Explanatory notes for BIPM comparisons EM-K13 and EM-K14

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The worldwide adoption of the values R_{K-90} and K_{J-90} for use with the familiar quantum representations of the ohm and volt (based on the quantum Hall and Josephson effects respectively) has been so successful that it is easy to forget that these are not true realisations of the SI units. Fortunately there are very few situations where this poses any practical problem (evidently, otherwise the system would not have been allowed to develop in this way).

In the field of voltage metrology, problems are very unlikely to arise, as there is now no real competitor to the Josephson effect as a primary standard. In impedance metrology, the calculable capacitor remains competitive against a quantum Hall system in terms of both uncertainty and easy of use as a primary standard for deriving the unit of capacitance. This means that a top level capacitance comparison will most likely include NMIs using both of these traceability routes, and we are forced to consider the status of R_{K-90} within the SI.

The 1990 recommended values R_{K-90} and K_{J-90} were arrived at by an analysis of all the available experimental data, as summarised in [1]. We should note (as described in section 3.1 of [1]) that this analysis was not the same as, and did not have the same goals as the more familiar CODATA least squares adjustment of all the fundamental constants. It is instructive to compare [1] with the latest full CODATA adjustment [2]. Clearly in almost 20 years, much has changed – some experiments have improved, new experiments have been added, theory has been developed, and in general uncertainties have been reduced. What has not changed, however, is the rationale for using R_{K-90} for traceability in impedance measurements, rather than the latest available CODATA value of R_K.

The consistency of R_{K-90} with the regular CODATA reports (now on a 4 year cycle) is monitored by those involved in quantum electrical metrology, and in particular by the CCEM. This process has resulted in a reassessment of the uncertainty that is recommended for the quantum Hall based representation of the ohm within the SI. In 2000, this was reduced from the original relative standard uncertainty of 0.2 μΩ/Ω to 0.1 μΩ/Ω [3]. It is this uncertainty that concerns us here in the analysis of comparisons that include both the quantum Hall effect and calculable capacitors.

There are several points to note:

- The quantum Hall effect for resistance calibrations has been comprehensibly demonstrated to be reproducible to a few nΩ/Ω
- The best traceability chains from the quantum Hall effect to capacitance have a total uncertainty of the order 0.01 μF/F
- The best calculable capacitors also operate at an uncertainly of around 0.01 μF/F
- Using travelling artefact resistance or capacitance standards it is possible (in the best cases) to achieve a transfer uncertainty between laboratories of around 0.01 μΩ/Ω or 0.01 μF/F
- R_{K-90} has a value of 25 812.807 Ω; the CODATA 2006 value for R_K is 25 812.807 557 (18) Ω (relative uncertainty 0.68 nΩ/Ω); the difference between these values is 0.56 mΩ (in relative terms 0.022 μΩ/Ω)

All this means that in the case of resistance or capacitance comparisons between NMIs, the SI uncertainty associated with the use of R_{K-90} can in a few cases be the dominant source of uncertainty, and the difference between R_{K-90} and R_K may be significant compared to the uncertainties of the measurement systems being compared.

It is therefore tempting to consider using the latest CODATA value of R_K, rather than R_{K-90} when analysing the comparison results. The approach taken by the BIPM in recent comparisons where this has been an issue is to explore the use of the CODATA value of R_K within the report, as a scientific exercise, but to maintain the use of R_{K-90} (with its associated uncertainty) for the final presentation of results.

There no single scientifically correct approach to this problem, but the insistence on the use of R_{K-90} (and importantly the inclusion of the related uncertainty) is consistent with the metrological purpose of comparisons, and with the desire to uphold the integrity of the SI. It is clear that the uncertainty of 0.1 μΩ/Ω is somewhat conservative, but this is inevitable from the original philosophy of the choice of value for R_{K-90} – that we should not need to adopt a new value within the 'foreseeable future'. It is also apparent, nearly 20 years on, that the choice of values for both R_{K-90} and K_{J-90} was a good one. We may now nearly be at the point where we need to abandon these values, but this could well be achieved by a redefinition of the SI rather than an adoption of new conventional values.

There is a particular difficulty in including the 0.1 μΩ/Ω uncertainty in comparison results – it does not belong to any individual laboratory's measurement equipment. In order for there to be a reference value for the comparison, it has to be included 'on one side or the other'. This has the effect of increasing some laboratories uncertainties, but this is unfortunately inevitable. With a proper treatment of correlations, the uncertainty is removed from the degrees of equivalence between laboratories that use the same type of reference.

In the case of BIPM comparisons EM-K13 and EM-K14, the comparison reference value is the BIPM value. (These comparisons are ongoing bilateral comparisons, and using the BIPM value provides continuity between the different bilateral elements.) Since the BIPM uses the quantum Hall effect and R_{K-90} for both resistance and capacitance, the 0.1 μΩ/Ω uncertainty has to be included in the comparison uncertainty for a participating laboratory using a calculable capacitor. This can have the unfair effect of making these laboratories uncertainties look worse, but is unfortunately unavoidable while R_{K-90} continues to be in use.

[1] B. N. Taylor and T. J. Witt, 'New International Electrical Reference Standards Based on the Josephson and Quantum Hall Effects', Metrologia 26, 47-62 (1989)

[2] P. J. Mohr, B. N. Taylor and B. D. Newell, 'CODATA recommended values of the fundamental physical constants: 2006', Rev. Mod. Phys., 80(2), 633-730 (2008)

[3] CIPM Resolution, 2000 (PV, 68, 101).