Comparison of the Josephson Voltage Standards of the SMD and the BIPM

(part of the ongoing BIPM key comparison BIPM.EM-K10.b)

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Abstract. A comparison of the 10 V Josephson array voltage standard of the Bureau International des Poids et Mesures (BIPM) was made with that of the Service Métrologie – Metrologische Dienst (SMD), Belgium, in November 2009. For this exercise, the option B of the BIPM.EM-K10.b comparison protocol was applied. The results of both participants are in very good agreement and the overall relative standard uncertainty is 1.3 parts in $10^{10}$.

1. Introduction

In the framework of CIPM-MRA key comparisons, the BIPM performed a direct Josephson voltage standard (JVS) comparison with that of the National Metrology Institute of Belgium (SMD), Belgium, in November 2009. The transportable BIPM JVS was shipped to SMD and the comparison was carried out on-site.

This comparison followed the technical protocol for BIPM.EM-K10.b comparisons and was carried out for option B of the protocol where the participating laboratory operates its own measurement setup that usually includes a digital voltmeter as a detector.

This article describes the technical details of the experiments which were carried out to achieve the final result of the comparison.
2. Comparison equipment

2.1 The BIPM JVS

The BIPM JVS used in this comparison comprises the cryoprobe, the microwave equipment and the bias source for the Hypres 10 V SIS array (S/N: 2538F-3). The Gunn diode frequency was stabilized using an EIP 578B counter, and an ETL/Advantest stabiliser. To visualize the array $I$-$V$ characteristics, while keeping the array floating from ground, an optical isolation amplifier was placed between the array and the oscilloscope; during the measurements, the array was disconnected from this instrument. An HP 34420A digital voltmeter (DVM) was inserted between the array voltage bias leads to measure the voltage in order to verify the step stability. As reported below, we could however perform the measurement without monitoring the voltage across the BIPM JVS, that is the BIPM JVS was stable enough for remaining on the selected step. The series resistance of the measurement leads was less than 1 $\Omega$ in total (both measurement leads), and the value of the thermal electromotive forces (EMFs) due to both JVS was found to range between 70 nV and 130 nV (Cf. Fig. 1). The insulation resistance between the measurement leads was greater than $1 \times 10^{11} \Omega$ (Cf. Appendix A).
2.2 The SMD JVS

- The SMD JVS is a fully automated and commercially available JVS manufactured by Supracon AG – Germany. The automatic process includes the step selection and the microwave power adjustment. This JVS was firstly developed by the Institute for Physical High Technology Jena (IPHT) for two main applications: calibration of DC voltage standards and the verification of the linearity of DC multimeters or nanovoltmeters;

The JVS is composed of the following parts:

1. The cryoprobe with the chip carrier for the JVS circuit with about 19 700 SIS Josephson junctions;
2. A liquid helium dewar;
3. The JVS electronics unit;
4. The microwave unit which includes a Gunn oscillator, an isolator, a directional coupler, a mixer and a voltage controlled attenuator;
5. A source locking microwave counter;
6. A nanovoltmeter;
7. A 3-Channel polarity reversal switch;
8. Sensors for recording the temperature and the relative humidity of the air as well as the atmospheric pressure;
9. A portable PC with the suitable LabView software for driving the above mentioned items;
10. An USB/IEEE 488 interface.

Other details of the SMD JVS are as follows:
- Resistance of both precision measurement leads: 1.46 \( \Omega \);
- Leakage resistance between the precision measurement leads: 2 \( \times 10^{10} \) \( \Omega \);
- Josephson junction array: 10 V – assembled in the cryoprobe Supracon AG – sn: 05;
- Source locking microwave counter – Phase Matrix Inc. – model 578B – sn: 2021-1561 with remote sensor 2030030 – sn: 613;
- Null detector: Keithley 2182A – sn: 1 116 423 - range: 10 mV;
- 10 V DVM: see null detector; range 10 V;
- Bias source : Supracon AG – sn: 05 ;
- Software : Labview based software upgraded by Supracon AG.

3. Comparison procedures - Option B
Under the option B, considered here, the BIPM only provides a reference voltage that has to be measured by the SMD using its JVS with its own measuring device. The BIPM array is floating from ground.

3.1 Preliminary measurements
Within the week allotted to the comparison, different experiments were carried out in order to achieve the lowest voltage difference between the two JVS.

Before trying to improve the results, a preliminary measurement was made in the configuration routinely used by SMD. The results of the experiments made to improve the situation are described in the next section.
On the very first attempt to measure the voltage difference between the two JVS, the level of noise on the apparatuses connected together was so high that it was impossible to carry out a measurement. An investigation was performed and we found out that the electrical adaptation of the filters in the two JVS measurement leads was poor. We had to replace the filter on the measurement leads of the BIPM JVS with another one that has characteristics comparable to the SMD JVS filter. All the details of this investigation and its solution are described in Appendix A.

Depending on the measurement configuration, the SMD JVS array was either connected (biasing current adjustment phase) or disconnected from its bias source (data acquisition phase).

The BIPM bias source was operated on batteries during the step adjustment sequence, and was then disconnected from the array during the data acquisition process. The reference ground was connected to the SMD JVS. The two arrays were connected in series-opposition via the switch controlled by the SMD JVS measurement setup. In this comparison scheme (option “B”), the SMD JVS measurement setup was used to measure the BIPM array voltage as if it were a Zener voltage standard, but without using the polarity reversal switch. During the comparison, only the biases of the two arrays were reversed (no mechanical switch reversal). This operation was done electronically on the SMD JVS and manually on the BIPM JVS. The polarity reversal could typically be achieved within 30 s depending on the stability recovery on the SMD array.

Once sufficiently stable conditions were found (Cf. Appendix A), the JVS were connected to the SMD measurement system and four measurement points were acquired following the procedure applied by the software which controls the SMD JVS.

The result was \( \frac{U_{\text{SMD}} - U_{\text{BIPM}}}{U_{\text{BIPM}}} = + 3.4 \times 10^{-11} \).

The experimental standard deviation of the mean was 3.3 nV on these 4 measurements points, that is \( 3.3 \times 10^{-10} \) in relative terms.

This comparison result shows the high reliability of the SMD Josephson Voltage Standard and validates the CMCs of the laboratory. Following the preliminary measurements, a larger series of measurements was carried out where we tried to vary the measurement conditions.

3.2 Description of the additional measurements (See also Appendix A)

Both JVS were connected to the same 230 V and 50 Hz uninterruptible power supply through a high isolation transformer followed by a network filter. The operating conditions in the underground
room were as usual and respectively 23 °C ± 1 °C for the air temperature and 40 % ± 10 % for the relative humidity.

### 3.2.1 Operating setup for the first measurements

The following is a brief description of the procedure used by SMD to obtain a single measurement of the voltage of the BIPM array. During the comparison, the BIPM current bias source was manually adjusted to select the same step after each polarity reversal. The *Supracon AG* software was used to control the detector and to record the data. The detector was a digital nanovoltmeter (Keithley 2182A set on its 10 mV range). Four sets of 20 readings taken at an integration time of 1 powerline cycle each (NPLC = 1) were taken, one set in the positive polarity of the bias of the two arrays, two sets in the negative polarity, a fourth set in the positive polarity of the bias of the two arrays. The measurement followed the scheme: +, -, -, +. The recorded data were transferred to the computer through a GPIB interface. The complete series of measurements (+, -, -, +), took from 1 to 4 minutes depending on the adjustability of the SMD JVS. The readings were stored in an ASCII data file and the values attributed to the BIPM standard were also calculated by the software. The nanovoltmeter gain of the 10 mV range was measured before every run and the correction factor was automatically applied by the software to the results.

### 3.2.2 Operating setup for the best measurements

For all measurements, no significant changes to the SMD JVS were made. The measurement loop was arranged in such a way that both positive polarities of the arrays were connected together and the nanovoltmeter was placed in between the two negative polarities of the arrays. The “High” of the nanovoltmeter always connected to the BIPM array. The 48 individual points used for the computation of the final result have been performed in the same configuration.

Both arrays were biased at the same frequency $f = 74.6$ GHz. A series of measurements consisted of 8 individual measurements of the voltage difference. Two series were performed and the gain of the nanovoltmeter on its 10 mV range was measured before each series. The selected data to compute the final result are:

- **16 points obtained on 20/11/2009**
- **16 points obtained on 23/11/2009**
- **16 points obtained on 24/11/2009**
3.2.3 Individual comparison measurements

The differences between the values measured by the SMD JVS and the theoretical value of the BIPM array voltage during the comparison are plotted on Fig. 2. A histogram showing the distribution of all 48 data points is shown in Fig. 3.

![Figure 2. Voltage difference between the measured values and the theoretical value of the BIPM array voltage.](image)

The solid line represents the mean value, the dashed lines (−−−) represent the experimental standard deviation, and the dotted-dashed lines (−−−−) are the experimental standard deviation of the mean.

3.2.4 Considerations on the Type A uncertainty

All the 48 points are strongly correlated as there was no change in the measurement setup between each series. This assumption is supported by the estimated Birge ratio. We have calculated the standard deviation based on internal consistency for the 6 series of measurements (also called a priori uncertainty) which is given by: $\sigma_i^{-1} = \sum_j w_j$, where $w_j = \sigma_{ij}^{-1}$ is the weight of the $j^{th}$ series, estimated by the reciprocal of the series’ sample variance. This result was compared to the standard deviation based on external consistency (also called a posteriori uncertainty) calculated from $\sigma_E = \left[ \sum_j w_j \times (x_j - m_w)^2 \right] / \left[ (N - 1) \sum_j w_j \right]$, where $N$ is the total number of series of...
measurements and $m_w$ is the weighted mean [1]. The Birge ratio is then given by $R_B = \sigma_E / \sigma_I$ and is equal to one if the consistency is perfect. In our case, $R_B = 0.31$ ($\sigma_E = 0.157 \text{nV}$ and $\sigma_I = 0.508 \text{nV}$). This result infers that all the six series of measurement values do belong to the same statistical population. Although no experimental parameter had a major influence on the day-to-day reproducibility, the Type A uncertainty can’t be calculated from the standard deviation of the mean of the 48 individual results.

![Histogram of the JVS comparison using option B protocol.](image)

**Fig. 3.** The histogram of the JVS comparison using option B protocol.

### 3.2.5 Impact of 1/f noise floor of DVM to Type A uncertainty.

To account for correlations among successive values of the voltage difference between the two arrays, the plot of the Allan deviation of the voltage values interpolated to equal time intervals would be a robust method to evaluate the Type A uncertainty. Unfortunately, there are not enough points to carry out such a statistical analysis. However it has been shown that the analysis of voltage values measured with a digital nanovoltmeter with a short circuit at the input usually leads to the same form of the Allan variance [2-4]. We have carried out this experiment with a Keithley 2182A and the results are presented in Fig. 4a. The plot suggests that the estimated Allan
variance, $A_{\text{var}}$, can be modelled as a mixture of white noise and $1/f$ noise of the form $A_{\text{var}}(\tau) = h_0/2\tau^1 + b$, where $\tau$ is the sampling time and $h_0$ and $b$ are constants related to the spectral density.

Fig. 4a. Allan deviation calculated from 8192 successive voltage measurements with equal time interval on a 2182A with its input short-circuited.

The resulting Allan deviation is plotted in Fig.4b as a curved line which shows that for this particular nanovoltmeter the $1/f$ noise floor is for $(A_{\text{var}})^{1/2} = 0.5$ nV. The Allan deviation corresponding to the total time of the measurements during a series (10 s corresponding to the 20 data acquisition in one polarity direction), taken as the integration time is 1.2 nV. It is taken as the Type A standard uncertainty of the comparison measurements.
3.3 Uncertainties and results

The sources of Type B uncertainty (Table 1) are: the frequency accuracy of the Gunn diodes, the leakage currents, and the detector gain and linearity. Most of the effects of detector gain and frequency stability are already contained in the Type A uncertainty. As both array polarities were reversed during the measurements, the effect of the residual thermal EMFs (i.e., non-linear drift) and electromagnetic interferences are also already contained in the Type A uncertainty of the measurements. Uncertainty components related to RF power rectification and sloped Shapiro voltage steps are considered negligible as no such physical effect was observed. This was tested by fixing the bias current on a voltage step at several values above and below the step center and observing any possible voltage variations. None were observed.
Table 1. Estimated Type B standard uncertainty components.

<table>
<thead>
<tr>
<th>Type</th>
<th>Uncertainty / nV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BIPM</td>
</tr>
<tr>
<td>Frequency</td>
<td>0.02₁</td>
</tr>
<tr>
<td>Leakage resistance²</td>
<td>0.10</td>
</tr>
<tr>
<td>Detector linearity and gain³</td>
<td>0.28</td>
</tr>
<tr>
<td>Uncompensated thermal EMFs or drifts</td>
<td></td>
</tr>
<tr>
<td>Total (RSS)</td>
<td>0.10</td>
</tr>
</tbody>
</table>

(¹) As both systems are referred to the same 10 MHz frequency reference, the accuracy of the frequency sources is fixed by the 10 MHz reference thus only a Type B uncertainty from the frequency measured by the EIP is included [5].

The frequency reference used for the comparison is issued by the intermediary of two high isolation distribution amplifiers from one of the four high performance cesium frequency standards which the performances are monthly published in the BIPM Circular T.

The performances for the frequency measurement mode of the SMD EIP source locking microwave counter between 60 GHz and 90 GHz have been verified before the comparison by means of a setup consisting of a swept frequency synthesizer ranging from 10 MHz up to 40 GHz, a microwave amplifier, a x4 frequency multiplier and microwave frequency counter operating up to 60 GHz with 1 mHz resolution. The 10 MHz references of each frequency counter and of the microwave synthesizer were connected to the same frequency distribution amplifier fed by a 10 MHz reference signal coming from a high performance cesium frequency standard.

(²) A detailed description is given in Appendix A.

(³) A large part of the detector uncertainty is already contained in the Type A uncertainty of the measurements. This component only expresses the effect of the uncertainty of the detector non-linearity correction. The uncertainty due to DVM gain and linearity errors was estimated by 0,28 nV.
due to the fact that the maximum voltage differences taken into account by the nanovoltmeter were fixed to two steps (approximately 155 µV each) and the uncertainty on the short term linearity did not exceed 2 ppm.

The Type A uncertainty is 1.2 nV as described in section 3.2.4.

The final Option B result, expressed as the relative difference between the values that would be attributed to the 10 V Josephson array standard by SMD ($U_{SMD}$) and its theoretical value ($U_{BIPM}$) is:

$$\frac{(U_{SMD} - U_{BIPM})}{U_{BIPM}} = -4.3 \times 10^{-11} \quad \text{and} \quad u_c / U_{BIPM} = 1.33 \times 10^{-10}$$

where $u_c$ is the combined standard uncertainty.

It is important to note that the uncertainty budget is derived to this comparison only. Effectively, the BIPM JVS was adjusted to the working parameters of the SMD measurement setup: identical RF frequency and compatible filter on the measurement leads.

6. Conclusion

The preliminary measurements demonstrated SMD’s ability to perform accurate and precise automatic 10 V measurements.

During the subsequent following days, a small number of basic experiments were carried out on various parts of the whole system.

The SMD JVS is a commercial automatic system which has shown a very good reliability. However, this system doesn’t allow easily in the variation of influence parameters (measurement procedure, frequency of the RF source, replacement of the DVM, removal of the array from the probe, etc.) as well as changes of its software. Actually this setup is dedicated to routine measurements like Zener calibrations and to automatic linearity measurements of DC multimeters and nanovoltmeters. It does fit perfectly the requirements for these purposes. It also allows direct comparisons to other JVS but for the BIPM comparison described in this paper, the software was modified and a dedicated filter was required on the BIPM JVS. We have shown that the location of

* In the situation of a Zener calibration, the contributions of the JVS setup should be recalculated on the basis that the voltage difference between the Device Under Test and the Josephson array will be significantly larger.
the nanovoltmeter in the measurement bay is not adequate for measurements performed with a high level of accuracy without respecting a sufficient warm up period of the source locking microwave counter. The instrument is indeed too close to at least this main source of heat. The nanovoltmeter gain changes by 10 ppm in a few hours after all instruments in the measurement rack have been switched on. However this effect can be reduced due to the fact that the gain can automatically by an absolute manner be determined before each series of measurements using the same JVS and also automatically included in the measurement results.

Further enhancement of the SMD JVS will include the use of an active hydrogen maser and a modification of the filter on the cryoprobe.

Furthermore, the filter on the measurement leads was changed on the BIPM standard in order to be able to carry out voltage difference measurements between the two JVS. The uncertainty budget has been revised consequently.

The final result shows that both JVS are in very good agreement within the stated uncertainties.

The authors are indebted to Marco Schubert and Michael Starkloff of Supracon AG for their work during the first two days of this comparison (see appendix A).
References


[5] Field Note 5, Rev 02/90, EIP Microwave Inc.

DISCLAIMER

Certain adequate equipment, instruments or materials are identified in this paper in order to specify the environmental and experimental procedures. Such identification does not imply recommendation or endorsement by the BIPM or the NIST, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.
Appendix A

This appendix describes the comparison measurements in a chronological manner.

19 November 2009

After having assembled the BIPM equipment, we cooled down the BIPM array (Hypres S/N 2538F-3) and obtained a suitable critical current of 90 µA on the first cooling process. We selected the proper BIPM Gunn source to bias the array at 74.6 GHz which is the frequency at which the SMD JVS is operated. We successfully checked for the proper height of the Shapiro steps (>20 µA) at 10 V and their stability.

Connection operations

The SMD JVS system is grounded through its DC biasing source during its adjustment sequence and disconnected from the ground and completely floating during the measurement process. The SMD Helium Dewar is always referred to the earth ground of the laboratory through the cryoprobe.

The BIPM JVS DC biasing source was operated on batteries during the adjustment phase then disconnected from the array biasing leads. It was found that the level of noise in the measurement loop increased significantly if the BIPM Dewar was referred to the earth ground of the laboratory.

The positive poles of the two quantum standards were connected together and the detector was inserted between the negative poles, with the high side of the detector connected to the BIPM JVS side. The JVS were directly connected in such a way that each polarity reversal of the standards had to be done at the same time.

Under these conditions, it was impossible for the SMD JVS to be stable enough to get reasonable measurement points as the level of noise in the measurement loop was greater than 1 µV.

Searching for the possible reasons, we assumed that the problem could come from a resonance between the filter installed on the BIPM JVS measurement leads and the impedance of the input of the nanovoltmeter.

Both JVS were equipped with the filters of the same type of structure (Π) on their measurement leads but their inductances values were far different (below 1 mH for the SMD system and larger than 20 mH for the BIPM system).

To verify this assumption, the BIPM array was biased on its zero volt step and a Keithley 2182A was directly plugged across its measurement leads. The noise level was similar to that observed on the measurement loop.
The 2182A was then installed to measure a short just after a \( \Pi \) filter of the same type of the one mounted on the BIPM JVS measurement leads. The measurements showed the same high level of noise.

We therefore decided to realise a \( \Pi \) filter equivalent to the one mounted on the SMD JVS and to install it on the BIPM JVS (Cf. Figure A1).

\[
\begin{align*}
L_1 &< 1 \text{ mH} \\
L_2 &= 8 \text{ mH} \\
C &= 100 \text{ nF}
\end{align*}
\]

**Figure A1:** structure of the filter on the measurement leads of the SMD JVS, where \( L_1 < 1 \text{ mH} \), \( L_2 = 8 \text{ mH} \) and \( C = 100 \text{ nF} \).

### 20 November 2009

**Software**

We started the day by the realisation of the new PI filter for the BIPM JVS. At the same time, a modification was made on the software that controls the acquisition of the voltage difference of the JVSs. The original software was written in such a way that the acquisition is run only once both arrays are on the same step.

The motivation behind this was to limit the effect of the variation of the gain of the detector. The detector gain had changed for more than 7 ppm since the measurement bay was turned on.

This condition “both arrays on the same step” was often difficult to achieve rapidly. It is well-known that if a measurement takes too long to be completed, the non-linear component of the thermal emfs will affect the final result by adding a detrimental offset.

We decided to allow a voltage difference within 1 mV, to correct for the gain and to measure it every new series of 8 points.

**Gain of the detector**
Finally, we found that after a decrease of 30 ppm between the morning and the afternoon, the gain remains stable within 3 ppm (Cf. Fig A2). Looking closer to the measurement bay, we realised that the position of the DVM in the measurement bay is in between the EIP counter (lower side) and the top of the bay (high side) where a laptop was installed to run the measurement program. In terms of temperature variations, the DVM was placed in a critical environment with a heating source on each side. It is thus evident that the variation of the gain is correlated to the evolution of the internal temperature of the DVM exposed to two sources of heat.

![Evolution of the detector gain](image)

**Figure A2**: Evolution of the gain of the detector on the 23rd of November since the measurement bay has been switched on.

Once the new filter was installed on the measurement leads of the BIPM JVS, the array was cooled down ($I_c = 95 \mu A$) and the new isolation resistance was measured to $1 \times 10^{10}$ ohms. A first set of 4 consecutive measurement points was successfully carried out. The results show an average value of $(U_{SMD} - U_{BIPM}) = 0.34 \times 10^{-9}$ V with a standard deviation of the mean of $\sigma_A = 3.34 \times 10^{-9}$ V.
Resistance of the measurement leads and uncertainty due to the leakage resistance

From the measurements of the resistance of each lead connected to the array (Cf. Fig. A3), it is possible to determine the resistance of the measurement leads \((R_E + R_F)\) for each of the two arrays (Cf. Table A1):

![Fig. A3: Detail of the finger board of an SIS Josephson array.](image)

<table>
<thead>
<tr>
<th>BIPM JVS</th>
<th>SMD JVS</th>
</tr>
</thead>
<tbody>
<tr>
<td>((R_E + R_A) = 1.4 \Omega; (R_C + R_B) = 1.4 \Omega)</td>
<td>((R_E + R_A) = 2.3 \Omega; (R_C + R_B) = 2.3 \Omega)</td>
</tr>
<tr>
<td>((R_D + R_A) = 2.0 \Omega; (R_C + R_F) = 1.9 \Omega)</td>
<td>((R_D + R_A) = 3.15 \Omega; (R_C + R_F) = 3.15 \Omega)</td>
</tr>
<tr>
<td>((R_E + R_D) = 1.5 \Omega; (R_B + R_F) = 1.5 \Omega)</td>
<td>((R_E + R_D) = 2.3 \Omega; (R_B + R_F) = 2.3 \Omega)</td>
</tr>
</tbody>
</table>

Table A1: Details of the series resistance measurement.

\((R_E + R_F)(\text{BIPM}) = 0.95 \ \Omega \text{ and } (R_E + R_F)(\text{SMD}) = 1.46 \ \Omega\)

Therefore, the uncertainty \(u_L\) on the voltage provided by the JVS due the leakage resistance \(R_L\), will be expressed by the following equation if we assume a rectangular statistical distribution:

\[
u_L = \Delta V / \sqrt{3} = ((R_E + R_F) / R_L) \times 10 \times \sqrt{3} = 0.42 \text{ nV}
\]

23 November 2009

A large number of series of eight points were carried out on that day, essentially to check for the repeatability of the measurements.
We performed two series of eight measurement points inserting a resistance in series in the measurement loop to seek for an effect of a change in the leakage current paths from the “normal conditions”:

- one with a 1 kΩ
- the second one with a 500 Ω.

Both series showed a significant increase of the noise level. It was thus impossible to identify a clear voltage offset introduced by the resistance.

In the afternoon, we tried to vary the biasing frequency of the SMD system from 74.580 GHz to 74.620 GHz but no other frequency than 74.600 GHz was found to be stable enough to perform the comparison.

**24 November 2009**

Three series of 8 measurements were carried out at $f = 74.6$ GHz. The results were very similar to the ones obtained during the previous days. The SMD measurement setup remained switched on during the night and the detector gain didn't vary for more than 1 ppm during the series of measurements.

During the afternoon, we unsuccessfully tried to carry out long series of measurements (8192 consecutive points) on the zero voltage steps in order to be able to perform an electrical noise statistical analysis of the setup but too many jumps occurred during the series and it was not possible to record a suitable number of data. Therefore, this experiment was carried out with a short-circuit at the input of a similar detector.