Linking the results of the regional key comparison APMP.AUV.V-K1 to those of the CIPM key comparison CCAUV.V-K1

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Abstract

During 1996 and 1997, eight national metrology institutes (NMI) took part in a vibration accelerometer comparison, identifier APMP.AUV.V-K1. Two NMIs ultimately withdrew from the comparison and the results of the remaining six NMIs have been approved by the CCAUV. Four NMIs subsequently took part in the 2001 CIPM key comparison for the same quantity, identifier CCAUV.V-K1. The results of these four CIPM participants have been used to link the results of the remaining two NMIs to the results in the CIPM key comparison using the reference frequency of 160 Hz. The CCAUV nominated the PTB to propose the methodology for the link and subsequently approved the linked results as presented in this report. The degrees of equivalence between each result and the key comparison reference value (KCRV), and between each NMI have been calculated and the results are given in the form of a matrix and graph for six NMIs. As two results from the APMP can now be linked to the published CCAUV.V-K1 comparison, the updated graph for the key comparison database is also given.

1. Introduction

As part of a major comparison programme, the APMP conducted a vibration accelerometer comparison using a back-to-back accelerometer type 8305 S/N 1610202 with charge amplifier type 2626, S/N 1242511. Eight laboratories participated in this APMP comparison that took place between February 1996 and September 1997. In addition to seven participants from the APMP, the PTB was invited to undertake a peer review and to participate as a final check laboratory. Two of the participating laboratories that had applied the comparison calibration method withdrew their results for various reasons. The other six laboratories are listed in Table 1.

Each participant applied the interferometer method in accordance with the ISO standard 16063-11 [1]. At the PTB, the calibrations were performed within two days in the presence of a representative of CMS/ITRI who had hand-carried the artifact to Germany and, after the calibration, back to Chinese Tapei. The PTB made major contributions to the analysis of the results and to the Final Report of the APMP comparison (Project number APMP-IC-4-95) and consequently, the project leader Dr. Shing Chen (CMS/ITRI) included Dr. H.-J. von Martens (PTB) as the co-author of the Final report of January 2001 [2]. At the meeting of the CCAUV in October 2002, it was agreed to confer the status of a regional key comparison on this APMP comparison (APMP.AUV.V-K1) and to link it to the CCAUV.V-K1 comparison that has well established key comparison reference values [3].

In the current report, the CIPM key comparison CCAUV.V-K1 is referred to as the CIPM comparison and the RMO key comparison APMP.AUV.V-K1 is referred to as the RMO comparison.

***Table 1*. Details of the participants in the APMP.AUV.V-K1**

NMI	Full name	Country	Regional metrology organization
CMS/ITRI (pilot)	Center for Measurement Standards of the Industrial Technology Research Institute	Chinese Tapei	APMP
CSIRO-NML	The National Measurement Laboratory in the Commonwealth Scientific and Industrial Research Organisation	Australia	APMP
KRISS	Korea Research Institute of Standards and Science	Republic of Korea	APMP
NMIJ*	National Metrology Institute of Japan	Japan	APMP
SIRIM	Standards and Industrial Research Institute of Malaysia	Malaysia	APMP
PTB	Physikalisch-Technische Bundesanstalt	Germany	EUROMET

* previously known as the National Research Laboratory of Metrology, Tsukuba (Japan)

The link for the RMO comparison has focused on the results obtained in both the CIPM comparison and the RMO comparison at the frequency of 160 Hz for the following reasons:

- 160 Hz is the reference frequency which is of special importance (cf. ISO 16063-11 and Annex C of the MRA, CMCs for AUV, area vibration)
- the frequency series applied to the RMO comparison was different from that applied to the CIPM comparison. The stated frequency range for the RMO comparison was 30 Hz to 10 kHz and deviated from the third octave frequency series in the CIPM comparison
- the reference frequency of 160 Hz was given preference in both the CIPM and the RMO comparison.

Only two laboratories, the CMS/ITRI and the SIRIM, participated in the RMO comparison but not in the CIPM comparison. The other four participants of the RMO comparison, the CSIRO, KRISS, NMIJ and the PTB participated in both comparisons and have been used as the linking laboratories.

2. Model for the linkage

The results of the APMP comparison need to be linked to those of CIPM comparison. The model for the linkage is as follows, cf. also [4].

The measurand in the CIPM comparison is denoted by X and this is the charge sensitivity of the back-to-back accelerometer type 8305, S/N 1483337, manufacturer Brüel & Kjær. The values x_1, $u(x_1)$, ..., x_N, $u(x_N)$ denote the best estimates and associated standard uncertainties of the laboratories.

The measurand in the RMO comparison is denoted by Y and this is the amplified sensitivity of the back-to-back accelerometer type 8305, S/N 1610202, with amplifier type 2626, S/N1242511, manufacturer Brüel & Kjær. The values y_1, $u(y_1)$, ..., y_M, $u(y_M)$ denote the best estimates and associated standard uncertainties of the laboratories.

Furthermore, $G = \{1,..., p\}$ ($p \le \min(N,M)$) is the index set of the linking laboratories. The laboratories are labelled such that any number within G denotes the same laboratory in both comparisons.

The value $R = X/Y$ denotes the linking coefficient between the two measurands to make the link between the two comparisons. The linking coefficient is estimated using the KCRV of the CIPM comparison and the combined results in the RMO comparison of the linking laboratories. The estimated linking coefficient is then applied to the results of the RMO comparison.

Since no information about correlations is available, the estimators X_1, ..., X_N, Y_1, ..., Y_M are treated as being uncorrelated. Let x denote the KCRV of the CIPM comparison and y the weighted mean of the linking laboratories in the RMO comparison

$$x = \sum_{l=1}^{N} \frac{x_l}{u^2(x_l)} \Big/ \sum_{l=1}^{N} \frac{1}{u^2(x_l)} \qquad u^2(x) = \frac{1}{\sum_{l=1}^{N} \frac{1}{u^2(x_l)}} \tag{1}$$

$$y = \sum_{l \in G} \frac{y_l}{u^2(y_l)} \Big/ \sum_{l \in G} \frac{1}{u^2(y_l)} \qquad u^2(y) = \frac{1}{\sum_{l \in G} \frac{1}{u^2(y_l)}} . \tag{2}$$

Then R is estimated according to

$$r = \frac{x}{y} \qquad u^2(r) = \frac{u^2(x)}{y^2} + \frac{x^2}{y^4} u^2(y) \tag{3}$$

$Z = RY$ denotes the linked measurand of the RMO comparison and

$$z_l = ry_l \qquad u^2(z_l) = y_l^2 u^2(r) + r^2 u^2(y_l) + 2ry_l u(r, y_l), \qquad l = 1, \ldots, M$$

(4)

$$u(r, y_l) = \begin{cases} -\frac{x}{y^2} u^2(y), & l \in G \\ 0, & \text{otherwise} \end{cases}$$

are the corresponding estimates including the associated uncertainties.

The degrees of equivalence are defined as the differences between the linked results of the RMO comparison and the KCRV of the CIPM comparison

$$d_i = z_i - x, \quad i = 1, \ldots, M \tag{5}$$

and the uncertainties associated with these differences where

$$u^2(d_i) = u^2(z_i) + \left[1 - 2\frac{z_i}{x}\right] u^2(x), \quad i = 1, \ldots, M. \tag{6}$$

Furthermore, the degrees of equivalence between the laboratories are defined as the differences

$$d_{ij} = x_i - z_j, \quad i = 1, \ldots, N, \quad j = 1, \ldots, M \tag{7}$$

and the uncertainties associated with these differences where

$$u^2(x_i - z_j) = u^2(x_i) + u^2(z_j) - 2\frac{z_j}{x} u^2(x), \qquad i = 1, \ldots, N, \quad j = 1, \ldots, M \tag{8}$$

3. Results

The results of the RMO comparison have been linked to the CIPM comparison at 160 Hz for the reasons already described. According to the CIPM-MRA, the results are given with expanded uncertainties (k = 2), $U_i = 2u(d_i)$ [5]. The degrees of equivalence of each NMI with the KCRV are shown in Table 2 and Figure 1.

	D_i	U_i
	pC/(m/s²)x10^{-4}	
NMIJ	0.554	3.759
KRISS	-0.336	3.624
CSIRO-NML	2.207	7.526
PTB	-0.081	0.560
CMS/ITRI	-4.530	13.423
SIRIM	0.300	3.966

Table 2: Differences D_i = d_i of the linked results of the RMO comparison and the KCRV of the CIPM comparison, with the corresponding expanded uncertainties $U_i = 2u(d_i)$

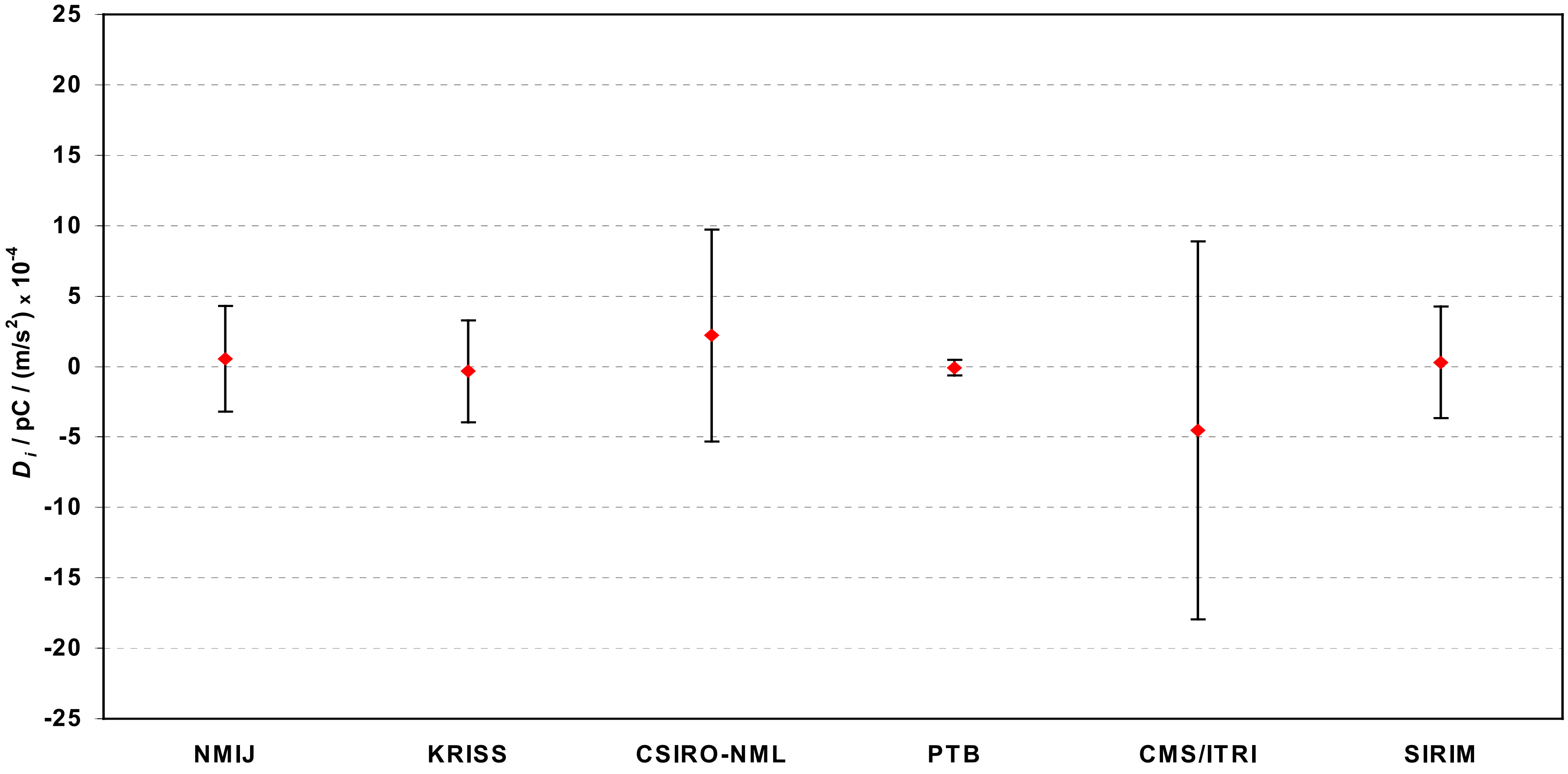


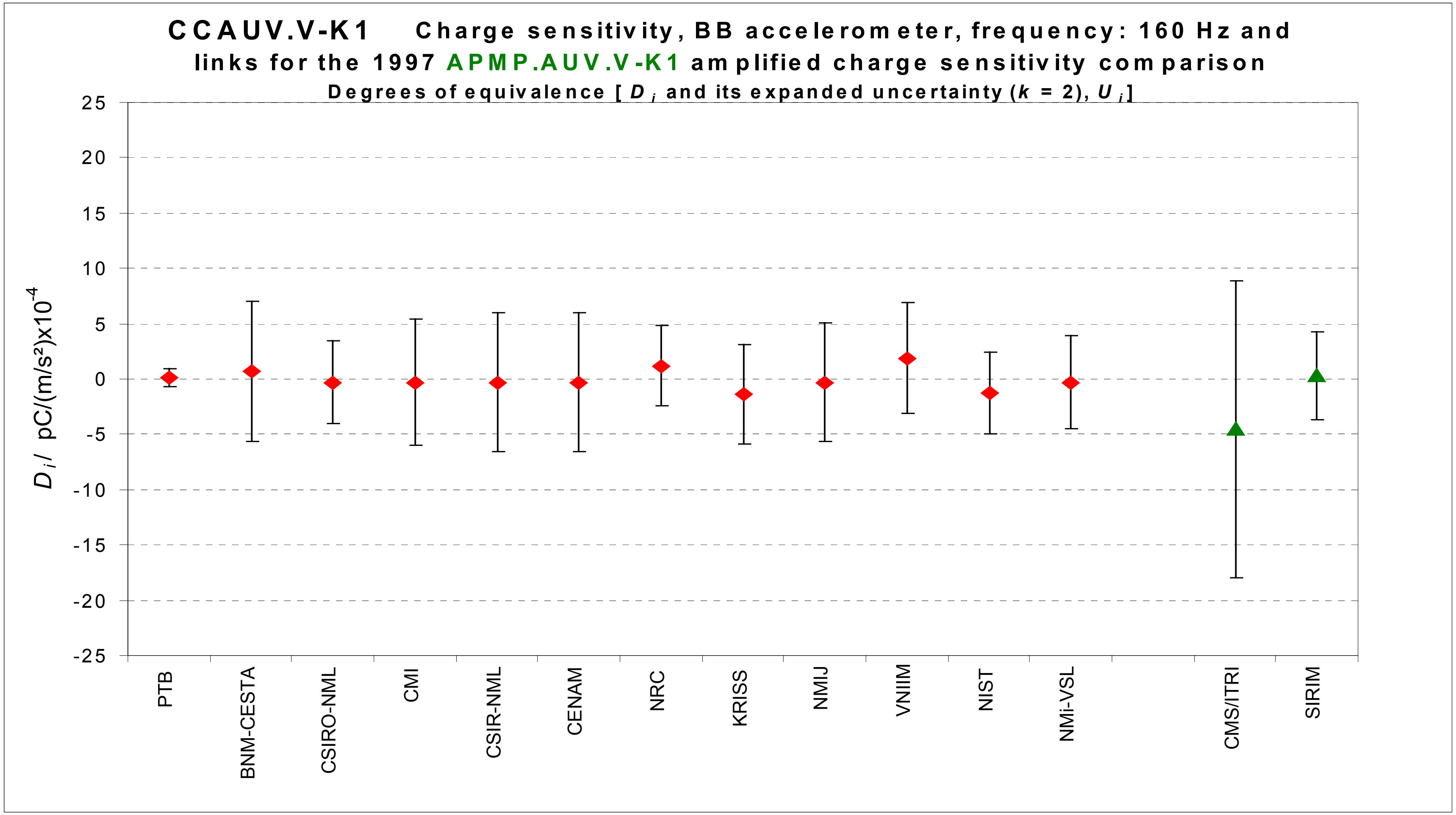
Figure 1: Differences $D_i = d_i$ of the linked results of the RMO comparison and the KCRV of the CIPM comparison, with the corresponding expanded uncertainties $U_i = 2u(d_i)$

Lab *j* ⇨

Lab *i* ⇩	NMIJ		KRISS		CSIRO-NML		PTB		CMS/ITRI		SIRIM	
	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}
	/ pC/(m/s²)x10^{-4}		/ pC/(m/s²)x10^{-4}		/ pC/(m/s²)x10^{-4}		/ pC/(m/s²)x10^{-4}		/ pC/(m/s²)x10^{-4}		/ pC/(m/s²)x10^{-4}	
NMIJ	**-0.862**	6.653	**0.028**	6.577	**-2.514**	9.315	**-0.226**	5.517	**4.223**	14.502	**-0.608**	6.772
KRISS	**-1.962**	5.828	**-1.072**	5.742	**-3.614**	8.745	**-1.326**	4.489	**3.123**	14.142	**-1.708**	5.964
CSIRO-NML	**-0.862**	5.259	**0.028**	5.163	**-2.514**	8.376	**-0.226**	3.720	**4.223**	13.917	**-0.608**	5.409
PTB	**-0.462**	3.852	**0.428**	3.720	**-2.114**	7.572	**0.174**	1.008	**4.623**	13.449	**-0.208**	4.054
BNM-CESTA	**0.138**	7.305	**1.028**	7.236	**-1.514**	9.791	**0.774**	6.288	**5.223**	14.812	**0.393**	7.414
CMI	**-0.862**	6.866	**0.028**	6.793	**-2.514**	9.468	**-0.226**	5.773	**4.223**	14.601	**-0.608**	6.982
CSIR-NML	**-0.862**	7.301	**0.028**	7.232	**-2.514**	9.788	**-0.226**	6.283	**4.223**	14.810	**-0.608**	7.409
CENAM	**-0.862**	7.301	**0.028**	7.232	**-2.514**	9.788	**-0.226**	6.283	**4.223**	14.810	**-0.608**	7.409
NRC	**0.638**	5.262	**1.528**	5.166	**-1.014**	8.378	**1.274**	3.724	**5.723**	13.919	**0.893**	5.412
VNIIM	**1.338**	6.242	**2.228**	6.162	**-0.314**	9.026	**1.974**	5.015	**6.423**	14.318	**1.593**	6.369
NIST	**-1.862**	5.257	**-0.972**	5.161	**-3.514**	8.375	**-1.226**	3.717	**3.223**	13.917	**-1.608**	5.407
NMi-VSL	**-0.862**	5.636	**0.028**	5.546	**-2.514**	8.618	**-0.226**	4.236	**4.223**	14.064	**-0.608**	5.775

Table 3: Matrix of equivalence determined by the differences $D_{ij} = d_{ij}$ and by the uncertainties $U_{ij} = 2u(d_{ij})$ associated with these differences where laboratory *i* is in the CCAUV comparison and laboratory *j* in the RMO comparison

Figure 2: Degrees of equivalence determined with the key comparison reference value for the CCAUV.V-K1 comparison and the linked RMO comparison APMP.AUV.V-K1



The matrix in Table 3 shows the degree of equivalence between any pair of laboratories. In this matrix laboratory *i* is a laboratory in the CIPM comparison and laboratory *j* is a laboratory in the RMO comparison. The results for the linking laboratories are shown as both *i* and *j* to analyse and demonstrate the consistency (long-term stability and reproducibility) of the linking laboratories' results (i.e. the calibration and measurement capabilities) over the period of both the RMO and CIPM comparison. The agreement shown illustrates the robustness of the link.

The results of the CCAUV key comparison that are already published are more recent than the RMO comparison and consequently, the four linking laboratories already have results in the KCDB [3]. This leaves the two APMP participants, the SIRIM and the CMS/ITRI to be linked into the KCDB Appendix B results. This has been realized using the data in Table 2. The linking coefficient calculated in accordance with (3) is 0.12710 (u = 0.06 %) pC/mV. The degrees of equivalence with the KCRV are shown in Figure 2 where the results from the two laboratories to be added from the APMP comparison are indicated with green triangles.

4. Discussion

Having transformed the results of the two laboratories, CMS/ITRI and SIRIM, which participated in the RMO comparison APMP.AUV.V-K1 only, the degrees of equivalence with respect to the KCRV as well as the matrix of equivalence between the participants of the RMO comparison and all participants of the CIPM comparison CCAUV.V-K1 were computed. In addition to the two laboratories which participated in the RMO comparison only, the four linking laboratories were included in the presentation of the degrees of equivalence (Figure 1, Table 2 and Table 3), with respect to their results obtained in the RMO comparison. The latter approach was to analyse and demonstrate the consistency (long-term stability, reproducibility) of the linking laboratories' results (i.e. the calibration and measurement capabilities) over the period of both the RMO and CIPM comparison.

As can be seen from Figure 1 and Tables 2 and 3, for all six laboratories participating in the RMO comparison, the deviations D_i from the reference value and D_{ij} between all laboratories which participated in the CIPM comparison were well below the respective expanded uncertainties U_i, U_{ij} calculated for the coverage factor k = 2.

5. Conclusions

The linking problem consists of relating the result of a laboratory that has participated only in the RMO comparison to the results in the CIPM comparison.

This has been achieved by computing a linking coefficient in terms of the ratio of the KCRV and the weighted mean in the RMO comparison of the four laboratories that also participated in the CCAUV key comparison. This linking coefficient has been used to convert the RMO results to CIPM equivalent results.
The analysis and the results presented in this report provide a robust link between the results of the RMO key comparison APMP.AUV.V-K1 and the CIPM key comparison CCAUV.V-K1. The matrix of degrees of equivalence for both

comparisons has been approved by the CCAUV for publication in the key comparison database.

References

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[2] Shin Chen, von Martens H.-J., 2001, APMP.AUV.V-K1 Report, APMP.AUV.V-K1 Regional key comparison of standard accelerometer Final Report, 87 pages.

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[4] Elster C., Link A., Wöger W., Proposal for linking the results of CIPM and RMO key comparisons, 2003, *Metrologia*, **40**, 189-194.

[5] MRA: *Mutual recognition of national measurement standards and of calibration and measurement certificates issued by national metrology institutes*, International Committee for Weights and Measures, 1999, 45 pp. http://www.bipm.org/pdf/mra.pdf.