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Guide to
the Realization of the ITS-90

Interpolating Constant-Volume Gas Thermometry

Consultative Committee for Thermometry
under the auspices of the
International Committee for Weights and Measures

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P P M Steur, Istituto Nazionale di Ricerca Metrologica, Turin, Italy

B Fellmuth, Physikalisch-Technische Bundesanstalt, Berlin, Germany

O Tamura, National Metrology Institute of Japan, AIST, Tskuba, Japan

ABSTRACT

This paper is a part of guidelines, prepared on behalf of the Consultative Committee for Thermometry, on the methods how to realize the International Temperature Scale of 1990.

It discusses the major issues linked to interpolating constant-volume gas thermometry for the realization of the International Temperature Scale of 1990 at low temperatures.

1. Introduction

In the range from 3 K to the triple point of neon (24.5561 K), the ITS-90 is defined in terms of a ^{3}He or ^{4}He constant-volume gas thermometer, CVGT, calibrated at three temperatures.

Background information on absolute constant-volume gas thermometry can be found in the reports of four modern gas thermometry experiments [Berry 1979, Steur and Durieux 1986, Kemp *et al.* 1986/87, Astrov *et al.* 1989], which provide details of current good practice, and form the basis for the numerical values assigned to the low-temperature portions of ITS-90. These experiments demonstrate in different ways the very great care required in apparatus design and data analysis when a helium gas thermometer is used to determine thermodynamic temperatures in terms of a single reference temperature.

Measurement difficulties can be reduced to trivial levels, however, if the single reference temperature is replaced by two or three designated calibration temperatures, provided that these calibration temperatures lie within the measurement range, and that the measurement range itself be not overly wide. These requirements are met in the case of the ITS-90 gas thermometer: with appropriate design and choice of gas density (the criteria are not too restrictive), measurements accurate to the order of 0.2 mK are not difficult to obtain.

The difficulty of filling the temperature gap between 5.2 K, the upper end of the ^{4}He helium vapour-pressure scale, and 13.8033 K, the lower end of the platinum resistance thermometer (PRT) range, stems from the nature of the resistance-temperature relationships of available stable materials, the necessity for a great number of calibration points to describe these relationships (the only viable resistance thermometer for this range, the rhodium-iron resistance thermometer (RIRT) may require as many as 10 calibration points for the range between 5.2 K and 13.8 K), and the absence of satisfactory fixed points of definition. As a last resort, during the preparation of the ITS-90 in the late 1980's, Barber's original idea [Barber 1971] of a (constant-volume) gas thermometer calibrated at two fixed points, 4 K and 20 K, and with an expected accuracy of about 0.5 mK, was taken up again. To fill the temperature gap, he turned his attention to the then available primary thermometers: the (constant-volume) gas thermometer, the acoustic gas thermometer and the magnetic thermometer. He chose the CVGT as being the one on which most experience is available. The reason for Barber to use two fixed points may have been that the relative CVGT, with only a single reference point, was considered not accurate enough for direct use in a temperature scale. Barber's proposal regarded a ^{4}He CVGT correcting, before interpolation, explicitly for dead space and implicitly for non-ideality. Barber reasoned that "the error in temperature measurement due to the gas imperfections is zero to first order if the usual form of the second virial coefficient is adopted of $B = a/T + b/T^2$, where a and b are constants" [Barber 1971]. All other correction terms were intended to be calibrated out.

The ITS-90 gas thermometer goes somewhat further than Barber's proposal, with three calibration points instead of two, and correcting, before interpolation, explicitly for non-ideality only. This turned out to be necessary because of the non-quadratic behaviour of the ^{3}He and ^{4}He virial correction in the range between 3.0 K and 24.6 K. The definition of the interpolating CVGT (ICVGT) makes no mention of a correction for dead space, it just states that "p is the pressure in the gas thermometer", which has been cause of some discussion [Steur and Pavese 2005]. Using three fixed points, it was the intention to define an apparatus "not as complicated as the CVGT".

2. Definition of the scale

In the range from 4.2 K to 24.6 K using ^{4}He, the ITS-90 temperature T_{90} is defined by the relation

$$T_{90} = a + b\,p + c\,p^2, \qquad (1)$$

where p is the measured pressure and a, b and c are coefficients, the numerical values of which are obtained from measurements made at the three defining fixed points given in Section 3.2 of the text of the ITS-90, but with the further restriction that the lowest one of these points lies between 4.2 K and 5.0 K.

For a ^{3}He ICVGT in the range from 3.0 K to 24.6 K, and for a ^{4}He ICVGT, when the lowest defining fixed point is at a temperature below 4.2 K, the non-ideality of the gas must be accounted for explicitly, using the appropriate second virial coefficient $B_3(T_{90})$ or $B_4(T_{90})$. In these two cases, T_{90} is defined over the whole range from 3 K (or from the lowest defining fixed point) to 24.6 K by the relation:

$$T_{90} = \frac{a + b\,p + c\,p^2}{1 + B_x(T_{90})N/V_{\mathrm{B}}}, \qquad (2)$$

where p is the measured pressure, a, b and c are coefficients, the numerical values of which are obtained from measurements at the three defining fixed points given in Section 3.2 of the text of the ITS-90, N/V_{B} is the molar density of the gas with N being the amount of gas (number of moles) and V_{B} the volume of the bulb, x is 3 or 4 according to the isotope used, and the values of the second virial coefficients are given by the relations (6a) and (6b) of the text of the ITS-90.

3. General design considerations

There are very few explicit restrictions on the design of an interpolating gas thermometer to implement the ITS-90. The implicit requirement in the above discussion, that the errors of the quadratic interpolation should be of the order of 0.1 mK, can be achieved for a wide range of gas densities, bulb volumes and capillary geometries. See the designs of realized ICVGTs [Sakurai 1992, Meyer and Reilly 1997, Hill 2002, Kang *et al.* 2002, Sakurai 2002, Steur *et al.* 2003, Steur *et al.* 2005, Tamura *et al.* 2008, Peruzzi *et al.* 2010] for illustrations of how the conflicting requirements that both the thermomolecular pressure differences and dead space effects should be small may be satisfied.

It has been shown [Pavese and Steur 1987, Steur and Pavese 1989, Swenson 1989] that for an interpolating gas thermometer the major low-temperature deviations from a quadratic interpolation function are associated with the temperature dependence of the second virial coefficient, $B(T)$, of helium. However, in the ITS-90 these deviations are not significant (< 0.02 mK) for ^{4}He between 4.2 K (if this is one of the three calibration points) and 24.6 K. For ^{3}He and ^{4}He, when used below 4.2 K, this problem is dealt with in the ITS-90 by departing from the simple quadratic equation and including in the scale definition explicit corrections for $B(T)$, see Equation (2), which are based on recent results for ^{4}He [Steur *et al.* 1987] and ^{3}He [Matacotta *et al.* 1987]. The most recent *ab initio* calculations [Hurly and Mehl 2007, Mehl 2007, Bich *et al.* 2007] agree within 0.3 cm^3/mol with the ITS-90 equations for ^{4}He and ^{3}He. The latest experimental data deviate between 4.2 K and 25 K not more than 0.1 cm^3/mol from the ITS-90 equations for ^{4}He [Gaiser *et al.* 2010], and for ^{3}He not more than 0.3 cm^3/mol at 2.4 K [Gaiser Thesis 2008] as well as not more than 0.2 cm^3/mol between 3 K and 25 K [Tamura *et al.* 2011]. The temperature effect of differences in virial coefficient is, however, equivalent to less than 0.1 mK at a gas density of 160 mol/m^3, thus confirming Equation (2) well within the level of uncertainty of 0.5 mK originally foreseen for the ICVGT. This confirmation is important with respect to the possible magnitude of the non-uniqueness of the ITS-90. To improve pressure resolution, one would like to use relatively high gas densities. However, the differences in temperature due to the virial coefficient change linearly with density. Therefore, the maximum density to be used for the ICVGT is considered to be about 300 mol/m^3.

A typical interpolating gas thermometer (see Figure 1 for a schematic representation) has a measuring bulb, see Subsection 5, with a working volume of about 1 L, made of oxygen-free high-conductivity (OFHC) copper. The working gas is typically confined by a room-temperature pressure gauge (diaphragm manometer), see [Berry 1979, Sakurai 1982, Kemp *et al.* 1986/87, Steur and Durieux 1986, Astrov *et al.* 1989 and 1995, Tamura *et al*. 2011] for CVGT experiments as well as [Sakurai 1992, Meyer and Reilly 1997, Hill 2002, Kang *et al*. 2002, Sakurai 2002, Tamura *et al*. 2008 and Peruzzi *et al.* 2010] for ICVGT experiments, or, directly on the bulb, by a low-temperature pressure gauge [Steur *et al.* 2003, Steur *et al.* 2005]. (In the latter case, interpolation was done using room-temperature pressure values; only virial corrections were applied.) Pressure measurement can be performed through a capillary (pressure-sensing tube) with a diameter of about 1 mm. The purpose of a large bulb volume and a small capillary diameter at low temperatures is to keep the dead space contribution relatively small.

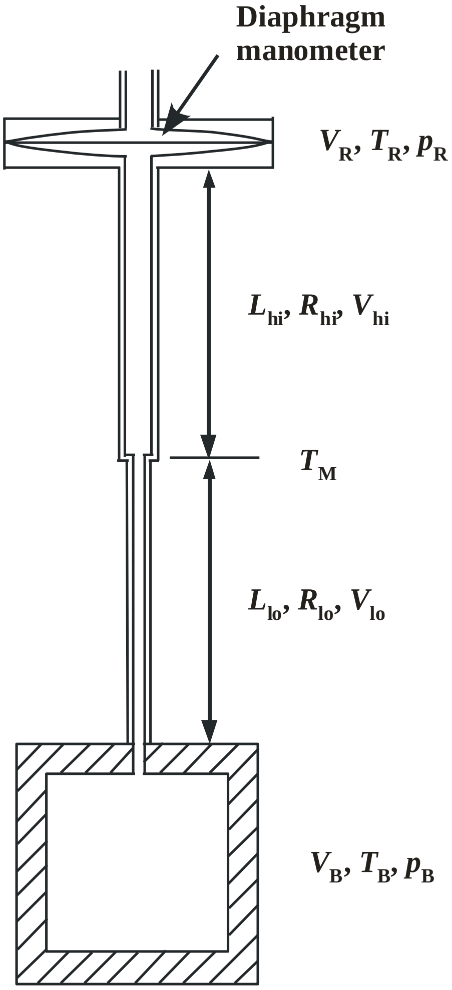

Figure 1. A schematic representation (not to scale) of a typical gas thermometer experiment, with V_B, T_B and p_B the volume, temperature and pressure of the bulb, T_R room temperature, p_R pressure measured at T_R, V_R and V_{hi}, V_{lo} dead-space volumes at T_R and in the two portions of the capillary, respectively, L_{hi}, L_{lo}, R_{hi}, R_{lo} length and inner radius of the higher and lower portion. The surrounding thermal shields and vacuum jacket are not shown. If, as is true for many metallic alloys, the thermal conductivity is nearly constant above T_M (about 80 K) and roughly proportional to T at lower temperatures, and in the absence of thermal anchoring [Steur 1999], the temperature distribution for the high and low temperature portions of the capillary can be approximated by

$$T^2 = T_B^2 + (T_M^2 - T_B^2)\,(L/L_{lo}) \qquad T < T_M$$

$$T = T_M + (T_R - T_M)\,(L/L_{hi}) \qquad T > T_M$$

where T_R is the room temperature and L is measured from the bottom of each portion upward.

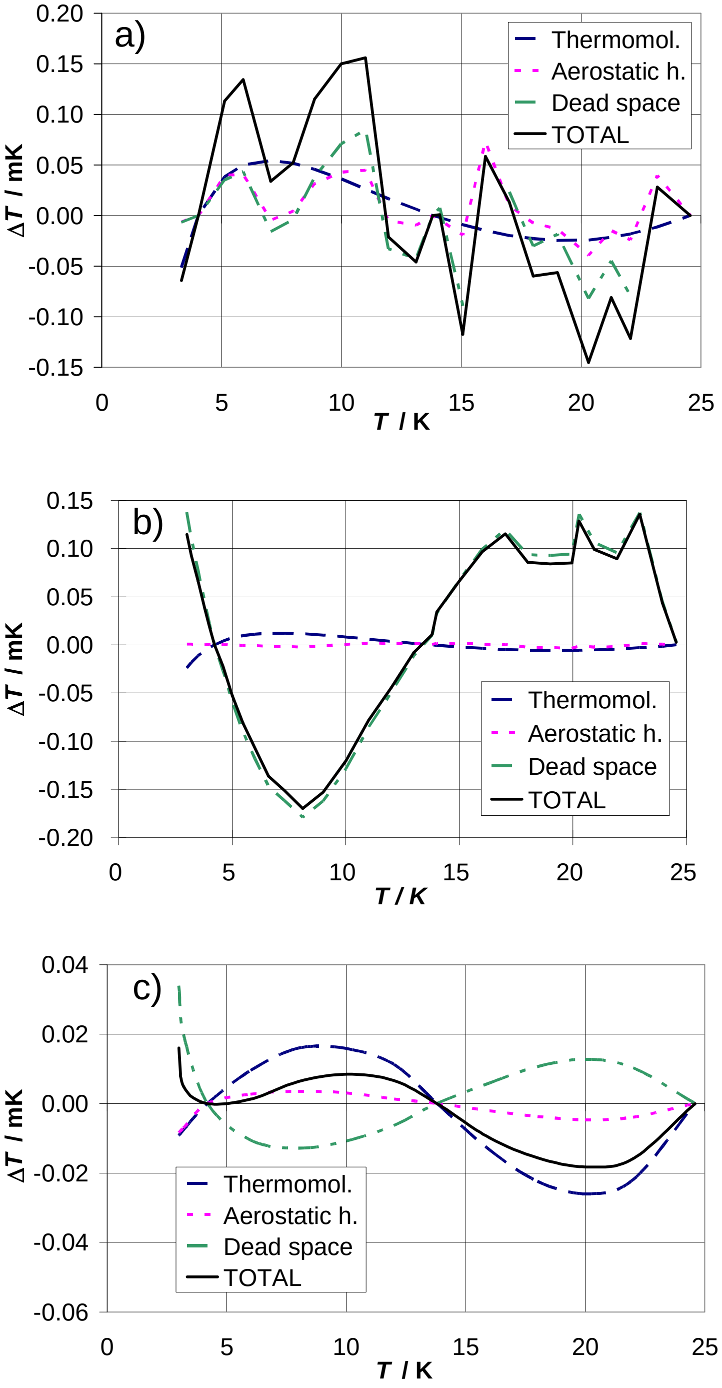

Figure 2. 'Residuals' from the quadratic interpolation between the values at the three calibration temperatures for three principle correction terms and for their sum that have been obtained in two recent experiments: (a) [Peruzzi *et al.* 2010]: For the dead space corrections, two clear outliers have been omitted. (b): [Tamura *et al.* 2013]: The 'residuals' have been obtained for the ICVGT described in [Tamura *et al.* 2008]. For comparison, calculated values, (c), are shown for the design of Figure 1, for typical parameters (T_R = 295 K, T_M = 80 K, lowest calibration point at T_B = 4.2 K, R_{lo} = 0.5 mm, R_{hi} = 1.0 mm, V_B = 1000 cm^3, V_R = 5 cm^3 and $L_{hi} = L_{lo}$ = 25 cm).

Contributions to non-ideal (i.e. imperfect quadratic interpolating) behaviour for an interpolating gas thermometer designed for the realization of the ITS-90 include dead space, aerostatic head, thermomolecular pressure difference and virial effects of the working gas (either ^{3}He or ^{4}He). Figure 2 shows how imperfect the quadratic interpolation of these correction terms is (without the virial effect, taken explicitly into account before interpolation), for some of the realized experiments. In each case, the 'residuals' are the deviations from a quadratic interpolation between the values at the three calibration temperatures.

As defined in the ITS-90, the values of p at the three fixed points (a point between 3.0 K and 5.0 K, 13.8033 K, 24.5561 K) are to be used to generate a quadratic interpolation function (Equation (2)), giving values T_{90}. The definition states that "p is the pressure in the gas thermometer". Often, p is interpreted to be the pressure inside the gas bulb, and one applies all the correction terms of traditional gas thermometry to the pressure measured at room temperature, thus making the interpolation results largely independent of the design. However, this requires the measurement of the temperature distribution inside the capillary, both for the determination of the dead-space effect and the aerostatic-head correction. On the other hand, it has been shown [Pavese and Steur 1987] that for a relatively wide range of design parameters, results well within the 0.5 mK limit can be obtained without applying *any* correction at all (except the virial correction, to be applied by definition), using for interpolation the pressure values measured at room temperature directly. As a consequence, however, the results become to some extent design dependent. This implementation (i.e. not correcting for the above corrections) was applied by Hill (2002). For a comparison of this experiment with other implementations see Figure 3 in [Rusby *et al* 2006].

Usually, the choice of applying or not some correction terms affects the interpolation results only within the foreseen limit of 0.5 mK [Steur and Pavese 2005, Tamura *et al*. 2003], provided that the temperature distribution along the pressure-sensing tube is stable during measurements and, above all, monotonically increasing towards room temperature. If the temperature distribution along the tube is not monotonic due to cooling by thermal anchoring, the interpolation applying the dead-space correction yields better results [Meyer and Reilly 1997, Steur 1999].

Any design for an interpolating gas thermometer should be tested with model calculations so that the magnitudes and temperature dependencies of the various contributions can be assessed. On the experimental side, good practice also requires adequate thermal isolation (small heat leaks, good isolation vacuum) and stability, as well as careful monitoring of various system temperatures to ensure that they are repeatable as the bulb temperature is cycled.

The essentials of gas thermometry can be classified under four headings: (1) the working fluid, (2) the thermometer bulb, (3) the determination of the pressure of the gas in the bulb, and (4) the effect of the volumes of the pressure-sensing tube and the room-temperature manometer system, the so-called dead space. These topics will be considered in the following subsections.

4. The working fluid

The very large, non-quadratic contribution of the second virial coefficient, $B(T)$, which is of paramount importance in primary gas thermometry, is removed *explicitly* in the ICVGT calibration procedure for ITS-90 based upon Equation (2). At low temperatures and for large gas densities, the temperature dependence of the third virial coefficient, $C(T)$ [Steur *et al.* 1987], can become significant. For this reason and in order not to amplify too much possible deviations of the ITS-90 values from the temperature resulting from an ideal-gas interpolation, the gas density N/V_B for ^{4}He should be less than 300 mol/m^3 for the highest accuracy thermometry. A similar limit probably applies to ^{3}He in the temperature regions especially below 4 K, where the third virial coefficient is likely to become significant [Matacotta *et al.* 1987]. While $B(T)$ is multiplied by N/V_B in the virial expansion, $C(T)$ is multiplied by $(N/V_B)^2$. The extent to which the choice of ^{3}He or ^{4}He as working fluid leads to different values of T_{90} depends upon the accuracy of the expressions adopted for $B_3(T)$ and $B_4(T)$ and on the gas density used in the thermometer. *Ab initio* calculations suggest that the expressions in the ITS-90 for $B_4(T)$ and $B_3(T)$ are sufficiently accurate; see Subsection 3. Namely, as already discussed above, the resulting interpolation errors with respect to the true gas behaviour will amount only to a few 0.1 mK for a density of 160 mol/m^3. (These interpolation errors influence the non-uniqueness of the ITS-90.)

Gas purity is also important, with hydrogen and neon impurities most significant for ^{4}He, and, in addition, ^{4}He impurities for ^{3}He. Significant levels of hydrogen and/or neon will show up as a discontinuity in the results due to condensation effects, but ^{4}He in ^{3}He will not. While impurity levels of less than 1 atomic part per million are the goal in primary gas thermometry, the gas thermometer calibration at three points allows perhaps 10 atomic parts per million, while 100 parts per million of ^{4}He in ^{3}He would be acceptable [Pavese and Steur 1987].

5. The thermometer bulb

The volume of the bulb, V_B, is generally large, typically one litre, to reduce dead-space effects (see Subsection 6) and to render adsorption effects negligible. The bulb is made of high-purity copper, usually oxygen-free high-conductivity (OFHC) copper, and is surrounded by an isothermal shield that is maintained at the bulb temperature so as to minimise temperature gradients. The thermal expansion of the copper bulb has only a very small effect [Kroeger and Swenson 1977]. It has been suggested that Kroeger's values may be in error by as much as 5 parts per million, at least in the temperature range below 25 K [Pitre and Moldover 2006]. Such an absolute difference in length does not, however, affect the working of a traditional gas thermometer in this temperature range, and even less that of the ICVGT. The bulb volume also will increase, slightly, with the gas pressure, an effect which is proportional to the pressure (and hence the temperature), but which is entirely compensated for in the calibration procedure. Only small effects due to gas adsorption have been detected in the gas thermometry experiments involving either gold-plated copper surfaces [Berry 1979] or clean copper surfaces, which were baked at high temperature [Astrov *et al.* 1989]. In each instance, the systems were evacuated thoroughly to remove adsorbed surface layers, and this practice is always advisable.

6. Pressure measurement

The pressure-measuring system is isolated from the working gas by a calibrated flexible diaphragm, which may be at room temperature or at the bulb temperature. (Figure 1 depicts a diaphragm manometer reading p_R directly; this is for illustration only.) In practice, the pressure p_R measured at room temperature is the sum of measurements made with some absolute manometer and a differential (diaphragm) manometer. The considerable advantages of a diaphragm at the lower temperature are to some extent offset by the need for a design that is stable despite temperature cycling [Steur *et al.* 2003, Steur *et al.* 2005]. In the experiments with low-temperature diaphragms, the maximum pressure was limited to 5 kPa (differential) and 60 kPa (absolute). In the CVGT experiments described in [Berry 1979, Kemp *et al.* 1986/87, Steur and Durieux 1986, Astrov *et al.* 1989 and 1995], differential diaphragm manometers having a full scale between 400 Pa and 1300 Pa were used at room temperature. In many of the cited ICVGT realizations, only an absolute diaphragm manometer was used, combined with a sufficiently high gas density. In order to reduce non-linearity effects as well as zero instabilities, a diaphragm is to be used only in the positive sense in most cases, i.e. without passing through zero, and by limiting pressure differences well below 30% of full scale. Pressure measurement at room temperature can be based, with high *absolute* accuracy, on a mercury manometer using optical interferometry or capacitive detection to measure differences in the heights of two mercury columns. On the other hand, pressure balances are preferred in gas thermometry for their ease of use and absence of mercury. High-quality devices can also have an *absolute* uncertainty of order a few parts per million. But for interpolating gas thermometry, the very high *relative* accuracy of pressure balances is of special interest. This means, a pressure-independent error of their effective area influences only the coefficients of the quadratic interpolation function, but not the interpolation results. The pressure sensitivity of a gas thermometer is proportional to the filling density, e.g. a sensitivity of about 2500 Pa/K corresponds to a density of 300 mol/m^3. Results have been reported for gas densities as small as 44 mol/m^3 (366 Pa/K) and as large as 460 mol/m^3 (3800 Pa/K). Pressure measurement resolution is typically 0.01 Pa for diaphragm gauges, with uncertainties close to 0.1 Pa for mercury manometers and pressure balances. For further information, see also the *Guide* Section 3 *Vapour-Pressure Thermometry and Pressure Measurement*.

6.1. Aerostatic head correction

The aerostatic pressure generated by the column of gas in the pressure-sensing tube (the aerostatic head) causes the room temperature pressure to be less than that in the bulb. To evaluate the aerostatic-head correction, the temperature distribution along the tube must be known. This can be measured directly, or it can be approximately calculated either from a knowledge of the temperature dependence of the thermal conductivity λ (T) of the tube material (assuming perfect adiabatic conditions around the tube), or it can be estimated, as a poorer approximation, from simple expressions λ (T) for temperatures above and below T_M [Kemp *et al.* 1986/87], where T_M ($\approx$ 80 K) is the approximate temperature, at which λ (T) changes from roughly proportional to T, below

T_M, to roughly constant, above T_M. These authors and Astrov *et al.* [1989] have simplified the calculation of aerostatic-head effects with a design in which temperature gradients occur only in horizontal tube parts. In practice, it is preferable to monitor the actual temperature distribution to ensure consistency and repeatability during an experiment. The aerostatic head correction is proportional to the molar mass of the gas, so its values will be 25 % smaller if the working gas is ^{3}He rather than ^{4}He for the same design of gas thermometer. With vertical tubes it will also be proportional to the length of the sensing tube. A pressure gradient also will occur inside the bulb, with T_B corresponding to the average (midpoint) pressure. This correction is independent of temperature (or pressure) and does not affect the calibration.

6.2. Thermomolecular pressure correction

A thermomolecular pressure difference Δp results from a temperature gradient along the pressure-sensing tube if the diameter of the tube is not large compared with the mean free path of the gas molecules. The pressure at the higher-temperature end (frequently at room temperature) is greater than the cryogenic bulb pressure. The magnitude of this pressure difference depends on the temperatures at the ends of the tube, on the tube diameter, on the pressure, and on the accommodation coefficient of the surface, which is a function of both the material and its surface conditions. Unfortunately, a straightforward and elementary discussion of these effects does not exist, see the overview in [Pavese and Molinar Min Beciet 2013]. For borosilicate-glass tubes, many results can be approximated to within a few percent by the relation [Swenson 1989]

$$(p_H - p_L)/p_L = (2\times10^{-9})\ (R\,p_L/(\mathrm{Pa\cdot m}))^{-1.99}\ ((T_H/\mathrm{K})^{2.27} - (T_L/\mathrm{K})^{2.27}), \qquad (3)$$

where p_H, p_L, T_H and T_L refer to the pressures and temperatures at the high and low temperature extremities, respectively, of a sensing tube of inner radius R. Equation (3) could be used also for tubes of other materials, but an additional uncertainty of at least about 10% should be considered if no special tests are performed. Such an additional uncertainty can also be caused by the dependence on the physical-chemical conditions of the inner surface of the tube, which may vary with time. Another expression is given by the so-called Weber-Schmidt equation [Weber *et al.* 1936]. For commonly used pressure-sensing tubes (see e.g. Figure 1), this expression yields estimates that coincide with those obtained using Equation (3) within about a few percent in the pressure range being of interest here for ICVGT.

For a typical ITS-90 gas thermometer the effects of thermomolecular pressure differences are shown, as deviations from interpolated values, in Figure 2 (c) for a pressure-sensing tube subdivided in an upper part, 2 mm diameter, and a lower part, 1 mm diameter, of Figure 1. Since more than 90 % of the thermomolecular pressure difference for a uniform capillary occurs between room temperature and 80 K (a somewhat arbitrary choice for T_M related to the use of liquid nitrogen), the total contribution is reduced by using a sensing tube that is wider above T_M than below, because $\Delta p\,/\,p$ is approximately proportional to $(R\,p)^{-2}$, see Equation (3). The same result in $\Delta p\,/\,p$ could be achieved by doubling the pressure (the gas density) for a uniform 1 mm diameter tube, but with increased non-ideality contributions. The total thermomolecular pressure difference for the gas thermometer depends on the bulb temperature, T_B, primarily through the pressure dependence in Equation (3), and therefore manifests itself mostly at temperatures below about 10 K. The use of a smaller diameter tube at temperature below T_M is necessary to reduce dead-space effects (see Subsection 7), but results also in longer equilibration times. In contrast with primary gas thermometry, corrections for the thermomolecular pressure difference need not be made provided its magnitude is small enough for the quadratic interpolation to be valid to within the desired uncertainty.

7. Dead space correction

Temperature-dependent changes occur in the quantity of gas in the bulb if the diaphragm separating the thermometer and the pressure-measurement device is not at the bulb. The effects of the room temperature volume, V_R, are most important for the highest bulb temperatures. The effect of the pressure-sensing tube dead space depends on the temperature distribution, and the length and cross-sectional area of the tube. For a uniform bore (and vertical tube), these contributions would be proportional to the aerostatic-head correction. The values shown in Figure 2 represent typical magnitudes. Again, the important requirement is that these effects be not so large that changes in the temperature distribution (due to refrigerant levels changing with time, for instance) can introduce significant systematic errors. Furthermore, especially due to the dead-space effects, the thermal anchoring of the pressure-sensing tube has to be given very careful consideration [Steur 1999]. In the simulations performed in this paper, the best results were obtained if the temperature of the tube increased monotonically between bulb and room temperatures, for all T_B values.

8. Estimation of uncertainty

Uncertainty components result for the ICVGT from: (i) pressure measurement, (ii) necessary corrections if the pressure in the bulb is used, (iii) calibration at the three fixed points applying resistance thermometers as transfer standards, and thermal conditions in the measuring system, such as possible temperature gradients, and (iv) other influences as purity of the measuring gas, adsorption effects as well as thermal expansion and pressure dilatation of the bulb. Examples for ICVGT uncertainty budgets are given in [Meyer and Reilly 1997, Steur *et al.* 2005, Tamura *et al.* 2008]. For some of the components it has to be considered that due to the calibration, not the whole uncertainty has to be considered, but only the effects causing a temperature dependence, which cannot be described sufficiently by the quadratic interpolation equations of the ITS-90. For instance, a relative error in the pressure measurement, being constant over the whole pressure range of interest, has no influence on the interpolation results. For instance, such an error is caused by an incorrect value of the effective area of a pressure balance if this value is independent of pressure. Table 1 gives the typical order of magnitude of the uncertainty contributions due to the different effects if state-of-the-art equipment is used. An entirely different set of components, not strictly related to the ICVGT, originates from the need to transfer the measured temperatures to a convenient thermometer, such as a resistance thermometer.

Table 1. State-of-the-art uncertainty budget for an ICVGT with a helium gas density of 300 mol/m^3 (estimates given in μK). The abbreviations have the following meaning: PB pressure balance, CDG capacitive diaphragm gauge used in differential mode near zero reading, RIRT rhodium-iron resistance thermometer used as transfer standard for the traceable calibration of the ICVGT at the fixed points of the ITS-90

Component	Min (near 4 K)	Max (near 25 K)
Pressure measurement		
Resolution of PB (0.1 Pa)	40	40
Effective area of PB, due to pressure (1 ppm)	4	25
Masses and temperature of PB (0.03 Pa)	15	15
Residual pressure of PB (0.01 Pa)	4	4
CDG in differential mode (0.1 Pa)	40	40
Pressure corrections		
Thermomolecular pressure difference	15	15
Aerostatic head	15	15
Dead space	60	60
Propagated calibration uncertainty		
Calibration of RIRTs at the fixed points	100	100
Temperature instability and inhomogeneity	50	50
Resistance ratio bridge	3	30
Instability of reference resistor	1	10
Self-heating correction	30	30
Pressure measurement during calibration	60	60
Propagation of ICVGT calibration uncertainty	80	80
Other components		
Gas purity	30	30
Thermal expansion	30	30
Pressure dilatation	10	10
Combined standard uncertainty	182	186

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