Final Report

CCEM Comparison of 10 pF Capacitance Standards

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Abstract

Comparison of electrical standards must be carried out periodically between the National Metrology Institutes (NMIs) to establish the relationship between their practical electrical units. This comparison of 10 pF capacitors was initiated by the Comité Consultatif d'Électricité et Magnétisme, to establish the relationship between the practical capacitance units of the NMIs from several of the metrology regions. The results of this comparison are described.

1. Introduction

A mutual recognition arrangement (MRA) between nations has been drawn up by the Comité International des Poids et Mesures (CIPM) and signed by the directors of the national metrology institutes (NMIs). The CIPM consists of individuals elected by the Conférence Genérale des Poids et Mesures (CGPM), which consists of delegates from the member states that have signed the Convention du Mètre. The MRA will facilitate trade between nations and will be based on a continuing set of key comparisons. The key comparisons for each metrology area will be organized by the Consultative Committees of the CIPM and the Bureau International des Poids et Mesures (BIPM). The Comité Consultatif d'Électricité et Magnétisme (CCEM) is the committee responsible for electrical measurements and the comparison of 10 pF capacitors is one of the areas that was selected for a key comparison.

The most recent CCEM comparison of 10 pF capacitors began in 1996 and lasted 3 years. Invitations to participate were sent to participants at the 1995 CCEM meeting. So that the comparison could be completed in a reasonable time, only a few laboratories were selected from each metrology region. Each region will then perform their own comparison, thereby establishing links with a much larger number of NMIs. The key comparisons will test the principal techniques in each field of metrology and check the uncertainty of independent primary realizations of the units of the SI. With this in mind, preference was given to laboratories with an independent realization of the farad.

2. Participants

There were 11 participants from 4 regions, as listed in Table 1. Over the course of the comparison, one participant withdrew. The pilot laboratory was the National Institute of Standards and Technology (NIST). NIST was responsible for providing and preparing the standards and the coordination of the schedule. NIST was also responsible for collecting and analyzing the comparison data and preparing the draft reports.

Table 1. List of participants.

Laboratory	Country	Region
NIST- National Institute of Standards and Technology - Pilot	USA	SIM
BIPM- Bureau International des Poids et Mesures	International	-
BNM-LCIE – Bureau National de Métrologie, Laboratoire Central des Industries Électriques	France	EUROMET
CSIRO-NML –Commonwealth Scientific and Industrial Research Organization – National Measurement Laboratory	Australia	APMP
MSL – Measurement Standards Laboratory	New Zealand	APMP
NIM – National Institute of Metrology	China	APMP
NMi - Nederlands Meetinstituut	Netherlands	EUROMET
NPL – National Physical Laboratory	UK	EUROMET
NRC – National Research Council	Canada	SIM
PTB – Physikalisch-Technische Bundesanstalt	Germany	EUROMET/ COOMET
VNIIM – D. I. Mendeleyev Institute for Metrology	Russia	COOMET

3. Capacitance Standards

The traveling standards are 10 pF fused silica dielectric capacitors in hermetically sealed dry nitrogen filled metal containers with British Post Office (BPO) connectors. The capacitors were made at NIST and are described in Ref. [1]. The capacitors have a large temperature coefficient and require immersion in an oil bath for temperature control. An air bath is also acceptable. The stability needed for the oil bath will depend on the uncertainty with which the capacitors are measured. The capacitors have temperature coefficients of approximately 10 (μF/F) /K so the oil bath should be stable to about 1 mK if the capacitance value is to be measured with a relative uncertainty of 0.01 μF/F.

A prescription of temperature cycling was developed to remove the effects of temperature hysteresis. In a previous comparison that employed these standards, large shifts in value were seen after transport. An investigation showed that these were due to temperature hysteresis and not mechanical shock. The temperature cycling consists of three cycles, each cycle being 25 °C to

50 °C and back to 25 °C. The capacitors are held at each temperature for approximately 48 hours. If it is necessary to do the cycling in a separate temperature bath than the one used for measurement, the transfer time between baths should be as short as possible. The cycling causes the capacitor values to drift and the capacitors should be allowed at least three weeks to settle down to a stable value (fractional fluctuations within 0.05 μF/F).

Two sets of two capacitance standards were used for the comparison. Capacitors with serial numbers S/N 108 and S/N 185 were sent to 8 of the 10 participants. Capacitor S/N 108 was part of an original set that was used for international comparisons and there is much history of its behavior. Capacitor S/N 185 was built in the late 1980's and monitored for several years. Capacitors S/N 190 and S/N 193 were introduced half way through the comparison to ease the tight schedule of the comparison and were sent to 2 out of the 10 participants. They were part of a set of capacitors completed in the early 1990s and were chosen because they displayed a predictable drift rate after temperature cycling. All capacitors were subjected to several sets of cycling and were monitored before they were sent out.

4. Measurement

4.1 Measurement parameters

Participants were asked to measure the capacitors at an applied voltage of 100 V and at a frequency of 1592 Hz. The capacitors were to be placed in a stable temperature bath at ≈ 25 °C for measurement and the 25 Ω resistance thermometer inside each capacitor measured to within 0.01 mΩ (corresponding to 0.1 mK) at a current of 1 mA. This measurement should be made close to the time of each capacitance measurement so each capacitance measurement may be corrected for temperature. Since the temperature cycling causes the capacitors to have a very high drift rate, the capacitors should be allowed 3 weeks to settle down before measurement. Figures depicting the capacitors' behavior after cycling were sent to the participants so that they would know what to expect.

The capacitance measurement should be corrected to the reference resistance using the resistance coefficient (coefficient of dependence of capacitance on resistance) and the resistance measurement. This ensures that the capacitance measurements are compared at the same reference resistance and that there are no differences due to differences in the measurement temperatures. The resistance coefficients and the reference resistances for the capacitors are given in Table 2. The reference resistances are somewhat arbitrarily chosen. Therefore, the difference between the resistance reading of the thermometer and its reference resistance will not be the same for each capacitance standard.

Table 2 Resistance coefficients and the reference resistances for the capacitors.

Capacitor S/N	Resistance Coefficient $\frac{1}{C}\frac{\partial C}{\partial R}$ ($\mu\Omega^{-1}$)	Reference Resistance (Ω)
108	106.00	25.76300
185	115.84	25.54600
190	120.83	25.23700
193	135.32	24.84300

4.2 Behavior of capacitors

Figures 1-4 show the NIST measurements of the capacitors during the comparison. Each cluster of data points are measurements made at NIST in between the times the capacitors were sent to the other participants for measurement. The cycling causes the capacitances to drift and the capacitors should be allowed at least three weeks to settle down to a stable and sufficiently small drift rate. The effect of the drift is removed by using data only from a specified time period after the cycling. It was shown that selection of such a time period after cycling gives consistent measurements after each temperature cycle. This behavior is illustrated in Figs. 1-4, where the circled points are data taken in a specified time period, which was usually a two-week period 3 or 4 weeks after the end of the temperature cycling. A linear fit to the circled points shows how well the fit predicts the capacitors behavior after cycling. In Figs. 1 - 4, typical relative differences between the data and the linear fit of the data are also shown. A linear fit to the before and after data from a specified time period is used to predict a NIST value on the mean measurement date for each participant. At NIST, measurements outside of the selected time period were made to study the behavior of the capacitors.

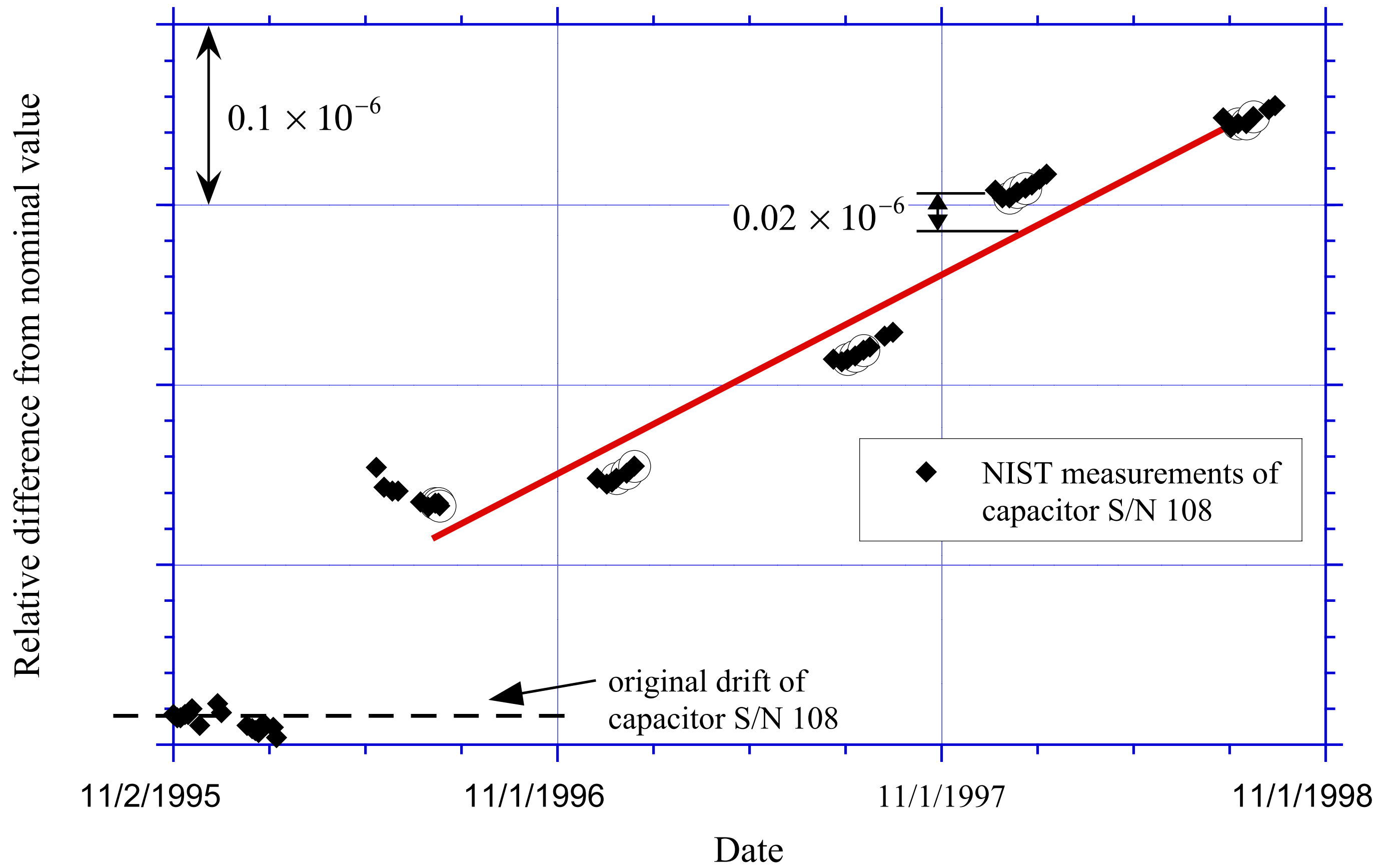

Figure 1. NIST measurements of capacitor S/N 108 (black diamonds) during the comparison. The solid line is a linear fit to the circled diamonds. These selected measurements are taken in a 2-week period starting 3 weeks after cycling. Indicated on the graph is the deviation of the measurements from the linear fit. The dashed line indicates the drift of the capacitor before the large shift in its value.

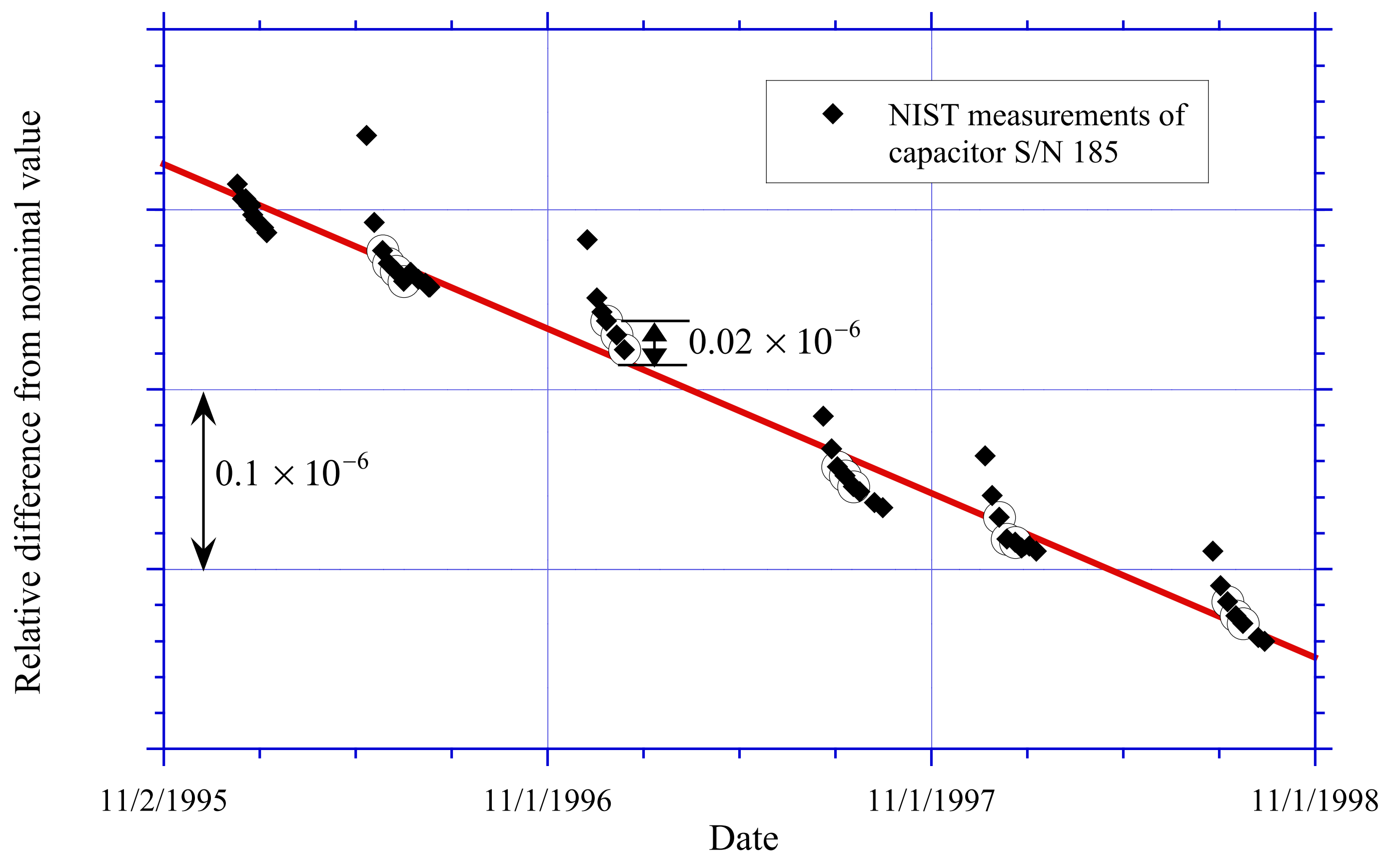

Figure 2. NIST measurements of capacitor S/N 185 (black diamonds) during the comparison. The solid line is a linear fit to the circled diamonds. These selected measurements are taken in a 2-week period starting 3 weeks after cycling. Indicated on the graph is the deviation of the measurements from the linear fit.

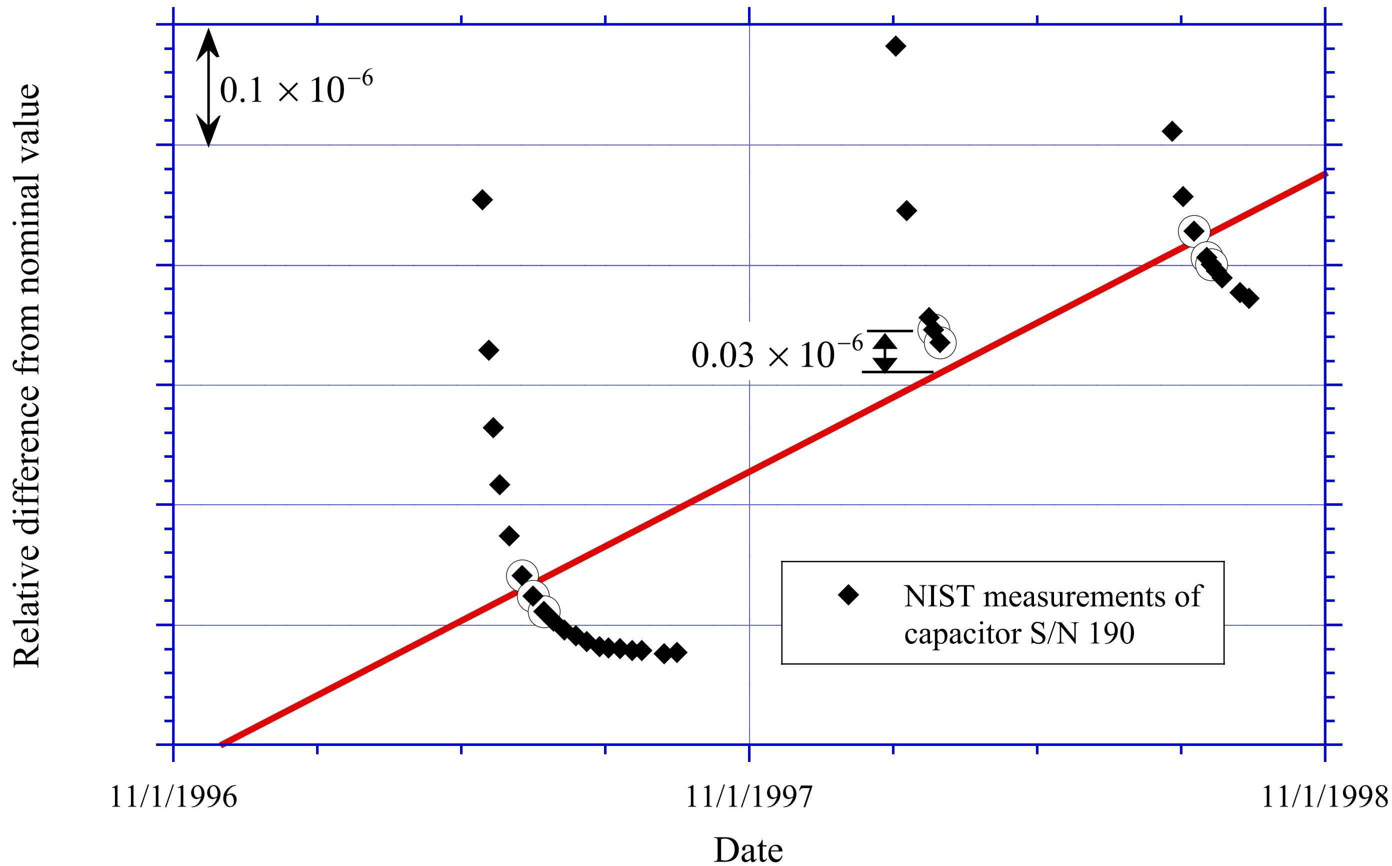

Figure 3. NIST measurements of capacitor S/N 190 (black diamonds) during the comparison. The solid line is a linear fit to the circled diamonds. These selected measurements are taken in a 2-week period starting 4 weeks after cycling. Indicated on the graph is the deviation of the measurements from the linear fit.

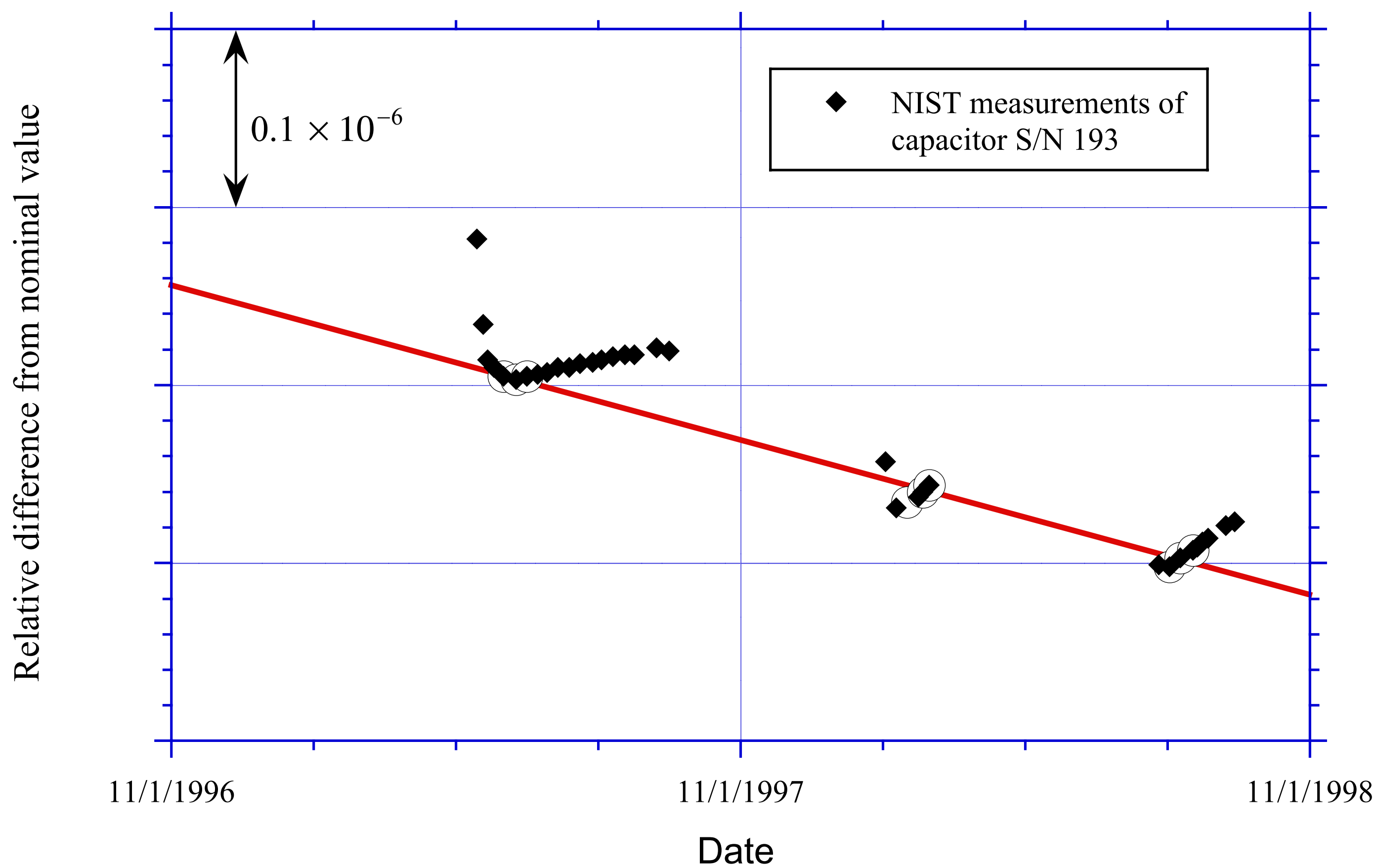

Figure 4. NIST measurements of capacitor S/N 193 (black diamonds) during the comparison. The solid line is a linear fit to the circled diamonds. These selected measurements are taken in a 2-week period starting 3 weeks after cycling.

Capacitor S/N 108, shown in Fig. 1, had a large shift in its value after its first trip to NML. From analysis of the data and comparison of results with capacitor S/N 185, it appears that the jump occurred on its way to the laboratory. Capacitor S/N 108 was not replaced for the next comparison since there was no replacement available. Since no such large jumps in its value were observed in the next comparison and its behavior after temperature cycling continued to be predictable, it was used for the rest of the comparisons. It is apparent, however, that the drift rate of the capacitance of S/N 108 changed significantly after the shift in its value.

The transport uncertainty for the comparison of capacitor S/N 108 with NML was not increased since the NIST measurements made before the jump were not used in the analysis. The relative difference between the NML and the NIST measurements of both capacitors S/N 108 and S/N 185 is within 2×10^{-9} if only the data from NIST obtained after capacitor S/N 108 returned from NML is used. This is an indication that this large shift occurred on the trip to NML and not during or after the measurements there. If the data before the jump is not used, the uncertainty should not be increased to account for it. The transport uncertainty of the capacitors is large enough to account for only using the NIST measurements made after the measurements at NML in the analysis.

Another reason for concern was that Capacitor S/N 108 did not settle down to a linear drift rate until the third set of measurements shown in Fig. 1 along with the rest of the data. The capacitors were sent to NRC and NPL between the second and third set of measurements. The capacitor's behavior between the second and third set does not appear linear and might be better predicted by a polynomial fit. The linear fit to the before and after data was compared to the polynomial fit to the same data. The value of the NIST measurement at the mean measurement dates for NRC and NPL predicted by both of these fits did not differ by more than a fractional difference of 4×10^{-9}. Since this is less than the expected transport uncertainty, the linear fit was thought to be sufficient.

5. Results

Each participant's capacitance unit is traceable to their calculable capacitor, quantum Hall standard, or to another national metrology institute. Details of the participants' traceability are given in Table 3. Most laboratories made the capacitance measurements at the specified conditions of 1592 Hz with 100 V applied to the 10 pF capacitors. There were a few exceptions that are listed in Table 3. Any differences due to voltage dependence are expected to be within the uncertainty of the comparison.

Table 3 Traceability and measurement conditions

Laboratory	Traceability	Voltage on 10 pF capacitor	Frequency
NIST	Calculable capacitor	100 V	1592 Hz
BIPM	QHR measured at 1 Hz	100 V	1592 Hz
BNM-LCIE	DC QHR and Calculable capacitor	3 V	1592 Hz
CSIRO-NML	Calculable capacitor	100 V	1592 Hz
IRL	Calculable capacitor	100 V	1592 Hz
NIM	Calculable capacitor	100 V	1592 Hz
NMi	Calculable capacitor	100 V	1592 Hz
NPL	DC QHR	100 V	1592 Hz
NRC	NIST	100 V	1592 Hz
PTB	Calculable capacitor	100 V	1592 Hz
VNIIM	Calculable capacitor	90 V	1592 Hz

At NIST, the capacitance unit is traceable to a calculable capacitor and is described in Ref. [2]. The relative combined standard uncertainty in assigning a value to the 10 pF bank of capacitors from the calculable capacitor is 0.019×10^{-6}. The uncertainty in assigning a value to the travelling standards from the 10 pF bank depends on the relative measurement uncertainty of the transformer bridge (0.005×10^{-6}), and the relative uncertainty due to calculating the correction to the bank (0.002×10^{-6}). The total relative combined standard uncertainty of the measurement of capacitors from the NIST calculable capacitor is then 0.02×10^{-6}. The von Kiltzing constant, the resistance of the $i = 1$ plateau of the quantized Hall resistance (QHR), has also been measured in terms of the NIST calculable capacitor and the most recent value assigned to it is 25 812.808 31(62) Ω [2].

The results of the comparison are obtained as differences from the NIST measurements of the capacitors. This is necessary as all the capacitors drift with time and it is only by assuming consistency of the values measured by NIST throughout the comparison that the effect of the drift can be eliminated. At NIST, the capacitors are compared against a bank of four 10 pF capacitors that have a linear drift rate of 2×10^{-7} pF per year. The bank is measured against the NIST calculable capacitor two to three times per year.

Each laboratory reports a value for each capacitor measured on a mean measurement date along with a measurement uncertainty, u_L. The uncertainty budget for each laboratory from which u_L is derived is shown in Appendix B. The value reported by each laboratory is an average of the measurements taken in a specific time period after cycling. The NIST predicted value for that date is found from a linear fit to the before and after NIST data in a similar time window after cycling as used by the laboratory. The final result for each laboratory is the average of the two differences from the NIST predicted values for the two capacitors.

Table 4 gives each laboratory's measurements of the capacitors, m_{lab}, on their mean measurement date and the NIST measurement, m_{NIST}, extrapolated to the same date. The values m_{lab} and m_{NIST} are relative deviations from the nominal value of 10 pF and the corresponding values C_{lab} and C_{NIST} in pF are given by $C_{lab} = 10\ (1 + m_{lab})$ pF and $C_{NIST} = 10\ (1 + m_{NIST})$ pF, respectively. Also given in Table 4 is the difference between the laboratory value and the NIST value for each capacitor as well as the average of the differences of the two capacitors that each participant measured. The BIPM values for each capacitor are the average of measurements made on two mean measurement dates. BNM-LCIE reported two values, one from the QHR and one from a calculable capacitor. The value from the calculable capacitor is still under investigation and is given for information only; no uncertainty is reported.

Table 4. Laboratory's measurements m_{lab} of capacitors S/N 108 and S/N 185 or S/N 190 and S/N 193. Measurements are reported as fractional differences from the nominal value of 10 pF. The NIST measurement of each capacitor, m_{NIST}, extrapolated to the same measurement date, is also shown. The difference, m_{lab}- m_{NIST}, between each laboratory's measurement and NIST's measurement is listed. The measurement uncertainty, u_L, reported by each laboratory is also listed.

Laboratory	**Mean measurement date**	m_{lab} S/N 108 (*or S/N 190) ($\times 10^{-6}$)	m_{NIST} S/N 108 (*or S/N 190) ($\times 10^{-6}$)	m_{lab}- m_{NIST} ($\times 10^{-6}$)	m_{lab} S/N 185 (*or S/N 193) ($\times 10^{-6}$)	m_{NIST} S/N 185 (*or S/N 193) ($\times 10^{-6}$)	m_{lab}- m_{NIST} ($\times 10^{-6}$)	u_L ($\times 10^{-6}$)	Average m_{lab}- m_{NIST} ($\times 10^{-6}$)	**Average** m_{lab}- m_{NIST} ($\times 10^{-6}$)
CSIRO-NML	03/29/96	-13.019	-13.058	**0.039**	10.362	10.325	**0.037**	0.033	**0.038**	**0.04**
NPL	08/28/96	-12.891†	-13.067	**0.176†**	10.524†	10.298	**0.226†**	0.047	**0.201†**	**0.20†**
NRC	11/01/96	-13.014	-13.058	**0.044**	10.325	10.289	**0.036**	0.16	**0.040**	**0.04**
NMi	03/01/97	-13.79	-13.032	**-0.758**	9.48	10.259	**-0.779**	0.6	**-0.768**	**-0.77**
VNIIM	05/29/97	-13.32	-12.999	**-0.321**	9.92	10.230	**-0.310**	0.2	**-0.316**	**-0.32**
MSL	11/18/97	-12.92	-12.912	**-0.008**	10.16	10.199	**-0.039**	0.061	**-0.024**	**-0.02**
BIPM-1	11/16/97	-40.625*†	-40.626*	0.001	32.732*†	32.722*	0.010	-	-	-
BIPM-2	01/04/98	-40.625*†	-40.590*	-0.035	32.674*†	32.711*	-0.037	-	-	-
BIPM	12/10/97	-	-	**-0.017†**	-	-	**-0.014†**	0.040	**-0.015†**	**-0.02†**
PTB	03/8/98	-12.85	-12.863	**0.013**	10.15	10.166	**-0.016**	0.045	**-0.002**	**0.00**
NIM	05/28/98	-40.56*	-40.486*	**-0.074**	32.68*	32.680*	**0.000**	0.13	**-0.037**	**-0.04**
BNM-LCIE	06/21/98	-13.063†	-12.847	**-0.216†**	9.942†	10.153	**-0.211†**	0.031	**-0.214†**	**-0.21†**
	06/21/98	-12.563‡	-12.847	**0.284‡**	10.432‡	10.153	**0.279‡**	-	**0.282‡**	**0.28‡**

† Measurements derived from R_{K-90}. ‡BNM-LCIE's measurement from their calculable capacitor.

Because laboratories sometimes selected different time periods after cycling in which to make measurements, not all of the NIST data were used in the linear fit to predict the NIST value corresponding to each laboratory's measurement date. However, a linear fit to all the NIST measurements from a selected time period after cycling was used to estimate the relative standard uncertainty due to transport variability. The estimated value, $u_T = 0.02 \times 10^{-6}$, was obtained as described in the following.

For each capacitor, the transport uncertainty is based on a linear fit to NIST data taken throughout the comparison. The data for each capacitor in a 2-week period three weeks after cycling is reduced to one point for each 2-week period by averaging the data in that time period. A linear fit, as a trend with respect to time, is made from this set of reduced data. From a linear regression to this data, the residual standard deviation from the fit is obtained. (Alternately, this value is sometimes labeled "standard error" by software.) This standard deviation accounts for natural variation of actual capacitance on a given day about the straight-line model. In addition, the standard uncertainty of the predicted value for a particular date during the comparison is also computed. This second term accounts for uncertainty in predicting the expected value of capacitance on a given day, due to estimation of the slope and intercept of the regression line. The transport uncertainty is defined as the root-sum-of-squares of the residual standard deviation and the standard uncertainty of the predicted value. The relative transport uncertainty for each capacitor is given below:

$$u_T(108) = 0.03 \times 10^{-6}$$

$$u_T(185) = 0.01 \times 10^{-6}$$

$$u_T(190) = 0.04 \times 10^{-6}$$

$$u_T(193) = 0.003 \times 10^{-6}$$

Note that the first set of NIST measurements for capacitor 108 before its shift in value was not included in this set of reduced data. Since the result from each laboratory is the average of the results from two capacitors, the transport uncertainty, u_T, for each individual comparison is found from either of the equations below.

$$u_T = \frac{\sqrt{u_T(108)^2 + u_T(185)^2}}{2}$$

or

$$u_T = \frac{\sqrt{u_T(190)^2 + u_T(193)^2}}{2}$$

Conveniently the computed value of u_T for each set of capacitors is the similar and is

$$u_T = 0.02 \times 10^{-6}.$$

Key comparison reference value

While a reference value is not needed to obtain the results, the CCEM has requested that one be reported. The results are found as differences from the NIST measurements in order to remove the effect of the drift of the capacitors. Since all the capacitance standards drift, a reference value that is the average of the capacitance values will represent a value that the standards no longer have.

The calculation of the reference value was done as follows: A NIST mean comparison value was found. Using a linear fit to the reduced set of NIST data described above, the value of each capacitor on the mean comparison date is found. The mean comparison date is the average of the mean dates for the time periods in which both sets of capacitors were used. Capacitors S/N 108 and 185 were used from 1/11/96 to 9/4/98 giving an average measurement date of 5/8/97. Capacitors S/N 190 and 193 were used from 5/27/97 to 9/14/98 giving an average measurement date of 1/19/98. The average of these 2 dates is 9/13/97. The average of the values of all four capacitors on 9/13/97 is the NIST mean comparison value and is $C_{\text{NIST-Mean}} = 10\ (1 - 2.672 \times 10^{-6})$ pF. The differences from the NIST value for each participant was applied to this mean comparison value to generate a set of values, C'_{lab} in picofarads. The weighted average of these values from participants who derive their capacitance unit from a calculable capacitor was taken to be the reference value. The values were weighted by $1/\ u_C^2$ where u_C is the combined standard uncertainty given by

$$u_C = \sqrt{(u_L^2 + u_T^2)}\ .$$

The transport uncertainty used to weight the NIST value would be expected to be smaller than the transport uncertainty used to weight other participants values since NIST has made several measurements of the capacitors while each participant has made one. The NIST transport uncertainty u_T(NIST) would then be

$$u_T(\text{NIST}) = \frac{u_T}{\sqrt{n}},$$

where n is the number of NIST measurements. For all capacitors [n = 5 (S/N 108), n = 6 (S/N 185) and n = 3 (S/N 190 and S/N 193)], this gives

$$u_T(\text{NIST}) = 0.01 \times 10^{-6}.$$

The key comparison reference value C_{ref} is then arbitrarily given the nominal value of 10 pF and C'_{lab} for each participant shifted accordingly. In capacitance measurements, values are usually reported as the deviation from nominal value so we write $C_{ref} = 10\,(1 + m_{ref})$ pF. The key comparison reference value, m_{ref}, is then

$$m_{ref} = 0,$$

and m'_{lab} is the deviation of each participant from m_{ref}. The relative standard uncertainty of the reference value u_{ref} is found from the weighted average of the u_c for each laboratory whose result contributed to m_{ref} and is 0.017×10^{-6}.

The m'_{lab} and u_c for each participant is shown in Table 5 and plotted in Fig. 5. Included in the u_c for laboratories that derive their value from the $R_{K\text{-}90}$ is the uncertainty $u_{R_{K\text{-}90}}$ This is taken to be the difference between $R_{K\text{-}90}$ and the present value assigned to R_K by the 1998 CODATA publication [3] and is $u_{R_{K\text{-}90}} = 0.022 \times 10^{-6}$.

The weighted average $m_{R_{K\text{-}90}}$ of the results from laboratories that derive their value from $R_{K\text{-}90}$ can be used to estimate the coherence with the SI value achieved in this comparison. In the units described above (i.e. $m_{ref} = 0$),

$$m_{R_{\text{K-90}}} = -\,0.045 \times 10^{-6}\,.$$

The relative standard uncertainty of $m_{R_{K\text{-}90}}$ is given by the weighted average of the u_c for each laboratory whose result contributed to $m_{R_{\text{K-90}}}$ and is 0.028×10^{-6}.

Appendix B of the MRA requires giving the degree of equivalence with the reference value and between pairs of laboratories. These are shown in Table 6. The degree of equivalence with the reference value is given by m'_{lab} - m_{ref} which is just m'_{lab} since $m_{\text{ref}} = 0$. For the seven laboratories whose results contribute to the reference value (the weighted mean of results from laboratories with independent calculable capacitors) correlation between m'_{lab} and m_{ref} is accounted for by using the expression $u^2(m'_{\text{lab}} - m_{\text{ref}}) = u^2_{\text{C}}(\text{lab}) - u^2(m_{\text{ref}})$. The value of the degree of equivalence between pairs of laboratories is given by m'_{lab1} - m'_{lab2}. The uncertainty in the degree of equivalence is given by the expression $u^2(m'_{\text{lab1}} - m'_{\text{lab2}}) = u^2_{\text{C}}(\text{lab1}) + u^2_{\text{C}}(\text{lab2}) - 2\text{cov}(\text{lab1},\text{lab2})$. In cases where lab1 and lab2 both derive their capacitance standards from $R_{\text{K-90}}$ the covariance term becomes $-2\,u^2_{R_{\text{K-90}}}$ and its effect is included in Table 6. A covariance term enters into the uncertainty in the degree of equivalence between the NRC and the NIST but its influence is below one part in 10^8, which is negligible. In Table 6, standard uncertainties, u_{Lab}, are given in the third row and the third column while the remaining uncertainties correspond to a coverage factor of k = 2.

Table 5. Laboratory's relative deviation from the reference value, m′$_{lab}$. † Measurements derived from R_{K-90}. The difference between R_{K-90} and the present value assigned to R_K by the 1998 CODATA publication is taken to be the uncertainty in R_{K-90} and is $u_{RK} = 0.022 \times 10^{-6}$. . * This value is correlated with that of the NIST and is not used in calculating the key comparison reference value.

Laboratory	m'_{lab} ($\times 10^{-6}$)	u_L ($\times 10^{-6}$)	u_T ($\times 10^{-6}$)	u_C ($\times 10^{-6}$)	$[u_L^2 + u_T^2 + u_{RK}^2]^{1/2}$ ($\times 10^{-6}$)
BIPM	-0.018†	0.040	0.02	-	0.050
BNM-LCIE	-0.216†	0.031	0.02	-	0.043
BNM-LCIE	0.278	-	0.02	-	-
CSIRO-NML	0.035	0.033	0.02	0.039	-
MSL	-0.026	0.061	0.02	0.064	-
NIM	-0.040	0.13	0.02	0.132	-
NIST (pilot)	-0.003	0.020	0.01	0.022	-
NMi	-0.772	0.6	0.02	0.600	-
NPL	0.198†	0.047	0.02	-	0.056
NRC	0.037*	0.16	0.02	0.161	-
PTB	-0.004	0.045	0.02	0.049	-
VNIIM	-0.318	0.2	0.02	0.201	-

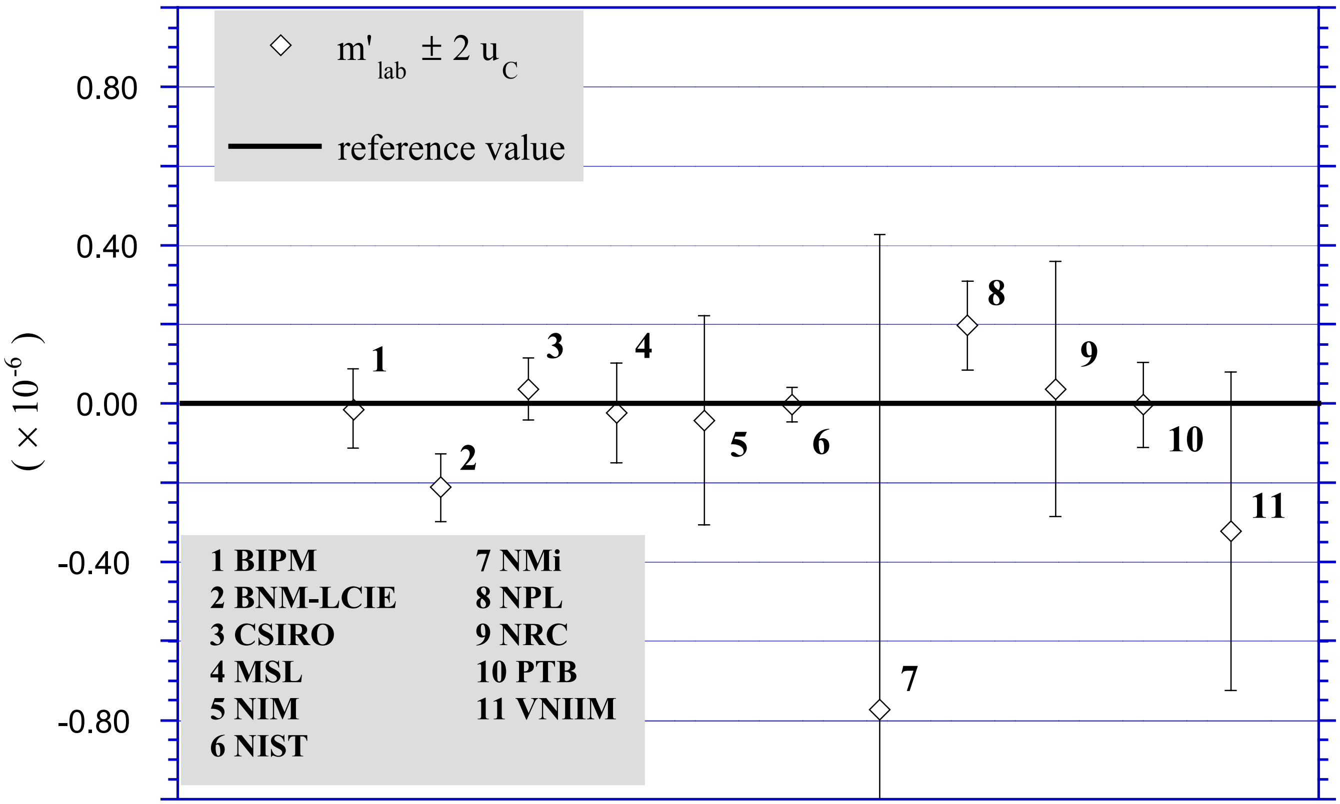

Figure 5. Measurements of each participant, m'_{lab}, relative to the key comparison reference value. †Measurements derived from R_{K-90}

Table 6 The degree of equivalence between each laboratory and the reference value, with its uncertainty U_i (for $k = 2$); the value of the degree of equivalence between pairs of laboratories (shown in the darker gray cells); and the uncertainty of each pairwise degree of equivalence (given in the lighter gray cells).

	Lab			m_{ref}	BIPM*	BNM-LCIE *	CSIRO -NML	MSL	NIM	NIST	NMi	NPL*	NRC	PTB	VNIIM
Lab		**m'_{lab}**		**0.000**	**-0.018**	**-0.216**	**0.035**	**-0.026**	**-0.040**	**-0.003**	**-0.772**	**0.198**	**0.037**	**-0.004**	**-0.318**
	m'_{lab}		u_{Lab}	0.017	0.040	0.031	0.033	0.061	0.13	0.020	0.6	0.047	0.16	0.045	0.2
		u_{Lab}	$\underline{U_i}$	0.034	0.105	0.092	0.069	0.124	0.261	0.029	1.200	0.116	0.324	0.092	0.401
m_{ref}	**0.000**	0.017	0.034		0.018	0.216	-0.035	0.026	0.040	0.003	0.772	-0.198	-0.037	0.004	0.318
BIPM*	**-0.018**	0.040	0.105	0.10		0.20	-0.05	0.01	0.02	-0.02	0.75	-0.22	-0.06	-0.01	0.30
BNM-LCIE *	**-0.216**	0.031	0.092	0.09	0.12		-0.25	-0.19	-0.18	-0.21	0.56	-0.41	-0.25	-0.21	0.10
CSIRO -NML	**0.035**	0.033	0.069	0.07	0.13	0.12		0.06	0.08	0.04	0.81	-0.16	0.00	0.04	0.35
MSL	**-0.026**	0.061	0.124	0.12	0.16	0.15	0.15		0.01	-0.02	0.74	-0.22	-0.06	-0.02	0.29
NIM	**-0.040**	0.13	0.261	0.26	0.28	0.28	0.27	0.29		-0.04	0.73	-0.24	-0.08	-0.04	0.28
NIST	**-0.003**	0.020	0.029	0.03	0.11	0.10	0.09	0.14	0.27		0.77	-0.20	-0.04	0.00	0.32
NMi	**-0.772**	0.6	1.200	1.20	1.20	1.20	1.20	1.21	1.23	1.20		-0.97	-0.81	-0.77	-0.45
NPL*	**0.198**	0.047	0.116	0.12	0.14	0.13	0.14	0.17	0.29	0.12	1.21		0.16	0.20	0.52
NRC	**0.037**	0.16	0.324	0.32	0.34	0.33	0.33	0.35	0.42	0.33	1.24	0.34		0.04	0.36
PTB	**-0.004**	0.045	0.092	0.09	0.14	0.13	0.13	0.16	0.28	0.11	1.20	0.15	0.34		0.31
VNIIM	**-0.318**	0.2	0.401	0.40	0.41	0.41	0.41	0.42	0.48	0.40	1.27	0.42	0.52	0.41	

*Measurements derived from R_{K-90} that have an associated relative uncertainty of 0.022×10^{-6}.

There are several alternative methods to calculating the degree of equivalence between laboratories. Two of these methods are presented in Appendix A.

5. Conclusion

This comparison establishes the relationships among the capacitance standards of laboratories in four regional metrology organizations and of the BIPM. The capacitors used in the comparison appear to have performed satisfactorily in spite of a large shift in value of one of the standards near the start of the comparison. There appear to be a few differences between capacitance units, but for the majority of participants, there is agreement within the 95 % confidence level.

6. References

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Appendix A: Other estimates of the Degree of Equivalence

Two other methods of calculating the degree of equivalence between participants will be presented. One estimation of the degree of equivalence, Δ, between the pilot laboratory and each participant is given by the equation below.

$$\Delta = |\ m'_{\mathrm{lab}} - m'_{\mathrm{NIST}}\ | + t \sqrt{(u_L^2 + u_{\mathrm{NIST}}^2 + u_T^2)} \qquad (1)$$

where t = Students t value which is taken to be 2 for a 95 % confidence level

The degree of equivalence between any pair of laboratories 1 and 2 is given by,

$$\Delta = |\ (m'_{\mathrm{lab1}} - m'_{\mathrm{NIST}}) - (m'_{\mathrm{lab2}} - m'_{\mathrm{NIST}})\ | + t \sqrt{(u_{L1}^2 + u_{L2}^2 + 2(u_{\mathrm{NIST}}^2 + u_T^2))} \qquad (2)$$

where m'_{lab1} = laboratory 1 mean value of the capacitor at their mean measurement date,

m'_{lab2} = laboratory 2 mean value of the capacitor at their mean measurement date,

u_{L1} = standard uncertainty for the laboratory 1's measurement and

u_{L2} = standard uncertainty for the laboratory 2's measurement.

Equation 1 is a more general version of the following equation, which is an estimation of the degree of equivalence at the 95 % confidence level. The derivation of this equation is described in [4].

$$\Delta^* = |\ m'_{\mathrm{lab}} - m'_{\mathrm{NIST}}\ | + \{1.645 + 0.3295\ [\exp(-4.05\ |m'_{\mathrm{lab}} - m'_{\mathrm{NIST}}\ |\ /\ u_p)]\}\ u_p \qquad (3)$$

where $u_p = \sqrt{(u_L^2 + u_{\mathrm{NIST}}^2 + u_T^2)}$.

Equation 3 will also give the degree of equivalence for any pair of laboratories if $u_p = \sqrt{(u_{L1}^2 + u_{L2}^2 + 2(u_{\mathrm{NIST}}^2 + u_T^2))}$ is used.

It could be argued that using u_{NIST} in the u_p above is an over-estimation of the degree of equivalence since the Type B components of u_{NIST} are stable and do not change for each comparison with NIST and another laboratory. However, if only the type A components of u_{NIST} were used, u_{p} would only have a fractional change of 0.028×10^{-6}, so this separation was not made.

The estimation of the degree of equivalence Δ using Eq. (1) between NIST and the other participants is shown in Table 1A and is plotted in Fig. 1A. The estimation of the degree of equivalence for any pair of laboratories is also shown in Table 1A.

Table 1A. Array showing the degree of equivalence between all pairs of laboratories which are reported as relative values ($\times 10^{-6}$). Numbers in the dark gray cells are the degree of equivalence between pairs of laboratories and are found using Eq. (2). except for the bolded numbers which is the case where one of the laboratories is the pilot laboratory so Eq. (1) is used. Numbers in the light gray cells are the degree of equivalence found using Eq. (3) with $u_p = \sqrt{(u_{L1}{}^2 + u_{L2}{}^2 + 2(u_{NIST}{}^2 + u_T{}^2))}$. Bolded numbers in the light gray cells are the degree of equivalence between the pilot and each laboratory and are found using Eqn. (3) with $u_p = \sqrt{(u_L{}^2 + u_{NIST}{}^2 + u_T{}^2)}$.

	Lab			**BIPM†**	**BNM-LCIE †**	**CSIRO-NML**	**MSL**	**NIM**	**NIST (pilot)**	**NMi**	**NPL†**	**NRC**	**PTB**	**VNIIM**
Lab		m'_{lab}		**-0.018**	**-0.216**	**0.035**	**-0.026**	**-0.040**	**-0.003**	**-0.772**	**0.198**	**0.037**	**-0.004**	**-0.318**
	m'_{lab}		u_L	0.04	0.031	0.033	0.06	0.13	0.02	0.6	0.047	0.16	0.05	0.2
		u_L	$2 \times u_c$	0.100	0.086	0.077	0.126	0.263	0.045	1.201	0.111	0.322	0.108	0.402
BIPM†	**-0.018**	0.04	0.100		0.341	0.191	0.179	0.309	0.123	1.959	0.376	0.398	0.171	0.718
BNM-LCIE †	**-0.216**	0.031	0.086	0.316		0.380	0.353	0.459	0.308	1.760	0.566	0.592	0.361	0.517
CSIRO-NML	**0.035**	0.033	0.077	0.168	0.357		0.220	0.355	0.125	2.011	0.310	0.338	0.184	0.767
MSL	**-0.026**	0.06	0.126	0.168	0.324	0.193		0.311	0.156	1.954	0.402	0.414	0.197	0.717
NIM	**-0.040**	0.13	0.263	0.283	0.409	0.310	0.292		0.303	1.962	0.529	0.497	0.325	0.762
NIST (pilot)	**-0.003**	0.02	0.045	0.109	0.291	0.110	0.138	0.270		1.970	0.319	0.365	0.116	0.719
NMi	**-0.772**	0.6	1.201	1.747	1.551	1.798	1.740	1.745	1.758		2.177	2.053	1.974	1.720
NPL†	**0.198**	0.047	0.111	0.348	0.539	0.284	0.371	0.478	0.298	1.963		0.507	0.367	0.937
NRC	**0.037**	0.16	0.322	0.352	0.532	0.331	0.366	0.438	0.327	1.833	0.447		0.386	0.874
PTB	**-0.004**	0.05	0.108	0.156	0.334	0.161	0.177	0.292	0.113	1.761	0.338	0.346		0.734
VNIIM	**-0.318**	0.2	0.402	0.644	0.453	0.693	0.642	0.677	0.648	1.507	0.863	0.782	0.660	

† Measurements derived from R_{K-90} that has an associated relative uncertainty of 0.022×10^{-6}.

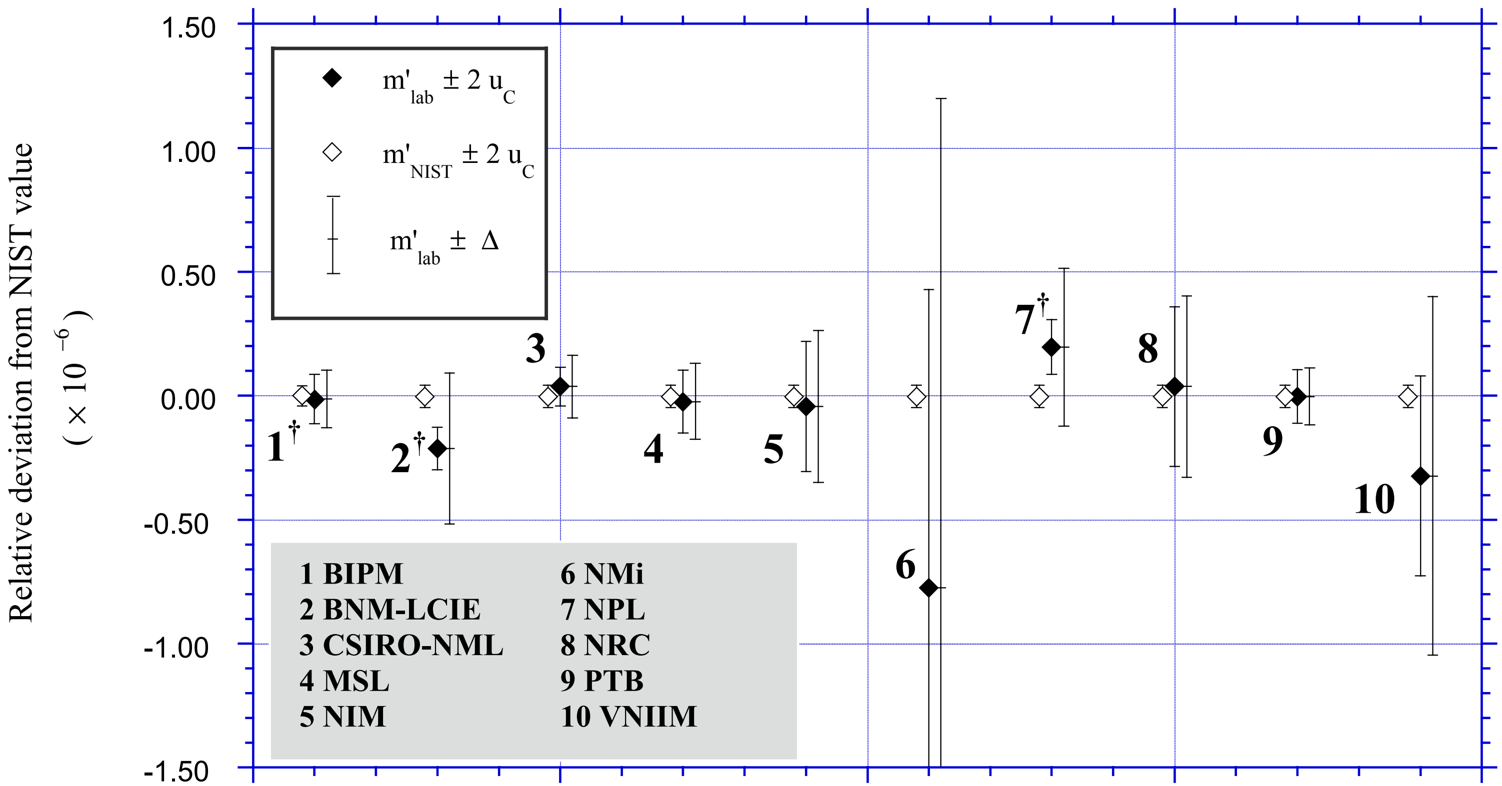

Figure 1A. The estimation of the degree of equivalence Δ [from Eq. (1)] between each participant and NIST is shown and is centered at m'_{lab}. Each laboratory's measurement, m'_{lab}, the NIST measurement, m'_{NIST}, and $\pm u_c$ are shown as a reference. † Measurements derived from $R_{\mathrm{K-90}}$ that has an associated relative uncertainty of 0.022×10^{-6}.

Appendix B

BIPM uncertainty budget

Source of uncertainty	**Relative standard uncertainty**	**Type**
Variability of repeated observations	0.010×10^{-6}	B
Measurement of thermometer resistance	0.007×10^{-6}	B
Comparison with reference group	0.010×10^{-6}	B
10 pF/2000 pF link	0.020×10^{-6}	B
Quadrature bridge	0.015×10^{-6}	B
Measurement of 51.6 kΩ / R_K ratio	0.010×10^{-6}	B
1 Hz – 1541 Hz difference for 51.6 kΩ resistors	0.020×10^{-6}	B
1541 Hz – 1592 Hz difference of reference group	0.015×10^{-6}	A
Combined relative standard uncertainty	$\mathbf{0.040 \times 10^{-6}}$	

BNM-LCIE uncertainty budget

Source of uncertainty	**Relative standard uncertainty**	**Type**
DC transfer resistor	0.0034×10^{-6}	B
Variation in AC-DC transfer resistor	0.012×10^{-6}	B
Quadrature bridge		
Frequency	0.0002×10^{-6}	B
Efficiency of choke	0.001×10^{-6}	B
Efficiency of lead compensation	0.015×10^{-6}	B
10 nF definition (10 nH uncertainty)	0.012×10^{-6}	B
10 pF measurements		
10 nF voltage variation	0.000×10^{-6}	B
1000 pF voltage variation	0.002×10^{-6}	B
100 pF voltage variation	0.001×10^{-6}	B
10 nF to 10 pF transfer (10:1 ratios)	0.007×10^{-6}	B
Variability in 10 pF measurements	0.02×10^{-6}	A
Combined relative standard uncertainty	$\mathbf{0.031 \times 10^{-6}}$	

CSIRO-NML uncertainty budget

Source of uncertainty	**Relative standard uncertainty**	**Type**
Calculable capacitor		
Geometrical imperfections	0.03×10^{-6}	B
Gaps between bars	0.001×10^{-6}	B
Gaps between bars and guard tube	0.0005×10^{-6}	B
Eccentricity of guard bar spikes	0.007×10^{-6}	B
Optical alignment	0.0003×10^{-6}	B
Obliquity (telescope aperture)	0.005×10^{-6}	B
Close approach	0.001×10^{-6}	B
Residual gas pressure	0.0001×10^{-6}	B
Laser length standard	0.002×10^{-6}	B
Frequency correction	0.006×10^{-6}	B
Residual loading on transformer	0.002×10^{-6}	B
Capacitance build-up to 10 pF		
Transformer ratio	0.002×10^{-6}	B
Loading corrections	0.003×10^{-6}	B
Bridge balance injection	0.002×10^{-6}	B
2-port definition	0.002×10^{-6}	B
Variability of repeated observations	0.006×10^{-6}	A
Combined relative standard uncertainty	$\mathbf{0.033 \times 10^{-6}}$	

MSL uncertainty budget

Source of uncertainty	**Relative standard uncertainty**	**Type**
Measurement of 0.5 pF (ES 14)capacitor in terms of the calculable capacitor		
Gaps between bars	0.0003×10^{-6}	B
Close approach of guard tubes	0.0003×10^{-6}	B
Bar non-uniformity	0.0007×10^{-6}	B
Frequency correction	0.0130×10^{-6}	B
Voltage coefficient	0.0060×10^{-6}	B
Interferometer alignment	0.0002×10^{-6}	B
Wavelength uncertainty	0.0004×10^{-6}	B
Vacuum	0.0010×10^{-6}	B
Diffraction	0.0120×10^{-6}	B
ES-14 apparent short term stability	0.0500×10^{-6}	A
Combined standard uncertainty	0.054×10^{-6}	
Transformer ratio uncertainty *(Transformer stability included in capacitor measurements)*		
Permutable dial resolution	0.0017×10^{-6}	B
Permutable lead correction	0.0058×10^{-6}	B
Combined standard uncertainty ($\times$ 2)	0.012×10^{-6}	
5 pF (ES13) vs. 0.5 pF (ES14) bridge uncertainty		
5 pF dial resolution	0.0058×10^{-6}	B
Voltage dependence	0.0100×10^{-6}	B
Lead corrections	0.0023×10^{-6}	B
Combined standard uncertainty	0.0120×10^{-6}	
5 pF (ES16) vs. 0.5 pF (ES14) bridge uncertainty		
5 pF dial resolution	0.0058×10^{-6}	B
Voltage dependence	0.0100×10^{-6}	B
Lead corrections	0.0021×10^{-6}	B
Combined standard uncertainty	0.0120×10^{-6}	
100 pF (GR 100) vs. 10 pF (ES13 + ES14) bridge uncertainty		
100 pF bridge resolution	0.0040×10^{-6}	B
Voltage dependence	0.0100×10^{-6}	B
Lead corrections	0.0066×10^{-6}	B
Combined standard uncertainty	0.0130×10^{-6}	
100 pF (GR 100) vs. 10 pF (NIST 185) bridge uncertainty		
100 pF bridge resolution	0.0040×10^{-6}	B
Temperature correction	0.0111×10^{-6}	B
Voltage dependence	0.0100×10^{-6}	B
Lead corrections	0.0065×10^{-6}	B
Combined standard uncertainty	0.0170×10^{-6}	
Total combined relative standard uncertainty	$\mathbf{0.061\times 10^{-6}}$	

NIM uncertainty budget

Source of uncertainty	**Relative standard uncertainty**	**Type**
Reproducibility of SI capacitance unit from the cross-capacitor	0.10×10^{-6}	B
Comparison of the 10 pF capacitor to the cross-capacitor	0.08×10^{-6}	B
Temperature measurement	0.03×10^{-6}	B
Variability in the capacitance and temperature measurement	0.005×10^{-6}	A
Combined relative standard uncertainty	$\mathbf{0.13 \times 10^{-6}}$	

NIST uncertainty budget

Source of uncertainty	**Relative standard uncertainty**	**Type**
<u>Measurement of 10 pF bank from the calculable capacitor</u>		A
Variability of repeated observations	0.002×10^{-6}	B
Geometrical imperfections	0.015×10^{-6}	B
Laser/Interferometer alignment	0.003×10^{-6}	B
Frequency (loading) corrections	0.004×10^{-6}	B
Microphonic coupling	0.005×10^{-6}	B
Voltage dependence	0.005×10^{-6}	B
Transformer ratio measurement	0.002×10^{-6}	B
Bridge linearity and phase adjustment	0.003×10^{-6}	B
Detector uncertainties	0.002×10^{-6}	B
Drift between calibrations/ failure to close	0.006×10^{-6}	B
Coaxial choke effectiveness	0.001×10^{-6}	B
Temperature corrections for 10 pF capacitors	0.002×10^{-6}	B
<u>Measurement of travelling standards from the 10 pF bank</u>		
Variability of repeated observations	$< 0.001 \times 10^{-6}$	A
2-pair transformer bridge	0.005×10^{-6}	B
Correction to account for drift rate of 10 pF	0.002×10^{-6}	B
Combined relative standard uncertainty	$\mathbf{0.020} \times 10^{-6}$	

NMi uncertainty budget

Source of uncertainty	**Relative standard uncertainty**	**Type**
Measurement of NIST 10 pF capacitors		
Bridge uncertainty	0.2×10^{-6}	B
- Calibration of internal standards		
- Temperature drift of internal standards		
- Resolution of bridge readings		
Cable contributions	0.02×10^{-6}	B
Standard deviation in measurements	0.03×10^{-6}	A
Combined standard uncertainty	0.2×10^{-6}	
Measurement of reference 10 pF capacitor (C118) in terms of the calculable capacitor in 1988	0.2×10^{-6}	B
Uncertainty in drift of C118 since last measurement by the calculable capacitor	0.5×10^{-6}	B
Uncertainty in temperature measurement of reference capacitor, C118	0.02×10^{-6}	B
Uncertainty in temperature measurement of NIST capacitors	0.03×10^{-6}	B
Combined relative standard uncertainty	$\mathbf{0.6 \times 10^{-6}}$	

NPL uncertainty budget

Source of uncertainty	**Relative standard uncertainty**	**Type**
Measurements linking quantum Hall device to DC value of 1000 Ω quadrifilar resistor	0.012×10^{-6}	B
Measurements 1000 Ω quadrifilar resistor to 10 pF NPL Primary Capacitor	0.044×10^{-6}	B
Resistance measurement of thermometer	0.001×10^{-6}	B
Repeatability	0.010×10^{-6}	A
Combined relative standard uncertainty	$\mathbf{0.047 \times 10^{-6}}$	

NRC uncertainty budget

Source of uncertainty	**Relative standard uncertainty**	**Type**
NRC Farad (uncertainty of the mean of three NRC reference capacitors in reference to the NIST SI unit in 1990)		
Oil bath temperature measurements	0.010×10^{-6}	B
Standard deviation of internal comparisons 3 × 2	0.020×10^{-6}	A
Stability of NRC 10 pF capacitors (yearly)	0.020×10^{-6}	B
Calculable reference	0.000×10^{-6}	B
Temperature hysteresis from transfer of NIST unit	0.030×10^{-6}	B
Intercomparisons (2)	0.020×10^{-6}	B
Transportations (3)	0.029×10^{-6}	B
Combined standard uncertainty	0.055×10^{-6}	
Measurements in this comparison (1996)		
Internal comparison of capacitor 107 to three NRC reference capacitors	0.009×10^{-6}	A
Comparison of capacitor 107 to NIST travelling capacitors (108 & 185)	0.020×10^{-6}	A
Linear fit variance	0.003×10^{-6}	A
Oil bath temperature measurements	0.016×10^{-6}	A + B
Temperature correction to mean of three NRC reference capacitors	0.002×10^{-6}	B
Combined standard uncertainty	0.027×10^{-6}	
Drift of mean of three reference capacitors	0.14×10^{-6}	B
Total combined relative standard uncertainty	$\mathbf{0.16 \times 10^{-6}}$	

PTB uncertainty budget

Source of uncertainty	**Relative standard uncertainty**	**Type**
Calculable capacitor		
Laser length standard	0.008×10^{-6}	B
Laser/interferometer alignment	0.002×10^{-6}	B
Residual gas pressure	0.002×10^{-6}	B
Geometrical imperfections in the calculable capacitor	0.009×10^{-6}	B
Transformer ratio measurement	0.012×10^{-6}	B
Bridge linearity and phase adjustment	0.005×10^{-6}	B
Loading corrections	0.002×10^{-6}	B
Frequency correction	0.005×10^{-6}	B
Temperature correction	0.010×10^{-6}	B
Relative standard deviation	0.030×10^{-6}	A
1:1 capacitance bridge		
Drift of the transfer between the determine with the determination with the calculable capacitor and the international comparison	0.020×10^{-6}	B
Bridge linearity and phase adjustment	0.005×10^{-6}	B
Temperature correction of the primary standard	$0.010 \times 10^{-6-}$	B
Temperature correction of the unknown standard	0.010×10^{-6}	B
Relative standard deviation ($\times$ 2)	0.009×10^{-6}	A
Total combined standard uncertainty	$\mathbf{0.045 \times 10^{-6}}$	
Total combined standard uncertainty (rounded)	$\mathbf{0.050 \times 10^{-6}}$	

VNIIM uncertainty budget

Source of uncertainty	**Relative standard uncertainty**	**Type**
Repeatability	0.06×10^{-6}	A
Calculable capacitor *Geometrical imperfections* *Laser interferometer* *Transformer bridge* *Imperfections in detector* Combined standard uncertainty	0.18×10^{-6}	B
Maintenance of capacitance unit by primary group *Short term instability* *Hysteresis* *Voltage dependence* *Loading of the transformer bridge* Combined standard uncertainty	0.02×10^{-6}	B
Temperature measurement of the 10 pF fused silica capacitors *Temperature instability of the oil bath* *Calibration of temperature sensors* *Measurement of temperature sensor' resistance* Combined standard uncertainty	0.06×10^{-6}	B
Total combined relative standard uncertainty	$\mathbf{0.2 \times 10^{-6}}$	