Comparison of the Josephson Voltage Standards of the VNIIM and the BIPM

(part of the ongoing BIPM key comparison BIPM.EM-K10.b)

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Abstract. A comparison of the 10 V Josephson array voltage standard of the Bureau International des Poids et Mesures (BIPM) was made with that of the D.I. Mendeleyev Institute for Metrology (VNIIM), Russian Federation, in November 2010. For this exercise, option A of the BIPM.EM-K10.b comparison protocol was applied. This option required the VNIIM to provide a reference voltage for measurement by the BIPM using its Josephson standard with its own measuring device. The BIPM array is kept floating from ground.

The final results are in very good agreement with the combined relative standard uncertainty of 2.0 parts in $10^{10}$ for the nominal voltage of 10 V.

1. Introduction

In the framework of CIPM MRA key comparisons, the BIPM performed a direct Josephson voltage standard (JVS) comparison with that of the VNIIM, Russian Federation, in November 2010.

This comparison followed the technical protocol for BIPM.EM-K10.b comparisons and was carried out following option A of the protocol, whereby the BIPM measured the voltage of the VNIIM Portable Josephson Voltage Standard (PJVS) using its own measurement chain. The PJVS was first compared to the VNIIM primary Josephson voltage standard on 7-8 September 2010. The PJVS was then shipped to the BIPM headquarters in Sèvres, France, on the 8 November 2010 where an on-site direct comparison was carried out from 8-12 November 2010.
For option A of the protocol, the VNIIM PJVS only provided a reference voltage that was to be measured by the BIPM using its Josephson standard and its own measuring device. The BIPM array is kept floating from ground.

This article describes the technical details of the experiments which were carried for the comparison.

2. Comparison equipment

2.1 The BIPM JVS

The BIPM JVS used in this comparison comprised the cryoprobe with a Hypres 10 V SIS array (S/N: 2548E-6), microwave equipment and the bias source for the array. The Gunn diode frequency was stabilized using an EIP 578B counter, and an ETL/Advantest stabilizer. To visualize the array I-V characteristics, while keeping the array floating from ground, an optical isolation amplifier was placed between the array and the oscilloscope. During the measurements, the array was disconnected from this instrument. The measurements were carried out without monitoring the voltage across the BIPM JVS. The VNIIM PJVS produced a unique voltage output corresponding to 69350 Josephson SINIS junctions biased by an RF signal of $f = 69.731880$ GHz. The BIPM JVS biasing frequency was adjusted to minimize the voltage difference between the 2 JVS to below 1 µV. An analogue nanovoltmeter on its 3 µV range was used to record the voltage difference in such a way that if a jump occurred on the BIPM SIS array, the software would have stopped the acquisition because of the overload of the detector.

The series resistance of the measurement leads was less than 4 Ω in total, and the value of the thermal electromotive forces (EMFs) was found to be lower than 100 nV and are eliminated by the polarity reversal of the arrays. The leakage resistance between the measurement leads was greater than $1 \times 10^{11}$ Ω for the BIPM JVS.

2.2 The VNIIM Portable JVS

A complete description of the standard is given in the report related to the direct comparison between the PJVS and the VNIIM primary JVS (cf. Appendix A). In this paragraph, we have summarized the main characteristics of the PJVS. Details of the VNIIM PJVS are as follows:
• Resistance of the precision measurement leads: 11.5 Ω in total (measured on site: see 4.3.2. for more details)
• Leakage resistance between the precision measurement leads: 5 x 10^{10} Ω (measured on site at the temperature of liquid helium: see 4.3.2 for more details)
• Josephson junction array: SINIS 10 V (S/N: ALD-365/4)
• Bias source: VNIIM designed bias source, powered from two 9 V batteries in series which provide a current in the range 2.03 mA to 2.64 mA at 10 V and 0.98 mA to 1.35 mA at 0 V.

The I-V characteristic of the PJVS is shown in Figure 1.

![Graph showing I-V characteristic](image)

**Fig. 1**: Voltage-Current characteristic of the 10 V SINIS VNIIM PJVS. The output power of the RF source is 12.96 dBm. The width of the step #69350 is 100 μA in the current range from 2.19 mA to 2.29 mA.

The I-V characteristic of this array is unusual compared to what is normally observed for that type of programmable array. An assumption which can be made to explain this behaviour is that some of the Josephson junctions might be in parallel rather than in series. Effectively, the array does not show the expected total number of Josephson junctions (69351 instead of the theoretical value of 69632). Two consequences are that the Shapiro step is narrower than expected and that the programmable array is not sensitive to noise induced by phase slipping and thus the step cannot be sloped [1].
3. Comparison procedures - Option A at the 10 V level

3.1 First measurements

After the VNIIM PJVS was set up and checked for trapped flux, the voltage-current characteristic was determined (cf. Fig.1). The PJVS was then connected to the BIPM measurement system and three measurements were taken following the option A procedure of the BIPM protocol (http://kcdb.bipm.org). The VNIIM array was biased with a current \( I = 2.29 \text{ mA} \) on the step \( n = 69350 \) at \( f = 69.731 \text{ 880 GHz} \) and the BIPM array was set to the step number \( n = 64400 \) at the frequency \( f = 75.091 \text{ 710 GHz} \) in order to reduce the theoretical voltage difference to:

\[
d = (U_{PJVS} - U_{BIPM})_{\text{theo.}} = -0.50868 \mu\text{V}.
\]

The result was \( (U_{PJVS} - U_{BIPM}) / U_{BIPM} = 1.09 \times 10^{-10} \) with an experimental standard deviation of the mean of \( 2.6 \times 10^{-10} \).

This comparison result shows that the two standards were in very good agreement. The stability achieved on the two pieces of apparatus, even when they were connected together, was satisfactory and many grounding configurations were tested to confirm this preliminary result. The details of these measurements are described in the following paragraph.

3.2 Description of the BIPM measurement procedure

The BIPM array was always disconnected from its bias source during the data acquisition process. The reference ground of the chassis of the instruments that constitute the BIPM JVS was connected to the laboratory Earth potential. The two arrays were connected in series-opposition via a dedicated BIPM polarity switch. In this comparison scheme (option A), the BIPM JVS measurement set-up was used to measure the PJVS voltage as if it were a Zener voltage standard. During the comparison, only the biases of the two arrays were reversed (no mechanical switch reversal). This operation was carried out manually on both JVS. The polarity reversal was typically completed in less than 10 s.

The measurement loop was arranged in such a way that both positive polarities of the arrays were connected together and the nanovoltmeter was placed in between the two negative polarities of
the arrays. The “High” of the nanovoltmeter was connected to the BIPM array. A measurement point was acquired according to the following procedure:

1- Positive array polarity and reverse position of the detector;
2- Data acquisition;
3- Positive array polarity and normal position of the detector;
4- Data acquisition;
5- Negative array polarity and reverse position of the detector;
6- Data acquisition;
7- Negative array polarity and normal position of the detector;
8- Data acquisition;
9- Negative array polarity and reverse position of the detector;
10- Data acquisition
11- Negative array polarity and normal position of the detector;
12- Data acquisition;
13- Positive array polarity and reverse position of the detector;
14- Data acquisition;
15- Positive array polarity and normal position of the detector;
16- Data acquisition;

The reversal of the detector polarity is carried out to compensate for the non-unity gain of the isolation amplifier placed in between the analogue detector output and the DVM input. This operation also cancels out the thermo-electromotive forces at the detector level.

Each “data acquisition” step consists of 10 preliminary points followed by 30 measurement points. Each of these should not differ from the mean of the preliminary points by more than twice their standard deviation, otherwise the data are rejected and the acquisition is restarted. The “data acquisition” sequence lasts 25 s and corresponds to the time period during which the BIPM array is to stay on the selected step. The total measurement time (including polarity reversals and data acquisition) is approximately 5 minutes.

During the comparison, the BIPM current bias source was adjusted to select manually the same step after each polarity reversal. BIPM software, written in HP Basic, was used to monitor the detector and record the data. The detector was an analogue nanovoltmeter (EM N1a) set to its 3 µV range. The recorded data were transferred to the computer through a GPIB interface. After each polarity reversal we waited for 10 seconds before beginning the data acquisition in order to avoid the effects of filter capacitor discharge.
4. Description of the measurements

The measurement setup was not modified for the following series of measurements. Only slight changes were made to the grounding conditions.

The 48 individual measurements (cf. Fig. 2) used for the computation of the final result have been performed in the following configurations:

![Graph showing individual results](image_url)

**Fig. 2:** Individual results obtained to calculate the final comparison result at the level of 10 V: The solid line represents the mean value, the dotted dashed lines (---) represent the experimental standard deviation, and dotted lines (- - -) are the experimental standard deviation of the mean.
4.1 35 measurements performed on 9 November 2010

Note that points #1 to #3 are considered as preliminary measurements.

Points #4 to #8:

The PJVS array was set to a bias current $I = 2.24$ mA and the BIPM bias source was powered from the mains. An increase of the electrical noise level was clearly visible on the BIPM voltage steps from the scope screen. However, once the bias source was disconnected, the stability of the step was not affected.

Points #9 to #13:

The PJVS array was set to the same bias current ($I = 2.24$ mA) and the BIPM bias source was operated using the batteries in the same way as during the preliminary measurements.

Points #14 to #18:

An RF amplifier was inserted on the RF source of the PJVS: millimetre wave synthesizer (MWS). According to the technical specifications, the MWS should deliver 80 mW instead of 45 mW. This modification should shift the $I$-$V$ characteristics of the array in the $I$-$V$ plane and modify the length of the step. However the $I$-$V$ curve of the array remained almost unchanged: the step was only slightly extended* (cf. Fig 3). In accordance with the new $I$-$V$ characteristic, the bias current was changed to $I = 2.23$ mA, which corresponds approximately to the centre of the step.

* the power supply parameters of the amplifier were actually not properly set for the MWS to deliver 80 mW (Cf.4.3.1)
Fig. 3: Voltage-Current characteristic of the 10 V SINIS array of the VNIIM PJVS. The output power of the RF source is 13.56 dBm. The width of the step #69350 is 170 µA in the current range from 2.15 mA to 2.32 mA.

Points #19 to #23:

The full scale of the 3 µV range of the detector was calibrated by varying the RF frequency of the signal applied on the BIPM array. As we did not know how this correction factor evolves over time, we decided to take the corresponding mean value of the error on the five measurement of the voltage difference, \( u = 0.7 \, \text{nV} \) as the uncertainty of the readings from the nanovoltmeter.

Points #24 to #27:

The PJVS was disconnected from the ground reference point. The BIPM bias source was still powered from the mains and the noise on the BIPM step decreased. There was no visible difference when the BIPM bias source was powered using batteries, independent of whether or not the PJVS was connected to the ground.

The MWS is powered from a switching power supply which provides 2.8 mA under 9 V. This type of device is known to produce a high level of noise, in particular because of the high frequency of
the chopping module. There was no visible difference to the results when we reversed the polarity and filtered the power supply from the mains.

It was necessary to re-bias the PJVS a second or third time after almost each polarity reversal of the arrays, because after the first operation, the array spontaneously set on the half integer voltage step (cf. step #69350.5 on Fig.1).

The PJVS was biased alternatively to both ends of the determined voltage step in order to evaluate the flatness of the SINIS step.

Points #28 to #30: the PJVS was biased to one end of the determined voltage step corresponding to \( I = 2.28 \) mA.

Points #31 to #32: for the same reason, the working bias point was moved to the other end of the step: \( I = 2.18 \) mA.

Points #33 to #35: the working bias point was moved forward to: \( I = 2.28 \) mA.

Points #36 to #38: the working bias point was moved back to: \( I = 2.18 \) mA.

The results of these four experiments are in excellent agreement and show that the PJVS Shapiro step is not sloped within the stated uncertainties.

4.2 13 measurements performed on 10 November 2010

We attempted to cool down the BIPM’s PTB 10 V SINIS array (S/N: ADL 355/3) so that it would operate at the same frequency as the PJVS. Unfortunately, no step was detected at this frequency. The behaviour of the array was tested at \( f = 70.158 \) GHz where the array is known to work correctly [2]. A small step of 180 µA was found for a voltage output of \( U = 10.1007 \) V. The BIPM measurement set-up is not appropriate to measure this large theoretical voltage difference and thus the array was changed for a conventional SIS array (Hypres S/N: 2538F-3).

In order to investigate the effect of the noise coming from the power supply of the PJVS MWS, we carried out 3 measurements at the level of 0 V in the following configurations:

- MWS not powered, BIPM bias source powered from the mains, VNIIM set-up not connected to the ground.
- MWS not powered, BIPM bias source powered from batteries, VNIIM set-up not connected to the ground.

- MWS powered, BIPM bias source powered from batteries, VNIIM set-up connected to the ground.

- MWS powered, BIPM bias source powered from the mains, VNIIM set-up not connected to the ground.

- MWS powered, BIPM bias source powered from batteries, VNIIM set-up not connected to the ground.

- MWS powered, BIPM bias source powered from batteries, VNIIM set-up connected to the ground.

The results are presented in Fig.4. The mean value of the voltage difference is:

\[(U_{PJVS} - U_{BIPM}) = -0.01 \text{ nV}\]

with a Type A uncertainty (calculated as the standard deviation of the mean of all the measurements) equal to 0.38 nV. This dispersion is comparable to the one obtained at the 10 V level and might indicate that this noise level corresponds to the noise of the measurement set-up dominated by the white noise coming from the detector.
Fig. 4: Voltage difference between the BIPM JVS and the VNIIM PJVS at the level of 0 V within different configurations (see the text).

Points #39 to #42: the BIPM JVS was biased at $f = 74.163510 \text{ GHz}$ and the bias current of the PJVS was adjusted to $I = 2.21 \text{ mA}$. The BIPM bias source was powered from the mains and the PJVS set-up was not connected to the voltage reference point.

Note: we had to replace the BIPM RF source in the middle of the series as it displayed some frequency instability. This was probably due to an uncontrolled shift in the dimensions of the cavity in the Gunn diode. The millimetre adjustment screw, that had become loose, might have been responsible for this instability.

Points #43 to #45: For this series, the BIPM bias source was still powered from the mains but the PJVS was grounded.

Points #46 to #48: For these three measurements, the BIPM bias source was powered from batteries and the PJVS was not grounded.

Points #49 to #51: For this series, the BIPM bias source was still powered from batteries but the PJVS was grounded.
4.3 Experiments performed on the 11 November 2010

Experiments to support the uncertainty budget of the comparison were performed on 11 November 2010.

4.3.1 Power measurement

No significant change in the $I$-$V$ curve was seen when the RF amplifier was mounted on the MWS. As a result, we decided to carry out a series of power measurements at the output of the waveguide, at room temperature and when the MWS was operated at the top of the probe. The results are presented in Table 1:

<table>
<thead>
<tr>
<th>Voltage of the MWS amplifier power supply (V)</th>
<th>Amplifier present</th>
<th>Power at the output of the waveguide (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>NO</td>
<td>12.96</td>
</tr>
<tr>
<td>2</td>
<td>YES</td>
<td>13.56</td>
</tr>
<tr>
<td>5</td>
<td>YES</td>
<td>16.98</td>
</tr>
</tbody>
</table>

Table 1: Measurement of the power delivered by the MWS in terms of the power supply level of the RF amplifier.

When the amplifier was biased at its minimum voltage (2 V), the increase in the power, compared to the reading obtained without the amplifier, was negligible showing a difference of only 0.6 dBm. This explained the very slight changes between the two configurations in the $I$-$V$ curve of the PJVS array (cf. Fig. 1 and 3).

In a second experiment, the output power was recorded as a function of the voltage of the MWS power supply. The results are shown in Figure 5.
4.3.2 Resistance of the measurement wires and leakage resistance measurement of the PJVS

The resistance of the two measurement wires was measured at the temperature of liquid helium to \( r = 11.5 \) ohms. The leakage resistance was measured twice using a direct measurement method (Keithley 500 Megaohmeter).

The first measurement was performed at the temperature of the laboratory for which \( R_L = 5 \times 10^9 \) \( \Omega \). This value would imply a voltage drop of 23 nV from the theoretical value of the PJVS. This offset was not observed. As the probe is operated at the temperature of liquid helium, the probe was cooled down and a second measurement was performed, for which \( R_L = 5 \times 10^{10} \) \( \Omega \).

To explain this shift of one order of magnitude, we assume that the isolation of the printed circuit board, which was fixed to the ground close to the array, showed a lower resistance at room temperature than at helium temperature. In this configuration, the limiting leakage resistance came from the capacitors in the filter. The influence of the resistance of the output wires, which consisted of TiNb covered with copper, is lower at liquid helium temperature than at room temperature because of their superconducting state. Leakage measurements of the PJVS filter carried out at the VNIIM after the comparison showed that the capacitors have a dielectric absorption effect. The value of the absorption resistance changed over time from \( 3 \times 10^{10} \) \( \Omega \) after 10 s to \( 5 \times 10^{11} \) \( \Omega \) after 180 s. The value retained for the uncertainty component corresponds to a time constant of 20 s and is equal to \( R_L = 5 \times 10^{10} \) \( \Omega \) (cf. Appendix A).
5. Uncertainties and results

5.1 Final result and Type A uncertainty
As different experimental conditions (RF frequency of the BIPM JVS, bias current of the PJVS array, etc.) were tested, we considered that the 48 measurements used in the calculation of the final result were not strongly correlated and decided to take the standard deviation of the mean of the 48 measurements as the Type A uncertainty. The comparison result is therefore:

\[
\frac{U_{\text{PJVS}} - U_{\text{BIPM}}}{U_{\text{BIPM}}} = -1.05 \times 10^{-11} \quad \text{and} \quad \frac{u_A}{U_{\text{BIPM}}} = 4.3 \times 10^{-11},
\]

where \(u_A\) is the Type A uncertainty.

5.2 Type B uncertainty components
The sources of Type B uncertainty (Table 2) are: the frequency accuracy of the Gunn diode and the MWS, the leakage currents, and the detector gain and linearity. Most of the effects of detector noise and frequency stability are already contained in the Type A uncertainty. As both array polarities were reversed during the measurements, the effect of the residual thermal EMFs (i.e. non-linear drift) and electromagnetic interferences are already contained in the Type A uncertainty of the measurements. Uncertainty components related to RF power rectification and sloped Shapiro voltage steps are considered negligible as no such physical effect was observed.

<table>
<thead>
<tr>
<th>Source</th>
<th>Type</th>
<th>Relative uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BIPM</td>
</tr>
<tr>
<td>Frequency offset (^{(A)})</td>
<td>B</td>
<td>(8.0 \times 10^{-13})</td>
</tr>
<tr>
<td>Leakage resistance (^{(B)})</td>
<td>B</td>
<td>(5.0 \times 10^{-11})</td>
</tr>
<tr>
<td>Detector (^{(C)})</td>
<td>B</td>
<td>(7.0 \times 10^{-11})</td>
</tr>
<tr>
<td>Total (RSS)</td>
<td>B</td>
<td>(8.6 \times 10^{-11})</td>
</tr>
</tbody>
</table>

Table 2: Estimated Type B relative standard uncertainty components.

\(^{(A)}\) As both systems referred to the same 10 MHz frequency reference, only a Type B uncertainty from the frequency measured by the EIP is included. The frequency reference used for the comparison was produced by a Hydrogen maser belonging to the BIPM Time, Frequency and
Gravimetry Department. Its frequency uncertainty is negligible compared to the frequency stability of the RF sources.

BIPM JVS: It has been demonstrated on many occasions that the EIP-578B is a very good frequency locker and the accuracy of the frequency can reach 0.1 Hz. Assuming a rectangular distribution, the relative uncertainty for the offset of the frequency can be calculated from the formula: 

$$u_f = \left( \frac{1}{\sqrt{3}} \right) \times \left( \frac{0.1}{74} \right) \times 10^{-9} = 8 \times 10^{-13}$$

VNIIM PJVS: According to the PJVS synthesizer specifications that include a SLSM4 frequency synthesizer and multiplier, its frequency stability and accuracy is similar to that of the input 10 MHz reference signal. The uncertainty offset of the PJVS synthesizer was estimated by a frequency meter (CH3-66) and corresponds to a relative uncertainty for the voltage of $5 \times 10^{-11}$.

(8) Assuming a rectangular statistical distribution, the relative uncertainty contribution of the leakage resistance $R_L$ can be calculated from the formula: 

$$u_f = \left( \frac{1}{\sqrt{3}} \right) \times \left( \frac{r}{R_L} \right).$$

The values attributed to the resistances have been measured during the comparison exercise on both JVS. Note: the measured value of $r = 11.5 \, \Omega$ for the VNIIM PJVS was modified to $r = 8.2 \, \Omega$ and this value is justified in Appendix A.

(9) A large proportion of the detector uncertainty is already contained in the Type A uncertainty of the measurements. This component only expresses the effect of the uncertainty of the detector non-linearity correction. The uncertainty due to the DVM gain and linearity errors was estimated by calibrating the 3 μV range of the detector by changing the RF frequency of the BIPM JVS. The uncertainty on the gain is calculated as the maximum voltage difference due to the difference between the scale multiplying coefficient before and after its calibration: $u = 7 \times 10^{-10} \, V$.

5.3 Results at 10 V

The result using option A, expressed as the relative difference between the values attributed to the 10 V VNIIM PJVS ($U_{P,\text{PJVS}}$) and its theoretical value ($U_{BIPM}$) is:

$$\left( U_{P,\text{PJVS}} - U_{BIPM} \right) / U_{BIPM} = -1.05 \times 10^{-11} \quad \text{and} \quad u_c / U_{BIPM} = 1.44 \times 10^{-10}$$

where $u_c$ is the combined standard uncertainty.
5.4 Linking the results to the VNIIM Primary Josephson Voltage Standard

In September 2010, a direct comparison between the VNIIM PJVS and the VNIIM primary Josephson voltage standard was carried out. The results are detailed in the comparison report in the Appendix A. This result allows the establishment of a link between the VNIIM primary JVS and the BIPM primary standard. The result can thus be published in the BIPM Key Comparison Data Base (KCDB).

The degrees of equivalence of the VNIIM JVS, with respect to the BIPM JVS, is given in Table 3 by the following equations, both expressed in nV:

\[
\begin{align*}
d_{\text{VNIIM-BIPM}} &= d_{\text{VNIIM-PJVS}} + d_{\text{PJVS-BIPM}} \\
u^2_{\text{VNIIM-BIPM}} &= u^2_{\text{VNIIM-PJVS}} + u^2_{\text{PJVS-BIPM}}
\end{align*}
\]

<table>
<thead>
<tr>
<th></th>
<th>Voltage difference (nV)</th>
<th>Total combined uncertainty (nV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VNIIM – PJVS</td>
<td>0</td>
<td>1.80</td>
</tr>
<tr>
<td>PJVS – BIPM</td>
<td>-0.105</td>
<td>0.96 *</td>
</tr>
<tr>
<td>VNIIM – BIPM</td>
<td>-0.105</td>
<td>2.04</td>
</tr>
</tbody>
</table>

*The correlated uncertainties related to the PJVS are not included in this component as it has already been taken into account in the VNIIM/PJVS direct comparison uncertainty. The correlated uncertainties are all of Type B in this case.*

Table 3: Degrees of equivalence of VNIIM with respect to BIPM.
6. Discussion and conclusion

The results of the comparison are as follows:

- the preliminary comparison result:
  \( (U_{PJVS} - U_{BIPM}) / U_{BIPM} = 1.09 \times 10^{-10} \) and \( u_c / U_{BIPM} = 2.94 \times 10^{-10} \)

- the final comparison result \( (U_{PJVS} - U_{BIPM}) / U_{BIPM} = -1.05 \times 10^{-11} \) and \( u_c / U_{BIPM} = 1.44 \times 10^{-10} \)

- the comparison result between BIPM and VNIIM : \( (U_{VNIIM} - U_{BIPM}) / U_{BIPM} = -1.05 \times 10^{-11} \) and \( u_c / U_{BIPM} = 2.04 \times 10^{-10} \)

Even if the preliminary measurements had not varied from the later measurements, experiments were carried out on various parts of the whole system, particularly on the grounding of the assembly of both JVS in order to identify possible sources of electrical noise. From these investigations, we observed that when the PJVS was connected to the potential reference point of the laboratory, an increase in the level of noise could be clearly seen on the BIPM JVS, although it was not large enough to affect the stability of the SIS array voltage steps.

Any erroneous adjustment of either of the two JVS was easily detectable before running the data acquisition because the BIPM measurement set-up operates with an analogue detector. Therefore, adjustments were allowed before the acquisition. Under those circumstances, we noticed that after almost each polarity reversal of the PJVS, the array “jumps” on an intermediate half-step. The PJVS had to be re-biased a couple of times before the selected step \( n = 69350 \) was achieved.

Both standards were found to be in very good agreement, however, the Type A uncertainty of the direct comparison of the PJVS and the VNIIM primary standards could be significantly reduced if the detector noise on the negative polarity could be explained and therefore corrected.

Acknowledgement

The authors would like to thank PTB for providing the array of the VNIIM PJVS and in particular Dr Ralf Behr for his constant support.
References


DISCLAIMER

Certain commercial equipment, instruments or materials are identified in this paper in order to adequately specify the environmental and experimental procedures. Such identification does not imply recommendation or endorsement by the BIPM or VNIIM, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.
Appendix A: VNIIM primary standard – VNIIM PJVS direct comparison report

1. Introduction
The reference voltage across the VNIIM portable Josephson voltage transfer standard is measured using the VNIIM primary Josephson array voltage standard (JVS). This scheme allows the direct comparison via the routine measurement technique used for calibrations. This report describes the comparison of the VNIIM primary 10 V standard with the portable Josephson voltage transfer standard (PJVS) of the D.I. Mendeleyev Institute for Metrology (VNIIM), Russian Federation, carried out at the VNIIM in September 2010.

2. Comparison equipment

2.1 The VNIIM primary JVS
The primary VNIIM JVS comprises a cryoprobe with a PTB 10 V SIS array (S/N Me-168/10), microwave equipment and the bias source for the array. The custom-made microwave synthesizer was referred to a 5 MHz frequency signal provided by a rubidium frequency standard. An amplifier was placed between the array and the oscilloscope to monitor the array characteristic. During the measurements the array was disconnected from this instrument. To verify the step stability, a Keithley 2182 digital voltmeter (DVM) was used to measure the voltage difference between the two standards. The total series resistance of the measurement leads was 10 Ω, and the mean value of the thermal electromotive forces (EMFs) was found to be less than 300 nV. The leakage resistance between the measurement leads was greater than 5×10^{11} Ω.

2.2 The VNIIM PJVS
The transportable VNIIM JVS comprises a cryoprobe with a PTB 10 V SINIS array (S/N ALD-365/4), microwave equipment and a bias source for the array. The custom-made microwave synthesizer was stabilized using a 5 MHz (or 10 MHz) frequency signal from an external source. The total series resistance of the measurement leads was close to 10 Ω. The leakage resistance between the measurement leads was greater than 5×10^{10} Ω.

3. Grounding configuration
The grounding configuration of the two JVS can considerably affect the final result because of its effect on the dispersion of the measurement results selected to calculate the final comparison result. Both synthesizers were isolated from the cryoprobes. The bias source of the primary JVS
was disconnected during the measurements. The bias source of the transportable JVS was battery operated and was insulated from the ground. All equipment shields, including the DVM, were grounded to one point in the shielding room.

4. Comparison procedures

The two arrays were connected in series-opposition. A VNIIM low thermal-EMF switch allowed the polarity of the detector to be changed.

During the measurements, the primary VNIIM JVS was disconnected from its bias source, while the low potential of the bias source output was connected to the ground. The transportable VNIIM JVS was operated on batteries and was thus floating from ground. During the comparison, the polarities of the two arrays as well as the detector input were reversed.

4.1 Description of the measurements

The measurements carried out from 6-7 September 2010 had no voltage difference. Within a standard deviation of the mean of 1.4 nV. This value is taken as the Type A uncertainty.

The following is a brief description of the procedure used by VNIIM to obtain a single measurement of the voltage of the transportable VNIIM JVS. The voltage difference was measured by a Keithley 2182A nanovoltmeter (S/N 1103360), and the DVM was used to monitor the voltage across the PJVS array when the voltage of the primary array was equal to zero.

Parameters of the Keithley 2182 nanovoltmeter are as follows:
- range – 10 mV,
- rate – 1 plc,
- analogue filter – off,
- digital filter – on,
- number of measurements – 10.

The mean and the standard deviation of these measurements were calculated from the readings of the DVM.

The time required to perform a measurement is about 40 s and consists of 10 s for the acquisition of the positive polarity DVM readings, 20 s to change the polarity of the 10 V SIS array to the same step and 10 s for the acquisition of the negative polarity DVM readings. The second point starts with 10 s for the acquisition of the negative DVM readings, 20 s to change the polarity of the 10 V SIS array to the same step and 10 s for the acquisition of the positive DVM readings and so on. Six sets of 10 measurements were taken, three sets in the positive polarity of the detector (D+) and three sets in negative polarity (D-). These measurements were made according to the
following scheme: U+D+, U-D+, U+D+, U-D+, U-D+, U-D+, U-D+. The readings were imported into an Excel spreadsheet where the value attributed to the transportable VNIIM JVS was calculated. The first, second and sixth series (10 measurements each) were performed with the positive polarity of the detector and the third, fourth and fifth series were performed with the negative polarity.

The conventional array (VNIIM primary standard) was biased on its 67098th step at a frequency of \( f = 72.0728 \text{ GHz} \).

The programmable array (VNIIM transportable standard) was biased on its 69350.5th step at a frequency of \( f = 69.73188 \text{ GHz} \).

The differences between the measured values of the transportable JVS and the theoretical value of the primary VNIIM JVS from the expected difference during the comparison are plotted in Fig. A1.

Fig. A1: Differences between the measured values and the theoretical value of the VNIIM primary array voltage. The solid lines (———) represent the standard deviation of the mean of the measurements, and the dotted lines (– – –) represent the standard deviation of the measurements.

Note: the detector noise is considerably different depending on its selected polarity. The dispersion of the measurements in the positive polarity of the detector is at least 2 times lower than that of the
negative polarity. This effect was not investigated but could be due to a voltage common mode effect.

4.2 Uncertainties and results

The sources of Type B uncertainty (Table A1a and A1b) are as follows: absolute value of the frequency produced by the synthesizers (i.e. frequency offset), leakage resistance, and detector gain. Most of the effects of the frequency stability are already contained in the Type A uncertainty. As both array polarities were reversed during the measurements, the effect of the residual thermal EMFs (i.e. non-linear drift) is already contained in the Type A uncertainty of the measurements.

The Type A uncertainty was taken as the standard deviation of the mean and is equal to 1.4 nV.

<table>
<thead>
<tr>
<th>Uncertainty components of the VNIIM primary JVS</th>
<th>Type</th>
<th>Contribution / nV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (*)</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>Leakage resistance</td>
<td>B</td>
<td>0.12</td>
</tr>
<tr>
<td>Digital nanovoltmeter (**)</td>
<td>B</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>(0.01 mV reading w/Keithley 2182)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Type B uncertainties.</td>
<td></td>
<td>0.51</td>
</tr>
</tbody>
</table>

Table A1a: Estimated Type B standard uncertainty components of the primary VNIIM JVS.

(*) As both systems use the same 5 MHz frequency reference, only the Type B uncertainty of the synthesizer is included.

(**) As the VNIIM primary JVS array was always biased on the same step, the theoretical voltage difference was about 20 nV. The influence of the gain and linearity of the detector is therefore negligible. A large proportion of the detector uncertainty is already contained in the Type A uncertainty of the measurements.

<table>
<thead>
<tr>
<th>Type B uncertainties components of the VNIIM PJVS</th>
<th>At 10 V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microwave frequency.</td>
<td>0.5 nV</td>
</tr>
<tr>
<td>Circuit Leakage</td>
<td>0.9 nV</td>
</tr>
<tr>
<td>(8 Ω/50 GΩ @ 20 s)</td>
<td></td>
</tr>
<tr>
<td>RMS Type B</td>
<td>1.0 nV</td>
</tr>
</tbody>
</table>

Table A1b: The estimated type B standard uncertainty components in the transportable VNIIM JVS.

The result expressed as the relative difference between the values that would be attributed to the 10 V transportable VNIIM JVS \( U_{\text{transfer}} \) and its theoretical value \( U_{\text{primary}} \) is zero within a relative combined standard uncertainty \( u_c \), where \( u_c / U_{\text{primary}} = 1.8 \times 10^{-10} \).
5. Presentation of PJVS.

As shown on Figure A2 the system consists of three parts: the synthesizer; the cryoprobe comprising the array and LF filters; and the bias circuit.

The system was designed to use the 10 V SINIS or SNS Josephson array produced at the PTB. The array generates a constant voltage step close to 10 V. The peak-to-peak current width of the Shapiro step, selected for the comparison, is typically not less than 100 µA. The array is enclosed in a cryoperm magnetic shield.

The cryoprobe is constructed from a 20 mm stainless steel tube through which the waveguide and three pairs of electrical leads pass. One pair is for the current supply and the other two pairs are dedicated to the voltage measurement. These leads are connected to the measuring circuit through LF filters that were constructed with both ordinary and superconducting inductance coils made of superconducting leads and cryoperm. These inductance coils are immersed in a helium bath close to the array. The output voltage leads have 10 Ω series resistance. A 14 mm oversized circular waveguide, which was made in the laboratory, is inserted in the cryoprobe to irradiate the array with millimetre waves originating from the synthesizer. The length of the waveguide is about 1 m and the insertion loss is of the order of 2.5 dB. The order of magnitude of the thermal EMFs is usually lower than 250 nV.

The output power of the synthesizer can vary from 45 mW up to 80 mW (by inserting a dedicated RF amplifier) and its frequency is tunable within the range 68 GHz to 72 GHz with a minimum resolution of 400 kHz. During the comparison, the frequency $f = 69.73188$ GHz was selected to produce an output voltage near the nominal value of 10 V. The X-band reference signal is generated by a custom-made microwave synthesizer that can be locked to a 5 MHz or 10 MHz reference signal. The microwave source was electrically insulated from the cryoprobe.

A bias circuit with a variable internal resistance was custom-built for the 10 V step and to allow the current to be varied. The bias circuit was constructed in the laboratory and is used to manually control the step selection sequence. The bias circuit is equipped with terminals that can be connected to monitor the bias current and the voltage across the array. The bias circuit polarity switch allows polarity reversal of the biasing current and the output voltage terminals. The bias circuit is isolated from the screen with a terminal connected to the ground.

During transportation, the PJVS was carried in a rigid case. The sample holder was especially designed to allow the length of case to be reduced to 80 cm.

After measuring the leakage resistance of the filter at the BIPM (see 4.3.2), an additional investigation of the PJVS filter was carried out at the VNIIM. A schematic of the filter is shown in Fig. A3.

The leakage was measured by determining the current that flows between the output leads of the filter for an applied voltage $E$ (cf. Fig.4). The current is calculated from the measured value of the voltage drop on the resistor $R_0$ that is connected in series with the leakage resistor $R_L$. The voltage drop $U(t)$ is measured with a nanovoltmeter. The leakage resistance is then calculated from the formula: $R_L(t) = R_0(E/U(t)-1)$. 

Fig. A2: Schematic diagram of the PJVS
The filter consists of three pairs of capacitors of 0.1 µF with a leakage resistance $R$, four induction coils of 0.1 µH having a series resistance of 1.4 Ω each, two output leads of 0.25 Ω and two leads of 2.7 Ω connected to the superconducting induction coil of 50 mH. $R_p$ is the isolation of the printed circuit board. This board is fixed to the ground point of the array holder and its influence on the value of $R_p$ is negligible at the liquid helium temperature.

![Fig. A3: Scheme of PJVS filter.](image)

![Fig. A4: Measuring circuit of $R_L$.](image)
According to the scheme on Fig.2:

\[ R_L = R/3 + R/3 = 2R/3, \]

\[ R_{\text{leads}} = (0.25 \Omega \times 2) + (1.4 \Omega \times 4) + (2.7 \Omega \times 2) = 11.5 \Omega. \]

\[ \delta(\text{leakage}) = (\text{leads resistance})/(\text{leakage resistance})/\sqrt{3} \]

\[ \delta(\text{leakage}) = (1/\sqrt{3}) \times \{ (2.7 \times 2/2R) + [(2.7 \times 2 + 1.4 \times 2)/2R] + [(2.7 \times 2 + 1.4 \times 4)/2R] \} = \]

\[ \delta(\text{leakage}) = 12.3/R/\sqrt{3} = 8.2/R_L/\sqrt{3} = r_e/R_L/\sqrt{3}, \]

where \( r_e = 8.2 \Omega \) is the equivalent resistance of the leads, \( R_L \) is the measured leakage resistance of the filter.

Figure A5 shows that \( R_L \) increases from \( 2.9 \times 10^{10} \Omega \) at 10 s up to \( 5 \times 10^{10} \Omega \) at 18 s and reaches the value of \( 5 \times 10^{11} \Omega \) after 180 s.

![Graph showing the absorption resistance over time](image)

**Fig. A5**: Results of the measurement of the absorption resistance \( R_L \) when the voltage across the filter is switched from 0 V to 9 V and from -9 V to 9 V.

7. Conclusion

The results of the comparison demonstrate the high quality of the transportable VNIIM JVS at the 10 V level and the ability to use the VNIIM transportable standard for comparisons of JVS at 10 V.