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Report on EURAMET key comparison of multiples and submultiples of the kilogram (EURAMET.M.M-K2.1)

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ABSTRACT

This report summarises the results of comparison EURAMET.M.M-K2.1 of multiples and submultiples of the kilogram carried out between eleven laboratories. The transfer standards comprised five weights of nominal mass 10 kg, 500 g, 20 g, 2 g and 100 mg. The majority of the results of the participants are consistent with each other and with the key comparison reference value (KCRV) of comparison CCM.M-K2 to which this comparison has been linked.

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Approved on behalf of the Managing Director, NPL,
by Martyn Sené, Director of Operations

Contents

1 Introduction

The comparison EURAMET.M.M-K2.1, described in this report, is the follow on comparison to EURAMET.M.M-K2 [1]. Regional key comparison EURAMET.M.M-K2 was carried out between twenty-five National Measurement Institutes (NMIs), the measurements taking place between August 2002 and June 2003. Its aim was to provide a link to the CCM key comparison CCM.M-K2 [2]. Following completion of the measurements for this comparison, an additional six NMIs expressed an interest in participating in a further comparison over the same range, whilst five NMIs who had taken part in EURAMET.M.M-K2 wished to repeat their participation, either because of a significant change in calibration method or claimed uncertainty or due to anomalous results. It was therefore agreed that a further loop to the comparison, designated EURAMET.M.M-K2.1 be carried out to incorporate the additional eleven participants.

The comparison was carried out using mass standards of the following five nominal values:

100 mg, 2 g, 20 g, 500 g and 10 kg.

The transfer standards were provided by SP(Sweden), INRIM (Italy) agreed to act as pilot laboratory and NPL (UK) took on the role of collation and analysis of the results.

The participating laboratories are listed in Table 1.

Table 1: List of Participating Laboratories

Laboratory		Country
State Office for Metrology	DZM	Croatia
National Institute of Standard	NIS	Egypt
AS Metrosert	METROSERT	Estonia
Hellenic Institute of Metrology	EIM	Greece
National Physical Laboratory of Israel	INPL	Israel
Latvijas Nacionālais Metroloģijas Centrs	LNMC	Latvia
Malta Standards Authority	MSA-NMS	Malta
VSL	VSL	Netherlands
Justervesenet	JV	Norway
Directorate of Measures and Precious Metals	DMPM	Serbia
Tübitak Ulusal Metroloji Enstitüsü	UME	Turkey

The transfer standards comprised one weight of each of the five nominal values. They were circulated in a petal fashion, returning twice to the pilot laboratory during the comparison: the pilot laboratory also carried out mass determinations at the beginning and end of the process. The data from these determinations were used to provide a measure of the stability of the transfer standards.

The comparison measurements were carried out between November 2005 and August 2008. The dates of participation are given in Table 2.

Table 2: Dates of Participation

Participant	Date of participation
INRIM (pilot)	July 2005
MSA-NMS	November 2005
INPL	December 2005
DZM	February 2006
UME	May 2006
VSL	August 2006
INRIM (pilot)	October 2006
Metrosert	December 2006
LNMC	January 2007
DMPM	March 2007
JV	April 2007
INRIM (pilot)	July 2007
NIS	September 2007
EIM	June 2008
INRIM (pilot)	August 2008

This report details and analyses the results obtained by the participants. In order to provide consistency with EURAMET.M.M-K2 the reference value was chosen to be the median of the participants' results and the uncertainty was calculated according to the method described by Müller [3].

2 Description of the transfer standards

The transfer standard set supplied by SP comprised five weights made from non-magnetic stainless steel and with the form and quality recommended by OIML [4] for weights of accuracy Class E_1.

3 Summary of results reported by the participants

3.1 Values of mass and uncertainty

The results and uncertainties provided by each of the participants for each of the nominal mass values are shown in Table 3. Each result is shown as the difference between the mass determined by the participant (m) and the nominal mass value (m_o), in mg. The uncertainties (u_c) are given in mg at $k = 1$.

3.2 Stability of the transfer standards

Results obtained by INRIM for each of the transfer standards are shown in Table 4 [5]. As in Table 3, the results are shown as the difference in mg between the mass determined by INRIM (m) and the nominal mass value (m_o), together with the associated uncertainty (u_c) at $k = 1$.

For the 10 kg weight, INRIM reported problems with their balance for their first set of weighings [5].

4 Mass differences

Each participant reported their measured mass difference from the nominal value together with an associated uncertainty for each of the five weights. In order to compare the values from all of the participants it is necessary to link them to initial reference values obtained from the measurements of the pilot laboratory, INRIM. An estimate of the pilot laboratory's mass value is the mean of these four (three, at 10 kg due to the reported problems with their first measurement) measurements. The associated uncertainty was taken to be the standard deviation of these measurements.

The mass difference between participant A and the pilot laboratory P is calculated from:

$$\Delta m_{A,P} = m_A - \frac{m_{P1} + m_{P2} + m_{P3} + m_{P4}}{4} \qquad (1)$$

5 Calculation of reference value and uncertainty

For the purposes of this comparison, the reference value has been taken to be the median of the calculated differences between each participant and the pilot laboratory's mean value. As described in the CCM report [2], a major consideration for adopting this approach is its reduced sensitivity to outliers and the fact that it does not require the exclusion of data, as would be the case when calculating a mean value only from data showing a positive *t*-test.

The reference value m_{ref} can therefore be defined as:

$$m_{ref} = median(\Delta m_{i,P})_{i=1\ to\ n} \qquad (2)$$

The uncertainty in the reference value has been calculated according to the method described by Müller [3]. The five reference values and their associated uncertainties are shown in Table 5.

Table 5: Reference values and associated uncertainties for EURAMET comparison

Nominal mass	Reference value /mg	Uncertainty (k=1) /mg
10 kg	0.30	0.20
500 g	0.025	0.011
20 g	0.001	0.002
2 g	0.001 0	0.000 7
100 mg	-0.000 2	0.000 3

6 Mass difference and uncertainty between participants and reference value

The mass difference between a participant and the reference value is calculated from:

$$\Delta m_{\mathrm{A,ref}} = \Delta m_{\mathrm{A,P}} - m_{\mathrm{ref}} \qquad (3)$$

The uncertainties have been calculated in accordance with the Guide to the Expression of Uncertainty in Measurement [6]. The uncertainty of the difference between a participant's measurement and the reference value is made up of four components:

- the uncertainty in the participant's measurement
- the uncertainty due to the stability of the transfer standard
- the uncertainty in the pilot laboratory's measurement of the stability
- the uncertainty in the reference value.

The uncertainty due to the instability of the transfer standard was taken to be the standard deviation of the pilot laboratory's measurements and the uncertainty in the reference value is as calculated in Section 5. The other uncertainties are as provided by the participants and the pilot laboratory.

The uncertainty is therefore calculated from:

$$u_{\mathrm{A}}(\Delta m_{\mathrm{A,ref}}) = \sqrt{u^2(\Delta m_{\mathrm{A,P}}) + u^2(m_{\mathrm{ref}})} \qquad (4)$$

where

$$u^2(\Delta m_{\mathrm{A,P}}) = u^2(m_{\mathrm{A}}) + u^2(\Delta m_{\mathrm{P}}) + u^2(m_p) \qquad (5)$$

The differences between each participant and the reference value, together with their associated uncertainties, are given in Table 6 and shown graphically in Figures 1 to 5.

7 Mass differences and uncertainties between participants

Mass differences between participants A and B are calculated by subtracting the difference between participant B and the reference value from the difference between participant A and the reference value. These differences and their associated uncertainties are given in Table 7 to Table 11.

8 Linkage to key comparison CCM.M-K2

The results of this comparison have been linked to the results of the key comparison CCM.M-K2 using a method based upon that of Sutton [7] as described in Appendix A.

9 References

[1] Perkin M. Report on EUROMET key comparison of multiples and submultiples of the kilogram (EUROMET.M.M-K2), NPL Report ENG 13, 2009

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[3] Müller J W. Possible advantages of a robust evaluation of comparisons. *J Res Natl Inst Stand Technol* **105** 551-555, 2000

[4] Weights of classes E_1, E_2, F_1, F_2, M_1, M_{1-2}, M_2, M_{2-3} and M_3, International recommendation OIML R111-1, OIML, 2004

[5] Bich W and Goria R. Key Comparison M.M-K2.1 EUROMET Project 786 on five mass standards – results of the INRIM, INRIM Technical Report 142, 2008

[6] Guide to the Expression of Uncertainty in Measurement, BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 1993

[7] Sutton C M, Metologia, **41** 271-277, 2004

10 Tables of Results

Table 3: Reported results for the transfer standards, shown as the difference between mass, m, and nominal mass, m_0, and standard uncertainty (k = 1).

	10 kg		500 g		20 g		2 g		100 mg	
Laboratory	m-m_0/mg	u_c/mg	m-m_0/mg	u_c/mg	m-m_0/mg	u_c/mg	m-m_0/mg	u_c/mg	m-m_0/mg	u_c/mg
MSA-NMS	-0.10	1.20	-0.024	0.060	-0.010 8	0.008 5	+0.003 5	0.003 0	-0.001 4	0.001 5
INPL	+1.30	0.50	-0.005	0.010	+0.000 0	0.003 0	+0.005 8	0.000 9	-0.000 8	0.000 3
DZM	-0.38	1.45	-0.088	0.063	-0.002 3	0.004 0	+0.006 1	0.001 4	-0.001 1	0.000 8
UME	+4.50	0.80	-0.019	0.016	+0.006 0	0.002 0	+0.006 5	0.000 4	+0.000 6	0.000 3
VSL	+0.16	0.75	-0.050	0.040	+0.005 0	0.004 0	+0.005 0	0.002 0	-0.000 3	0.000 8
Metrosert	+0.61	0.65	-0.037	0.020	-0.001 9	0.002 8	+0.004 1	0.001 2	-0.001 3	0.000 6
LNMC	+0.81	0.79	-0.084	0.014	-0.005 1	0.002 4	+0.004 7	0.001 0	+0.000 3	0.000 5
DMPM	+0.60	0.94	-0.063	0.046	-0.006 0	0.005 0	+0.002 7	0.002 4	+0.000 4	0.000 9
JV	-	-	-0.031	0.017	-	-	-	-	-	-
NIS	+0.51	0.30	-0.026	0.020	-0.003 0	0.020 0	+0.005 1	0.001 1	-0.000 5	0.001 0
EIM	+0.80	0.47	-0.012	0.017	+0.001 9	0.001 1	+0.003 7	0.000 8	-0.000 8	0.000 5

Table 4: Results obtained by pilot laboratory, INRIM, for each of the transfer standards, shown as the difference between mass, m, and nominal mass, m_0, and standard uncertainty ($k = 1$).

	10 kg		500 g		20 g		2 g		100 mg	
Date	m-m_0/mg	u_c/mg	m-m_0/mg	u_c/mg	m-m_0/mg	u_c/mg	m-m_0/mg	u_c/mg	m-m_0/mg	u_c/mg
July 2005	+0.45	0.40	-0.042	0.007	-0.000 6	0.001 0	+0.004 3	0.000 7	+0.000 1	0.000 3
October 2006	+0.28	0.40	-0.057	0.007	-0.002 3	0.001 0	+0.003 2	0.000 7	-0.000 8	0.000 3
July 2007	+0.24	0.40	-0.063	0.007	-0.002 6	0.001 0	+0.004 3	0.000 7	-0.000 6	0.000 3
August 2008	+0.23	0.40	-0.061	0.007	-0.005 1	0.001 0	+0.003 5	0.000 7	-0.000 6	0.000 3

Table 6: Differences between each participant and the reference value, together with their associated uncertainties ($k = 2$)

	10 kg		**500 g**		**20 g**		**2 g**		**100 mg**	
Laboratory	m_{AP}-m_{ref}/mg	u_A/mg	m_{AP}-m_{ref}/mg	u_A/mg	m_{AP}-m_{ref}/mg	u_A/mg	m_{AP}-m_{ref}/mg	u_A/mg	m_{AP}-m_{ref}/mg	u_A/mg
MSA-NMS	-0.70	2.43	+0.007	0.122	-0.008 7	0.017 5	-0.001 3	0.006 1	-0.000 8	0.003 1
INPL	+0.70	1.08	+0.026	0.030	+0.002 1	0.007 4	+0.000 9	0.002 2	-0.000 1	0.000 8
DZM	-0.98	2.92	-0.057	0.128	-0.000 2	0.009 1	+0.001 3	0.003 1	-0.000 4	0.001 6
UME	+3.89	1.65	+0.012	0.039	+0.008 1	0.005 9	+0.001 7	0.001 5	+0.001 3	0.000 9
VSL	-0.44	1.55	-0.019	0.083	+0.007 1	0.009 1	+0.000 1	0.004 2	+0.000 3	0.001 6
Metrosert	+0.00	1.36	-0.006	0.046	+0.000 2	0.007 1	-0.000 7	0.002 8	-0.000 6	0.001 3
LNMC	+0.21	1.62	-0.053	0.036	-0.003 0	0.006 4	-0.000 1	0.002 4	+0.000 9	0.001 2
DMPM	-0.00	1.92	-0.032	0.095	-0.003 9	0.010 9	-0.002 1	0.005 0	+0.001 0	0.001 9
JV	-	-	+0.000	0.041	-	-	-	-	-	-
NIS	-0.09	0.72	+0.005	0.046	-0.000 9	0.040 2	+0.000 3	0.002 6	+0.000 1	0.002 1
EIM	+0.20	1.02	+0.019	0.041	+0.004 0	0.004 8	-0.001 1	0.002 1	-0.000 1	0.001 2

11 Figures

Figure 1: Differences between participants' results and reference value, and uncertainty ($k = 2$), for 10 kg

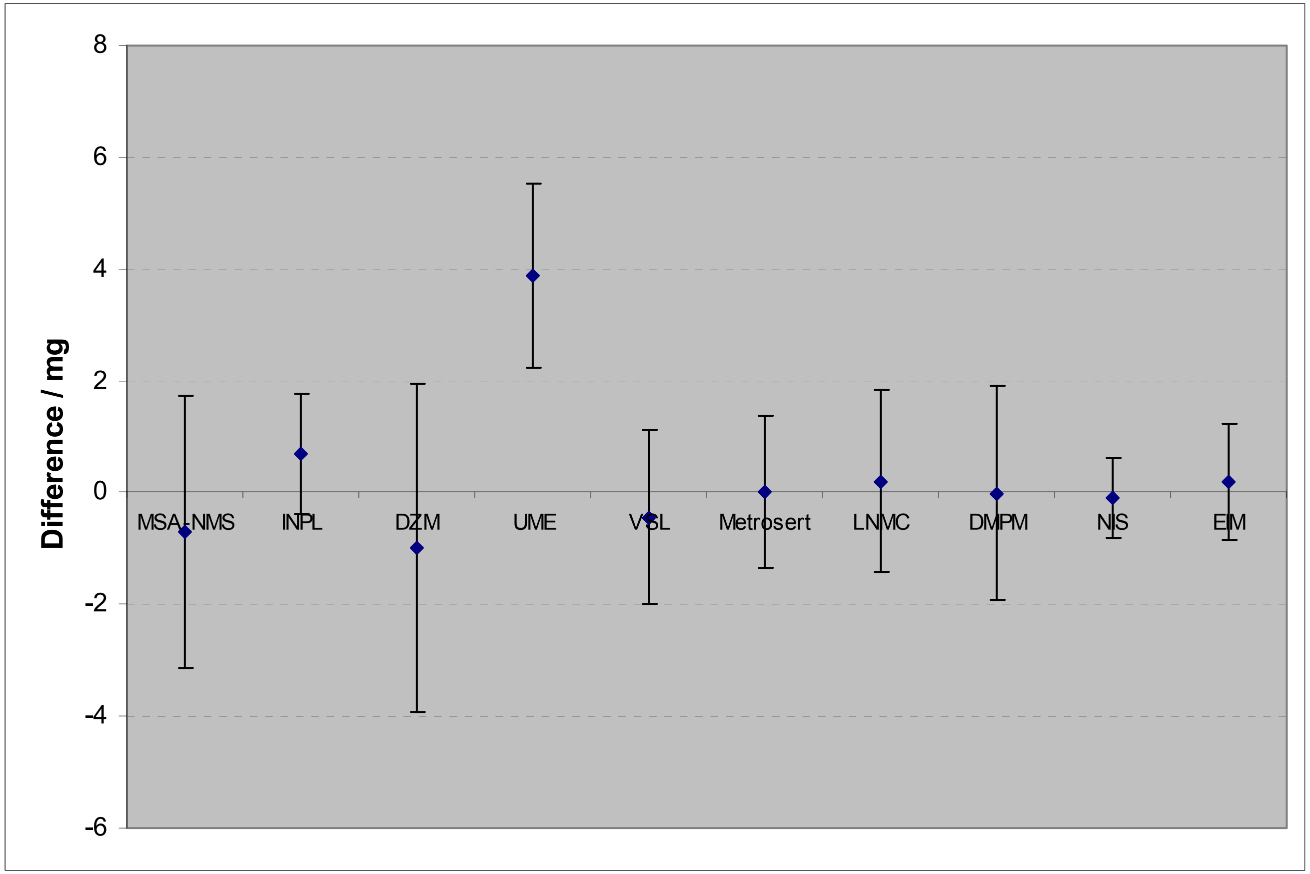

Figure 2: Differences between participants' results and reference value, and uncertainty (k = 2), for 500 g

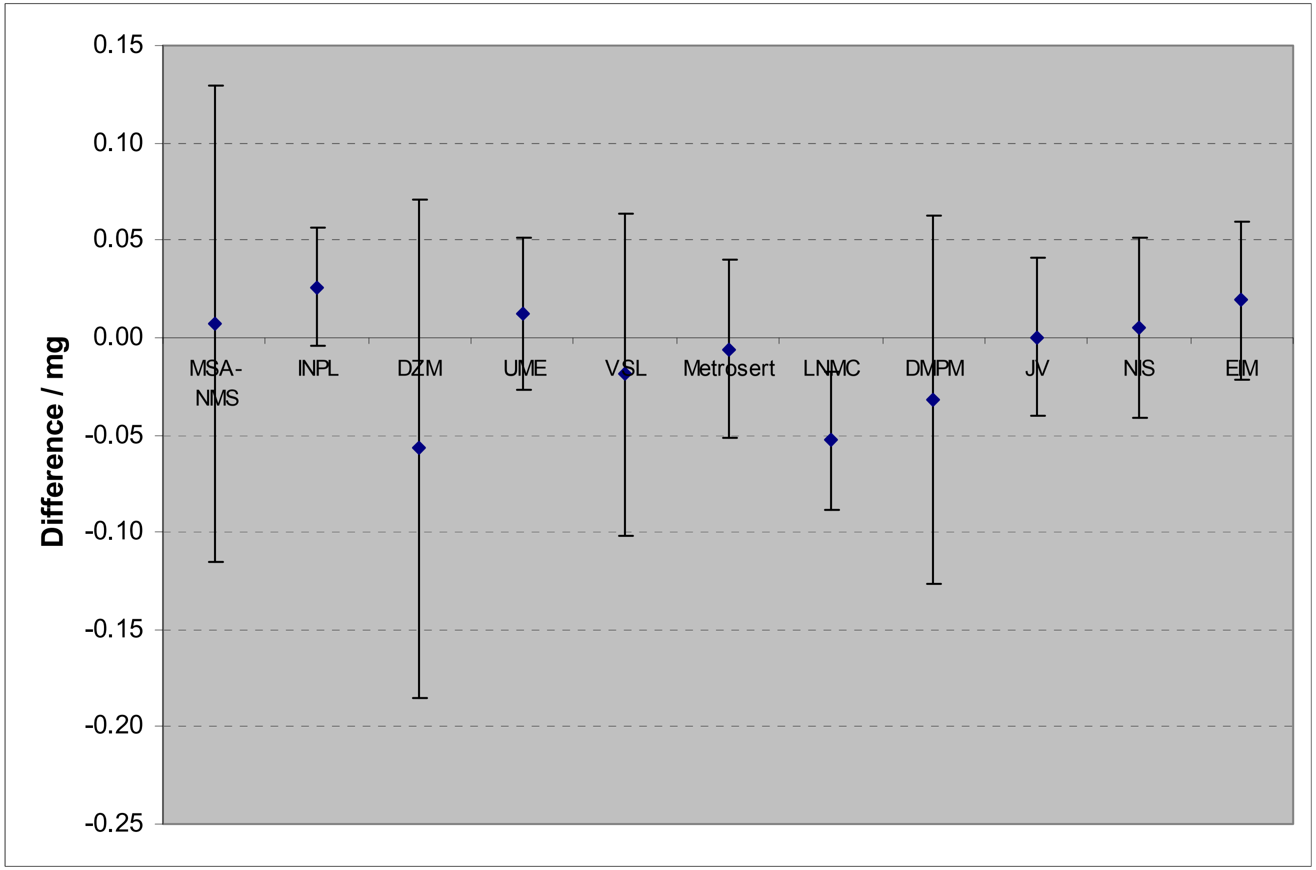

Figure 3: Differences between participants’ results and reference value, and uncertainty ($k = 2$), for 20 g

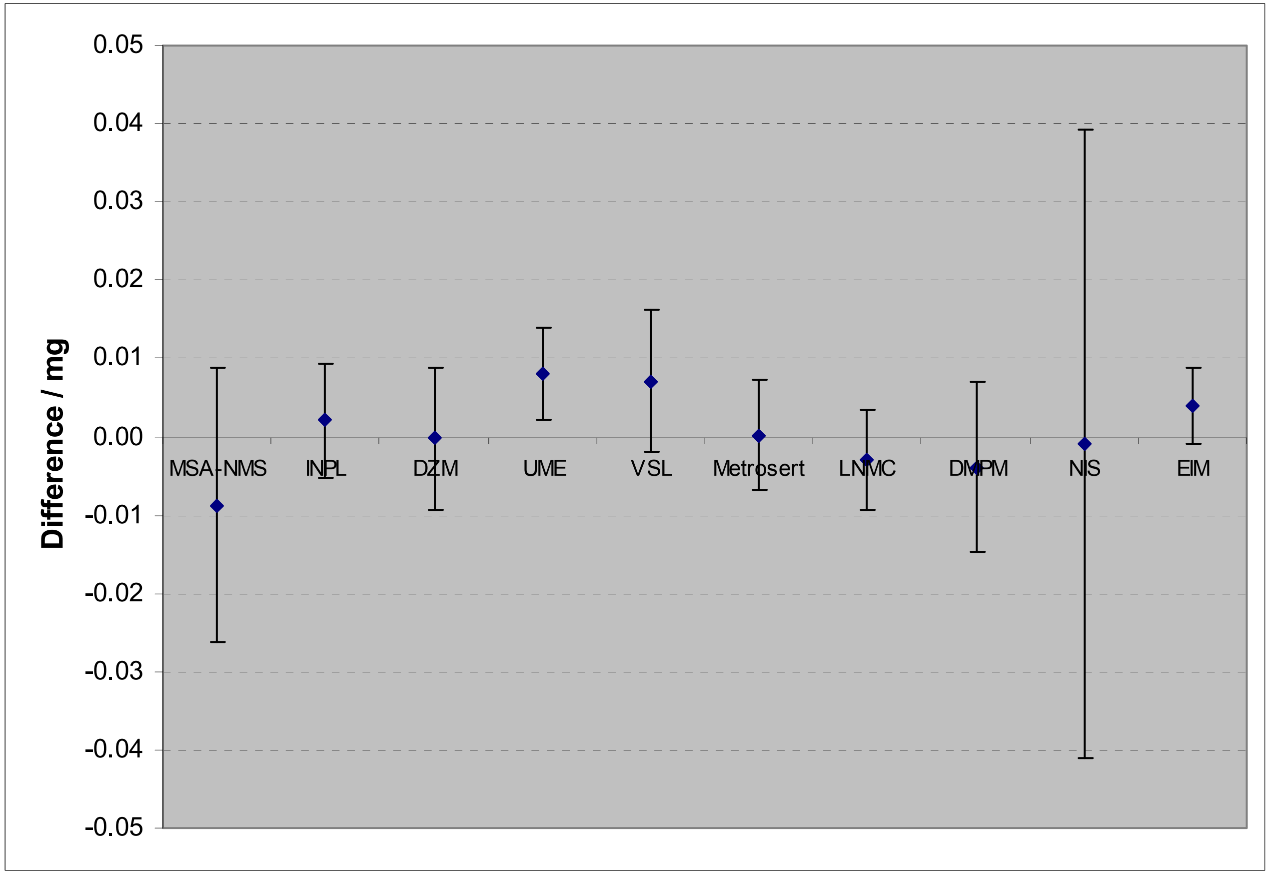

Figure 4: Differences between participants' results and reference value, and uncertainty ($k = 2$), for 2 g

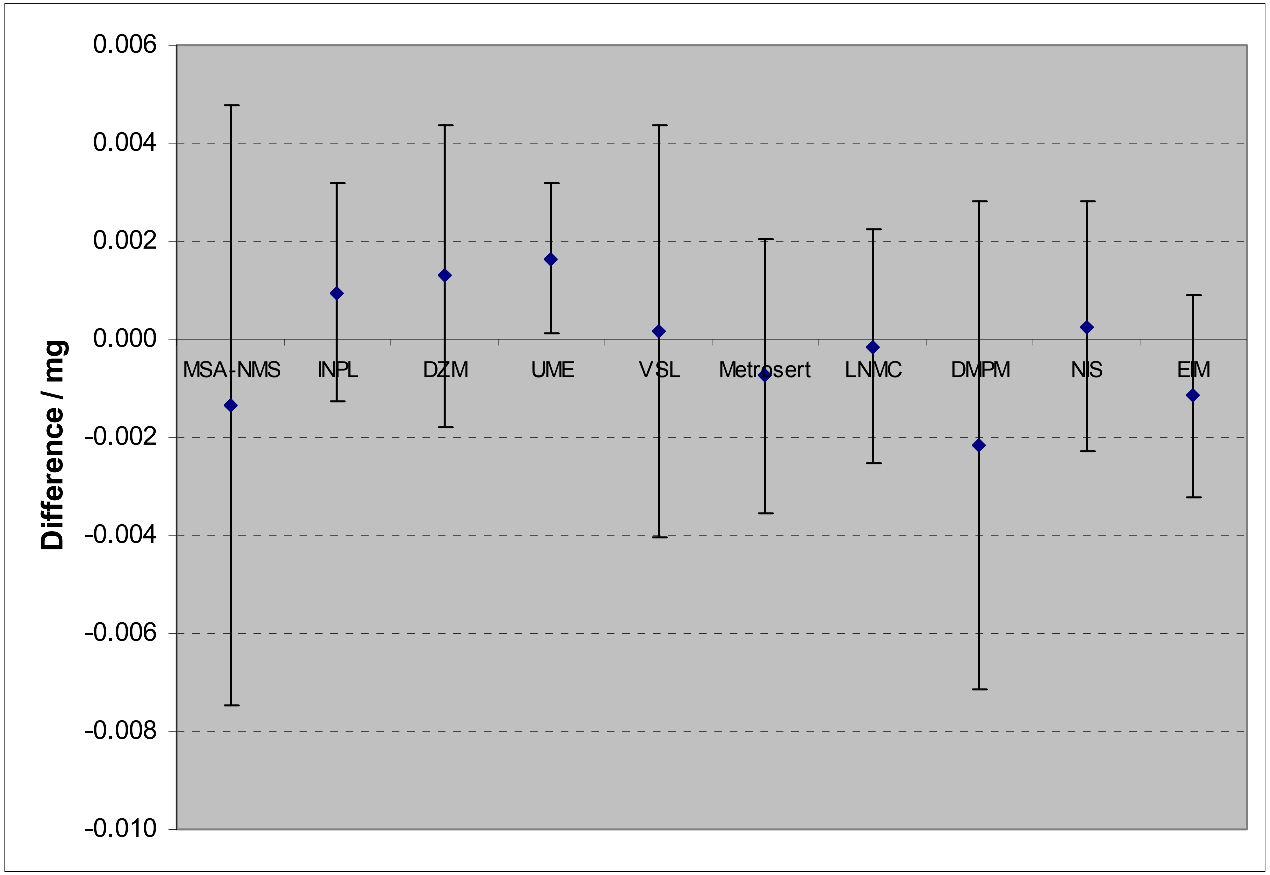

Figure 5: Differences between participants' results and reference value, and uncertainty ($k = 2$), for 100 mg

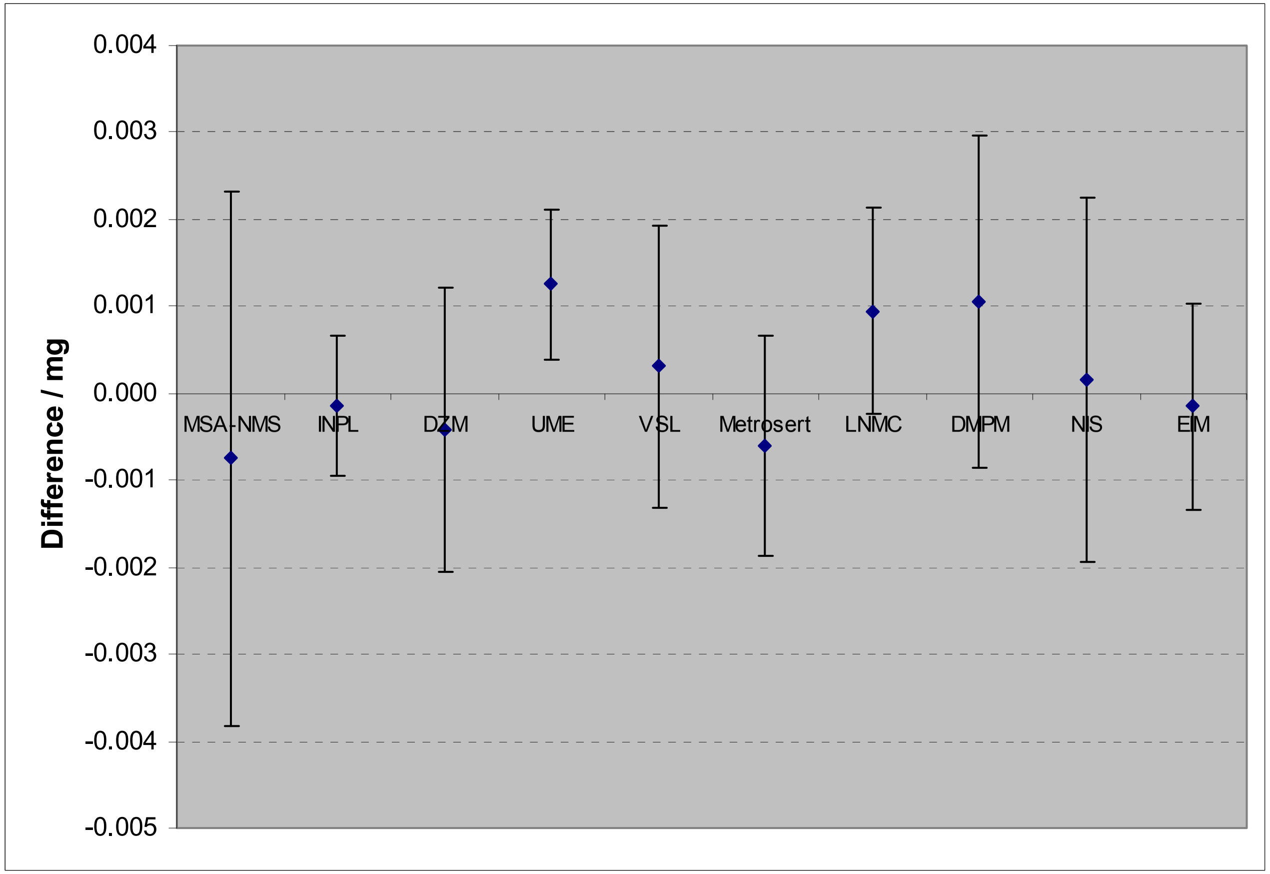

Table 7: Differences *Δm* (top) in assigned values between laboratory A (left column) and laboratory B (top row) and expanded uncertainties at $k = 2$ (bottom) for 10 kg

Δm /mg	MSA-NMS	INPL	DZM	UME	VSL	Metrosert	LNMC	DMPM	NIS	EIM
MSA-NMS		-1.40	0.28	*-4.60*	-0.26	-0.71	-0.91	-0.70	-0.61	-0.90
INPL	1.40		1.68	*-3.20*	1.14	0.69	0.49	0.70	0.79	0.50
DZM	-0.28	-1.68		*-4.88*	-0.54	-0.99	-1.19	-0.98	-0.89	-1.18
UME	*4.60*	*3.20*	*4.88*		*4.34*	*3.89*	*3.69*	*3.90*	*3.99*	*3.70*
VSL	0.26	-1.14	0.54	*-4.34*		-0.45	-0.65	-0.44	-0.35	-0.64
Metrosert	0.71	-0.69	0.99	*-3.89*	0.45		-0.20	0.01	0.10	-0.19
LNMC	0.91	-0.49	1.19	*-3.69*	0.65	0.20		0.21	0.30	0.01
DMPM	0.70	-0.70	0.98	*-3.90*	0.44	-0.01	-0.21		0.09	-0.20
NIS	0.61	-0.79	0.89	*-3.99*	0.35	-0.10	-0.30	-0.09		-0.29
EIM	0.90	-0.50	1.18	*-3.70*	0.64	0.19	-0.01	0.20	0.29	

$U_{\Delta m}$ /mg	MSA-NMS	INPL	DZM	UME	VSL	Metrosert	LNMC	DMPM	NIS	EIM
MSA-NMS		2.61	3.77	2.89	2.84	2.74	2.88	3.06	2.48	2.59
INPL	2.61		3.07	1.90	1.81	1.65	1.87	2.14	1.18	1.39
DZM	3.77	3.07		3.32	3.27	3.18	3.30	3.46	2.96	3.05
UME	2.89	1.90	3.32		2.20	2.07	2.25	2.48	1.72	1.87
VSL	2.84	1.81	3.27	2.20		2.00	2.18	2.41	1.63	1.78
Metrosert	2.74	1.65	3.18	2.07	2.00		2.05	2.29	1.45	1.62
LNMC	2.88	1.87	3.30	2.25	2.18	2.05		2.46	1.70	1.84
DMPM	3.06	2.14	3.46	2.48	2.41	2.29	2.46		1.98	2.11
NIS	2.48	1.18	2.96	1.72	1.63	1.45	1.70	1.98		1.13
EIM	2.59	1.39	3.05	1.87	1.78	1.62	1.84	2.11	1.13	

Table 8: Differences *Δm* (top) in assigned values between laboratory A (left column) and laboratory B (top row) and expanded uncertainties at *k* = 2 (bottom) for 500 g

Δm/mg	MSA-NMS	INPL	DZM	UME	VSL	Metrosert	LNMC	DMPM	JV	NIS	EIM
MSA-NMS		-0.019	0.064	-0.005	0.026	0.013	0.060	0.039	0.007	0.002	-0.012
INPL	0.019		0.083	0.014	0.045	0.032	*0.079*	0.058	0.026	0.021	0.007
DZM	-0.064	-0.083		-0.069	-0.038	-0.051	-0.004	-0.025	-0.057	-0.062	-0.076
UME	0.005	-0.014	0.069		0.031	0.018	*0.065*	0.044	0.012	0.007	-0.007
VSL	-0.026	-0.045	0.038	-0.031		-0.013	0.034	0.013	-0.019	-0.024	-0.038
Metrosert	-0.013	-0.032	0.051	-0.018	0.013		0.047	0.026	-0.006	-0.011	-0.025
LNMC	-0.060	*-0.079*	0.004	*-0.065*	-0.034	-0.047		-0.021	*-0.053*	*-0.058*	*-0.072*
DMPM	-0.039	-0.058	0.025	-0.044	-0.013	-0.026	0.021		-0.032	-0.037	-0.051
JV	-0.007	-0.026	0.057	-0.012	0.019	0.006	*0.053*	0.032		-0.005	-0.019
NIS	-0.002	-0.021	0.062	-0.007	0.024	0.011	*0.058*	0.037	0.005		-0.014
EIM	0.012	-0.007	0.076	0.007	0.038	0.025	*0.072*	0.051	0.019	0.014	

$U_{\Delta m}$/mg	MSA-NMS	INPL	DZM	UME	VSL	Metrosert	LNMC	DMPM	JV	NIS	EIM
MSA-NMS		0.122	0.175	0.125	0.145	0.127	0.124	0.152	0.125	0.127	0.125
INPL	0.122		0.128	0.040	0.083	0.046	0.037	0.095	0.041	0.046	0.041
DZM	0.175	0.128		0.131	0.150	0.133	0.130	0.157	0.131	0.133	0.131
UME	0.125	0.040	0.131		0.087	0.053	0.044	0.098	0.048	0.053	0.048
VSL	0.145	0.083	0.150	0.087		0.090	0.086	0.123	0.088	0.090	0.088
Metrosert	0.127	0.046	0.133	0.053	0.090		0.050	0.101	0.054	0.058	0.054
LNMC	0.124	0.037	0.130	0.044	0.086	0.050		0.097	0.046	0.050	0.046
DMPM	0.152	0.095	0.157	0.098	0.123	0.101	0.097		0.099	0.101	0.099
JV	0.125	0.041	0.131	0.048	0.088	0.054	0.046	0.099		0.054	0.050
NIS	0.127	0.046	0.133	0.053	0.090	0.058	0.050	0.101	0.054		0.054
EIM	0.125	0.041	0.131	0.048	0.088	0.054	0.046	0.099	0.050	0.054	

Table 9: Differences *Δm* (top) in assigned values between laboratory A (left column) and laboratory B (top row) and expanded uncertainties at $k = 2$ (bottom) for 20 g

Δm /mg	MSA-NMS	INPL	DZM	UME	VSL	Metrosert	LNMC	DMPM	NIS	EIM
MSA-NMS		-0.011	-0.009	-0.017	-0.016	-0.009	-0.006	-0.005	-0.008	-0.013
INPL	0.011		0.002	-0.006	-0.005	0.002	0.005	0.006	0.003	-0.002
DZM	0.009	-0.002		-0.008	-0.007	0.000	0.003	0.004	0.001	-0.004
UME	0.017	0.006	0.008		0.001	*0.008*	*0.011*	*0.012*	0.009	0.004
VSL	0.016	0.005	0.007	-0.001		0.007	*0.010*	0.011	0.008	0.003
Metrosert	0.009	-0.002	0.000	*-0.008*	-0.007		0.003	0.004	0.001	-0.004
LNMC	0.006	-0.005	-0.003	*-0.011*	*-0.010*	-0.003		0.001	-0.002	*-0.007*
DMPM	0.005	-0.006	-0.004	*-0.012*	-0.011	-0.004	-0.001		-0.003	-0.008
NIS	0.008	-0.003	-0.001	-0.009	-0.008	-0.001	0.002	0.003		-0.005
EIM	0.013	0.002	0.004	-0.004	-0.003	0.004	*0.007*	0.008	0.005	

$U_{\Delta m}$ /mg	MSA-NMS	INPL	DZM	UME	VSL	Metrosert	LNMC	DMPM	NIS	EIM
MSA-NMS		0.018	0.019	0.018	0.019	0.018	0.018	0.020	0.044	0.018
INPL	0.018		0.011	0.008	0.011	0.009	0.009	0.012	0.041	0.007
DZM	0.019	0.011		0.010	0.012	0.010	0.010	0.013	0.041	0.009
UME	0.018	0.008	0.010		0.010	0.008	0.007	0.011	0.040	0.006
VSL	0.019	0.011	0.012	0.010		0.010	0.010	0.013	0.041	0.009
Metrosert	0.018	0.009	0.010	0.008	0.010		0.008	0.012	0.041	0.007
LNMC	0.018	0.009	0.010	0.007	0.010	0.008		0.012	0.040	0.006
DMPM	0.020	0.012	0.013	0.011	0.013	0.012	0.012		0.041	0.011
NIS	0.044	0.041	0.041	0.040	0.041	0.041	0.040	0.041		0.040
EIM	0.018	0.007	0.009	0.006	0.009	0.007	0.006	0.011	0.040	

Table 10: Differences *Δm* (top) in assigned values between laboratory A (left column) and laboratory B (top row) and expanded uncertainties at $k = 2$ (bottom) for 2 g

Δm /mg	MSA-NMS	INPL	DZM	UME	VSL	Metrosert	LNMC	DMPM	NIS	EIM
MSA-NMS		-0.002 3	-0.002 6	-0.003 0	-0.001 5	-0.000 6	-0.001 2	0.000 8	-0.001 6	-0.000 2
INPL	0.002 3		-0.000 3	-0.000 7	0.000 8	0.001 7	0.001 1	0.003 1	0.000 7	0.002 1
DZM	0.002 6	0.000 3		-0.000 4	0.001 1	0.002 0	0.001 4	0.003 4	0.001 0	0.002 4
UME	0.003 0	0.000 7	0.000 4		0.001 5	0.002 4	0.001 8	0.003 8	0.001 4	*0.002 8*
VSL	0.001 5	-0.000 8	-0.001 1	-0.001 5		0.000 9	0.000 3	0.002 3	-0.000 1	0.001 3
Metrosert	0.000 6	-0.001 7	-0.002 0	-0.002 4	-0.000 9		-0.000 6	0.001 4	-0.001 0	0.000 4
LNMC	0.001 2	-0.001 1	-0.001 4	-0.001 8	-0.000 3	0.000 6		0.002 0	-0.000 4	0.001 0
DMPM	-0.000 8	-0.003 1	-0.003 4	-0.003 8	-0.002 3	-0.001 4	-0.002 0		-0.002 4	-0.001 0
NIS	0.001 6	-0.000 7	-0.001 0	-0.001 4	0.000 1	0.001 0	0.000 4	0.002 4		0.001 4
EIM	0.000 2	-0.002 1	-0.002 4	*-0.002 8*	-0.001 3	-0.000 4	-0.001 0	0.001 0	-0.001 4	

$U_{\Delta m}$/mg	MSA-NMS	INPL	DZM	UME	VSL	Metrosert	LNMC	DMPM	NIS	EIM
MSA-NMS		0.006 4	0.006 7	0.006 2	0.007 3	0.006 6	0.006 4	0.007 8	0.006 5	0.006 3
INPL	0.006 4		0.003 5	0.002 3	0.004 5	0.003 3	0.002 9	0.005 2	0.003 1	0.002 7
DZM	0.006 7	0.003 5		0.003 1	0.005 0	0.003 9	0.003 6	0.005 7	0.003 7	0.003 4
UME	0.006 2	0.002 3	0.003 1		0.004 2	0.002 8	0.002 4	0.005 0	0.002 6	0.002 1
VSL	0.007 3	0.004 5	0.005 0	0.004 2		0.004 8	0.004 6	0.006 3	0.004 7	0.004 5
Metrosert	0.006 6	0.003 3	0.003 9	0.002 8	0.004 8		0.003 4	0.005 5	0.003 5	0.003 2
LNMC	0.006 4	0.002 9	0.003 6	0.002 4	0.004 6	0.003 4		0.005 3	0.003 2	0.002 8
DMPM	0.007 8	0.005 2	0.005 7	0.005 0	0.006 3	0.005 5	0.005 3		0.005 4	0.005 2
NIS	0.006 5	0.003 1	0.003 7	0.002 6	0.004 7	0.003 5	0.003 2	0.005 4		0.002 9
EIM	0.006 3	0.002 7	0.003 4	0.002 1	0.004 5	0.003 2	0.002 8	0.005 2	0.002 9	

Table 11: Differences *Δm* (top) in assigned values between laboratory A (left column) and laboratory B (top row) and expanded uncertainties at *k* = 2 (bottom) for 100 mg

Δm /mg	MSA-NMS	INPL	DZM	UME	VSL	Metrosert	LNMC	DMPM	NIS	EIM
MSA-NMS		-0.000 6	-0.000 3	-0.002 0	-0.001 1	-0.000 1	-0.001 7	-0.001 8	-0.000 9	-0.000 6
INPL	0.000 6		0.000 3	*-0.001 4*	-0.000 5	0.000 5	-0.001 1	-0.001 2	-0.000 3	0.000 0
DZM	0.000 3	-0.000 3		-0.001 7	-0.000 7	0.000 2	-0.001 4	-0.001 5	-0.000 6	-0.000 3
UME	0.002 0	*0.001 4*	0.001 7		0.000 9	*0.001 9*	0.000 3	0.000 2	0.001 1	0.001 4
VSL	0.001 1	0.000 5	0.000 7	-0.000 9		0.000 9	-0.000 6	-0.000 7	0.000 2	0.000 5
Metrosert	0.000 1	-0.000 5	-0.000 2	*-0.001 9*	-0.000 9		-0.001 6	-0.001 7	-0.000 8	-0.000 5
LNMC	0.001 7	0.001 1	0.001 4	-0.000 3	0.000 6	0.001 6		-0.000 1	0.000 8	0.001 1
DMPM	0.001 8	0.001 2	0.001 5	-0.000 2	0.000 7	0.001 7	0.000 1		0.000 9	0.001 2
NIS	0.000 9	0.000 3	0.000 6	-0.001 1	-0.000 2	0.000 8	-0.000 8	-0.000 9		0.000 3
EIM	0.000 6	0.000 0	0.000 3	-0.001 4	-0.000 5	0.000 5	-0.001 1	-0.001 2	-0.000 3	

$U_{\Delta m}$/mg	MSA-NMS	INPL	DZM	UME	VSL	Metrosert	LNMC	DMPM	NIS	EIM
MSA-NMS		0.003 1	0.003 5	0.003 2	0.003 4	0.003 3	0.003 3	0.003 6	0.003 7	0.003 3
INPL	0.003 1		0.001 8	0.001 1	0.001 8	0.001 4	0.001 4	0.002 0	0.002 2	0.001 4
DZM	0.003 5	0.001 8		0.001 8	0.002 3	0.002 0	0.002 0	0.002 5	0.002 6	0.002 0
UME	0.003 2	0.001 1	0.001 8		0.001 8	0.001 5	0.001 4	0.002 1	0.002 2	0.001 4
VSL	0.003 4	0.001 8	0.002 3	0.001 8		0.002 0	0.002 0	0.002 5	0.002 6	0.002 0
Metrosert	0.003 3	0.001 4	0.002 0	0.001 5	0.002 0		0.001 7	0.002 3	0.002 4	0.001 7
LNMC	0.003 3	0.001 4	0.002 0	0.001 4	0.002 0	0.001 7		0.002 2	0.002 4	0.001 6
DMPM	0.003 6	0.002 0	0.002 5	0.002 1	0.002 5	0.002 3	0.002 2		0.002 8	0.002 2
NIS	0.003 7	0.002 2	0.002 6	0.002 2	0.002 6	0.002 4	0.002 4	0.002 8		0.002 4
EIM	0.003 3	0.001 4	0.002 0	0.001 4	0.002 0	0.001 7	0.001 6	0.002 2	0.002 4	

Appendix A: Linking EURAMET.M.M-K2.1 to CCM.M-K2

A1. Background

This Appendix describes the method used to link EURAMET.M.M-K2.1 to CCM.M-K2 which is based upon that described by Sutton [7]. This method of linking is consistent with that used to link EURAMET.M.M-K2 to CCM.M-K2 [1].

A2. Data

A.2.1 EURAMET.M.M-K2.1

EURAMET.M.M-K2.1 is a European Key Comparison of five mass standards (10 kg, 500 g, 20 g, 2 g and 100 mg) in stainless steel comprising eleven participating laboratories and INRIM (the pilot laboratory).

A single set of five transfer standards was circulated to all the participating laboratories. The pilot laboratory measured each standard four times, and the measured data obtained was used as the basis of investigating the stability of the standards.

Information about the measurements made of the transfer standards is provided in Table 3. The information includes (a) the nominal mass of the standard, (b) the laboratory name, (c) the measured mass difference from nominal value, and (d) the standard uncertainty associated with the measured mass difference. Information about the measurements made by the Pilot laboratory during the lifetime of the comparison can be found in Table 4.

No information is provided about the correlation associated with pairs of measured values. For the purpose of the analysis described here, the following ‘simple rules’ are applied:

- The correlation coefficient associated with pairs of measured values provided by the same laboratory (such as the Pilot laboratory) is set as 0.8;
- There is no correlation associated with pairs of measured values provided by different laboratories.

A.2.2 CCM.M−K2

CCM.M−K2 is a CIPM Key Comparison of five mass standards (10 kg, 500 g, 20 g, 2 g and 100 mg) in stainless steel, comprising fourteen laboratories. One of the laboratories (VSL) participated in the EURAMET.M-K2.1 Key Comparison and another (INRIM) acted as the Pilot laboratory for EURAMET.M-K2.1. These two laboratories are used as the basis of linking the two Key Comparisons.

Table 11 of the final CCM.M-K2 report [2] contains the degrees of equivalence for the linking laboratories obtained from the CCM.M−K2 Key Comparison. The degrees of equivalence are expressed as a value with an associated uncertainty reported for a 95 % coverage probability.

For the purposes of linking EURAMET.M-K2.1 and CCM.M−K2 it is necessary to account for the correlation associated with pairs of measured values provided in the two comparisons. In the absence of information about such correlations, the following ‘simple rules’ are applied:

- The correlation coefficient associated with a mass difference measured by VSL (or INRIM) in EURAMET.M-K2.1 and the value component of the degree of equivalence for the laboratory obtained in CCM.M−K2 is set as 0.4;
- The correlation coefficient associated with the value components of the degrees of equivalence for VSL and INRIM obtained in CCM.M−K2 is set as 0.4.

The values used for the correlation coefficients are based on the results of discussions between NPL and BIPM metrologists.

A3. Model

Consider one of the masses (transfer standards) with nominal mass value m_0.

Let D_i, i = 1, …, 11 (or 12), denote the value component of the degree of equivalence for laboratory i, where i = 1, …, 10 (or 11), identify the laboratories participating in EURAMET.M-K2.1, and i = 11 (or 12) identifies the Pilot laboratory of that comparison. (Generally, ten laboratories participated in EURAMET.M-K2.1, except for the 500 g transfer standard for which an additional laboratory, *viz.* JV, also participated.)

Let $\varDelta$ denote the mass difference of the transfer standard from the nominal value m_0, and X_i the mass difference measured by laboratory i. Then, a model for X_i in terms of D_i and $\varDelta$ is

$$X_i = D_i + \varDelta. \qquad (1)$$

Tables 3 and 4 contain measured values x_i for X_i for i = 1, …, 11 (or 12), with associated standard uncertainties $u(x_i)$. Table 11 in CCM.M-K2 report contains measured values d_i for D_i for the linking laboratories identified by i = 5 (VSL) and i = 11 (or 12), which is the Pilot laboratory. Assuming that all measurement results are included in the analysis, there are 12 (or 13) parameters (11 or 12 parameters D_i and $\varDelta$) to be determined and 16 (or 17) measured values (10 or 11 measurements relating to the laboratories participating in EURAMET.M-K2.1, 4 measurements relating to the Pilot laboratory and 2 relating to degrees of equivalence for the linking laboratories).

If we denote the vector of parameters by **Y** and that of measured quantities by **X**, then

$$\mathbf{X} = \mathbf{AY},$$

where **A** is a 16 × 12 (or 17 × 13) matrix determined by the relationships (1) and the information provided by CCM.M−K2. Given an estimate **x** of **X** with the associated uncertainty matrix $\mathbf{U_x}$, an estimate **y** of **Y** with the associated uncertainty matrix $\mathbf{U_y}$ is found as the solution **z** = **y** to the generalised least-squares problem

$$\min_{\mathbf{z}} \quad (\mathbf{x} - \mathbf{Az})^{\mathrm{T}} \mathbf{U}_{\mathbf{x}}^{-1} (\mathbf{x} - \mathbf{Az}).$$

The components of **y** contain estimates of the value components of the degree of equivalence for the laboratories (including the Pilot laboratory) and an estimate of the mass difference for the transfer standard. The diagonal elements of $\mathbf{U_y}$ contain the variances (squared standard uncertainties) associated with the estimates **y**.

A4. Model and data consistency

For the model described in section A3 to constitute a valid description of the measurement data, it is necessary to show that the model is consistent with the data taking account of the uncertainties and correlations associated with the data.

For the case of two measured values (z_1 and z_2, say), the values are judged consistent (and therefore realizations of the same quantity) if the magnitude of the difference $r = z_1 - z_2$ is not too large compared with the standard uncertainty $u(r)$ associated with the difference, where $u^2(r) = u^2(z_1) + u^2(z_2) - 2\rho(z_1, z_2)u(z_1)\ u(z_2)$ and $\rho(z_1,\ z_2)$ is the correlation coefficient associated with z_1 and z_2. Regarding the quantities involved as Gaussian, the values are regarded as consistent, at the 95 % level, if

$$|r| < 2u(r).$$

For the case of a model fitted to a general number of measured values, a measure of consistency is the sum of squares of the (uncertainty-) weighted model residuals: the 'observed chi-squared value'. If the value of this measure is no greater than an appropriate percentile of a chi-squared distribution, here chosen to be the 95 percentile, the model is judged consistent with the data, and inconsistent otherwise. For the model described in section A3, the (uncertainty-) weighted model residuals are

$$\mathbf{r} = \mathbf{L}^{-1}(\mathbf{x} - \mathbf{A}\mathbf{y}), \qquad \mathbf{U}_{\mathbf{x}} = \mathbf{L}\mathbf{L}^{\mathrm{T}},$$

and the chi-squared distribution has degrees of freedom equal to the difference between the number of measured values (e.g., 16 or 17) and the number of parameters (e.g., 12 or 13).

Tests of consistency are applied to:

1. the ten (or eleven) measured values provided for each transfer standard by the laboratories participating in EURAMET.M-K2.1;
2. the four measured values provided for each transfer standard by the Pilot laboratory of EURAMET.M-K2.1;
3. the two degrees of equivalence provided by the linking laboratories participating in CCM.M−K2.

These tests are used to investigate the consistency of subsets of the measured data. In the case that a subset of the measured data is found to be inconsistent, the results of the test are used as the basis of removing measured values from the subset to ensure consistency. Table 12 records the results of these tests (in terms of which measured data are removed) for each nominal mass value.

A test of consistency is also applied to the model described in section A3 and the complete set of measured data (with data removed as described in Table 12). This test is used to investigate whether the model constitutes a valid description of the 'union' of the different subsets of the measured data. For the results indicated in section 5, this test of consistency is passed.

For a laboratory participating in EURAMET.M-K2.1 that has been removed from the calculation of the reference value (e.g., UME for the 10 kg mass standard), the value component of the degree of equivalence for that laboratory is evaluated from

$$d = x - x_{\text{ref}},$$

with associated standard uncertainty $u(d)$ given by

$$u^2(d) = u^2(x) + u^2(x_{\text{ref}}),$$

where x is the measured value provided by the laboratory with associated standard uncertainty $u(x)$ and x_{ref} is an estimate of the mass difference Δ for the transfer standard with associated standard uncertainty $u(x_{\text{ref}})$.

Nominal mass m_0	**Measured data removed from calculation of reference value**
10 kg	UME measured data from EURAMET.M-K2.1
500 g	LNMC measured data from EURAMET.M-K2.1
	First Pilot laboratory measured data from EURAMET.M-K2.1
	VSL degree of equivalence from CCM.M−K2
20 g	UME and LNMC measured data from EURAMET.M-K2.1
	First and fourth Pilot laboratory measured data from EURAMET.M-K2.1
2 g	Second Pilot laboratory measured data from EURAMET.M-K2.1
100 mg	UME measured data from EURAMET.M-K2.1
	First Pilot laboratory measured data from EURAMET.M-K2.1

Table 12 For each nominal mass value, the measured data removed from the calculation of the reference value to ensure data consistency.

A5. Results

The results from the linkage are given in Table 12. Using these data it is possible to calculate the degree of equivalence between any of the laboratories participating in EUROMET.M.M-K2.1 and those participating in CCM.M−K2.

Table 13: Differences between participants' results and CCM.M-K2 reference value, Δm, and associated k=2 uncertainties, $U_{\Delta m}$

		MSA-NMS	INPL	DZM	UME	VSL	Metrosert	LNMC	DMPM	JV	NIS	EIM
10 kg	Δm/mg	-0.67	0.73	-0.95	3.93	-0.41	0.04	0.24	0.03		-0.06	0.23
	$U_{\Delta m}$/mg	2.50	1.22	2.98	1.74	1.58	1.47	1.73	2.00		0.92	1.17
500 g	Δm/mg	0.044	0.063	-0.020	0.049	0.018	0.031	-0.016	0.005	0.037	0.042	0.056
	$U_{\Delta m}$/mg	0.121	0.026	0.127	0.036	0.082	0.043	0.032	0.093	0.038	0.043	0.038
20 g	Δm/mg	-0.013	-0.002	-0.005	0.004	0.002	-0.004	-0.007	-0.008		-0.005	0.000
	$U_{\Delta m}$/mg	0.019	0.010	0.011	0.009	0.009	0.009	0.009	0.013		0.041	0.008
2 g	Δm/mg	-0.002 9	-0.000 6	-0.000 3	0.000 1	-0.001 6	-0.002 3	-0.001 7	-0.003 7		-0.001 3	-0.002 7
	$U_{\Delta m}$/mg	0.006 9	0.003 9	0.004 5	0.003 6	0.005 1	0.004 2	0.004 0	0.005 9		0.004 1	0.003 8
100 mg	Δm/mg	-0.000 7	-0.000 1	-0.000 4	0.001 3	0.000 8	-0.000 6	0.001 0	0.001 1		0.000 2	-0.000 1
	$U_{\Delta m}$/mg	0.003 3	0.001 5	0.002 1	0.001 5	0.001 7	0.001 9	0.001 7	0.002 3		0.002 4	0.001 7