

## Link between EUROMET.EM.M-K1 and CCEM.M-K1

### *Measurement of the dc coil constant*

Three institutes<sup>1</sup>, CMI, NPL and PTB, applying NMR methods and thus providing the lowest uncertainty budgets, participated in both comparisons. Based on their results obtained in the particular comparison, the ratios  $F_{r,I} = k_{dc,I,CCEM} / k_{dc,I,EUROM}$  can be established to relate to each other the values of the dc-coil constants of the different standards circulated within the particular comparison. By means of the same weighting procedure as used for the weighted mean of the coil constants  $k_{wm}$ , the mean value  $F_{r,wm}$  is determined from the ratios obtained individually by the three institutes, and thus the dc transfer factor becomes

$$F_{r,wm} = \sum_I F_{r,I} \cdot \frac{\frac{1}{u_{k,I,CCEM}} + \frac{1}{u_{k,I,EUROM}}}{\sum_I \frac{1}{u_{k,I,CCEM}} + \frac{1}{u_{k,I,EUROM}}}.$$

The  $1\sigma$ -uncertainty of  $F_{r,wm}$  was determined according to

$$u_{F_{r,wm}} = \sqrt{\text{DIF}_{\text{CCEM}}^2 + \text{DIF}_{\text{EUROM}}^2 + \sum_I \left( \frac{F_{r,I} - F_{r,wm}}{F_{r,wm}} \cdot \frac{\frac{1}{u_{k,I,CCEM}} + \frac{1}{u_{k,I,EUROM}}}{\sum_I \frac{1}{u_{k,I,CCEM}} + \frac{1}{u_{k,I,EUROM}}} \right)^2},$$

where the DIF are the relative deviations of the weighted mean of the values reported by the three institutes from the  $k_{wm}$  value reported in the particular comparison. The final result with the standard deviation expanded by  $k=2$  is

$$F_{r,wm} = \mathbf{1.000\ 186_1 \pm 0.000\ 017_8}$$

by which the individual  $k_{dc}$  values obtained in the EUROMET comparison can be transferred to the CCEM comparison.

Fig. A1 presents the results reported by the institutes which participated in the CCEM comparison as well as the results obtained in the EUROMET comparison and transferred by applying  $F_{r,wm}$ .

The error bars of the EUROMET values are with respect to  $u_{F_{r,wm}}$ .

All results reported by the participants in the EUROMET comparison cover within their uncertainties the mean value  $k_{wm}$  of the CCEM comparison.

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<sup>1</sup> The results of IEN were not used for the link between the two comparisons, because they are not based on NMR measurements and therefore exhibit a larger uncertainty and a much lower weight.

### ***Measurement of the coil constants dependence on frequency***

Since the frequency response of the standard coil used in the CCEM comparison was adjusted so that it differed from the response of the standard used in the EUROMET comparison, it is of little importance to relate the results to each other.

Nevertheless, for each comparison the values of the quotient  $Q_I(f) = (k(f)/k_{dc})_I$  over a frequency range up to 20 kHz<sup>2</sup> were reported by the participants (I). Four institutes took place in both comparisons - NPL, IEN, CMI and PTB. Again, based on the results  $Q_I(f)$ , obtained in the particular comparison, the ratios  $F_{ac,I}(f) = Q_{I,CCEM}(f) / Q_{I,EUROMET}(f)$  were established to evaluate a transfer function  $F_{ac,fit}(f)$  by means of partial polynomial least square adjustment of all values  $F_{ac,I}(f)$ . The upper part of Fig. A2 shows the values  $F_{ac,I}(f)$  and the transfer function  $F_{ac,fit}(f)$  together with the lines representing its relative combined uncertainty  $u_{Fac,fit}$  expanded by  $k=2$ , which was evaluated by polynomial fitting of the relative uncertainty of all individual values  $F_{ac,I}(f)$  according to

$$u_{Fac,I} = \sqrt{[F_{ac,I}(f) - F_{ac,fit}(f)]^2 + u_{Q(f),I,CCEM}^2 + u_{Q(f),I,EUROMET}^2}.$$

The resulting uncertainty of the function  $F_{ac,fit}(f)$  as well as the  $u_{Fac,I}$  values are shown in the lower part of Fig. A2.

The frequency response of the standard coil used in the EUROMET comparison has been reported as the polynomial of second order

$$P(f)_{EUROMET} = 1 + 8,84 \cdot 10^{-10} \cdot f/\text{Hz} + 6 \cdot 10^{-11} \cdot (f/\text{Hz})^2.$$

By multiplying this function with the transfer function  $F_{ac,fit}(f)$  the EUROMET result agrees very well with the CCEM result as shown in Fig. A3 by the grey plotted curve. The  $2\sigma$  lines represent again the expanded uncertainty of the frequency response as obtained in the CCEM comparison

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<sup>2</sup> In the EUROMET comparison only one laboratory made measurements at 20 kHz. Therefore, 5 kHz was chosen as the upper frequency for the tables with the degrees of equivalence.

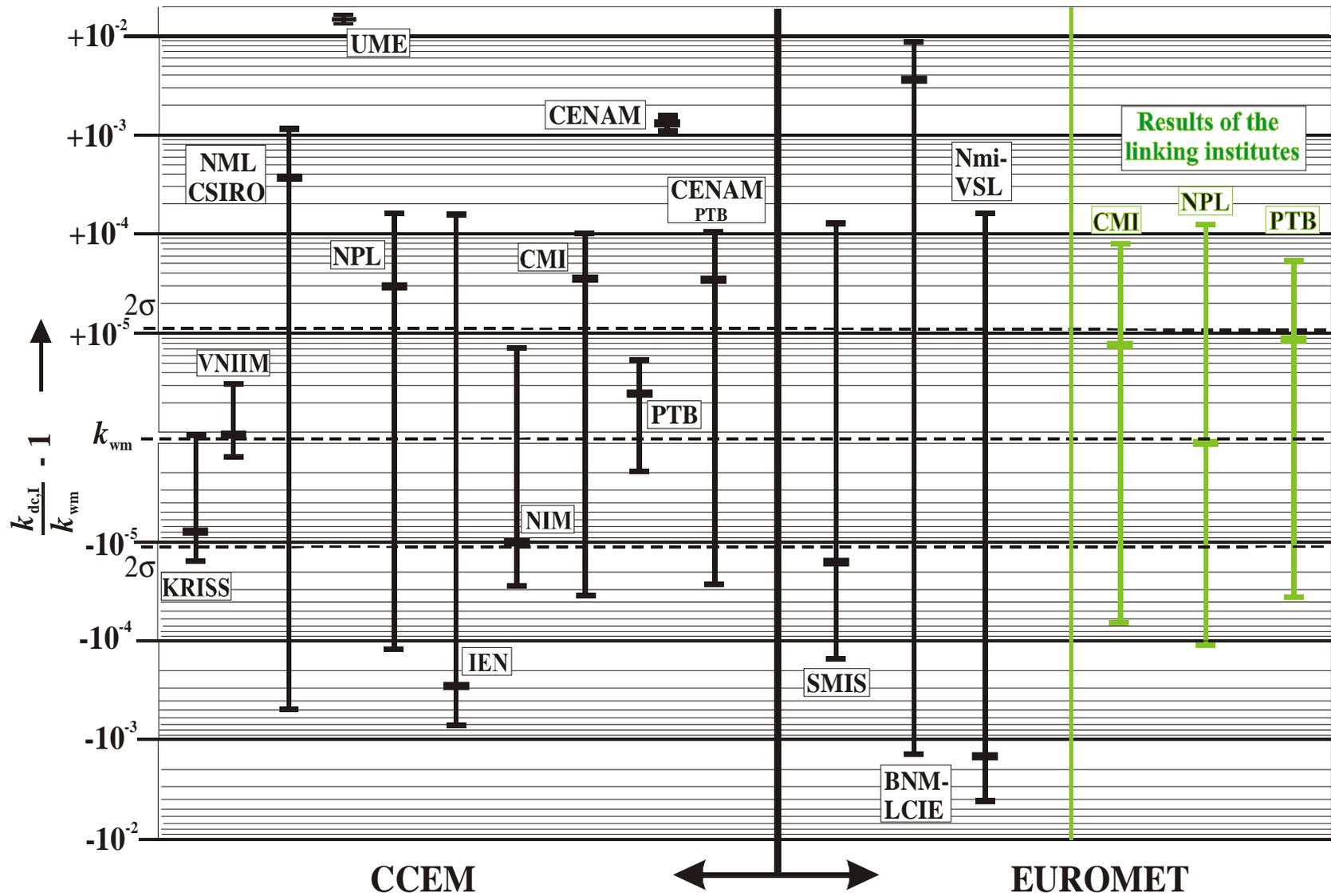


Fig. A1 DC coil constant determination obtained by the various institutes participating in the CCEM and in the EUROMET comparison, whereby the values reported within EUROMET are transferred by multiplying with  $F_{r,wm}$ . The  $2\sigma$  lines represent the uncertainty of the weighted mean  $k_{wm}$  of the CCEM comparison.

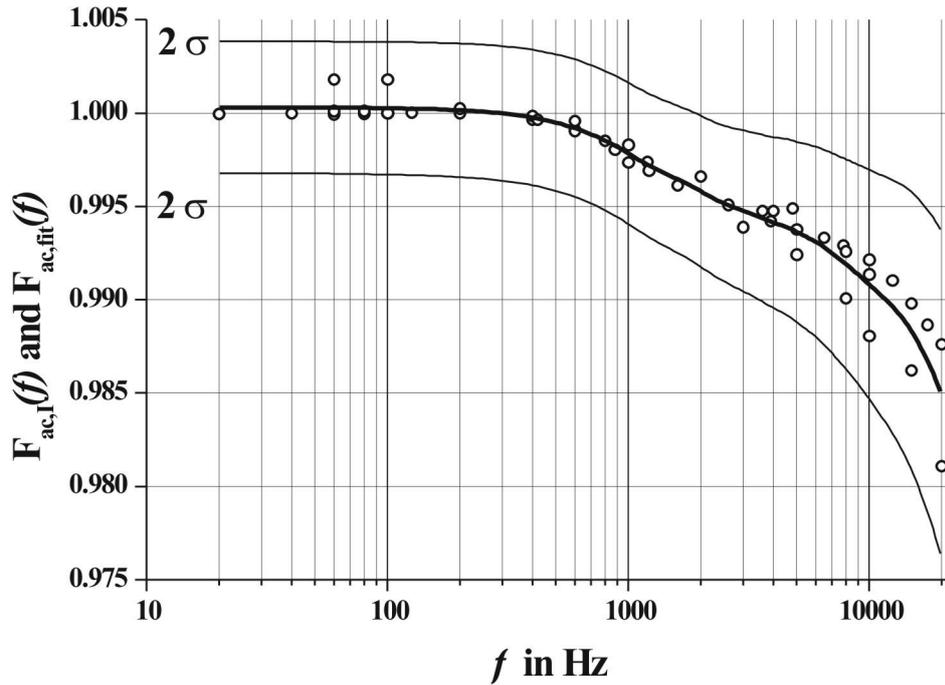
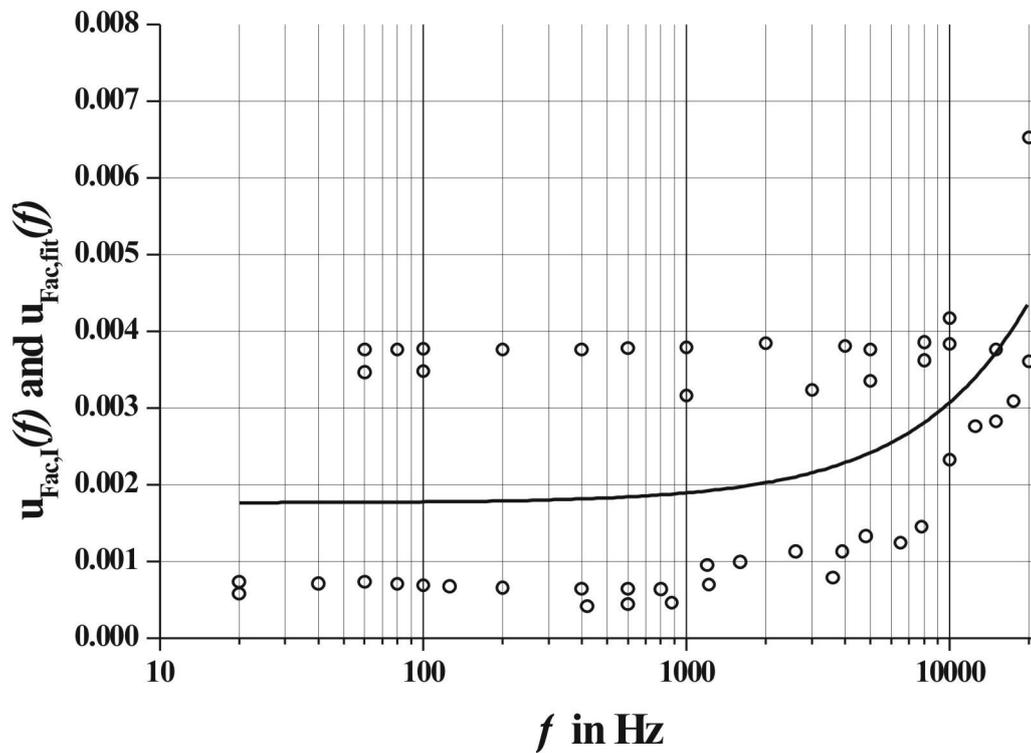


Fig. A2

Upper diagram: frequency dependence of the transfer factor  $F_{ac,fit}(f)$  as evaluated from the individual values  $F_{ac,I}(f) = Q_{I,CCEM}(f) / Q_{I,EUROMET}(f)$  by means of partial polynomial fitting.

Lower part:  $1\sigma$  uncertainty of the transfer function  $F_{ac,fit}(f)$  evaluated from the individual uncertainties of the  $Q_{I,CCEM}(f) / Q_{I,EUROMET}(f)$  values reported by NPL, IEN, CMI and PTB.



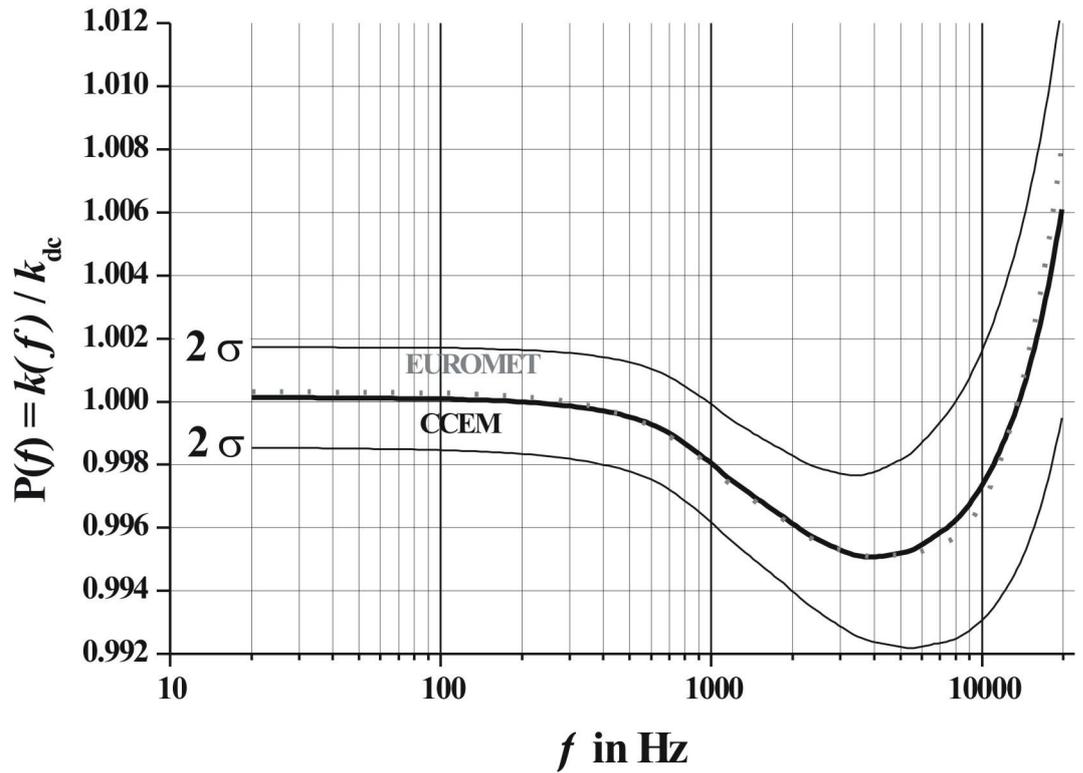


Fig. A3 Frequency dependence of the coil constant as measured by all institutes participating in the CCEM comparison - black line, and by multiplying the polynomial fit curve obtained as the final result in the EUROMET comparison with the transfer function  $F_{ac,fit}(f)$  - grey line; the  $2\sigma$  lines are according to these obtained in the CCEM comparison.