The role of the international prototype of the kilogram after redefinition of the SI units

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Alain Picard
Estefanía de Mirandés
The present definition of the kilogram

3rd General Conference
of Weights and Measures, 1901:

“The kilogram is the unit of mass;
it is equal to the mass of the international prototype of the kilogram.” [IPK]

(IPK manufactured in 1880s, put into service in 1889)
Properties of the IPK and its copies

- Chemically inert: Alloy of platinum (Pt) and iridium (Ir) ✓
- Sufficient hardness: 90% Pt / 10% Ir ✓
- High density (small volume): 21 530 kg m\(^{-3}\) ✓
- Electrically conducting ✓
- Non-magnetic ✓
- Convenient shape for manufacture and use: cylinder ✓
- Minimum surface area: height = diameter (about 39 mm), with polished surfaces ✓
- Reproducible cleaning method ✓
- Relatively expensive 😞
What the present definition of the kilogram implies

The mass in kilograms of any object X is given by:

\[
\{ m_X \} [\text{kg}] = \left\{ \frac{m_X}{m_{IPK}} \right\} [\text{kg}]
\]

This ratio represents a measurement having an experimental uncertainty.

\[
\{ m_X \} [\text{kg}] = \left\{ \frac{m_X}{m_{n}} \right\} \cdot \left\{ \frac{m_{n}}{m_{n-1}} \right\} \cdot \ldots \cdot \left\{ \frac{m_{1}}{m_{IPK}} \right\} [\text{kg}]
\]

\( \div 10^{26} \) \hspace{1cm} \times 10^{1} \]
What happens to the IPK after redefinition?

• We have the precedent of the international prototype of the metre, another artefact, which defined the unit of length from 1889 until 1960.

• What became of the metre?

From the SI Brochure:

The original international prototype of the metre, which was sanctioned by the 1st CGPM in 1889, is still kept at the BIPM under conditions specified in 1889.
The international prototype of the metre, before and after
...but there is more to the kilogram story...
The current draft resolution for the new SI

On the date of the redefinition of the kilogram:

- the mass of the international prototype of the kilogram \( m(K) \) will be exactly 1 kg but with a relative uncertainty equal to that of the recommended value of \( h \) just before redefinition and that subsequently its value will be determined experimentally.

What exactly does this mean?
Quick review (if needed):

see talk by P. Becker:

\[ N_A \equiv \frac{0.012 \text{ kg mol}^{-1}}{m^{(12)C}}; \quad m^{(12)C} = \left\{ \frac{m^{(12)C}}{m_{\text{IPK}}} \right\} \text{[kg]} \]

see talk by M. Stock:

\[ \frac{m_{\text{IPK}}}{h} = \frac{ff'}{4} \frac{1}{g\nu} \quad \text{(SI unit: s m}^{-2}) \]

\( h/m^{(12)C} \) is already known to very high accuracy
(You can look it up in CODATA 2006)
The draft resolution deconstructed

• The SI second and metre are **already** defined in terms of two physical constants having fixed numerical values.

• The numerical value of \( h \) will be fixed **to ensure continuity with the present definition of the kilogram**:

\[
\frac{m_{\text{IPK}}}{h} = \{Q\} \left[ \text{s m}^{-2} \right]
\]

\( \{Q\} \) has an experimental uncertainty (watt balances or “Avogadro” or…)}
Transition to the new SI

\[
\frac{m_{\text{IPK}}}{h} = Q
\]

Just before redefinition
- relative uncertainty of \( Q = u_r(Q) \)
- value of \( m_{\text{IPK}} = 1 \text{ kg} \)
- relative uncertainty of \( m_{\text{IPK}} = 0 \)
- relative uncertainty of \( h = u_r(Q) \)

Just after redefinition
- relative uncertainty of \( Q = u_r(Q) \)
- value of \( m_{\text{IPK}} = 1 \text{ kg} \)
- relative uncertainty of \( m_{\text{IPK}} = u_r(Q) \)
- relative uncertainty of \( h = 0 \)
Consequences

• In future, measurements of $Q$ will determine the value of $m_{\text{IPK}}$ where today such measurements determine $h$.

• Future “$Q$-like” measurements will disseminate the new SI kilogram to other artefacts besides the IPK.

• The history of determinations of the value of Planck’s constant ends on the day the kilogram is redefined (just as the much longer history of determinations of the value of the speed of light ended in 1983 with the redefinition of the metre).

• Classical metrology is best served if $u_r(Q)$ is negligible.
Is the cure worse than the disease?

<table>
<thead>
<tr>
<th>Source of h</th>
<th>(\frac{h}{h_{\text{CODATA06}}} - 1) \times 10^{-9}</th>
<th>(u_r(h)/10^{-9})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recommended in 2006</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>NIST 1998</td>
<td>-7.5</td>
<td>87</td>
</tr>
<tr>
<td>NIST 2007</td>
<td>-7.5</td>
<td>36</td>
</tr>
<tr>
<td>“Avogadro” 2011</td>
<td>168</td>
<td>30</td>
</tr>
<tr>
<td>NPL 2010</td>
<td>300</td>
<td>200</td>
</tr>
</tbody>
</table>
How does mass metrology deal with the present SI?

By strict adherence to the definition of the kilogram

\[ m_{\text{IPK}} = 1 \text{ kg (exactly)} \] which implies that \[ u_{\text{IPK}} = 0 \text{ (exactly)} \]

metrologists insist on standards being traceable to \( m_{\text{IPK}} \) but are less concerned that \( m_{\text{IPK}} \) might be changing with respect to the constants of physics. (As long as there is no experimental evidence for such a change!)
The marketplace is interesting

Taken from a manufacturer’s webpage for highest class, stainless steel weights

<table>
<thead>
<tr>
<th>Mass of weight</th>
<th>Price</th>
<th>Tolerance (Tolerance +/-) according to OIML</th>
</tr>
</thead>
<tbody>
<tr>
<td>1kg</td>
<td>425.00 €</td>
<td>0.50mg</td>
</tr>
<tr>
<td>Wooden box</td>
<td>35.00 €</td>
<td></td>
</tr>
<tr>
<td>Calibration certificate</td>
<td>191.00 €</td>
<td></td>
</tr>
</tbody>
</table>

“Off the shelf”

Weight specification: 1 kg ± 500×10⁻⁹

Price:
- 425 € for the weight
- 35 € for the container
- 191 € for certification
The ideal: dissemination of the mass unit from $h$

$h \equiv 6.62606 \times 10^{-34}$ joule second
But we are not yet there

- Excellent work has been done, but today we have only two published results which provide a value of $h$ with standard uncertainty less than 50 parts in $10^9$.
- These two results disagree by about 175 parts in $10^9$.
- One might hope: that this difference will be resolved; that many new watt balances now under development will come on line; and that all results will agree.
- If this ideal situation is not achieved, the redefinition of the kilogram might nevertheless proceed under certain conditions and under a different dissemination scheme.
There may be a role for the IPK after redefinition

\[ h \equiv 6.62606 \times 10^{-34} \text{ joule second} \]
According to this plan

• The mass value assigned to the IPK depends on a weighted mean of the available experimental calibrations. This helps to assure that the SI mass assigned to $m_{\text{IPK}}$ will have the smallest possible uncertainty.

• Macroscopic mass determinations will still be traceable to one macroscopic source (the IPK) thereby maintaining the coherence of the present system.

• All mass measurements traceable to the IPK acquire the same component of relative uncertainty: $u_r(m_{\text{IPK}})$. This component disappears from comparison measurements between any two mass standards traceable to the IPK.
But we can do even better

• Instead of the IPK, whose alloy (Pt-10%Ir) was a product of advanced materials research (in the late 19th century), we can make a better artefact representation of the kilogram:
  • The average mass of a pool of about twelve 1 kg artefacts made of several different materials should replace the IPK in the previous scheme.
  • One of these materials can still be Pt-Ir; a second should be monocrystal silicon. Experience gained with monocrystal silicon shows this to be a very suitable artefact standard.
Storage facilities, measurement protocols and appropriate mathematical tools are already under development.

The BIPM is working on this project in close collaboration with a number of partners in national metrology institutes.
Two predictions:

1. The same fate which befell the international prototype of the metre awaits the international prototype of the kilogram (IPK):
   - The IPK will continue to be kept at the BIPM;
   - Ultimately, the IPK will become a curiosity of historical interest but of no metrological interest.

2. However, during a transition period of as-yet indeterminable length, the IPK (and, after some years of study, a pool of diverse modern artefacts) will play an important part in mass metrology.