The watt balance: determination of the Planck constant and redefinition of the kilogram

Michael Stock, BIPM

Royal Society Discussion Meeting: The new SI
Outline

▪ The present SI definition of the kilogram
  - shortcomings of the present definition
  - possible alternative: link to fundamental constants

▪ Watt balance experiments
  - principle of operation
  - existing watt balances

▪ Outlook to the redefinition of the kilogram
  - present knowledge of the Planck constant $h$
  - status of the redefinition
  - future dissemination of the kilogram
The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.

- represents the mass of 1 dm³ of H₂O at maximum density (4 °C)
- manufactured around 1880, ratified in 1889
- alloy of 90% Pt and 10% Ir
- cylindrical shape, \( \varnothing = h \sim 39 \text{ mm} \)
- kept at the BIPM in ambient air

The kilogram is the last SI base unit defined by a material artefact.
Calibration history of the oldest national prototypes

Variations of about 50 \( \mu \text{g} \left( 5 \times 10^{-8} \right) \) in the mass of the standards over 100 years, that is 0.5 \( \mu \text{g} / \text{year} \)

Masses of same material can be compared to within 1 \( \mu \text{g} \)

A drifting kg also influences the electrical units

Is the IPK losing mass or are the check standards getting heavier ? ?

► Redefinition of the kg in terms of a fundamental constant of nature, for example Planck constant \( h \) (advantageous for electrical metrology)

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Example of a possible new definition of the kg

“The kilogram, unit of mass, is such that the Planck constant $h$ is exactly equal to $6.626\,068\,96 \times 10^{-34}$ joule second:

$$h = 6.626\,068\,96 \times 10^{-34} \text{ J s}$$

The value of $h$ is fixed by nature.

The numerical value of $h$ is fixed by the definition of the kg.

The units m and s are defined in the SI.

The units $m^2 s^{-1}$.

The effect of this equation is to define 1 kg.

The numerical value needs to be determined in the present SI, to avoid significant discontinuities.
Why do we need watt balance experiments?

A watt balance allows to establish a link between $h$ and a macroscopic mass.

Watt balances are needed for several objectives:

- Determination of $h$ with uncertainty of the order of 1 part in $10^8$ in the present SI, several independent results desirable;

- Realization of the new definition of the kg after the redefinition (long-term task). Several instruments needed;

- Long-term study of the drift of the international prototype.
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The basic problem: linking the macroscopic and the microscopic world

- **macroscopic** masses at the level of 1 kg can be compared with an uncertainty of about 1 part in $10^9$ (1 μg).

- **atomic** masses can be compared with an uncertainty typically less than 1 part in $10^9$ in a range of [0.00055 $u$, 100 $u$].

- but: how to compare a macroscopic mass with a microscopic mass ($m_e$) or a fundamental constant ($h$)?

Solution: - macroscopic electrical quantum effects
- equivalence of electrical and mechanical power
Macroscopic electrical quantum effects

**Josephson effect (1962)**
(B. Josephson, Nobel Prize 1973)

\[ U_J(n) = \frac{n f}{K_J}, \quad K_J = \frac{2e}{h} \]

unc. of \( K_J \): 2.5 \( \times \) 10\(^{-8} \) (2006)
reproducibility at level of 10 V: < 10\(^{-10} \)

**Quantum-Hall effect (1980)**
(K. von Klitzing, Nobel Prize 1985)

\[ R_H(i) = \frac{R_K}{i}, \quad R_K = \frac{h}{e^2} \]

unc. of \( R_K \): 7 \( \times \) 10\(^{-10} \) (2006)
reproducibility at level of 100 \( \Omega \): approx. 10\(^{-9} \)

Both effects link macroscopic measurands (voltage, resistance) with fundamental constants (\( h \) and \( e \)).
Derivation of the watt balance equation

- Electrical power can be expressed as
  \[ P_{\text{el}} = U \cdot I = \frac{U_1 \cdot U_2}{R} \]
  Josephson effect
  \[ U_J(n) = n \cdot f \left( \frac{h}{2e} \right) \]
  quantum Hall effect
  \[ R_H(i) = \frac{1}{i} \left( \frac{h}{e^2} \right) \]

- Electrical power now depends on \( h \)
  \[ P_{\text{el}} = C_{\text{el}} \cdot f_1 \cdot f_2 \cdot h \]

- Electrical and mechanical power are equivalent
  - are quantities of the same type
  - are measured with the same unit
  \[ P_m (m, v, g, ...) = P_{\text{el}} = C_{\text{el}} \cdot f_1 \cdot f_2 \cdot h \]
  mass \( m \) and \( h \) appear in the same equation

- Avoid direct energy/power conversion!
Phase 1: static experiment

Weight of a test mass is compared with the force on a coil in a magnetic field.

\[ m g = -I \frac{d\Phi}{dz} \]

In a radial magnetic field, this can be simplified to

\[ m g = I L B \]

- \( F_{el} = I L B \)
- \( F_m = m g \)
- \( B \) (flux density)
- \( L \) (wire length)
- \( I \) (current)
- \( mg \) (force on mass)
Coil is moved through the magnetic field and a voltage is induced.

\[ U = -v \frac{d\Phi}{dz} \]

In a radial magnetic field, this can be simplified to

\[ U = BV \]

- **U**: induced voltage
- **B**: flux density
- **L**: wire length
- **v**: velocity

**Phase 2: dynamic experiment**
Watt balance equations

static phase: \[ mg = ILB \]

dynamic phase: \[ U = BVL \]

If \( L, B \) constant: \[ UI = mgv \]

\[ P_{el} = P_{mech} \]

Watt balance does not realize a direct conversion of electrical and mechanical energy.

Energy losses due to dissipative processes (friction,…) do not enter into the measurement equation.
Link between the kg and the Planck constant

$U$ and $R$ are measured using Josephson effect and the quantum Hall effect

$$\begin{align*}
U I &= \frac{U_1 U_2}{R} = C_{el} f_1 f_2 h \\
UI &= mgv
\end{align*}$$

A new definition of the kg requires the measurement of $h$ with an uncertainty of some parts in $10^8$. 
Another interpretation: weighing the electron

Watt balance equation:

\[ h = \frac{m g v}{C_{el} f_1 f_2} \]

Definition of the Rydberg constant:
 trope of hydrogen spectrum

\[ R_\infty = \alpha^2 \frac{m_e c}{2 h} \]

Most accurate determination of the electron mass to date!

\[ m_e = \frac{2 h R_\infty}{\alpha^2 c} \]

\( u_r(R) = 7 \times 10^{-12}, u_r(\alpha) = 7 \times 10^{-10} \)

\( (u_r(m_e) = u_r(h) = 5 \times 10^{-8}), \quad m_e = 9.109\,382\,15\,45 \times 10^{-31}\,\text{kg} \)
**Existing watt balance experiments**

<table>
<thead>
<tr>
<th>Year</th>
<th>Experiment</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976</td>
<td>NPL</td>
<td>“first watt balance”</td>
</tr>
<tr>
<td>1980</td>
<td>NIST</td>
<td>“biggest watt balance“, with superconducting magnet</td>
</tr>
<tr>
<td>1997</td>
<td>METAS</td>
<td>“smallest watt balance“</td>
</tr>
<tr>
<td>2001</td>
<td>LNE</td>
<td>“moving beam watt balance“</td>
</tr>
<tr>
<td>2003</td>
<td>BIPM</td>
<td>“single mode watt balance“, plans for superconducting watt bal.</td>
</tr>
<tr>
<td>2006</td>
<td>NIM</td>
<td>“mutual inductance joule balance“</td>
</tr>
<tr>
<td>2009</td>
<td>NRC</td>
<td></td>
</tr>
</tbody>
</table>
Existing watt balance experiments

- NPL, 1976, “first watt balance”
- NIST, 1980, “biggest watt balance“, with superconducting magnet
- NIM, 2006, “mutual inductance joule balance“
- NRC, 2009
Watt balance principle was proposed by B. Kibble, NPL in 1976
Work on Mark II started around 1990

- balance beam (1.2 m) on knife edge
- test mass 1 kg, 0.5 kg
- vacuum chamber
- coil
- permanent magnet radial field 0.42 T
- velocity drive coil

Courtesy of NPL
The NPL watt balance - Mark II

vacuum enclosure
balance beam 1.2 m
coil suspension
permanent magnet 0.42 T, radial field
velocity drive coil

Courtesy of NPL
The NPL watt balance - Mark II, 2007 result

$h = 6.626\ 070\ 95 (44) \times 10^{-34}\ Js$

$u_r (h, 2007) = 7 \times 10^{-8}$


$u_r (h, 2010) = 20 \times 10^{-8}$

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NPL watt balance, starting a new life as NRC watt balance

Shipped in summer 2009 from NPL, Teddington, to NRC, Ottawa

Operational at the several ppm level
Several parts being rebuild

$u_r < 10^{-7}$ expected for mid 2011
The NIST watt balance - started 1980

- balance wheel, 61 cm diameter (knife edge pivot)
- test mass, 1 kg
- velocity drive coil
- superconducting magnet, 0.1 T
- traveling coil, Ø 70 cm
- fixed compensation coil

Courtesy of NIST
The NIST watt balance

Courtesy of NIST

\[ u_r (h, 1998) = 9 \times 10^{-8} \quad u_r (h, 2005) = 5 \times 10^{-8} \quad u_r (h, 2007) = 3.6 \times 10^{-8} \]
The NIST 2007 result – the lowest published uncertainty

\[ u_r (h, 2007) = 3.6 \times 10^{-8} \]


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## NIST uncertainty budget

Improvements to the most significant type B uncertainty contributions

<table>
<thead>
<tr>
<th>Uncertainty contribution (nW/W)</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Resistance</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Local gravity acceleration</td>
<td>30</td>
<td>12</td>
</tr>
<tr>
<td>Wheel surface flatness</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>Electrical grounding</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>Laser wave front shear</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Mass std. magnetic susceptibility</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>Fitting order, plc change</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>16 others</td>
<td>13.1</td>
<td>12.3</td>
</tr>
</tbody>
</table>

**RSS combined**

<table>
<thead>
<tr>
<th></th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>46.8</td>
<td>33.2</td>
</tr>
</tbody>
</table>

*Improved gravity transfer*

*New support band, and improved investigation of effect*
New approach: no dynamic phase needed
Based on mutual inductance between two coils

The fixed coil carries a current $I_2$ to produce a magnetic field.
The second coil with current $I_1$ hangs on an arm of the balance.

$$mg = \frac{\partial M}{\partial z} I_1 I_2$$

geometric factor: To be determined by separate experiment
The NIM watt balance - started in 2006

\[ mg = \frac{\partial M}{\partial z} I_1 I_2 \]

Integration leads to:

\[ mg(z_2 - z_1) + \left[ M(z_1) - M(z_2) \right] I_1 I_2 = \int_{1}^{2} \Delta f_z(z) \, dz \]

Present state

- work was initially focused on measuring \( M \) using an innovative approach based on direct digital synthesis

- mutual inductance has been measured to 1 part in \( 10^7 \), but difficult to improve

- the magnetic field is very small, they plan to use superconducting coil

- balance has been purchased in mid-2010
Photo gallery of all watt balances

NPL
 Courtesy of NPL

NIST
 Courtesy of NIST

METAS
 Courtesy of METAS

LNE
 Courtesy of LNE

BIPM
 Photo: BIPM

NRC
 Courtesy of NRC

NIM
 Courtesy of NIM

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Available data for Planck constant

$N_A$ and $h$ are related by

$$N_A h = \frac{c}{2} \frac{A_r(e) \alpha^2}{R_\infty} M_u$$

$$u_r(N_A h) = 2 \times u_r(\alpha) = 1.4 \times 10^{-9}$$
Available data for Planck constant (only WB and Avogadro results)

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Requirements for a redefinition of the kilogram

The Consultative Committee for Mass (CCM) recommends that the following conditions be met:

- At least three independent experiments, including watt balances and int. Avogadro Coordination project with \( u_r \leq 5 \times 10^{-8} \)
- At least one of these shall have \( u_r \leq 2 \times 10^{-8} \)
- All results shall agree within 95% level of confidence
- A sufficient number of facilities for robust realization of the new definition are needed after the redefinition

\[ \iff \quad \text{Conditions are not fulfilled in 2011, no redefinition at 2011 CGPM} \]

Next occasion will be CGPM in 2015
## Future developments

<table>
<thead>
<tr>
<th>Organization</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internat. Avo. Collab. ((^{28}\text{Si-sphere})</td>
<td>(u_r) close to (2 \times 10^{-8}) planned for 2011-2012</td>
</tr>
<tr>
<td>NIST watt balance</td>
<td>Unc. not likely to improve a lot, new instrument being planned</td>
</tr>
<tr>
<td>NPL watt balance</td>
<td>Final publication in preparation</td>
</tr>
<tr>
<td>METAS watt balance</td>
<td>New instrument being developed</td>
</tr>
<tr>
<td>LNE watt balance</td>
<td>First measurements end 2011, objective (u_r) close to (2 \times 10^{-8}) in 2014</td>
</tr>
<tr>
<td>BIPM watt balance</td>
<td>First meas. made, (u_r &lt; 10^{-7}) planned for 2015</td>
</tr>
<tr>
<td>NIM joule balance</td>
<td>Under development</td>
</tr>
<tr>
<td>NRC watt balance</td>
<td>(u_r &lt; 10^{-7}) expected for mid-2011</td>
</tr>
</tbody>
</table>
Possible future dissemination of the kilogram

- Watt balance 1
- Watt balance 2
- Watt balance N
- Other techniques (XRCD, ...)

Comparison: degree of equivalence

BIPM watt balance

Pool of artefacts

Dissemination of the kg

NMI 1
NMI 2
NMI 3

- Long-term stability: watt balances
- Short-term stability: artefacts

More details in Richard Davis’ talk this afternoon