

Linking the Results of Key Comparison CCEM-K4 with the 10 pF Results of EUROMET Project 345

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Introduction

A practical problem in implementing the Mutual Recognition Arrangement (MRA) is to establish the link between the results of comparisons carried out under the auspices of the Comité International des Poids et Mesures (CIPM) and those carried out by the regional metrology organizations (RMOs). One of the very first examples is the link between the comparisons of 10 pF capacitance standards carried out by the Consultative Committee for Electricity and Magnetism (CCEM) from 1996 to 1999 (CCEM-K4) and by the European Metrology Cooperation (EUROMET) from 1995 to 1998 (EUROMET Project 345).

The input data for the proposed link is taken from [1] for CCEM-K4 and from [2] for EUROMET 345. For the latter, the results are calculated from [2], Table 7, using the difference “Reported value” – “Reference value” as the EUROMET comparison result. In three cases, two values are reported in [2] and we use the mean of the two values as the final value. Results and uncertainties are given in parts in 10^6 of the nominal 10 pF capacitance. Seven laboratories participated in both comparisons. Each was asked if its results should be used to link the comparison results and, if so, to provide a $1-\sigma$ estimate of the uncertainty corresponding to the imperfect *reproducibility* ([3], definition 3.7) of its measurements during the time including its measurements for the two comparisons. One of these participants, the BNM-LCIE¹, asked that its results not be used for linking the comparison results.

Model

The key comparison reference value (KCRV), derived from CCEM-K4, is used as the reference value. Results and uncertainties from CCEM-K4 are unaltered by the linking procedure. The following notation is used:

D_c : result from CCEM-K4 for a linking laboratory;

D'_c : result from EUROMET project 345 for a linking laboratory;

D'_e : result from a laboratory participating in EUROMET project 345 only.

The linking problem consists of evaluating the correction d to apply to D'_e so that the corrected result D_e represents the best estimate of what would have been the result from laboratory e had it actually participated in CCEM-K4, so

$$D_e = D'_e + d.$$

The quantity $d_c = D_c - D'_c$ provides an estimate of $d = D_e - D'_e$ because we assume that any possible bias in the results of laboratory c remains reasonably constant over the time period. Because of wide variations in uncertainty among linking laboratories, d is calculated as a weighted mean of the estimates of d from the C (six) linking laboratories:

$$d = \sum_{c=1}^C w_c \times d_c.$$

¹ BNM-LNE is now the denomination of the French laboratory responsible for these measurements.

The weight w_c is calculated from

$$w_c = (1/s_c^2) / \sum_c 1/s_c^2$$

where s_c , the uncertainty associated with d_c , is given by $s_c^2 = t_c^2 + t'_c{}^2 + 2r_c^2$ and where t_c is the transfer uncertainty in the CCEM comparison, t'_c is the transfer uncertainty in the EUROMET comparison, and r_c is the uncertainty associated with the imperfect reproducibility of the results of laboratory c in the time period spanning its two measurements (whence the factor of 2) in the CCEM-K4 and EUROMET comparisons. Values for r_c are those provided by the participants in March and April 2001 except for that of the CSIRO-NML which was submitted in May following a discussion of the meaning of “reproducibility” in this case.

Results

Table 1 lists the elements entering into the calculation of d . The result is $d = 0.007 \times 10^{-6}$, with a *standard deviation* of 0.0202×10^{-6} , calculated from $1/\text{var}(d) = \sum_c [1/\text{var}(d_c)]$. This is the standard deviation calculated on the basis of internal consistency. A second way of estimating the variance of d is to use $\text{var}_{\text{ex}}(d) = [\sum_c w_c \times (d_c - d)^2] / (C - 1)$, the variance based on external consistency. The value of the standard deviation based on external consistency is 0.0186×10^{-6} . The Birge ratio [4] (pp. 430-431) is the quotient of the standard deviation based on external consistency divided by the standard deviation based on internal consistency. Here its value is 0.9, which, being close to unity, indicates good consistency between the variations from laboratory to laboratory and the uncertainties estimated by the laboratories. Alternatively, one can test the null hypothesis H_0 : “there is no significant difference between the observed variance, $\text{var}_{\text{ex}}(d)$, and the variance deduced using the laboratories’ reproducibility estimates”. The value of $\chi^2 = (C-1) \text{var}_{\text{ex}}(d) / \text{var}(d) = 5 \times (0.0186 \times 10^{-6} / 0.0202 \times 10^{-6})^2 = 4.24$. From tables of χ^2 , H_0 is rejected at the 95 % confidence level if $\chi^2(5, 0.05) > 11.07$. This is not the case, so H_0 is not rejected.

Table 2 lists the elements entering into the calculation of D_e and its uncertainty σ_e for $k = 2$. The four uncertainty components are the uncertainty of the reference value in CCEM-K4, the uncertainty of d , the transfer uncertainty in the EUROMET comparison, and the laboratory measurement uncertainty in that comparison. Table 3 serves as an intermediate step in calculating uncertainties in the degrees of equivalence between pairs of laboratories. In Table 3, u_E and u_{K4} are the uncertainties reported by the laboratories in the two comparisons, respectively, and include neither the uncertainties in the reference values nor the uncertainties associated with transportation. Four categories are formed: (1) laboratories that participated in the EUROMET comparison, including linking laboratories; (2) laboratories that participated only in the CCEM comparison; (3) laboratories referenced to the quantized Hall resistance using R_{K-90} and for which (and only for these laboratories) a $k = 2$ uncertainty of 0.044×10^{-6} is added in quadrature to account for the uncertainty of R_{K-90} with respect to R_K (following [1] this was estimated as being twice the difference between R_{K-90} and the 1998 CODATA value for R_K); and (4) laboratories that participated more than once in the EUROMET comparison and for whom the transfer uncertainty, t_E , is assumed to be reduced by the square root of the number of times the laboratory participated.

Table 4 shows the proposed table of bilateral equivalences, with uncertainties for $k = 2$, between any laboratory i amongst the 21 participating laboratories and any laboratory j amongst the 10 laboratories who participated in EUROMET-345 only. The elements entering into the uncertainty in the degree of equivalence for any pair of NMIs take into account the categories to which each NMI belongs. Notice that the D_{ij} are calculated differently depending on whether or not both laboratories participated to the EUROMET comparison.

For example, the degree of equivalence between the BEV and the BIPM, a pair of laboratories who participated in the EUROMET comparison, is calculated by subtracting the BEV result with respect to the reference value in EUROMET project 345, Table 3 column 2, 0.400, from the BIPM result with respect to the reference value in EUROMET project 345, Table 3 column 2, 0.022; $0.022 - 0.400 = -0.378$, which rounds to -0.38 . The uncertainty for the BEV and that for the BIPM are the quadrature sums of the elements in the corresponding row of Table 3, columns 5, 6 and 7, i.e. the total uncertainties in the EUROMET comparison. (A convenient notation for the quadrature sum of n uncertainties v_1, v_2, \dots, v_n is

$Q(v_1, v_2, \dots, v_n) = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$.) The quadrature sum of the uncertainties associated with the BIPM and the BEV is the uncertainty in the degree of equivalence, 1.41.

A second example that illustrates the calculation of the value and uncertainty in the degree of equivalence between an NMI that participated only in the EUROMET comparison with an NMI that participated only in the CCEM comparison is the pair BEV and MSL. The entry in column 2 of Table 4 for this pair of laboratories is calculated by subtracting the BEV result referenced to the KCRV from Table 3, column 3 and from the MSL result obtained in the CCEM comparison, Table 3, column 4; $-0.026 - 0.407 = -0.43$. The uncertainty of the MSL value is the quadrature sum of the uncertainty reported by the MSL for the CCEM comparison and the transfer uncertainty for the CCEM comparison. This quadrature sum is given in Table 3, column 13. This, added in quadrature with the uncertainty for the BEV given in Table 3, column 8 and with the uncertainty in the difference d given in Table 3 column 9, gives the uncertainty in the degree of equivalence.

Finally the data in Tables 2 and 4 are used to complete the previously published matrix of equivalence and uncertainties for CCEM-K4 which, of course will not be changed by the EUROMET project 345 results. Table 5 shows this full completed matrix for the 21 participating laboratories. The degrees of equivalence with respect to the KCRV from CCEM-K4 (D_i and U_i in Table 5) are also shown under the form of a graph (Figure 1).

Conclusion

The proposed procedure has been accepted by the CCEM. It is not claimed to be a general one which should be followed each time a link between comparisons is established. In other circumstances one may adopt different procedures, in particular when it is desirable to reevaluate and hopefully improve previously published degrees of equivalence. Also, a full matrix of equivalence (such as Table 5) may become impractical to establish if more laboratories or quantities are involved. In such cases a better alternative would be to publish only the degrees of equivalence with respect to the KCRV.

References

- [1] A.-M. Jeffery, *Final Report CCEM Comparison of 10 pF Capacitance Standards*, May 2000-Revised March 2002, <http://www.bipm.org/kcdb>, Appendix B of the MRA.
- [2] J. H. Belliss, *Euromet Project reference No. 345, Intercomparison of 10 pF and 100 pF Capacitance Standards*, NPL Report CEM 7, 1999.
- [3] *International Vocabulary of Basic and General Terms in Metrology*, ISO, Geneva, 1993.
- [4] B.N Taylor, W.H. Parker, D.N. Langenberg, *Determination of e/h , using macroscopic quantum phase coherence in superconductors: implications for quantum electrodynamics and the fundamental physical constants*, *Rev. Mod. Phys*, 41, pp. 375-496, 1969.

Table 1. Values and uncertainties, for $k=2$, associated with the linking laboratories and the resulting weights.

lab	$D_c/10^{-6}$	$D'_c/10^{-6}$	$d_c/10^{-6}$	$t_c/10^{-6}$	$t'_c/10^{-6}$	$r_c/10^{-6}$	$s_c/10^{-6}$	w_c
BIPM	-0.018	0.022	-0.040	0.040	0.080	0.050	0.114	0.125
CSIRO-NML	0.035	0.009	0.026	0.040	0.08	0.010	0.090	0.199
NIST	-0.003	0.005	-0.008	0.020	0.056	0.006	0.060	0.447
NMi	-0.772	-0.570	-0.202	0.040	0.080	0.400	0.572	0.005
NPL	0.198	0.090	0.108	0.040	0.020	0.084	0.126	0.103
PTB	-0.004	-0.010	0.006	0.040	0.056	0.066	0.116	0.121

Table 2. Values, component uncertainties, for $k=2$, and total uncertainties for NMIs that participated only in EUROMET project 345. In bold are the values and uncertainties for the results for these NMIs referenced to the (CCEM) KCRV.

lab	$D'_c/10^{-6}$	$D_c/10^{-6}$	$s_{\text{ref}}/10^{-6}$	$s_d/10^{-6}$	$t'_c/10^{-6}$	$\sigma'_c/10^{-6}$	$\sigma_c/10^{-6}$
BEV	0.400	0.407	0.034	0.040	0.080	1.400	1.404
CEM	-0.020	-0.013	0.034	0.040	0.080	3.000	3.002
CMI	-0.250	-0.243	0.034	0.040	0.080	0.400	0.412
CSIR	0.320	0.327	0.034	0.040	0.080	2.500	2.502
GUM	-0.400	-0.393	0.034	0.040	0.080	0.800	0.806
IEN	0.310	0.317	0.034	0.040	0.080	0.800	0.806
METAS	-0.200	-0.193	0.034	0.040	0.080	62.000	62.000
MIKES/VTT	-0.920	-0.913	0.034	0.040	0.080	1.500	1.504
SP	-0.600	-0.593	0.034	0.040	0.080	1.800	1.802
UME	0.320	0.327	0.034	0.040	0.080	1.600	1.602

Table 3. Values from EUROMET Project 345 referenced to the EUROMET comparison, D' and referenced to the KCRV, D_e , values from CCEM-K4 and the associated uncertainty components for $k = 2$. NMIs listed in bold letters participated in EUROMET project 345.

	$D'/10^{-6}$	$D_e/10^{-6}$	$D_{\text{CCEM}}/10^{-6}$	$u_E/10^{-6}$	$u_{R_{K-90}}/10^{-6}$	$t_E/10^{-6}$	$u_{\text{CE}}/10^{-6} = Q(u_E, u_{R_{K-90}}, t_E)/10^{-6}$	$u_d/10^{-6}$	$Q(u_{\text{CE}}, u_d)/10^{-6}$	$u_{\text{K4}}/10^{-6}$	$t_{\text{K4}}/10^{-6}$	$Q(u_{\text{K4}}, t_{\text{K4}})/10^{-6}$
BEV	0.400	0.407		1.4		0.080	1.402	0.040	1.403			
BIPM	0.022			0.074	0.044	0.080	0.118					
BNM-LNE	0.110			0.052	0.044	0.080	0.105					
CEM	-0.020	-0.013		3		0.080	3.001	0.040	3.001			
CMI	-0.250	-0.243		0.4		0.080	0.408	0.040	0.410			
CSIR	0.320	0.327		2.5		0.080	2.501	0.040	2.502			
CSIRO-NML	0.009			0.078		0.080	0.112					
GUM	-0.400	-0.393		0.8		0.080	0.804	0.040	0.805			
IEN	0.310	0.317		0.8		0.080	0.804	0.040	0.805			
MSL			-0.026							0.12	0.04	0.13
NIM			-0.04							0.26	0.04	0.26
NIST	0.005			0.04		0.057	0.069					
NMi	-0.570			1.2		0.080	1.203					
NPL	0.090			0.1	0.044	0.020	0.111					
NRC			0.037							0.32	0.04	0.32
METAS	-0.200	-0.193		62		0.080	62.000	0.040	62.000			
MIKES/VTT	-0.920	-0.913		1.5		0.080	1.502	0.040	1.503			
PTB	-0.010			0.1		0.057	0.115					
SP	-0.600	-0.593		1.8		0.080	1.802	0.040	1.802			
UME	0.320	0.327		1.6		0.080	1.602	0.040	1.603			
VNIIM			-0.318							0.40	0.04	0.40

Table 4. Degrees of equivalence (D_{ij} and its uncertainty for $k = 2$, U_{ij}) deduced from linking CCEM-K4 and EUROMET Project 345.

Lab i	Lab j																			
	BEV		CEM		CMI		CSIR		GUM		IEN		METAS		MIKES/VTT		SP		UME	
	D_{ij} / 10^{-6}	U_{ij} / 10^{-6}	D_{ij} / 10^{-6}	U_{ij} / 10^{-6}	D_{ij} / 10^{-6}	U_{ij} / 10^{-6}	D_{ij} / 10^{-6}	U_{ij} / 10^{-6}	D_{ij} / 10^{-6}	U_{ij} / 10^{-6}	D_{ij} / 10^{-6}	U_{ij} / 10^{-6}	D_{ij} / 10^{-6}	U_{ij} / 10^{-6}	D_{ij} / 10^{-6}	U_{ij} / 10^{-6}	D_{ij} / 10^{-6}	U_{ij} / 10^{-6}	D_{ij} / 10^{-6}	U_{ij} / 10^{-6}
BEV			0.42	3.31	0.65	1.46	0.08	2.87	0.80	1.62	0.09	1.62	0.60	62.02	1.32	2.05	1.00	2.28	0.08	2.13
BIPM	-0.38	1.41	0.04	3.00	0.27	0.42	-0.30	2.50	0.42	0.81	-0.29	0.81	0.22	62.00	0.94	1.51	0.62	1.81	-0.30	1.61
BNM-LNE	-0.29	1.41	0.13	3.00	0.36	0.42	-0.21	2.50	0.51	0.81	-0.20	0.81	0.31	62.00	1.03	1.51	0.71	1.80	-0.21	1.61
CEM	-0.42	3.31			0.23	3.03	-0.34	3.91	0.38	3.11	-0.33	3.11	0.18	62.07	0.90	3.36	0.58	3.50	-0.34	3.40
CMI	-0.65	1.46	-0.23	3.03			-0.57	2.53	0.15	0.90	-0.56	0.90	-0.05	62.00	0.67	1.56	0.35	1.85	-0.57	1.65
CSIR	-0.08	2.87	0.34	3.91	0.57	2.53			0.72	2.63	0.01	2.63	0.52	62.05	1.24	2.92	0.92	3.08	0.00	2.97
CSIRO-NML	-0.39	1.41	0.03	3.00	0.26	0.42	-0.31	2.50	0.41	0.81	-0.30	0.81	0.21	62.00	0.93	1.51	0.61	1.81	-0.31	1.61
GUM	-0.80	1.62	-0.38	3.11	-0.15	0.90	-0.72	2.63			-0.71	1.14	-0.20	62.01	0.52	1.70	0.20	1.97	-0.72	1.79
IEN	-0.09	1.62	0.33	3.11	0.56	0.90	-0.01	2.63	0.71	1.14			0.51	62.01	1.23	1.70	0.91	1.97	-0.01	1.79
MSL	-0.43	1.41	-0.01	3.00	0.22	0.43	-0.35	2.50	0.37	0.82	-0.34	0.82	0.17	62.00	0.89	1.51	0.57	1.81	-0.35	1.61
NIM	-0.45	1.43	-0.03	3.01	0.20	0.49	-0.37	2.52	0.35	0.85	-0.36	0.85	0.15	62.00	0.87	1.53	0.55	1.82	-0.37	1.62
NIST	-0.40	1.40	0.03	3.00	0.26	0.41	-0.32	2.50	0.41	0.81	-0.31	0.81	0.21	62.00	0.93	1.50	0.61	1.80	-0.32	1.60
NMi	-0.97	1.85	-0.55	3.23	-0.32	1.27	-0.89	2.78	-0.17	1.45	-0.88	1.45	-0.37	62.01	0.35	1.92	0.03	2.17	-0.89	2.00
NPL	-0.31	1.41	0.11	3.00	0.34	0.42	-0.23	2.50	0.49	0.81	-0.22	0.81	0.29	62.00	1.01	1.51	0.69	1.81	-0.23	1.61
NRC	-0.37	1.44	0.05	3.02	0.28	0.52	-0.29	2.52	0.43	0.87	-0.28	0.87	0.23	62.00	0.95	1.54	0.63	1.83	-0.29	1.63
METAS	-0.60	62.02	-0.18	62.07	0.05	62.00	-0.52	62.05	0.20	62.01	-0.51	62.01			0.72	62.02	0.40	62.03	-0.52	62.02
MIKES/VTT	-1.32	2.05	-0.90	3.36	-0.67	1.56	-1.24	2.92	-0.52	1.70	-1.23	1.70	-0.72	62.02			-0.32	2.35	-1.24	2.20
PTB	-0.41	1.41	0.01		0.24	0.42	-0.33	2.50	0.39	0.81	-0.32	0.81	0.19	62.00	0.91	1.51	0.59	1.81	-0.33	1.61
SP	-1.00	2.28	-0.58	3.50	-0.35	1.85	-0.92	3.08	-0.20	1.97	-0.91	1.97	-0.40	62.03	0.32	2.35			-0.92	2.41
UME	-0.08	2.13	0.34	3.40	0.57	1.65	0.00	2.97	0.72	1.79	0.01	1.79	0.52	62.02	1.24	2.20	0.92	2.41		
VNIIM	-0.73	1.46	-0.31	3.03	-0.08	0.57	-0.65	2.53	0.07	0.90	-0.64	0.90	-0.13	62.00	0.59	1.56	0.27	1.85	-0.65	1.65

Fig.1: CCEM-K4 and EUROMET-345: capacitance at nominal value 10 pF
 Degrees of equivalence [D_i and its expanded uncertainty ($k = 2$), U_i]

