1 Background

This document describes the method, based upon that described by Sutton [1], used to link the EUROMET.M.M-K1 and CCM.M–K1 Key Comparisons.

2 Data

2.1 EUROMET.M.M-K1

EUROMET.M.M-K1 is a European Key Comparison of 1 kg standards in stainless steel comprising 10 laboratories and piloted by NPL.

Two sets of transfer standards were used, with each comprising two standards. One of the standards from each set were circulated amongst the laboratories, and the other two remained at the pilot laboratory. The pilot laboratory measured periodically both standards in each set, and the measured data obtained was used as the basis of investigating the stability of the standards.

Table 1 contains information about the measurement results provided by the laboratories for the first weight identified as PTB C. The information comprises (a) the laboratory name, (b) the time of the measurement (as a number of months with $t = 1$ month denoting January 1995), (c) the measured difference from a nominal value of m_0 = 1 kg, and (d) the standard uncertainty associated with the measured difference. Table 2 contains similar information to that of Table 1, but for the second weight identified as INM 11.

No information is provided about the correlation associated with pairs of measured values. For the purpose of the analysis described here, the following simple "rules" are applied:

- The correlation coefficient associated with a pair of measured values provided by the same laboratory is set as 0.8;
- The correlation coefficient associated with a pair of measured values provided by different laboratories is set as 0.4.

The values used for the correlation coefficients are based on the results of discussions between NPL and BIPM metrologists, and the consideration of common systematic effects on the measurements, including traceability to BIPM and the use of a common formula for air density used in the application of air buoyancy corrections, etc.

2.2 CCM.M–K1

CCM.M–K1 is a CIPM Key Comparison of 1 kg standards in stainless steel, comprising 14 laboratories and piloted by BIPM. Three of these laboratories (PTB, INRIM and NPL) participated in the EUROMET.M.M-K1 Key Comparison, and are used as the basis of linking the two Key Comparisons.

Table 3 contains information about the degrees of equivalence for the linking laboratories obtained from CCM.M–K1. The information comprises (a) the laboratory name, (b) the value component d of the degree of equivalence, and (c) the standard uncertainty $u(d)$ associated with d obtained by dividing the uncertainty component of the degree of equivalence for a 95 % coverage probability by two.

For the purposes of linking EUROMET.M.M-K1 and CCM.M–K1 it is necessary to account for the correlation associated with pairs of measured values provided in the two comparisons by the laboratories participating in EUROMET.M.M-K1 . In the absence of information about such correlations, the same rules as described for EUROMET.M.M-K1 are applied. For example:

- The correlation coefficient associated with a mass difference provided by PTB in EUROMET.M.M-K1 and the value component of the degree of equivalence for PTB obtained in CCM.M–K1 is set as 0.8;
- The correlation coefficient associated with the value components of the degrees of equivalence for PTB and NPL obtained in CCM.M–K1 is set as 0.4.

3 Model

Let D_i, $i = 1, \ldots, 10$, denote the bias for laboratory i, i.e., the quantity of which d_i is an estimate.

The results of a stability analysis carried out by the pilot laboratory indicated that the masses of the travelling standards were not subject to drift. Let Δ_k denote the difference of the mass of travelling standard k (PTB C and INM 11) from a nominal value of $m_0 = 1$ kg.

Let X_{ik} denote the difference of the mass of travelling standard k from a nominal value of $m_0 = 1$ kg measured by laboratory i. Then, a model for X_{ik} in terms of D_i and Δ_k is

$$X_{ik} = D_i + \Delta_k. \tag{1}$$

Table 1 contains measured values x_{ik} for X_{ik} for $i = 1, \ldots, 10$, and $k = 1$. Table 2 contains values x_{ik} for $i = 1, \ldots, 10$, and $k = 2$. Finally, Table 3 contains measured values d_i for D_i for $i = 5$, 6, and 7.

Let $\mathbf{x}$ denote a vector comprising these 23 measured values (10 relating to weight PTB C, 10 to weight INM 11, and 3 relating to degrees of equivalence from CCM.M–K1) with associated uncertainty matrix $\mathbf{U}_\mathbf{x}$ determined from the standard uncertainties given in the tables and the numerical values used for the correlation coefficients. Furthermore, let $\mathbf{Y}$ denote a vector comprising the 12 (10 D_i and 2 Δ_k) parameters to be estimated. Then,

$$\mathbf{X} = \mathbf{AY},$$

where $\mathbf{A}$ is a 23 × 12 matrix determined by the relationships (1) and the information provided by CCM.M–K1, and $\mathbf{X}$ is the vector of quantities for which the measured values $\mathbf{x}$ are estimates. An estimate $\mathbf{y}$ of $\mathbf{Y}$ with the associated uncertainty matrix $\mathbf{U}_\mathbf{y}$ is found as the solution $\mathbf{z} = \mathbf{y}$ to the generalised least-squares problem

$$\min_\mathbf{z} \quad (\mathbf{x} - \mathbf{Az})^\mathrm{T} \mathbf{U}_\mathbf{x}^{-1} (\mathbf{x} - \mathbf{Az}).$$

The components of $\mathbf{y}$ contain estimates of the value components of the degree of equivalence for the laboratories and information about the travelling standards. The diagonal elements of $\mathbf{U}_\mathbf{y}$ contain the variances (squared standard uncertainties) associated with the estimates $\mathbf{y}$.

Laboratory i	t_i /month	$x_{i1} = m_{PTBC} - m_0$ /µg	$u(x_{ik})$ /µg
JV	4	–577	23.5
SP	6	–589	18.0
MIKES	9–10	–596	17.2
DFM	11	–571	20.0
PTB	17	–589	14.8
INRIM	18–20	–559	13.6
NPL	21	–572	14.4
SMD	23	–599	21.0
BNM-LNE	25–26	–591	18.0
CEM	27–28	–600	17.0

Table 1 Measurement results provided by the laboratories in the EUROMET.M.M-K1 Key Comparison for the weights PTB C.

Laboratory i	t_i /month	$x_{i2} = m_{INM11} - m_0$ /µg	$u(x_{ik})$ /µg
JV	4	2428	21.5
SP	6	2411	18.0
MIKES	9–10	2393	17.1
DFM	11	2444	20.0
PTB	17	2422	13.2
INRIM	18–20	2451	13.9
NPL	21	2432	14.9
SMD	23	2401	21.0
BNM-LNE	24–26	2415	18.0
CEM	27–28	2412	17.1

Table 2 Measurement results provided by the laboratories in the EUROMET.M.M-K1 Key Comparison for the weight INM 11.

Laboratory *i*	d_i /µg	$u(d_i)$ /µg
PTB	−1	13
INRIM	0	14
NPL	2	16

Table 3 Information about the degrees of equivalence for the linking laboratories obtained from CCM.M−K1.

4 Results

The results of the linking are given in Table 4:

	JV	SP	MIKES	DFM	PTB	INRIM	NPL	SMD	BNM-LNE	CEM
Δm/µg	-2	-17	-30	9	-6	12	2	-27	-16	-22
$U_{\Delta m}$/µg	43	35	34	39	25	26	29	41	35	33

Table 4 Degree of equivalence between each EUROMET.M.M-K1 participant and the CCM.M-K1 reference value, Δm, and associated k=2 uncertainties, $U_{\Delta m}$.

5 Notes

The observed chi-squared value corresponding to the computed solution is (approximately) 11, which is equal to the expected value (equal to the number of degrees of freedom). It is concluded that there is no evidence (on the basis of this test) to doubt the consistency of the data (values **x** and associated uncertainty matrix $\mathbf{U_x}$) with the model.

The estimates of the mass differences for the travelling standards are −572 µg and 2433 µg, respectively. These compare well with the results reported in EUROMET.M.M-K1 .

6 References

[1] Sutton C M, Metrologia, **41** (2004) 272–277