Linking the Results of Key Comparisons CCEM-K8 and EUROMET.EM-K8

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1. Introduction

Key comparisons CCEM-K8 and EUROMET.EM-K8 on DC voltage ratio [1, 2] were carried out in parallel, both having IEN as the pilot laboratory. Two Datron 4902S voltage dividers were used as travelling standards, each divider being dedicated to one comparison. On these dividers the measurements of the ratios 1000 V / 10 V and 100 V / 10 V were mandatory and were used to evaluate the degrees of equivalence of the participants with respect to the reference value of their own comparison(1). The mandatory measurements were performed by all the 15 participants in CCEM-K8 and by 19 out of the 20 participants in EUROMET.EM-K8.

The reference value of a key comparison carried out by a Consultative Committee of the CIPM is called Key Comparison Reference Value (KCRV), and the degrees of equivalence of a regional comparison must be translated in terms of the KCRV.

2. Linking method

The comparisons under exam concern the measurement of voltage ratios, hence no national standard is involved. Then any participant in both comparisons can be considered equally able, in principle, to perform an unbiased measurement of the travelling standard. If the number of participants in each comparison is significant and different methods of measurement are used, it is unlikely that the corresponding comparison reference value is affected by systematic errors, because such errors should then be common to all participants. If the reference values of the two comparisons are not affected by systematic errors they are equivalent, and the degrees of equivalence with respect to the KCRV of the participants in the regional comparison are the same as those evaluated in that comparison.

Table B1 in Appendix 2 reports, for both comparisons and for ratios 1000 V / 10 V and 100 V / 10 V, the number of laboratories participating in the definition of the reference values, the reference values themselves, their standard uncertainty and the corresponding degrees of freedom. It is worth to notice that the reference values of the two comparisons, defined as the mean difference of the laboratory results with respect to the pilot result, coincide within their uncertainties, which also supports their equivalence.

Fig. A in Appendix A reports graphically the degrees of equivalence of the participants in both comparisons.

3. Verification of the reproducibility of the degrees of equivalence

To evaluate the reproducibility of the degrees of equivalence in the two comparisons, the results of the common participants, also called linking laboratories, can be used. The essential requirement of a linking laboratory is that any possible bias affecting its measurements remains reasonably constant over the time period of the two comparisons. An analysis of the results of the comparisons under exam shows that, for both mandatory ratios, NPL results can not be used, being quite far from the reference value in CCEM-K8 and compatible with the reference value in EUROMET.EM-K8. The same situation applies to CEM, even if the disagreement in CCEM-K8 is lower. Then we

(1) Following the Mutual Recognition Arrangement of the CIPM, the degree of equivalence of a laboratory with respect to the reference value is given by two numbers: the difference of the laboratory's result with respect to the reference value and the expanded uncertainty of this difference.

remain with LCIE, SP and IEN, from which the difference between the degrees of equivalence of the two comparisons and the related uncertainty can be evaluated.
Similarly to the model proposed in [3], the following symbols will be used:

D_e' : difference between the result of a laboratory that has participated in EUROMET.EM-K8 and the corresponding CRV;

D_e : estimate of the difference between the result of a laboratory that has participated in EUROMET.EM-K8 and the KCRV, had the laboratory actually participated in CCEM-K8.

The model equation is:

$$D_e = D_e' + d \tag{1}$$

An estimate d_k of d is given by the results of each linking laboratory. If the uncertainties $u_k(d)$ associated with d_k are trustworthy, the best estimate of d can be evaluated by the weighted mean of the d_ks:

$$d = \sum_{k=1}^{3} w_k d_k \; ; \qquad \frac{1}{u^2(d)} = \sum_{k=1}^{3} \frac{1}{u_k^2(d)} \tag{2}$$

where

$$w_k = \left(1/u_k^2(d)\right) \Big/ \sum_{k=1}^{3} \left(1/u_k^2(d)\right) \tag{3}$$

and the standard uncertainty associated with d_k , $u_k(d)$, is given by

$$u_k^2(d) = s_k^2 + s_k'^2 + 2u_{k,\mathrm{rep}}^2 + u_k^2(\varepsilon) + u_k'^2(\varepsilon) + u_k'^2(\zeta) \tag{4}$$

In eq. (4), the primed symbols refer to comparison EUROMET.EM-K8, s_k represents the instability of the travelling standard, $u_{k,\mathrm{rep}}$ represents the reproducibility of the measurements at a linking laboratory between the two comparisons, $u_k(\varepsilon)$ is the uncertainty associated with temperature and humidity corrections, $u_k'(\zeta)$ is the uncertainty associated with correction for the collective heating effect, only occurring in EUROMET.EM-K8 for ratio 1000 V / 10 V [2].

The values to be entered in eqs. (4), (3) and (2) and the intermediate results of the calculations are reported, for the two voltage ratios, in Tables B2 and B3 in Appendix B (for more information see also [1] and [2]). The linking laboratories reported to the comparison coordinator their estimate for the reproducibility $u_{k,\mathrm{rep}}$. The final results are given in Table 1.

Table 1. Values of d and $u(d)$ from the results of the linking laboratories using eq. (2).

	Ratio 1000 V / 10 V	Ratio 100 V / 10 V
$d/10^{-6}$	-0.126	0.042
$u(d)/10^{-6}$	0.065	0.066
ν_d	102	93

The uncertainty $u(d)$ given in Table 1 is based on the uncertainties $u_k(d)$ associated with each estimate of d following eq. (2), and is often referred to as the standard deviation of the mean by internal consistency. It can be compared with the weighted uncertainty based on the dispersion of the three estimates d_k, which is the standard deviation of the mean by external consistency:

$$u_{ext}^2(d) = \frac{1}{n-1}\sum_{k=1}^{n} w_k (d_k - d)^2 \qquad \text{with } n = 3 \tag{6}$$

The ratio $u_{ext}(d)/u(d)$ is often called Birge ratio R_B. It is:

$$R_{B,1000/10} = \frac{0.078}{0.065} = 1.19 \ ; \quad R_{B,100/10} = \frac{0.067}{0.083} = 0.80 \tag{7}$$

While for ratio 100 V / 10 V the value of R_B is lower than one, giving confidence that, for this ratio, $u(d)$ in Table 1 is not underestimated; this is not the case for ratio 1000 V / 10 V. Even if the difference from one, for this ratio, is not statistically significant(2), looking at Table B2 and comparing the values of d_k and $u_k(d)$, an underestimation of the reproducibility of LCIE is quite possible.

To have a quantitative evaluation of the significance of the difference d between the degrees of equivalence of the two comparisons, we can apply the t-test to the d-values obtained. The results are given in Table 2.

Table 2. t-test for the d-values of ratios 1000 V / 10 V and 100 V / 10

	Ratio 1000 V / 10 V $u(d)$ from eq. (2)	Ratio 1000 V / 10 V $u(d)$ from eq. (6)	Ratio 100 V / 10 V
$t_d = \lvert d / u(d) \rvert$	1.92	1.61	0.63
ν_d	102	2	93
prob. $t > t_d$ (%)	5.7	25	53

The result of the t-test for ratio 100 V / 10 V shows that the d-value for this ratio is not significantly different from zero. The same conclusion can be drawn for ratio 1000 V / 10 V if the uncertainty $u(d)$ from eq. (6) is used, which seems a more careful choice given a possible underestimation of the reproducibility of LCIE.

Conclusions

Because both comparisons CCEM-K8 and EUROMET.EM-K8 were participated by a significant number of laboratories and ratio measurements were requested, not involving units of measurement, they are considered equivalent and the degrees of equivalence obtained from the EUROMET comparison are assumed to be the same as if the participants had taken part in the CCEM comparison. This assumption is supported by the closeness of the reference values. It is also supported by the results of three linking laboratories for ratio 100 V / 10 V. The results of the linking laboratories for ratio 1000 V / 10 V supports the equivalence if the standard deviation of the mean by external consistency is used.

(2) A calculation would show that the probability of having R_B>1.19 is about 24%.

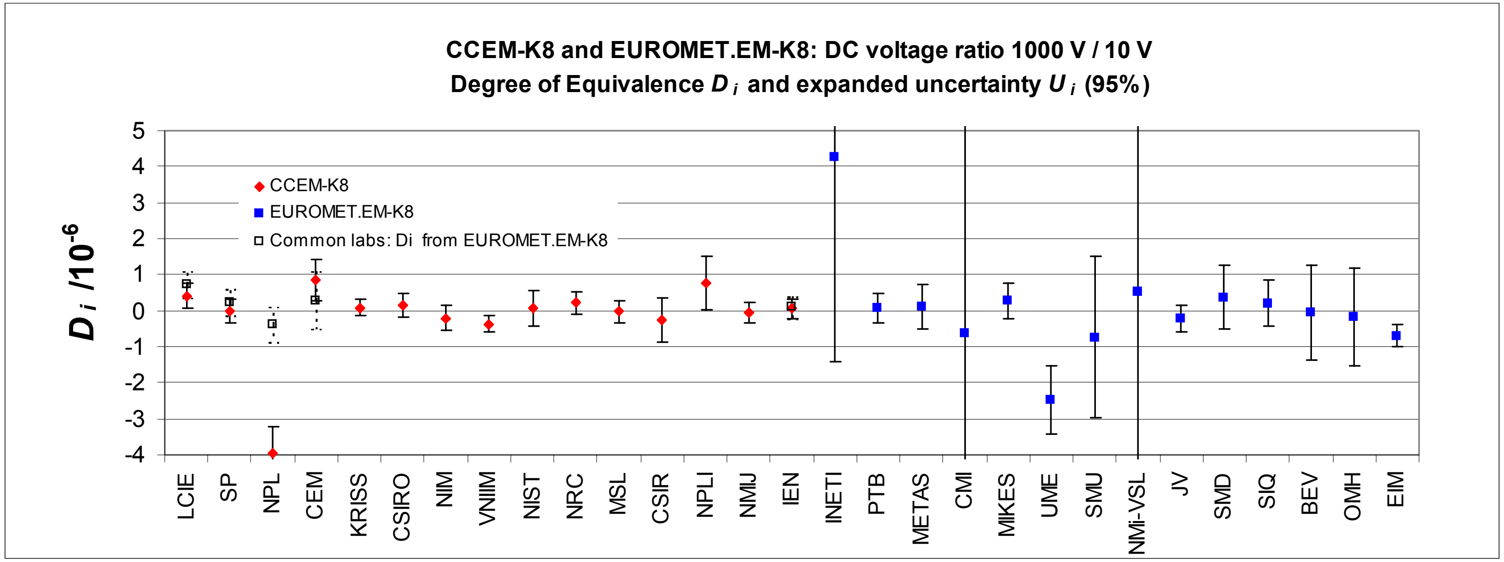
5. References

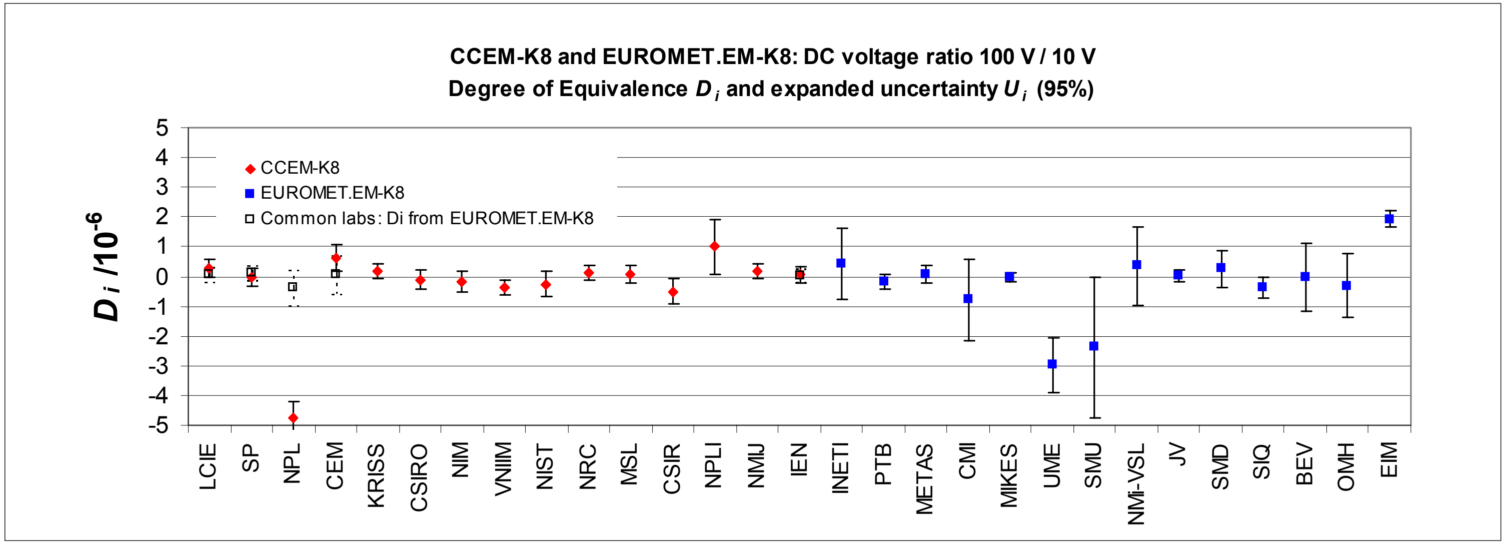
[1] G. Marullo Reedtz and R. Cerri, “Final Report of CCEM-K8 Comparison of DC Voltage Ratio”, IEN Technical Report 653, November 2002, published online in the *Key Comparison Data Base*: http://kcdb.bipm.fr/AppendixB/ (search for CCEM-K8).

[2] G. Marullo Reedtz and R. Cerri, “EUROMET.EM-K8 Comparison of DC Voltage Ratio (EUROMET Project 449)”, IEN Technical Report 670, December 2003, published online in the *Key Comparison Data Base*: http://kcdb.bipm.fr/AppendixB/ (search for EUROMET.EM-K8).

[3] F. Delahaye and T. J. Witt, “Linking the Results of Key Comparison CCEM-K4 with the 10 pF Results of EUROMET Project 345”, edition of 11 June 2002, published online in the *Key Comparison Data Base*: http://kcdb.bipm.fr/AppendixB/ (search for CCEM-K4).

Appendix A


CCEM-K8 and EUROMET.EM-K8: DC voltage ratio 1000 V / 10 V
Degree of Equivalence D_i and expanded uncertainty U_i (95%)
D_i /10^{-6}
CCEM-K8
EUROMET.EM-K8
Common labs: Di from EUROMET.EM-K8
5
4
3
2
1
0
-1
-2
-3
-4
LCIE
SP
NPL
CEM
KRISS
CSIRO
NIM
VNIIM
NIST
NRC
MSL
CSIR
NPLI
NMIJ
IEN
INETI
PTB
METAS
CMI
MIKES
UME
SMU
NMi-VSL
JV
SMD
SIQ
BEV
OMH
EIM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CCEM-K8
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5
4
3
2
1
0
-1
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-3
-4
-5
LCIE
SP
NPL
CEM
KRISS
CSIRO
NIM
VNIIM
NIST
NRC
MSL
CSIR
NPLI
NMIJ
IEN
INETI
PTB
METAS
CMI
MIKES
UME
SMU
NMi-VSL
JV
SMD
SIQ
BEV
OMH
EIM

Appendix B

Table B1. Number of participants, reference values Δ_R, corresponding standard uncertainties and degrees of freedom for CCEM-K8 and EUROMET.EM-K8 comparisons.

	CCEM-K8				EUROMET.EM-K8			
	N	Δ_R (10^{-6})	$u(\Delta_R)$ (10^{-6})	$\nu(\Delta_R)$	N	Δ_R (10^{-6})	$u(\Delta_R)$ (10^{-6})	$\nu(\Delta_R)$
Ratio 1000 V / 10 V	12	-0.048	0.062	11	17	-0.097	0.103	16
Ratio 100 V / 10 V	13	-0.050	0.083	12	16	-0.039	0.039	105

Table B2. Ratio 1000 V / 10 V. Values for calculation of d and $u(d)$. All values, except the degrees of freedom, are in units of 10^{-6}.

Lab	D_k	d_k	s_k	s_k'	$u_{k,\mathrm{rep}}$	$u_k(\varepsilon)$	$u'_k(\varepsilon)$	$u'_k(\zeta)$	$u_k(d)$
LCIE	0.415	-0.302	0.088	0.061	0.050	0.028	0.006	0.058	0.144
SP	-0.021	-0.246	0.092	0.061	0.105	0.010	0.008	0.000	0.185
IEN	0.048	-0.049	0.018	0.013	0.053	0.010	0.012	0.000	0.080

Lab	D_k'	w_k	$\nu_{k,\mathrm{s}}$	$\nu_{k,\mathrm{s}}'$	$\nu_{k,\mathrm{rep}}$	$\nu_{k,\varepsilon}$	$\nu_{k,\varepsilon}'$	$\nu_{k,\zeta}'$	$\nu_{\mathrm{eff}}(d_k)$
LCIE	0.717	0.206	6	19	inf.	3	2	2	25
SP	0.225	0.124	22	19	4	3	2	2	18
IEN	0.097	0.670	22	19	25	3	2	2	62

Table B3. Ratio 100 V / 10 V. Values for calculation of d and u(d). All values, except the degrees of freedom, are in units of 10^{-6}.

Lab	D_k	d_k	s_k	s_k'	$u_{k,\mathrm{rep}}$	$u_k(\varepsilon)$	$u'_k(\varepsilon)$	$u_k(d)$
LCIE	0.260	0.191	0.063	0.073	0.035	0.019	0.010	0.110
SP	-0.039	-0.173	0.083	0.073	0.064	0.057	0.010	0.154
IEN	0.050	0.012	0.016	0.015	0.043	0.070	0.012	0.096

Lab	D_k'	w_k	$\nu_{k,\mathrm{s}}$	$\nu_{k,\mathrm{s}}'$	$\nu_{k,\mathrm{rep}}$	$\nu_{k,\varepsilon}$	$\nu_{k,\varepsilon}'$	$\nu_{\mathrm{eff}}(d_k)$
LCIE	0.069	0.353	6	19	inf.	12	3	36
SP	0.134	0.181	22	19	3	12	3	35
IEN	0.038	0.466	22	19	14	12	3	34