R_2	25,26 Ω	0,012 Ω	Rect./B	$1{,}00 \cdot 10^{-5}$ mH/Ω	$1{,}20 \cdot 10^{-7}$ mH	∞
R	0,9998751 Ω	$2{,}9 \cdot 10^{-6}$ Ω	Normal/B	10,0 mH/Ω	$2{,}9 \cdot 10^{-5}$ mH	∞
f	16,6651 Hz	$2{,}0 \cdot 10^{-6}$ Hz	Rect./B	-0,6 mH/Hz	$-1{,}20 \cdot 10^{-6}$ mH	∞
δ	$1{,}6 \cdot 10^{-3}$	$1 \cdot 10^{-5}$	Rect./B	$1 \cdot 10^{-4}$ mH	$1 \cdot 10^{-9}$ mH	∞
$\frac{\delta U_{\mathrm{RFM}}}{U_{\mathrm{RFM}}}$	0	$5{,}8 \cdot 10^{-6}$	Rect./B	10 mH	$5{,}8 \cdot 10^{-5}$ mH	∞
$\frac{\delta U_{\mathrm{2FM}}}{U_{\mathrm{2FM}}}$	0	$5{,}8 \cdot 10^{-6}$	Rect./B	10 mH	$5{,}8 \cdot 10^{-5}$ mH	∞
$\frac{\partial M}{\partial T}$	0,0003 mH/°C	0,00005mH/°C	Rect./B	-0,6 °C	$-3{,}0 \cdot 10^{-5}$ mH	∞
T	23,6°C	0,058°C	Rect./B	-0,0003 mH/°C	$-1{,}8 \cdot 10^{-5}$ mH	∞
$k_{\mathrm{d}\Phi}$	9,99710 mH	$2{,}45 \cdot 10^{-4}$ mH	Normal/A	1	$2{,}45 \cdot 10^{-4}$ mH	9
$\boldsymbol{k_{d\Phi}}$	9,99710 mH				$\boldsymbol{u(k_{d\Phi})}$ $= 2{,}7 \cdot 10^{-4}$ mH	$\nu_{eff} = 11{,}8$

2. series

Quantity X_i	*estimate* x_i	*Standard uncertainty* $u(x_i)$	*Probability distribution / method of evaluation*(A,B)	*Sensitivity coefficient* c_i	*Uncertainty contribution* $u_i(k_{d\Phi})$	*Degree of freedom* ν_i
X_M	1,046854	$1\cdot10^{-6}$	Rect./B	9,55 mH	$9{,}55\cdot10^{-6}$ mH	∞
R_I	$1\cdot10^{6}$ Ω	$10\cdot10^{3}$ Ω	Rect./B	$-2{,}43\cdot10^{-10}$ mH/Ω	$-2{,}43\cdot10^{-6}$ mH	∞
R_2	25,26 Ω	0,012 Ω	Rect./B	$1{,}00\cdot10^{-5}$ mH/Ω	$1{,}20\cdot10^{-7}$ mH	∞
R	0,9998751 Ω	$2{,}9\cdot10^{-6}$ Ω	Normal/B	10,0 mH/Ω	$2{,}9\cdot10^{-5}$ mH	∞
f	16,6651 Hz	$2{,}0\cdot10^{-6}$ Hz	Rect./B	-0,6 mH/Hz	$-1{,}20\cdot10^{-6}$ mH	∞
δ	$1{,}6\cdot10^{-3}$	$1\cdot10^{-5}$	Rect./B	$1\cdot10^{-4}$ mH	$1\cdot10^{-9}$ mH	∞
$\dfrac{\delta U_\mathrm{RFM}}{U_\mathrm{RFM}}$	0	$5{,}8\cdot10^{-6}$	Rect./B	10 mH	$5{,}8\cdot10^{-5}$ mH	∞
$\dfrac{\delta U_\mathrm{2FM}}{U_\mathrm{2FM}}$	0	$5{,}8\cdot10^{-6}$	Rect./B	10 mH	$5{,}8\cdot10^{-5}$ mH	∞
$\dfrac{\partial M}{\partial T}$	0,0003 mH/°C	0,00005mH/°C	Rect./B	-0,6 °C	$-3{,}0\cdot10^{-5}$ mH	∞
T	23,6°C	0,058°C	Rect./B	-0,0003 mH/°C	$-1{,}8\cdot10^{-5}$ mH	∞
$k_{\mathrm{d}\Phi}$	9,99668 mH	$2{,}05\cdot10^{-4}$ mH	Normal/A	1	$2{,}05\cdot10^{-4}$ mH	9
$\boldsymbol{k_{d\Phi}}$	9,99668 mH				$\boldsymbol{u(k_{d\Phi})}$ $= 2{,}3\cdot10^{-4}$ mH	$\nu_{eff} = 13{,}2$

3. series

Quantity X_i	*estimate* x_i	*Standard uncertainty* $u(x_i)$	*Probability distribution / method of evaluation(A,*B)	*Sensitivity coefficient* c_i	*Uncertainty contribution* $u_i(k_{d\Phi})$	*Degree of freedom* ν_i
X_{M}	1,046872	$1\cdot10^{-6}$	Rect./B	9,55 mH	$9{,}55\cdot10^{-6}$ mH	∞
R_{I}	$1\cdot10^{6}$ Ω	$10\cdot10^{3}$ Ω	Rect./B	$-2{,}43\cdot10^{-10}$ mH/Ω	$-2{,}43\cdot10^{-6}$ mH	∞
R_2	25,26 Ω	0,012 Ω	Rect./B	$1{,}00\cdot10^{-5}$ mH/Ω	$1{,}20\cdot10^{-7}$ mH	∞
R	0,9998751 Ω	$2{,}9\cdot10^{-6}$ Ω	Normal/B	10,0 mH/Ω	$2{,}9\cdot10^{-5}$ mH	∞
f	16,6651 Hz	$2{,}0\cdot10^{-6}$ Hz	Rect./B	-0,6 mH/Hz	$-1{,}20\cdot10^{-6}$ mH	∞
δ	$1{,}6\cdot10^{-3}$	$1\cdot10^{-5}$	Rect./B	$1\cdot10^{-4}$ mH	$1\cdot10^{-9}$ mH	∞
$\frac{\delta U_{\mathrm{RFM}}}{U_{\mathrm{RFM}}}$	0	$5{,}8\cdot10^{-6}$	Rect./B	10 mH	$5{,}8\cdot10^{-5}$ mH	∞
$\frac{\delta U_{\mathrm{2FM}}}{U_{\mathrm{2FM}}}$	0	$5{,}8\cdot10^{-6}$	Rect./B	10 mH	$5{,}8\cdot10^{-5}$ mH	∞
$\frac{\partial M}{\partial T}$	0,0003 mH/°C	0,00005mH/°C	Rect./B	-0,5 °C	$-2{,}5\cdot10^{-5}$ mH	∞
T	23,5°C	0,058°C	Rect./B	-0,0003 mH/°C	$-1{,}8\cdot10^{-5}$ mH	∞
$k_{\mathrm{d}\Phi}$	9,99685 mH	$2{,}99\cdot10^{-4}$ mH	Normal/A	1	$2{,}99\cdot10^{-4}$ mH	9
$\boldsymbol{k_{d\Phi}}$	9,99685 mH				$\boldsymbol{u(k_{d\Phi})}$ $= 3{,}2\cdot10^{-4}$ mH	$\nu_{eff} = 10{,}8$

4. series

Quantity X_i	*estimate* x_i	*Standard uncertainty* $u(x_i)$	*Probability distribution / method of evaluation(A,*B)	*Sensitivity coefficient* c_i	*Uncertainty contribution* $u_i(k_{d\Phi})$	*Degree of freedom* ν_i
X_M	1,046788	$1\cdot10^{-6}$	Rect./B	9,55 mH	$9{,}55\cdot10^{-6}$ mH	∞
R_I	$1\cdot10^{6}$ Ω	$10\cdot10^{3}$ Ω	Rect./B	$-2{,}43\cdot10^{-10}$ mH/Ω	$-2{,}43\cdot10^{-6}$ mH	∞
R_2	25,26 Ω	0,012 Ω	Rect./B	$1{,}00\cdot10^{-5}$ mH/Ω	$1{,}20\cdot10^{-7}$ mH	∞
R	0,9998751 Ω	$2{,}9\cdot10^{-6}$ Ω	Normal/B	10,0 mH/Ω	$2{,}9\cdot10^{-5}$ mH	∞
f	16,6651 Hz	$2{,}0\cdot10^{-6}$ Hz	Rect./B	-0,6 mH/Hz	$-1{,}20\cdot10^{-6}$ mH	∞
δ	$1{,}6\cdot10^{-3}$	$1\cdot10^{-5}$	Rect./B	$1\cdot10^{-4}$ mH	$1\cdot10^{-9}$ mH	∞
$\frac{\delta U_\mathrm{RFM}}{U_\mathrm{RFM}}$	0	$5{,}8\cdot10^{-6}$	Rect./B	10 mH	$5{,}8\cdot10^{-5}$ mH	∞
$\frac{\delta U_\mathrm{2FM}}{U_\mathrm{2FM}}$	0	$5{,}8\cdot10^{-6}$	Rect./B	10 mH	$5{,}8\cdot10^{-5}$ mH	∞
$\frac{\partial M}{\partial T}$	0,0003 mH/°C	0,00005mH/°C	Rect./B	-0,5 °C	$-2{,}5\cdot10^{-5}$ mH	∞
T	23,5°C	0,058°C	Rect./B	-0,0003 mH/°C	$-1{,}8\cdot10^{-5}$ mH	∞
$k_{\mathrm{d}\Phi}$	9,99605 mH	$2{,}39\cdot10^{-4}$ mH	Normal/A	1	$2{,}39\cdot10^{-4}$ mH	9
$\boldsymbol{k_{d\Phi}}$	9,99605 mH				$\boldsymbol{u(k_{d\Phi})}$ = $2{,}6\cdot10^{-4}$ mH	ν_{eff} = 11,9

5. series

Quantity X_i	*estimate* x_i	*Standard uncertainty* $u(x_i)$	*Probability distribution / method of evaluation(A,*B)	*Sensitivity coefficient* c_i	*Uncertainty contribution* $u_i(k_{d\Phi})$	*Degree of freedom* ν_i
X_M	1,046816	$1\cdot10^{-6}$	Rect./B	9,55 mH	$9{,}55\cdot10^{-6}$ mH	∞
R_I	$1\cdot10^{6}$ Ω	$10\cdot10^{3}$ Ω	Rect./B	$-2{,}43\cdot10^{-10}$ mH/Ω	$-2{,}43\cdot10^{-6}$ mH	∞
R_2	25,26 Ω	0,012 Ω	Rect./B	$1{,}00\cdot10^{-5}$ mH/Ω	$1{,}20\cdot10^{-7}$ mH	∞
R	0,9998751 Ω	$2{,}9\cdot10^{-6}$ Ω	Normal/B	10,0 mH/Ω	$2{,}9\cdot10^{-5}$ mH	∞
f	16,6651 Hz	$2{,}0\cdot10^{-6}$ Hz	Rect./B	-0,6 mH/Hz	$-1{,}20\cdot10^{-6}$ mH	∞
δ	$1{,}6\cdot10^{-3}$	$1\cdot10^{-5}$	Rect./B	$1\cdot10^{-4}$ mH	$1\cdot10^{-9}$ mH	∞
$\dfrac{\delta U_\mathrm{RFM}}{U_\mathrm{RFM}}$	0	$5{,}8\cdot10^{-6}$	Rect./B	10 mH	$5{,}8\cdot10^{-5}$ mH	∞
$\dfrac{\delta U_\mathrm{2FM}}{U_\mathrm{2FM}}$	0	$5{,}8\cdot10^{-6}$	Rect./B	10 mH	$5{,}8\cdot10^{-5}$ mH	∞
$\dfrac{\partial M}{\partial T}$	0,0003 mH/°C	0,00005mH/°C	Rect./B	-0,4 °C	$-2{,}0\cdot10^{-5}$ mH	∞
T	23,4°C	0,058°C	Rect./B	-0,0003 mH/°C	$-1{,}8\cdot10^{-5}$ mH	∞
$k_{\mathrm{d}\Phi}$	9,99632 mH	$2{,}05\cdot10^{-4}$ mH	Normal/A	1	$2{,}05\cdot10^{-4}$ mH	9
$\boldsymbol{k_{d\Phi}}$	9,99632 mH				$\boldsymbol{u(k_{d\Phi})}$ $= 2{,}3\cdot10^{-4}$ mH	$\nu_{eff} = 12{,}9$

6. series

Quantity X_i	*estimate* x_i	*Standard uncertainty* $u(x_i)$	*Probability distribution / method of evaluation(A,*B)	*Sensitivity coefficient* c_i	*Uncertainty contribution* $u_i(k_{d\Phi})$	*Degree of freedom* ν_i
X_{M}	1,046834	$1\cdot10^{-6}$	Rect./B	9,55 mH	$9{,}55\cdot10^{-6}$ mH	∞
R_{I}	$1\cdot10^{6}$ Ω	$10\cdot10^{3}$ Ω	Rect./B	$-2{,}43\cdot10^{-10}$ mH/Ω	$-2{,}43\cdot10^{-6}$ mH	∞
R_2	25,26 Ω	0,012 Ω	Rect./B	$1{,}00\cdot10^{-5}$ mH/Ω	$1{,}20\cdot10^{-7}$ mH	∞
R	0,9998751 Ω	$2{,}9\cdot10^{-6}$ Ω	Normal/B	10,0 mH/Ω	$2{,}9\cdot10^{-5}$ mH	∞
f	16,6651 Hz	$2{,}0\cdot10^{-6}$ Hz	Rect./B	-0,6 mH/Hz	$-1{,}20\cdot10^{-6}$ mH	∞
δ	$1{,}6\cdot10^{-3}$	$1\cdot10^{-5}$	Rect./B	$1\cdot10^{-4}$ mH	$1\cdot10^{-9}$ mH	∞
$\frac{\delta U_{\mathrm{RFM}}}{U_{\mathrm{RFM}}}$	0	$5{,}8\cdot10^{-6}$	Rect./B	10 mH	$5{,}8\cdot10^{-5}$ mH	∞
$\frac{\delta U_{\mathrm{2FM}}}{U_{\mathrm{2FM}}}$	0	$5{,}8\cdot10^{-6}$	Rect./B	10 mH	$5{,}8\cdot10^{-5}$ mH	∞
$\frac{\partial M}{\partial T}$	0,0003 mH/°C	0,00005mH/°C	Rect./B	-0,6 °C	$-3{,}0\cdot10^{-5}$ mH	∞
T	23,6°C	0,058°C	Rect./B	-0,0003 mH/°C	$-1{,}8\cdot10^{-5}$ mH	∞
$k_{\mathrm{d}\Phi}$	9,99649 mH	$2{,}39\cdot10^{-4}$ mH	Normal/A	1	$2{,}39\cdot10^{-4}$ mH	9
$\boldsymbol{k_{d\Phi}}$	9,99649 mH				$\boldsymbol{u(k_{d\Phi})}$ $= 2{,}6\cdot10^{-4}$ mH	$\nu_{eff} = 12{,}0$

All measurements

Quantity X_i	*estimate* x_i	*Standard uncertainty* $u(x_i)$	*Probability distribution / method of evaluation*(*A*,B)	*Sensitivity coefficient* c_i	*Uncertainty contribution* $u_i(k_{d\Phi})$	*Degree of freedom* ν_i
X_M	1,046846	$1\cdot10^{-6}$	Rect./B	9,55 mH	$9{,}55\cdot10^{-6}$ mH	∞
R_I	$1\cdot10^{6}$ Ω	$10\cdot10^{3}$ Ω	Rect./B	$-2{,}43\cdot10^{-10}$ mH/Ω	$-2{,}43\cdot10^{-6}$ mH	∞
R_2	25,26 Ω	0,012 Ω	Rect./B	$1{,}00\cdot10^{-5}$ mH/Ω	$1{,}20\cdot10^{-7}$ mH	∞
R	0,9998751 Ω	$2{,}9\cdot10^{-6}$ Ω	Normal/B	10,0 mH/Ω	$2{,}9\cdot10^{-5}$ mH	∞
f	16,6651 Hz	$2{,}0\cdot10^{-6}$ Hz	Rect./B	-0,6 mH/Hz	$-1{,}20\cdot10^{-6}$ mH	∞
δ	$1{,}6\cdot10^{-3}$	$1\cdot10^{-5}$	Rect./B	$1\cdot10^{-4}$ mH	$1\cdot10^{-9}$ mH	∞
$\frac{\delta U_\mathrm{RFM}}{U_\mathrm{RFM}}$	0	$5{,}8\cdot10^{-6}$	Rect./B	10 mH	$5{,}8\cdot10^{-5}$ mH	∞
$\frac{\delta U_\mathrm{2FM}}{U_\mathrm{2FM}}$	0	$5{,}8\cdot10^{-6}$	Rect./B	10 mH	$5{,}8\cdot10^{-5}$ mH	∞
$\frac{\partial M}{\partial T}$	0,0003 mH/°C	0,00005mH/°C	Rect./B	-0,53 °C	$-2{,}7\cdot10^{-5}$ mH	∞
T	23,53°C	0,058°C	Rect./B	-0,0003 mH/°C	$-1{,}8\cdot10^{-5}$ mH	∞
$k_{\mathrm{d}\Phi}$	9,99658 mH	$1{,}02\cdot10^{-4}$ mH	Normal/A	1	$1{,}02\cdot10^{-4}$ mH	59
$\boldsymbol{k_{d\Phi}}$	9,99658 mH				$\boldsymbol{u(k_{d\Phi})}$ $= 1{,}4\cdot10^{-4}$ mH	$\nu_{eff} = 199$