Bilateral Comparison of 1.018 V and 10 V Standards
between the NSAI - NML (Ireland) and the BIPM,
March to April 2010
(part of the ongoing BIPM key comparison BIPM.EM-K11.a and b)

by O.Power*, S. Solve†, R. Chayramy* and M. Stock†

* National Standards Authority of Ireland - National Metrology Laboratory
  (NSAI - NML), Dublin, Ireland

† Bureau International des Poids et Mesures, Sèvres, France

Introduction

As a part of the ongoing BIPM key comparisons BIPM.EM-K11.a and b, a comparison of the 1.018 V and 10 V voltage reference standards of the BIPM and the National Standards Authority of Ireland – National Metrology Laboratory (NSAI - NML), Dublin, Ireland, was carried out from March to April 2010. Two BIPM Zener diode-based travelling standards (Fluke 732B), BIPM_A and BIPM_C, were transported by freight to NSAI-NML. At NSAI-NML, the reference standard for DC voltage is maintained at the 10 V level by means of a group of characterized Zener diode-based electronic voltage standards. The output EMF of each travelling standard, at the 10 V output terminals, was measured by direct comparison with the group standard. Measurements of the output EMF of the travelling standards at the 1.018 V output terminals were made using a potentiometer, standardized against the local 10 V reference standard.

At the BIPM, the travelling standards were calibrated at both voltages before and after the measurements at NSAI-NML, with the Josephson Voltage Standard. Results of all measurements were corrected for the dependence of the output voltages on internal temperature and ambient pressure.
Outline of the measuring method

NML-NSAI 10 V measurements
The EMF at the 10 V output terminals of the travelling standard is connected in series opposition to each individual member of the NSAI-NML group standard in turn, using a low thermal scanner. The EMF differences are measured using a digital nanovoltmeter. The measured voltage differences, together with the predicted values of the NSAI-NML standards are subjected to a weighted least squares adjustment procedure in order to arrive at a best estimate of the unknown EMF.

The travelling standard is isolated from the mains supply during the measurements. The GUARD and GROUND terminals are connected together and then connected to a common ground point. The internal thermistor resistance is monitored during the measurements.

NML-NSAI 1.018 V measurements
The travelling standard is isolated from the mains supply during the measurements.

The GUARD and GROUND terminals of the 1.018 V Zener standard are connected together and the GROUND terminal is connected to the same ground point as that of the measuring system.

The 1.018 V Zener standard is disconnected from the mains supply and allowed to stabilize.

An automated potentiometer based on the binary divider principle (Measurements Intl. Ltd Model 8000A) is standardized against a 10 V reference standard.

The output terminals of the 1.018 V standard are connected to an input channel of the potentiometer using low thermal connecting cable.

After a suitable stabilizing period, the Electromotive Forces (EMF) at the channel input is measured using the potentiometer \( U_s \).

The value of the internal thermistor resistance of the 1.018 V Zener standard is recorded. Ambient temperature, pressure and relative humidity are recorded.

The connecting cable is shorted at the low output terminal of the 1.018 V Zener standard and allowed to stabilize.

After a suitable stabilizing period, the EMF at the channel input is measured using the potentiometer \( U_s \).
The best estimate of the EMF at the output terminals of the 1.018 V Zener standard is then $U_A - U_B$.

**Measurements at BIPM for both 1.018 V and 10 V**

The output voltage of the Zener standard to be measured is connected to the BIPM Josephson Voltage Standard (in series opposition with the BIPM array) through a low thermal EMF switch. The binding posts “GUARD” and “CHASSIS” of the Zener standard are connected together and connected to a single point which is the grounding reference point of the measurement setup.

The measurements start after at least two hours since the mains plug at the rear of the Zeners has been disconnected.

The BIPM detector consists of an EM model N1a analog nanovoltmeter whose output is connected, via an optically-coupled isolation amplifier, to a pen recorder and a digital voltmeter (DVM) which is connected to a computer.

This computer is used to monitor measurements, acquire data and calculate results. Low thermal electromotive force switches are used for critical switching, such as polarity reversal of the detector input.

After the BIPM array biasing frequency has been adjusted to a value where the voltage difference between the primary and the secondary voltage standards is below 0.5 µV, the nanovoltmeter is set to its 3 µV or 10 µV range to perform measurements at 1.018 V and 10 V respectively. The measurement sequence can then be carried out. Three consecutive measurement points are acquired according to the following procedure:

1- Positive array polarity and reverse position of the detector;
2- Data acquisition;
3- Positive array polarity and normal position of the detector;
4- Data acquisition;
5- Negative array polarity and reverse position of the detector;
6- Data acquisition;
7- Negative array polarity and normal position of the detector;
8- Data acquisition;
9- Negative array polarity and reverse position of the detector;
10- Data acquisition
11- Negative array polarity and normal position of the detector;
12- Data acquisition;
13- Positive array polarity and reverse position of the detector;
14- Data acquisition;
15- Positive array polarity and normal position of the detector;
16- Data acquisition;

The reversal of the detector polarity is done to cancel out any detector offset error and internal linear thermo-electromotive forces.

Each “Data Acquisition” step consists of 10 preliminary points followed by 30 measurement points. Each of these should not differ from the mean of the preliminary points by more than twice their standard deviation. The “Data Acquisition” sequence lasts 25 s and is basically the time period during which both arrays are to stay on the selected step. The total measurement time (including polarity reversals and data acquisition) is approximately 5 minutes.

This procedure is repeated three times and the mean value corresponds to one point on the graph (Cf. Fig. 1).
Results at 10 V

Figure 1 shows the measured values obtained for the two standards by the two laboratories at 10 V and figure 2 presents the voltage evolution of the simple mean of the two standards which is used to compute the final result at 10 V. A linear least squares fit is applied to the results of the BIPM to obtain the results for both standards and their uncertainties at the mean date of the NSAI-NML measurements (2010/03/31).
Table 1 lists the results of the comparison and the uncertainty contributions for the comparison NSAI-NML/BIPM at 10 V. The relative value of the voltage noise floor due to flicker noise is about 1 part in $10^8$ and represents the ultimate limit of the stability of Zener voltage standards.

In estimating the uncertainty related to the stability of the standards during transportation, we have calculated the “a priori” uncertainty of the mean of the results obtained for the two standards (also called statistical internal consistency). It consists of the quadratic combination of the uncorrelated uncertainties of each result. We compared this component to the “a posteriori” uncertainty (also called statistical external consistency) which consists of the experimental standard deviation of the mean of the results from the two traveling standards\(^*\). If the “a posteriori” uncertainty is significantly larger than the “a priori” uncertainty, we assume that a standard has changed in an unusual way and we use the larger of these two estimates in calculating the final uncertainty.

\(^*\) With only two traveling standards, the uncertainty of the standard deviation of the mean is comparable to the value of the standard deviation of the mean itself.
In Table 1, the following elements are listed:

(1) the value attributed by NSAI-NML to each Zener $U_{NML}$, computed as the simple mean of all data from NSAI-NML;
(2) the Type A uncertainty due to the instability of the Zener at NSAI-NML;
(3) the uncertainty component arising from the maintenance of the volt at NSAI-NML: this uncertainty is completely correlated between the different Zeners used for a comparison;
(4-6) the corresponding quantities for the BIPM referenced to the mean date of NSAI-NML’s measurements;
(7) the uncertainty due to the combined effects of the uncertainties of the pressure and temperature coefficients and to the difference of the mean pressures and temperatures in the participating laboratories is calculated using the following assumption:

The uncertainty on the temperature correction is considered for the difference between the mean values of the temperature measured at both institutes which is then multiplied by the uncertainties of the temperature coefficients of each Zener standard.

$$u_{T,i} = U \times u(c_{T,i}) \times \Delta R$$

where $U = 10$ V, $u(c_{T,ZA}) = 1.4 \times 10^{-7}$ /kΩ, $u(c_{T,ZC}) = 0.7 \times 10^{-7}$ /kΩ and $\Delta R = -0.16$ kΩ for ZA and $\Delta R = -0.05$ kΩ for ZC.

The same procedure is applied for the uncertainty on the pressure correction for the difference between the mean values of the pressure measured at both institutes:

$$u_{P,i} = U \times u(c_{P,i}) \times \Delta P$$

where $U = 10$ V, $u(c_{P,ZA}) = 0.07 \times 10^{-9}$ /hPa, $u(c_{P,ZC}) = 0.085 \times 10^{-9}$ /hPa, $\Delta P = 21.5$ hPa for ZA and $\Delta P = 19$ hPa for ZC.

Note that the uncertainty on the measurement of the temperature and the pressure were neglected as being negligible.

(8) the difference $(U_{NML} - U_{BIPM})$ for each Zener, and (9) the uncorrelated part of the uncertainty;

(10) the result of the comparison is the simple mean of the differences of the calibration results for the different standards;

(11 and 12) the uncertainty related to the transfer, estimated by the following two methods:
(11) the *a priori* uncertainty, determined as described on page 3;
(12) the *a posteriori* uncertainty, which is the standard deviation of the mean of the two results;
(13) the correlated part of the uncertainty and
(14) the total uncertainty of the comparison, which is the root sum square of the correlated part of the uncertainty and of (11).

Table 2 summarizes the uncertainties related to the calibration of a Zener diode against the Josephson array voltage standard at the BIPM.

Table 3 lists the uncertainties related to the calibration of a Zener at the NSAI-NML. Note that the uncertainty of the temperature (3) and pressure (4) corrections and the Non-repeatability uncertainty (5) are given as an indication and do not appear in the final uncertainty budget as they are already contained in the comparison uncertainty budget.

The comparison result is presented as the difference between the value assigned to a 10 V standard by NSAI-NML, at NSAI-NML, $U_{\text{NML}}$, and that assigned by the BIPM, at the BIPM, $U_{\text{BIPM}}$, which for the reference date is

$$U_{\text{NML}} - U_{\text{BIPM}} = -1.03 \, \mu\text{V}; \quad u_c = 1.13 \, \mu\text{V} \quad \text{on 2010/03/31},$$

where $u_c$ is the combined standard uncertainty associated with the measured difference, including the uncertainty of the representation of the volt at NSAI-NML, at the BIPM (based on $K_{\text{J-90}}$), and the uncertainty related to the comparison.
### Table 1. Results of the NSAI-NML (Ireland)/BIPM bilateral comparison of 10 V standards using two Zener traveling standards: reference date 31 March 2010. Uncertainties are 1σ estimates.

<table>
<thead>
<tr>
<th>BIPM_A</th>
<th>BIPM_B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NSAI-NML (Ireland)</strong></td>
<td></td>
</tr>
<tr>
<td>$(U_z - 10 \text{ V})/\mu\text{V}$</td>
<td>-19.69</td>
</tr>
<tr>
<td><strong>Type A uncertainty/\mu\text{V}</strong></td>
<td>0.12</td>
</tr>
<tr>
<td>correlated unc. /\mu\text{V}</td>
<td>1.12</td>
</tr>
<tr>
<td><strong>BIPM (U_z - 10 \text{ V})/\mu\text{V}</strong></td>
<td>-18.72</td>
</tr>
<tr>
<td><strong>Type A uncertainty/\mu\text{V}</strong></td>
<td>0.1</td>
</tr>
<tr>
<td>correlated unc./\mu\text{V}</td>
<td>0.001</td>
</tr>
<tr>
<td>pressure and temperature correction uncertainty/\mu\text{V}</td>
<td>0.06</td>
</tr>
<tr>
<td>$(U_{\text{NSAI-NML}} - U_{\text{BIPM}})/\mu\text{V}$</td>
<td>-0.97</td>
</tr>
<tr>
<td>uncorrelated uncertainty/\mu\text{V}</td>
<td>0.17</td>
</tr>
<tr>
<td>$&lt; U_{\text{NSAI-NML}} - U_{\text{BIPM}} &gt;/\mu\text{V}$</td>
<td>-1.03</td>
</tr>
<tr>
<td><strong>a priori uncertainty/\mu\text{V}</strong></td>
<td>0.110</td>
</tr>
<tr>
<td><strong>a posteriori uncertainty/\mu\text{V}</strong></td>
<td>0.059</td>
</tr>
<tr>
<td>correlated uncertainty/\mu\text{V}</td>
<td>1.12</td>
</tr>
<tr>
<td><strong>comparison total uncertainty/\mu\text{V}</strong></td>
<td>1.13</td>
</tr>
</tbody>
</table>

The uncorrelated uncertainty is $w = [r^2 + t^2 + v^2]^{1/2}$, the expected transfer uncertainty (a priori uncertainty) is $x = \frac{1}{2} [w_A^2 + w_C^2]^{1/2}$, and the correlated uncertainty is $y = [s^2 + u^2]^{1/2}$, where:

- $r$ is the NML Type A uncertainty (2);
- $s$ is the NML Type B uncertainty, which is assumed to be correlated for both transfer standards (3);
- $t$ is the BIPM Type A uncertainty (5);
- $u$ is the BIPM Type B uncertainty, which is assumed to be correlated for both transfer standards (6);
- $v$ is the pressure and temperature coefficient correction uncertainty (7);
- $w_i$ is the quadratic combination of the uncorrelated uncertainties for the Zener (9);
- $x$ is the uncertainty of the mean based on internal consistency (11);
- $y$ is the quadratic combination of the correlated uncertainties (13).
Table 2. Estimated standard uncertainties for Zener calibrations with the BIPM equipment at the level of 10 V without the contribution of the Zener noise. The standard deviation of the mean of the BIPM daily measurement results is equal to 26 nV.

<table>
<thead>
<tr>
<th>JVS &amp; detector uncertainty components</th>
<th>Uncertainty/nV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual thermal electromotive forces</td>
<td>included in the Type A uncertainty</td>
</tr>
<tr>
<td>electromagnetic interference</td>
<td>0.86</td>
</tr>
<tr>
<td>detector gain</td>
<td>0.11</td>
</tr>
<tr>
<td>leakage resistance</td>
<td>3×10⁻²</td>
</tr>
<tr>
<td>frequency</td>
<td>3×10⁻²</td>
</tr>
<tr>
<td>pressure and temperature correction</td>
<td>included in the Zener unc. budget</td>
</tr>
<tr>
<td>total</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Note: We consider that the Type A uncertainty can’t be lower than the 1/f noise floor estimated at 100 nV.
Table 3. Estimated standard uncertainties for Zener calibrations with the NSAI-NML equipment at the level of 10 V.
The measurement model is: \( U_x = U_{\text{REF}} \cdot f(\delta U_i) + \delta p + \delta T \)

<table>
<thead>
<tr>
<th>Input Quantity</th>
<th>Symbol</th>
<th>Standard Uncertainty</th>
<th>Sensitivity Coefficient</th>
<th>Uncertainty Contribution</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMLI Reference</td>
<td>( U_x )</td>
<td>1 ( (\mu V) )</td>
<td>1</td>
<td>1</td>
<td>(1)</td>
</tr>
<tr>
<td>Voltage difference</td>
<td>( f(\delta U_i) )</td>
<td>0.5 ( (\mu V) )</td>
<td>1</td>
<td>0.5</td>
<td>(2)</td>
</tr>
<tr>
<td>Temperature correction</td>
<td>( \delta_t )</td>
<td>0.01 ( (\mu V) )</td>
<td>1</td>
<td>0.01</td>
<td>(3)</td>
</tr>
<tr>
<td>Pressure Correction</td>
<td>( \delta_p )</td>
<td>0.03 ( (\mu V) )</td>
<td>1</td>
<td>0.04</td>
<td>(4)</td>
</tr>
<tr>
<td>Non-repeatability</td>
<td></td>
<td>0.5 ( (\mu V) )</td>
<td>1</td>
<td>0.5</td>
<td>(5)</td>
</tr>
<tr>
<td></td>
<td>Combined Standard Uncertainty</td>
<td></td>
<td></td>
<td>1.22</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Expanded Uncertainty ((k=2))</td>
<td></td>
<td></td>
<td>2.4</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
(1) The uncertainty component includes the effects of drift, noise, and environmental influences on the ensemble reference standard.
(2) The uncertainty component includes the effects of uncompensated thermal voltage offsets, uncorrected errors in the detector reading, leakage effects, and common mode effects.
(3) A temperature coefficient of \( 3.5 \times 10^{-7} \text{k}\Omega^{-1} \) is used.
(4) A pressure coefficient of \( 2 \times 10^{-8} \text{kPa}^{-1} \) is used.
   An estimate of the \(1/f\) noise floor level is used as it is greater than the standard deviation of the mean.
(5)
Results at 1.018 V

Figure 3 shows the measured values obtained for the two standards by the two laboratories at 1.018 V and figure 4 presents the voltage evolution of the simple mean of the two standards which is used to compute the final result at 1.018 V. A linear least squares fit is applied to the results of the BIPM to obtain the results for both standards and their uncertainties at a common reference date corresponding to the mean date of the NSA-NML measurements (2010/03/29).

Figure 3. Voltage of BIPM_A (in red) and BIPM_C (in blue) at 1.018 V measured at both institutes, referred to an arbitrary origin, as a function of time, with a linear least-squares fit to the measurements of the BIPM.
Table 4 lists the results of the comparison and the uncertainty contributions for the comparison NSAI-NML/BIPM at 1.018 V. Experience has shown that flicker or 1/f noise ultimately limits the stability characteristics of Zener diode standards and it is not appropriate to use the standard deviation divided by the square root of the number of observations to characterize the dispersion of measured values. For the present standards, the relative value of the voltage noise floor due to flicker noise is about 1 part in $10^8$.

In estimating the uncertainty related to the stability of the standards during transportation, we have calculated the “a priori” uncertainty of the mean of the results and the “a posteriori” uncertainty which consists of the experimental standard deviation of the mean of the results from the two traveling standards. Then we applied the same methodology as described in the measurements at 10 V.

In Table 4, the following elements are listed:

(1) the value attributed by NSAI-NML to each Zener $U_{\text{NML}}$, computed as the simple mean of all data from NSAI-NML;
(2) the Type A uncertainty due to the instability of the Zener at NSAI-NML;
(3) the uncertainty component arising from the maintenance of the volt at NSAI-NML: this uncertainty is completely correlated between the different Zeners used for a comparison;

(4-6) the corresponding quantities for the BIPM referenced to the mean date of the NSAI-NML measurements;

(7) the uncertainty due to the combined effects of the uncertainties of the pressure and temperature coefficients and to the differences of the mean pressures and temperatures in the participating laboratories is calculated using the following assumption:

The uncertainty on the temperature correction is considered for the difference between the mean values of the temperature measured at both institutes which is then multiplied by the uncertainties of the temperature coefficients of each Zener standard.

\[ u_{T-i} = U \times u(c_{T-i}) \times \Delta R \]

where \( U = 1.018 \text{ V} \), \( u(c_{T-ZA}) = 0.7 \times 10^{-7} \text{ k\Omega} \), \( u(c_{T-ZC}) = 0.62 \times 10^{-7} \text{ k\Omega} \), \( \Delta R = -0.14 \text{ k\Omega} \) for ZA and \( \Delta R = -0.06 \text{ k\Omega} \) for ZC.

The same procedure is applied for the uncertainty on the pressure correction for the difference between the mean values of the pressure measured at both institutes:

\[ u_{P-i} = U \times u(c_{P-i}) \times \Delta P \]

where \( U = 1.018 \text{ V} \), \( u(c_{P-ZA}) = 0.043 \times 10^{-9} \text{ hPa} \), \( u(c_{P-ZC}) = 0.085 \times 10^{-9} \text{ hPa} \), \( \Delta P = 19.5 \text{ hPa} \) for both ZA and ZC.

Note that the uncertainty on the measurement of the temperature and the pressure were neglected as being negligible.

(8) the difference \( (U_{NML} - U_{BIPM}) \) for each Zener, and (9) the uncorrelated part of the uncertainty;

(10) the result of the comparison is the simple mean of the differences of the calibration results for the different standards;

(11 and 12) the uncertainty related to the transfer, estimated by the following two methods:

(11) the \textit{a priori} uncertainty, determined as described on page 6;

(12) the \textit{a posteriori} uncertainty, which is the standard deviation of the mean of the two results;

(13) the correlated part of the uncertainty and

(14) the total uncertainty of the comparison, which is the root sum square of the correlated part of the uncertainty and of the larger of (11) and (12).

\* The evaluation of the correction coefficients were performed in 1997.
Table 5 summarizes the uncertainties related to the calibration of a Zener diode against the Josephson array voltage standard at the BIPM and Table 6 lists the uncertainties related to the maintenance of the volt and the Zener calibration at NSAI-NML.

The result of the comparison is presented as the difference between the value assigned to a 1.018 V standard by NSAI-NML, at NSAI-NML, $U_{\text{NML}}$, and that assigned by the BIPM, at the BIPM, $U_{\text{BIPM}}$, which for the reference date is

$$U_{\text{NML}} - U_{\text{BIPM}} = 0.056 \, \mu\text{V}; \quad u_c = 0.283 \, \mu\text{V} \quad \text{on 2010/03/29},$$

where $u_c$ is the combined standard uncertainty associated with the measured difference, including the uncertainty of the representation of the volt at the BIPM, (based on $K_{\text{J-90}}$) and at NSAI-NML and the uncertainty related to the comparison.

$(U_Z - 1.018 \text{ V})$

Table 4. Results of the NSAI-NML (Ireland)/BIPM bilateral comparison of 1.018 V standards using two Zener traveling standards: reference date 29 March 2010. Uncertainties are $1\sigma$ estimates.

<table>
<thead>
<tr>
<th></th>
<th>BIPM A</th>
<th>BIPM C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{\text{NML}}$ (Ireland) $(U_Z - 1.018 \text{ V})/\mu\text{V}$</td>
<td>187.57</td>
<td>127.37</td>
</tr>
<tr>
<td>Type A uncertainty/\mu\text{V}</td>
<td>0.016</td>
<td>0.016</td>
</tr>
<tr>
<td>correlated unc./\mu\text{V}</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>$U_{\text{BIPM}}$ $(U_Z - 1.018 \text{ V})/\mu\text{V}$</td>
<td>187.43</td>
<td>127.40</td>
</tr>
<tr>
<td>Type A uncertainty/\mu\text{V}</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>correlated unc./\mu\text{V}</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>pressure and temperature correction uncertainty/\mu\text{V}</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>$(U_{\text{NML}} - U_{\text{BIPM}})/\mu\text{V}$</td>
<td>0.14</td>
<td>-0.03</td>
</tr>
<tr>
<td>uncorrelated uncertainty/\mu\text{V}</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>$&lt; U_{\text{NML}} - U_{\text{BIPM}} &gt;/\mu\text{V}$</td>
<td>0.056</td>
<td></td>
</tr>
<tr>
<td>$a \text{ priori}$ uncertainty/\mu\text{V}</td>
<td>0.022</td>
<td></td>
</tr>
<tr>
<td>$a \text{ posteriori}$ uncertainty/\mu\text{V}</td>
<td>0.086</td>
<td></td>
</tr>
<tr>
<td>correlated uncertainty/\mu\text{V}</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>comparison total uncertainty/\mu\text{V}</td>
<td>0.283</td>
<td></td>
</tr>
</tbody>
</table>

The uncorrelated uncertainty is $w = \sqrt{r^2 + t^2 + v^2}$, the expected transfer uncertainty ($a \text{ priori}$ uncertainty) is $x = \frac{1}{2} \sqrt{w_A^2 + w_C^2}$, and the correlated uncertainty is $y = \sqrt{s^2 + u^2}$, where:
$r$ is the NSAI-NML Type A uncertainty (2);
s is the NSAI-NML Type B uncertainty, which is assumed to be correlated for both transfer standards (3);
t is the BIPM Type A uncertainty (5);
u is the BIPM Type B uncertainty, which is assumed to be correlated for both transfer standards (6);
v is the pressure and temperature coefficient correction uncertainty (7);
w_i is the quadratic combination of the uncorrelated uncertainties for the Zener (9);
x is the expected uncertainty of the mean, based on internal consistency  (11);
y is the quadratic combination of the correlated uncertainties (13).

Table 5. Estimated standard uncertainties for Zener calibrations with the BIPM equipment at the level of 1.018 V without the contribution of the Zener noise. The standard deviation of the mean of the BIPM daily measurement results is equal to 9 nV. We consider that the Type A uncertainty can’t be lower than the 1/f noise floor estimated at 10 nV.

<table>
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<tr>
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<td>0.11</td>
</tr>
<tr>
<td>leakage resistance</td>
<td>$3 \times 10^{-3}$</td>
</tr>
<tr>
<td>frequency</td>
<td>$3 \times 10^{-3}$</td>
</tr>
<tr>
<td>pressure and temperature correction</td>
<td>included in the Zener unc. budget</td>
</tr>
<tr>
<td>total</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Methodology of the calculation of the uncertainties related to the maintenance of the volt and the Zener calibration at NSAI-NML.

The following simplified measuring equation applies:

$$U_x(R_{thx}, p_x) = U_{REF} \cdot (\rho_x - \rho_0 + \delta \rho) + \delta U_T$$

where $U_x$ is the EMF at the terminals of the 1.018 V output at the prevailing conditions of

- thermistor resistance $R_{thx}$ and ambient pressure $p_x$
- $U_{REF}$ is the EMF of the 10 V reference standard used to standardize the potentiometer,
- $\rho_x$ is the ratio reading of the potentiometer when its input channel is connected to the output terminals of the 1.018 V Zener standard,
- $\rho_0$ is the ratio reading of the potentiometer when its input channel is shorted at the LOW output terminal of the 1.018 V Zener standard,
\( \delta \rho \) is a correction to the ratio reading which includes the effects of errors in the self-calibration routine, standardization, short-term drift, temperature and non-linearity on the ratio reading, 
\( \delta U_T \) is a correction for uncompensated thermal offset voltages in the measuring leads.

The EMF at the output terminals of the 1.018 V standard at reference conditions \((R_{thR}, p_R)\) is calculated using:

\[
U_X(R_{thR}, p_R) = U_X(R_{thX}, p_X) \cdot [1 + \alpha_R \cdot (R_{thR} - R_{thX}) + \gamma_p \cdot (p_R - p_X)]
\]

where \( \alpha_R \) is the relative temperature coefficient (relative to the thermistor resistance), and \( \gamma_p \) is the relative pressure coefficient of the 1.018 V Zener standard.

Table 6. Uncertainties related to the maintenance of the volt and the Zener calibration at NSAI-NML.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Best Estimate</th>
<th>Standard Uncertainty</th>
<th>Sensitivity Coefficient</th>
<th>Uncertainty Contribution</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of 10 V reference standard</td>
<td>( U_{REF} )</td>
<td>10 V</td>
<td>2 ( \mu )V</td>
<td>0.1018</td>
<td>0.20 ( \mu )V</td>
<td></td>
</tr>
<tr>
<td>Ratio reading of potentiometer</td>
<td>( \rho_X - \rho_0 )</td>
<td>0.1018</td>
<td>1\text{x}10\text{\textsuperscript{-8}}</td>
<td>10 V</td>
<td>0.10 ( \mu )V</td>
<td>(1)</td>
</tr>
<tr>
<td>Correction to ratio reading</td>
<td>( \delta \rho )</td>
<td>0</td>
<td>1.5\text{x}10\text{\textsuperscript{-8}}</td>
<td>10V</td>
<td>0.15 ( \mu )V</td>
<td>(2)</td>
</tr>
<tr>
<td>Uncompensated thermal voltages</td>
<td>( \delta U_T )</td>
<td>0</td>
<td>0.08 ( \mu )V</td>
<td>1</td>
<td>0.08 ( \mu )V</td>
<td></td>
</tr>
<tr>
<td>Temperature correction</td>
<td>( \alpha_R \cdot (R_{thR} - R_{thX}) )</td>
<td>0</td>
<td>3.5\text{x}10\text{\textsuperscript{-9}}</td>
<td>1.018 V</td>
<td>0.004 ( \mu )V</td>
<td>(3)</td>
</tr>
<tr>
<td>Pressure correction</td>
<td>( \gamma_p \cdot (p_R - p_X) )</td>
<td>0</td>
<td>4 \text{x}10\text{\textsuperscript{-9}}</td>
<td>1.018 V</td>
<td>0.004 ( \mu )V</td>
<td>(4)</td>
</tr>
<tr>
<td>( U_X(R_{thR}, p_R) )</td>
<td></td>
<td></td>
<td>Combined Standard Uncertainty</td>
<td>0.28 ( \mu )V</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:  
(1) Type A evaluation of the standard uncertainty, based on repeated observations.  
(2) Standard uncertainty based on manufacturer’s accuracy specifications for 8000A potentiometer, supported by its calibration history.  
(3) Relative temperature coefficient of \( 3.5 \times 10^{-7} \) \( \text{K}^{-1} \) is assumed.  
(4) Relative pressure coefficient of \( 2 \times 10^{-6} \) \( \text{kPa}^{-1} \) is assumed.
Conclusion

The final result of the comparison is presented as the difference between the value assigned to DC voltage standard by NSAI-NML, at the level of 1.018 V and 10 V, at NSAI-NML, $U_{NML}$, and that assigned by the BIPM, at the BIPM, $U_{BIPM}$, at the reference dates of the 29th and 31st of March 2010, respectively.

\[
U_{NML} - U_{BIPM} = 0.056 \, \mu V; \quad u_c = 0.283 \, \mu V, \quad \text{at 1.018 V}
\]

\[
U_{NML} - U_{BIPM} = -1.03 \, \mu V; \quad u_c = 1.13 \, \mu V, \quad \text{at 10 V}
\]

where $u_c$ is the combined standard uncertainty associated with the measured difference, including the uncertainty of the representation of the volt at the BIPM and at NSAI-NML, based on $K_{J-90}$, and the uncertainty related to the comparison.

These are satisfactory results. The comparison results show that the voltage standards maintained by NSAI-NML and the BIPM were equivalent, within their stated expanded uncertainties, on the mean date of the comparison.