



LABORATOIRE NATIONAL D'ESSAIS
(BNM-LNE)

**EUROMET L - K1
Calibration of gauge blocks
by interferometry**

Final report

*Paris, December, 20, 2004
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1 INTRODUCTION

The mutual recognition of national measurements standards and of calibration and measurement certificates issued by national metrology institutes is, for the technical assessments, mainly based on key comparisons.

The Consultative Committees of the CIPM, the BIPM and the Regional Metrology Organizations (RMOs) carry out these key comparisons.

At its meeting in September 1997, the Consultative Committee for the Definition of the Metre (CCDM, today called Consultative Committee for Length, CCL) identified several key comparisons in the field of dimensional metrology.

One of these comparisons, named CCL-K1, consists in gauge blocks measurements by interferometry.

In the same time a regional comparison called EUROMET 471, now **EUROMET L- K1**, was planned.

This comparison is parallel to the Consultative Committee/BIPM key comparison CCL-K1 piloted by Metrology and Accreditation Switzerland [METAS].

The set of gauge blocks has exactly the same composition as for the CCL comparison, i.e. ten steel gauge blocks and ten tungsten carbide gauge blocks.

The measuring instructions are also the same as for the document prepared by the working group from BIPM (Ruedi Thalmann from METAS, Switzerland, Jennifer Decker from NRC, Canada and Nicholas Brown from CSIRO/NML, Australia) in order to avoid any distortion between the two comparisons.

BNM-LNE acts as pilot laboratory and participates with two other European laboratories (METAS and NPL) in the CCL-K1 key comparison.

Laboratories participating in both, the CIPM and the RMO comparisons establish the link between these comparisons and assure their equivalence.

2 ORGANISATION

2.1 PARTICIPANTS

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table 1 : Participating laboratories.

2.2 SCHEDULE

The schedule was respected until January 2000, and changed a little bit at the end, as some laboratories were not able to measure in the predefined period.

In fact the comparison lasted from October 1998 to October 2000, two years for 17 laboratories. The duration was of 3 months more than scheduled.

Country	Laboratory	Date
France	BNM-LNE	July 1998
Switzerland	METAS	October 1998
Germany	PTB	November 1998
United Kingdom	NPL	January 1999
Belgium	SMD	February 1999
Czech Republic	CMI	March 1999
Finland	MIKES	May 1999
Denmark	DFM	June 1999
<i>France</i>	<i>BNM-LNE</i>	<i>July 1999</i>
Norway	JV	August 1999
Sweden	SP	September 1999
Italy	IMGC	October 1999
Portugal	IPQ	January 2000
Austria	BEV	February 2000
Spain	CEM	March 2000
Hungary	OMH	April/May 2000
Netherlands	NMI-VSL	June 2000
Slovakia	SMU	Aug./Spt.1999
<i>France</i>	<i>BNM-LNE</i>	<i>October 2000</i>

table 2 :Time schedule.

3 STANDARDS

The set was composed of 10 steel gauge blocks and 10 tungsten carbide gauge blocks.

The gauge blocks were in accordance with the international standard ISO 3650.

The thermal expansion coefficient of the gauge blocks has been measured by the METAS laboratory (measurement uncertainties are stated as standard uncertainty) on the 80 mm, 90 mm, and 100 mm.

The other gauge blocks were assumed to have the same expansion coefficient.

Steel gauge blocks:

Serial number	Nominal length (mm)	Expansion coeff. (10^{-6} K^{-1})		Manufacturer
			u	
24.237 67	0,5	11,59	0,1	CARY
20.949 82	1,01	11,59	0,1	CARY
6.944 42	1,1	11,59	0,1	CARY
20.923 5	6	11,59	0,1	CARY
8.942 0	7	11,59	0,1	CARY
23.009 3	8	11,59	0,1	CARY
4.213 08	15	11,59	0,1	CARY
11.239 73	80	11,63	0,02	CARY
12.232 61	90	11,59	0,02	CARY
8.239 00	100	11,56	0,02	CARY

table 3a : Description of steel gauge blocks

Tungsten carbide gauge blocks :

Serial number	Nominal length (mm)	Expansion coeff. (10^{-6} K^{-1})		Manufacturer
			u	
C 6564	0,5	4,27	0,1	SELECT
C 6565	1	4,27	0,1	SELECT
C 6567	1,01	4,27	0,1	SELECT
C 6562	1,1	4,27	0,1	SELECT
C 6292	5	4,27	0,1	SELECT
C 6293	6	4,27	0,1	SELECT
C 6294	8	4,27	0,1	SELECT
C 6295	80	4,27	0,01	SELECT
C 6568	90	4,27	0,01	SELECT
C 6569	100	4.27	0.01	SELECT

Table 3b : Description of tungsten carbide gauge blocks.

4 MEASUREMENT INSTRUCTIONS AND REPORTING

Measurand was the central length of the gauge blocks, as defined in the International Standard ISO 3650. The gauge blocks had to be measured by interferometry, in their vertical position wrung to a flat plate, which was provided by each laboratory.

The central length of a gauge block is defined as the perpendicular distance between the centre point of the free measuring surface and the plane surface of an auxiliary plate of the same material and surface texture upon which the other measuring surface has been wrung.

The measurement result to be reported was the deviation of central length from nominal length,

$$\Delta l = l_{\text{measured}} - L_{\text{nominal}}$$

The measurands were :

- the results of the measurements on both sides - Δl_1 and Δl_2 - by wringing each measurement face in turn to the reference flat
- the average of Δl_1 and Δl_2

All the results had to be reported on the table in the annex A1 of the measurement instructions

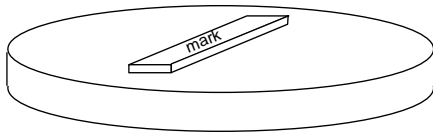


Figure 1a :
Position of the gauge block for Δl_1
The upper face is face 1

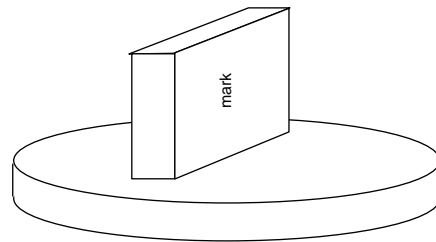


Figure 1b:
Position of the gauge block for Δl_1 ($L > 6$ mm)
The upper face is face 1

The measurement results had to be appropriately corrected to the reference temperature of 20 °C using the thermal expansion coefficients given in the technical instructions. Additional corrections had to be applied according to the usual procedure of the laboratory.

The uncertainty of measurement had to be estimated according to the *ISO Guide for the Expression of Uncertainty in Measurement*. In order to achieve optimum comparability, a mathematical model containing the principal influence parameters for gauge block calibration by interferometry was given in the measurement instructions.

5 MEASUREMENT METHODS AND INSTRUMENTS USED BY THE PARTICIPANTS

All laboratories measured the gauge blocks by optical interferometry, applying the method of fringe fractions.

Ten laboratories used NPL-TESA interferometer, five a Carl Zeiss Jena interferometer, one a Tsugami interferometer and one a Hilger & Watts interferometer.

Laboratory.	Manufacturer and type of interferometer
LNE	NPL-TESA Automatic gauge block interferometer, Twyman Green design
METAS	NPL-TESA Automatic gauge block interferometer, Twyman Green design
NPL	NPL-TESA Automatic gauge block interferometer, Twyman Green design
PTB	Michelson interferometer, modified TSUGAMI
SMD	NPL-TESA Automatic gauge block interferometer, Twyman Green design
CMI	NPL-TESA Automatic gauge block interferometer, Twyman Green design
MIKES	NPL-TESA Automatic gauge block interferometer, Twyman Green design
DFM	NPL-TESA Automatic gauge block interferometer, Twyman Green design
JV	NPL-TESA Automatic gauge block interferometer, Twyman Green design
SP	Carl Zeiss Jena interferometer, Köster type
IMGC	Hilger & Watts T 190 NPL-H&W (Fizeau interferometer)
IPQ	NPL-TESA Automatic gauge block interferometer, Twyman Green design
BEV	Carl Zeiss Jena interferometer, Köster type
CEM	NPL-TESA Automatic gauge block interferometer, Twyman Green design
OMH	Carl Zeiss Jena interferometer, Köster type
NMi	Carl Zeiss Jena interferometer, Köster type
SMU	Carl Zeiss Jena interferometer, Köster type

table 4 : Type and manufacturer of interferometer

Further details of the measurement procedures and conditions are summarized in *tables A1* and *A2* in the *appendix 1*.

6 MEASUREMENT RESULTS

Results given in this chapter, are taken from data sheet given by the laboratories, and do not integer any correction such as drift.

6.1 STEEL GAUGE BLOCKS : DEVIATION FROM NOMINAL LENGTH

laboratory	Steel gauge blocks : Nominal length in mm									
	0,5	1,01	1,1	6	7	8	15	80	90	100
BNM-LNE	-15	88	69	54	-22	3	38	-148	39	-127
METAS	-13	103	78	53	-14	8	56	-132	79	-115
PTB	-24	89	62	37	-31	-10	39	-160	54	-145
NPL	-23	90	73	45	-12	23	41	-157	64	-120
SMD	-14,3	88,6	83,0	48,8	-16,8	10,1	47,2	-146,6	69,8	-131,6
MIKES	-21	94	60	37	-19	-1	47	-160	54	-141
DFM	-17,3	95,2	75,4	45,8	-19,2	15,0	54,3	-148,1	80,7	-121,8
JV	-16	99	68	42	-22	-2	38	-206	7	-191
SP	-15	89	58	42	-21	3	42	-157	61	-143 (1)
IMGC	-22	81	63	42	-30	-4	55	-161	49	-149
IPQ	-14	87	65	38	-17	22	61	-105	104	-67
BEV	-18 (2)	88	71	34	-21	1	48	-167	53	-144 (2)
CEM	-20	93	83	47	-17	-4	56	-172	36	-154
OMH	3	24	9	31	-19 (1)	-11	39	-192	80	-89
NMI-VSL	-49	73 (1)	45	26	-39 (1)	-43	42	-167	49	-149 (1)
SMU	-31	85	69	40	-19	-15	36	-193	59	-149

table 5a : Steel gauge blocks ; deviation from nominal value (in nm) as reported by laboratories

Comments given by the participants :

- (1) Only one length has been measured, due to some scratches on one face ; in this case the reported value is not the mean of (Δl_1 , Δl_2) but the length corresponding to one face.
 (2) Δl_1 and Δl_2 measured but with no satisfactory wringing for one face.

6.2 STEEL GAUGE BLOCKS : COMBINED STANDARD UNCERTAINTY

laboratory	Steel gauge blocks : Nominal length in mm									
	0,5	1	1,1	6	7	8	15	80	90	100
BNM-LNE	10	10	10	10	10	10	10	14	15	16
METAS	9,6	9,6	9,6	8,2	8,2	8,3	8,9	11,0	12,0	12,0
PTB	8	8	8	8	8	8	8	10	10	11
NPL	14	14	14	14	14	14	15	28	31	33
SMD	9,7	9,7	9,7	9,7	9,7	9,7	9,8	13,0	13,8	14,6
MIKES	10,2	10,2	10,2	10,2	10,2	10,3	10,4	15,1	16,2	17,3
DFM	9,2	9,2	9,2	9,3	9,3	9,3	9,6	17,1	18,6	20,2
JV	13,1	13,1	13,1	13,1	13,1	13,1	13,3	20,4	21,9	23,5
SP	12,4	12,4	12,4	12,4	12,4	12,5	12,6	17,3	18,3	19,5
IMGC	9,1	9,1	9,1	9,1	9,1	9,2	9,4	16,4	17,8	19,2
IPQ	12,7	12,7	12,7	12,7	12,7	12,7	12,7	12,8	12,8	12,8
BEV	15 (1)	10	10	10	10	10	11	13	14	18 (1)
CEM	8,4	8,4	8,4	8,4	8,4	8,5	8,7	14,2	15,4	16,6
OMH	31	31	31	27	28	26	31	37	39	41
NMI-VSL	11,3	11,3	11,3	11,3	11,3	11,3	11,4	14,2	14,9	15,4
SMU	23,0	22,8	23,0	23,1	23,0	23,1	23,2	32,1	33,9	36,6

Table 5b : Steel gauge blocks ; combined standard uncertainty (in nm) as reported by laboratories.

Comments given by the participants :

(1) For these gauge blocks, uncertainty components were increased because of bad wringing conditions.

6.3 STEEL GAUGE BLOCKS : GRAPHICS

The following figures represent the results for each laboratory with error bars corresponding to one **standard uncertainty**.

Fig. 2a : 0,5 mm steel gauge block

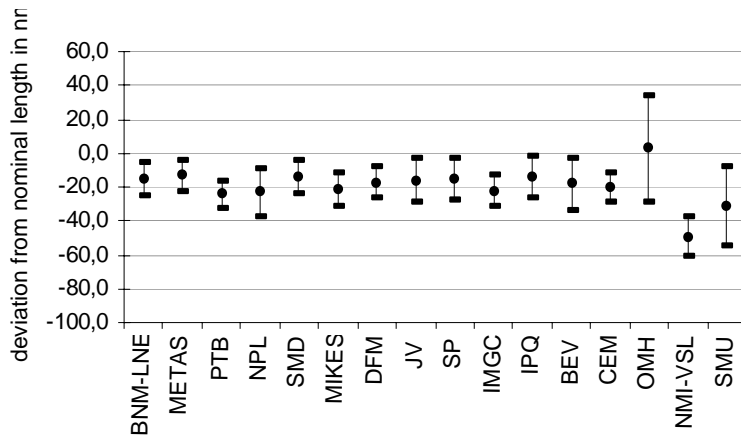


Fig. 2b : 1,01 mm steel gauge block

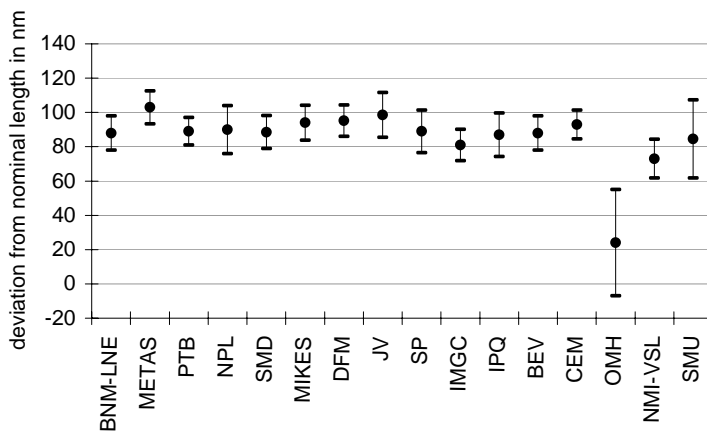


Fig. 2c : 1,1 mm steel gauge block

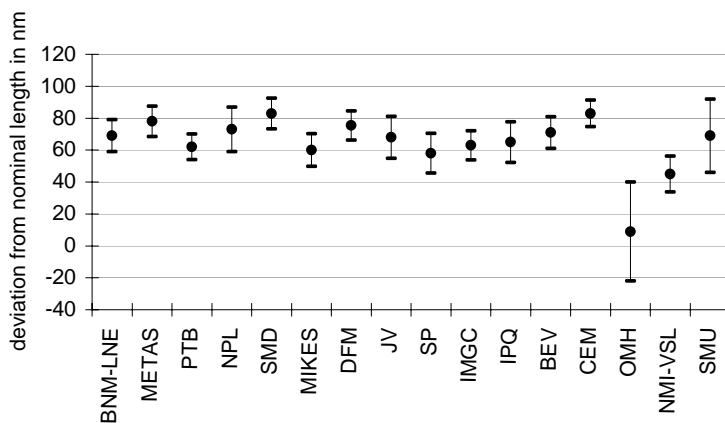


Fig. 2d : 6 mm steel gauge block

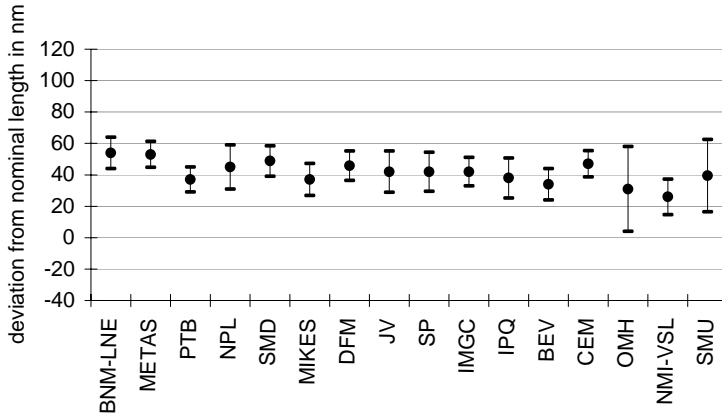


Fig. 2e : 7 mm steel gauge block

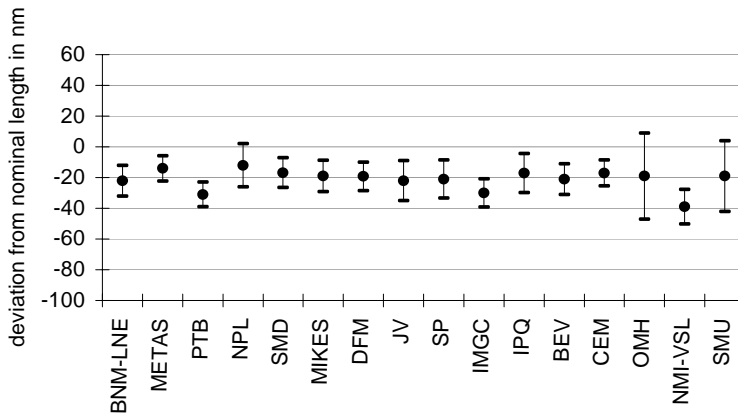


Fig. 2f : 8 mm steel gauge block

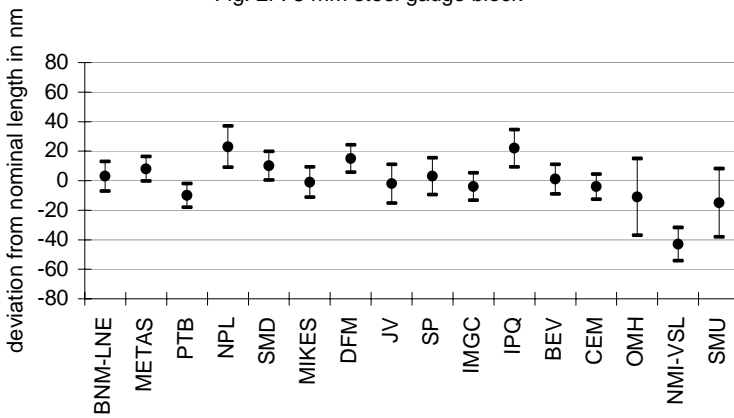


Fig. 2g : 15 mm steel gauge block

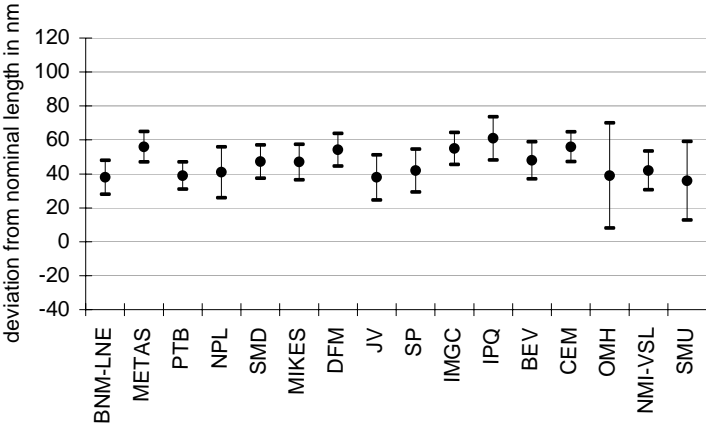


Fig. 2h : 80 mm steel gauge block

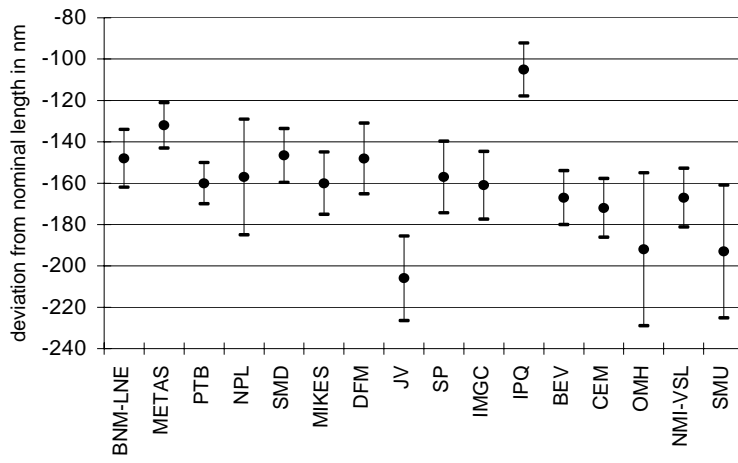


Fig. 2i : 90 mm steel gauge block

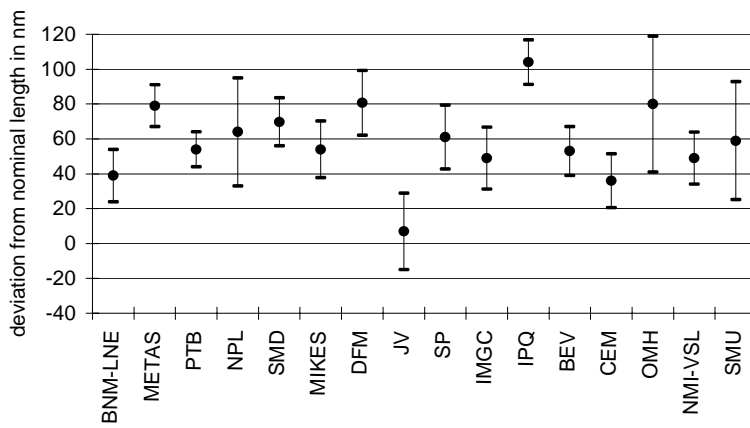
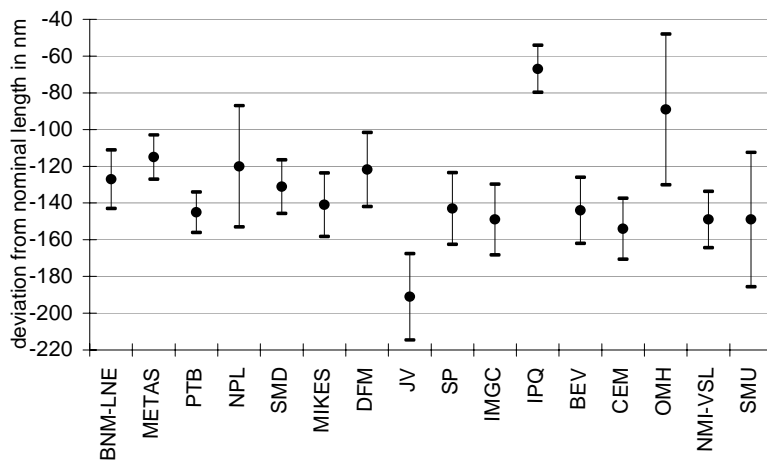


Fig. 2j : 100 mm steel gauge block



6.4 TUNGSTEN CARBIDE GAUGE BLOCKS : DEVIATION FROM NOMINAL LENGTH

laboratory	TC gauge blocks : Nominal length in mm									
	0,5	1	1,01	1,1	5	6	8	80	90	100
BNM-LNE	-5	31	101	78	26	30	2	62	34	-74
METAS	-9	22	92	62	7	14	-12	73	34	-64
PTB	-19	10	82	57	-6	5	-27	59	32	-76
NPL	-2	27	102	71	16	21	-7	64	29	-71
SMD	-16,6	12,0	83,9	51,8	-1,9	5,4	-18,9	43,3	5,7	-98,3
CMI	0	24	93	70	11	17	-11	60	16	-87
MIKES	-8	15	89	60	1	7	-20	60	25	-81
DFM	-0,3	31,4	93,9	63,4	7,2	5,8	-9,8	62,7	34,9	-66,7
JV	-14	12	91	63	-5	0	-22	26	-14	-124
SP	-12	5	81	59	3	5	-21 (1)	51	7	-82
IMGC	-15	-1	82	52	-1	1	-20	47	7	-99
IPQ	-43	-16	58	28	-11	-1	-24	48	23	-72
BEV	-2 (2)	28	100 (2)	77	6	13	-14	31	-11	-101
CEM	-11	13	83	58	-1	2	-23	36	-2	-120
NMI-VSL	-24	9	84	57	-1	-1	-17	43	-4	-104
SMU	-34	0	58	38	6	9	-33	14	-8	-121

table 6a : TC gauge blocks ; deviation from nominal value (in nm)

Comments given by the participants :

- (1) Only one length has been measured, due to some scratches on one face ; in this case the reported value is not the mean of (Δl_1 , Δl_2) but the length corresponding to one face.
- (2) Δl_1 and Δl_2 measured but with no satisfactory wringing for one face.

6.5 TUNGSTEN CARBIDE GAUGE BLOCKS : COMBINED STANDARD UNCERTAINTY

laboratory	TC gauge blocks : Nominal length in mm									
	0,5	1	1,01	1,1	5	6	8	80	90	100
BNM-LNE	10	10	10	10	10	10	10	10	11	11
METAS	9,6	9,6	9,6	8,2	8,2	8,2	8,3	10	10	11
PTB	8	8	8	8	8	8	8	9	9	9
NPL	14	14	14	14	14	14	14	17	17	19
SMD	9,7	9,7	9,7	9,7	9,7	9,7	9,7	12,5	13,2	13,9
CMI	8,1	8,1	8,1	8,1	8,1	8,1	8,2	13,2	14,2	15,3
MIKES	10,2	10,2	10,2	10,2	10,2	10,2	10,2	13,5	14,2	15
DFM	9,2	9,2	9,2	9,2	9,3	9,3	9,3	17,1	18,6	20,2
JV	14,1	14,1	14,1	14,1	14,1	14,1	14,1	16,6	17,2	17,9
SP	12,4	12,4	12,4	12,4	12,4	12,4	12,4	16,2	17	18
IMGC	9,1	9,1	9,1	9,1	9,1	9,1	9,1	10,5	10,9	11,3
IPQ	12,70	12,70	12,70	12,71	12,71	12,71	12,72	12,78	12,79	12,80
BEV	15 (1)	10	15 (1)	10	10	10	10	11	12	12
CEM	8,4	8,4	8,4	8,4	8,4	8,4	8,4	11	11,6	12,2
NMI-VSL	9,2	9,2	9,2	9,2	9,2	9,2	9,2	10,9	11,4	11,8
SMU	22,7	22,8	22,8	22,8	22,8	23,1	22,9	26,8	28,3	29

Table 6b : TC gauge blocks ; combined standard uncertainty (in nm) as reported by laboratories.

Comments given by the participants :

- (1) For these gauge blocks, uncertainty components were increased because of bad wringing conditions.

6.6 TUNGSTEN CARBIDE GAUGE BLOCKS : GRAPHICS

The following figures represent the results for each laboratory with error bars corresponding to one **standard uncertainty**.

Figure 3a : 0,5 mm TC gauge block

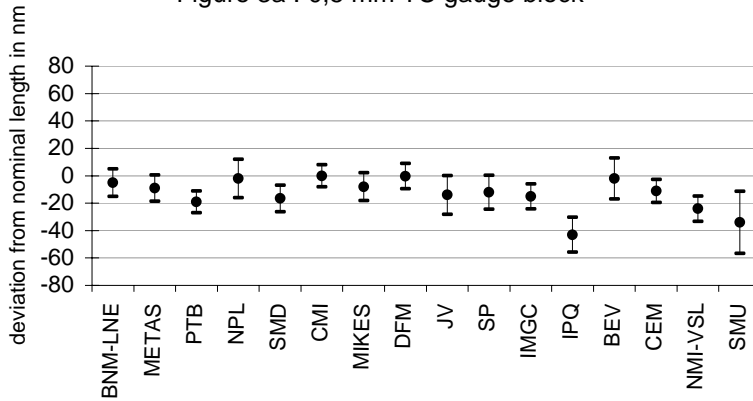


Figure 3b : 1 mm TC gauge block

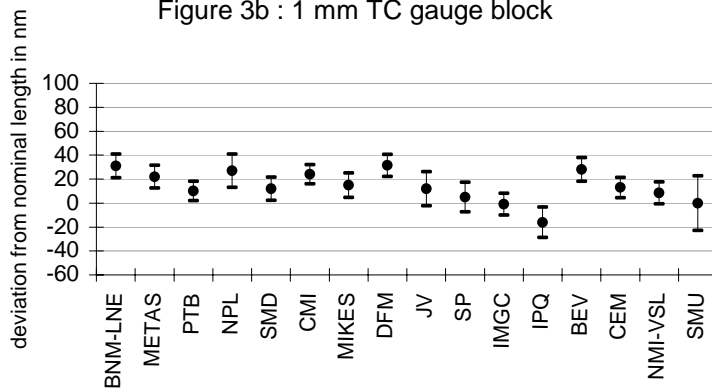


Figure 3c : 1,01 mm TC gauge block

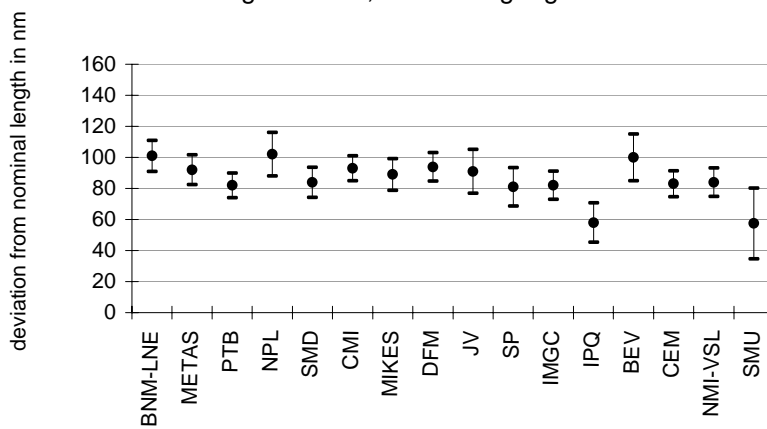


Figure 3d : 1.1 mm TC gauge block

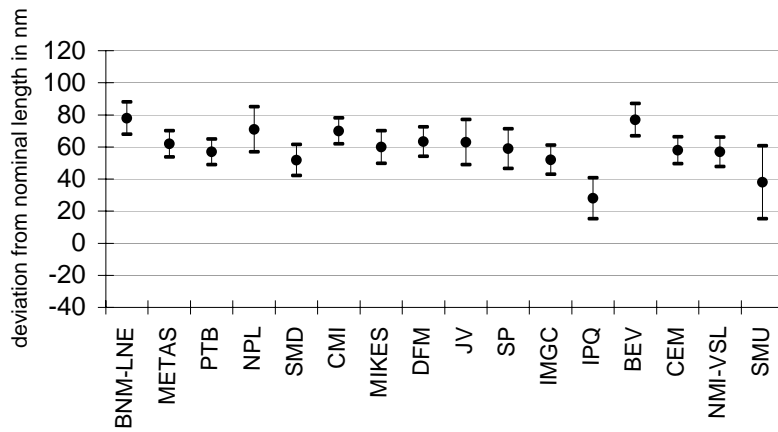


Figure 3e : 5 mm TC gauge block

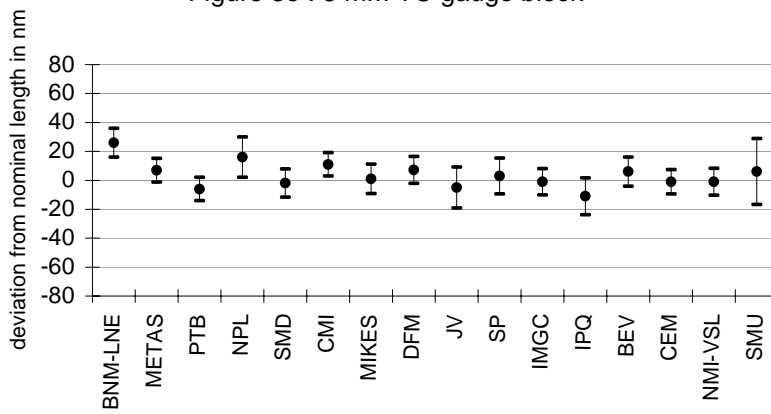


Figure 3f : 6 mm TC gauge block

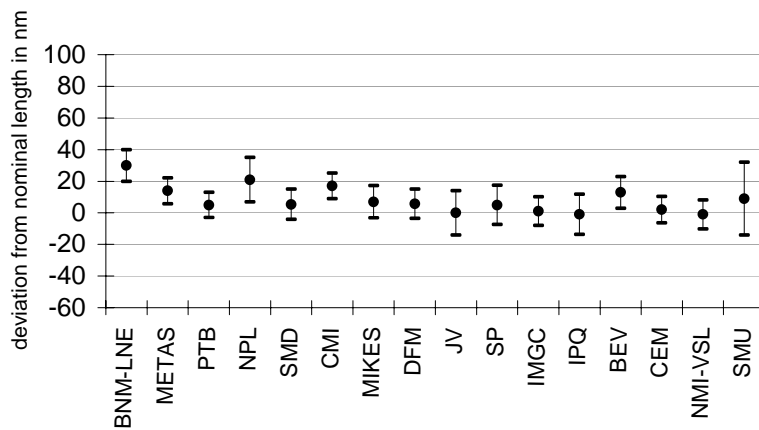


Figure 3g : 8 mm TC gauge block

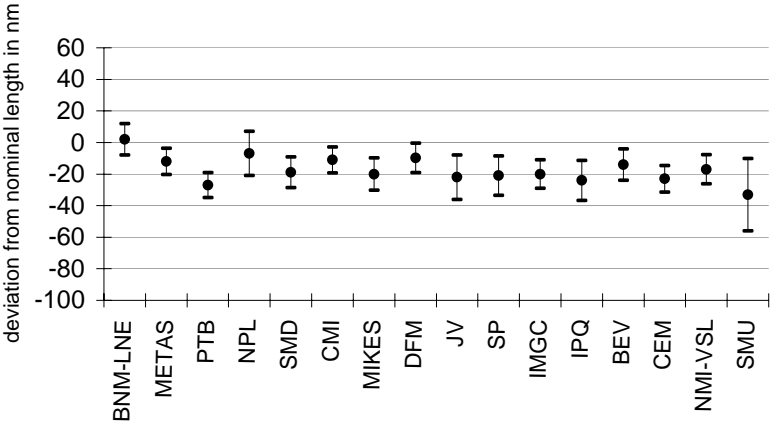


Figure 3h : 80 mm TC gauge block

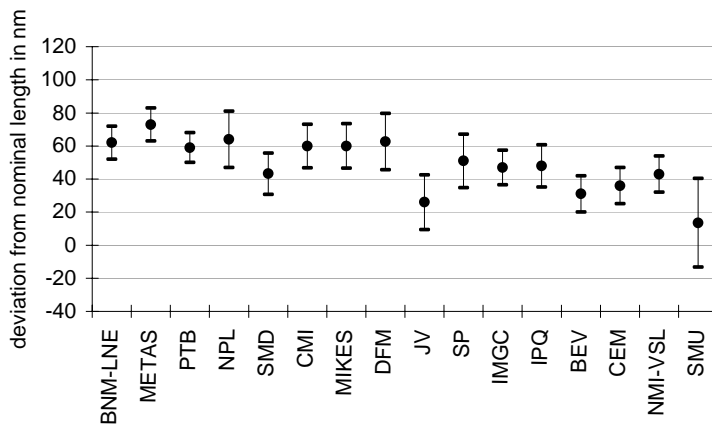


Figure 3i : 90 mm TC gauge block

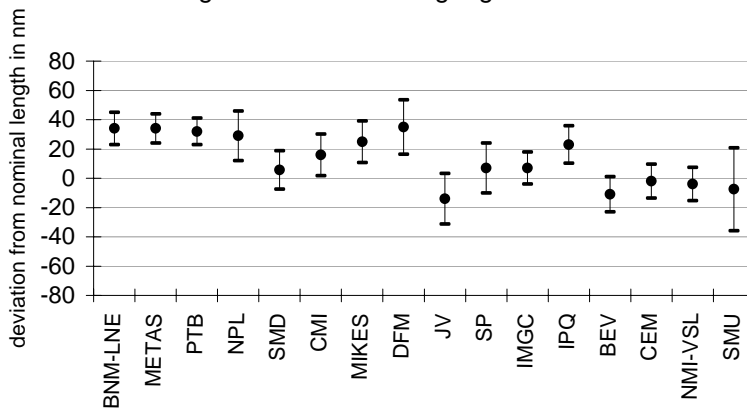
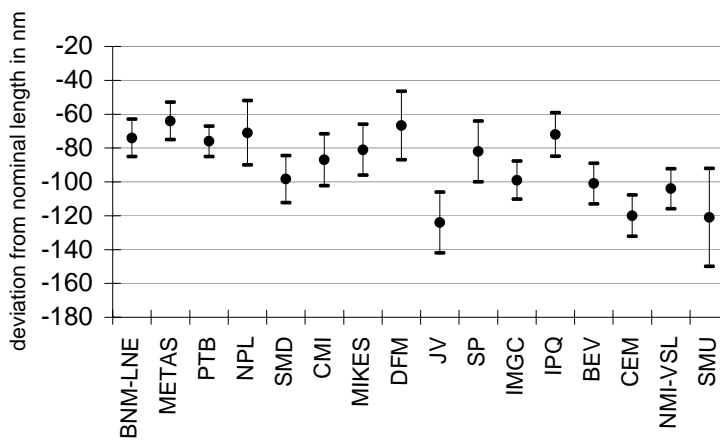


Figure 3j : 100 mm TC gauge block



6.7 DIFFERENCE BETWEEN THE TWO MEASUREMENTS ΔL_1 AND ΔL_2

laboratory	steel gauge blocks		TC gauge blocks	
	mean	std dev.	mean	std dev.
BNM-LNE	-0,5	7,4	-2,7	6,9
METAS	0,8	5,6	0,1	6,8
PTB	3,5	4,8	-0,5	6,7
NPL	1,5	15,0	1,2	5,2
SMD	-1,6	5,1	0,3	4,4
CMI	--	--	-3,2	5,8
MIKES	0,0	8,4	1,1	5,0
DFM	1,3	8,1	2,0	7,8
JV	8,6	8,5	2,0	8,6
SP	-2,6	2,8	-1,8	8,5
IMGC	-0,3	6,4	2,1	7,4
IPQ	-3,1	8,1	2,6	12,4
BEV	2,6	12,3	2,8	5,7
CEM	2,4	9,4	-1,0	6,5
OMH	-2,9	12,7	--	--
NMI-VSL	2,0	6,2	-3,6	7,6
SMU	-5,8	15,3	-9,2	16,0

table 7 : mean value and standard deviation of the differences between Δl_1 and Δl_2

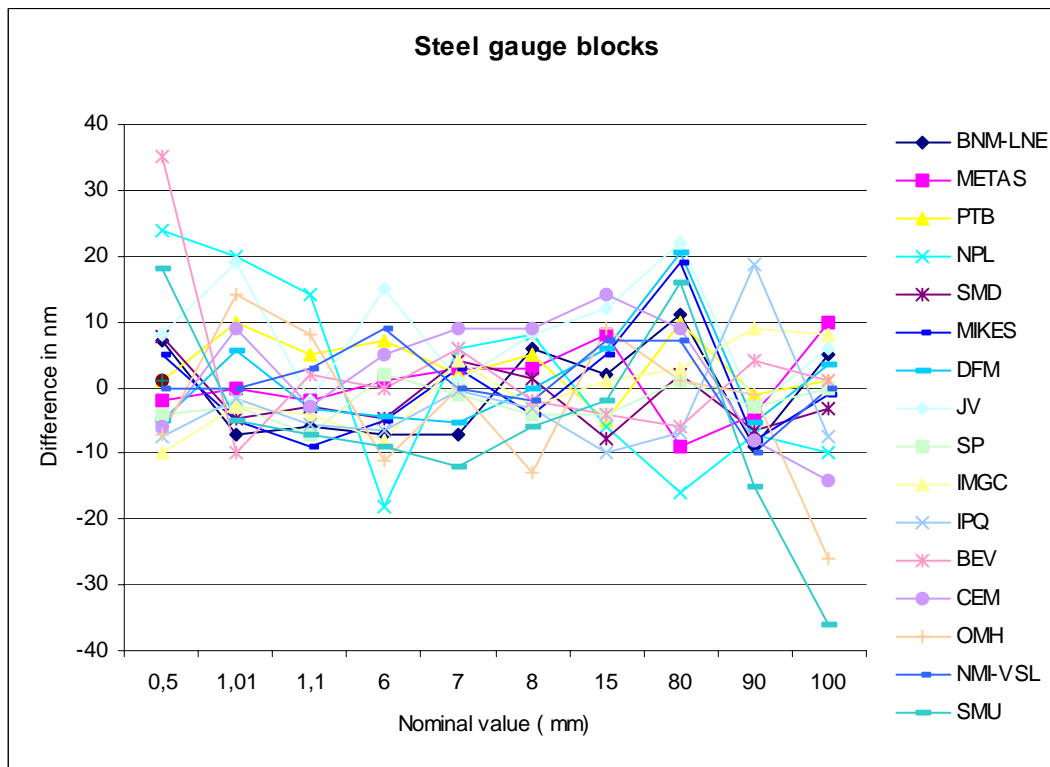


Figure 4a. Differences between Δl_1 and Δl_2 for all steel gauge blocks and all laboratories.

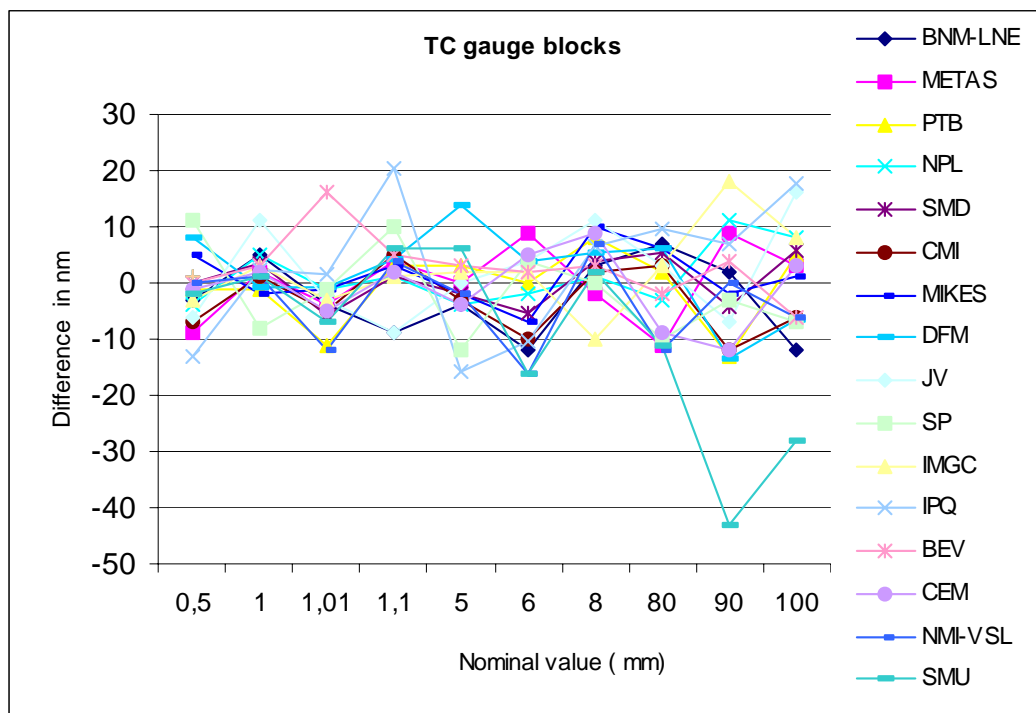


Figure 4b. Differences between Δl_1 and Δl_2 for all TC gauge blocks and all laboratories.

7 MEASUREMENT UNCERTAINTIES

7.1 DESCRIPTION OF THE COMPONENT AS GIVEN BY THE LABORATORIES

An example of a mathematical model was given in the protocol with a list (non exhaustive) of uncertainty sources.

$$l = \frac{1}{q} \sum_{i=1}^q (k_i + F_i) \frac{\lambda_i}{2n} + \Delta t_g \cdot \alpha \cdot L + \delta l_{\Omega} + \Delta l_s + \delta l_A + \delta l_G + \delta l_W + \Delta l_{\phi} \quad (1)$$

where :

l length of the gauge block at the reference temperature of 20 °C ;

L nominal length of the gauge block ;

q number of wavelengths used for the determination of the length based on the method of exact fractions ($i = 1, \dots, q$) ;

k_i integer part of number of half wavelengths within gauge block length (fringe order) ;

F_i fractional part of fringe order ;

λ_i vacuum wavelengths of the different light sources used ;

n index of refraction of the air ;

$\Delta t_g = (20 - t_g)$ is the difference of the gauge block temperature t_g in °C during the measurement from the reference temperature of 20 °C ;

α linear coefficient of thermal expansion of the gauge block ;

δl_{Ω} obliquity correction for the shift in phase resulting from the angular alignment errors of the collimating assembly, with zero expectation value $\langle \delta l_{\Omega} \rangle = 0$;

Δl_s aperture correction accounting for the shift in phase resulting from the finite aperture diameter s of the light source : $\Delta l_s = \frac{s^2}{16f^2} L$; f is the focal length of the collimating lens.

δl_A correction for wave front errors as a result of imperfect interferometer optics, with zero expectation value $\langle \delta l_A \rangle = 0$;

δl_G correction accounting for flatness deviation and variation in length of the gauge block, with zero expectation value $\langle \delta l_G \rangle = 0$;

δl_W length attributed to the wringing film, with zero expectation value $\langle \delta l_W \rangle = 0$, since the length of the gauge block is defined to include the wringing film ;

Δl_{ϕ} phase change accounting for the difference in the apparent optical length to the mechanical length.

In table 8, the uncertainty contributions are summarized for all the laboratories. The numerical values are standard uncertainties given for the case of a steel gauge block.

Table 9 gives for the 100 mm steel gauge block the numerical value of the contributions. Only major contributions are reported and the uncertainties are given for the 100 mm gauge blocks.

For some laboratories the uncertainty on the linear expansion coefficient is different from the values given in the technical protocol. In these cases, the uncertainty has been overestimated.

	LNE	METAS	PTB	NPL	SMD	CMI	MIKES	DFM	JV	SP
λ_i	$3 \cdot 10^{-9}$	$3 \cdot 10^{-9}$	$2 \cdot 10^{-9}$	$1 \cdot 10^{-8}$	$2 \cdot 10^{-9}$	$2 \cdot 10^{-8}$	$5 \cdot 10^{-9}$	$2 \cdot 10^{-9}$	$5 \cdot 10^{-9}$	$5 \cdot 10^{-9}$
F_i	0,015 fringe	0,015 fringe	0,003 fringe	0,0032 fringe	0,025 fringe	0,025 fringe	0,025 fringe	0,0144 fringe	0,025 fringe	0,05 fringe
n	$5,4 \cdot 10^{-8}$	$5,8 \cdot 10^{-8}$	$3 \cdot 10^{-8}$	$4,23 \cdot 10^{-8}$	$1 \cdot 10^{-8}$	$6 \cdot 10^{-8}$	$6 \cdot 10^{-8}$	$6 \cdot 10^{-8}$	$8,6 \cdot 10^{-8}$	$6 \cdot 10^{-8}$
t_G	8 mK	6 mK	5 mK	25,7 mK	5 mK	10 mK	10 mK	7,7 mK	5 mK	7 mK
α	$0,1 \cdot 10^{-6} K^{-1}$	$0,02 \cdot 10^{-6} K^{-1}$	$0,1 \cdot 10^{-6} K^{-1}$	$0,01 \cdot 10^{-6} K^{-1}$	$0,01 \cdot 10^{-6} K^{-1}$	$0,1 \cdot 10^{-6} K^{-1}$	$0,5 \cdot 10^{-6} K^{-1}$	$0,35 \cdot 10^{-6} K^{-1}$	$0,12 \cdot 10^{-6} K^{-1}$	négligeable
δ_{Ω}	~ 0	$0,6 \cdot 10^{-8}$	$0,8 \cdot 10^{-8}$	$0,58 \cdot 10^{-8}$	$1 \cdot 10^{-8}$	$1 \cdot 10^{-8}$	$5 \cdot 10^{-8}$	$0,58 \cdot 10^{-8}$	$1 \cdot 10^{-8}$	$1 \cdot 10^{-8}$
$S(\delta\Delta I_S)$	~ 0	$5 \cdot 10^{-9}$	$2 \cdot 10^{-9}$	$8,7 \cdot 10^{-9}$	$7 \cdot 10^{-9}$	$1 \cdot 10^{-8}$		$0,87 \cdot 10^{-8}$	$0,2 \cdot 10^{-6}$	$0,1 \cdot 10^{-6}$
δ_A	3 nm	3,7 nm	2,5 nm	5,2 nm	4 nm	3 nm	2,5 nm	3,5 nm	0,012 fringe	4 nm
δ_G	5 nm	1,5 nm	2,5 nm	3,5 nm	Individual values	2,5 nm	3 nm	1,5 nm	2,1 nm	2 nm
δ_w	5 nm	3,5 nm	4,3 nm	5 nm	5,1 nm	4 nm	5 nm	2,9 nm	6 nm	6 nm
ΔI_{ϕ}	3,4 nm	4,2 nm	1,5 nm	5 nm	7,27 nm	4 nm	6 nm	6,3 nm	6 nm	7 nm
$\Delta(I_{\phi} I_{\phi})$				10 nm*						
δ_R roughness correction gauge/platen			3.8 nm $\sqrt{2}$							
Other contributions		Individual roughness variation 3,5 nm Wringing problems for small g.b. 5 nm	Difference on optical phase shift on reflection : 1,5 nm	* Variation of phase correction within the group of g.b. : 10 nm Plus 2 nd order term (α, t_G)						

table 8 : Standard uncertainties quoted by the different laboratories for the uncertainty contributions given in the model of the technical protocols.

	IMGC	IPQ	BEV	CEM	OMH	NMi	SMU
λ_i	$1,3 \cdot 10^{-8}$	$2 \cdot 10^{-8}$	$2,4 \cdot 10^{-8}$	$0,5 \cdot 10^{-8}$		$3,2 \cdot 10^{-9}$	10^{-7}
F_i	0,01 fringe	0,025 fringe	0,01 fringe	0,02 fringe	0,02 fringe	5,4 nm	0,05 fringe
n	$2,6 \cdot 10^{-8}$		$1 \cdot 10^{-8}$	$7,5 \cdot 10^{-8}$	$2 \cdot 10^{-7}$	$3,3 \cdot 10^{-8}$	10^{-7}
t_G	15 mK	5 mK	7 mK	10 mK	30 mK	3 mK	20 mK
α	$0,02 \cdot 10^{-6} K^{-1}$	$0,02 \cdot 10^{-6} K^{-1}$	$0,02 \cdot 10^{-6} K^{-1}$	$0,35 \cdot 10^{-6} K^{-1}$	$0,1 \cdot 10^{-6} K^{-1}$	$0,66 \cdot 10^{-6} K^{-1}$	$0,1 \cdot 10^{-6} K^{-1}$
δ_{Ω}	$0,2 \cdot 10^{-8}$	$1 \cdot 10^{-8}$	$0,2 \cdot 10^{-8}$	$0,6 \cdot 10^{-8}$	$0,16 \cdot 10^{-8}$	$5 \cdot 10^{-8}$	$2 \cdot 10^{-8}$
$S (\delta\Delta l_s)$	$2 \cdot 10^{-9}$	$7 \cdot 10^{-9}$	$3 \cdot 10^{-9}$	$2 \cdot 10^{-9}$	$6 \cdot 10^{-9}$	$2 \cdot 10^{-9}$	$25 \cdot 10^{-9}$
δ_A	2 nm	0,5 nm	3,2 nm	3 nm	12 nm	3 nm	5 nm
δ_G	2 nm	3,6 nm	2 nm	2,5 nm	10 nm	2 nm	10 nm
δ_w	6 nm	6,5 nm	6 nm	3 nm	20 nm	7 nm	6 nm
Δl_{ϕ}	6 nm	5,8 nm	7 nm	5 nm	10 nm	6 nm	15 nm
Other contributions		roughness 6,4 nm			Flatness of platen 15 nm		

Table 8 , continuation.

	LNE	METAS	PTB	NPL	SMD	CMI	MIKES	DFM	JV	SP	IMGC	IPQ	BEV	CEM	OMH	NMi	SMU
λ_i	0,3	0,3	0,2	1	0,05	2	0,5	0,2	0,5	5	0,5	2	1,2	0,5		2,5	10
F_i	3,2	3,2	1	1	2,2	4,3	1	4,6	5,6	7	1,46	2,7	1,3	4,2	4,5	5,4	11,4
n	5,4	5,8	3	4,2	1	6	6	6	8,6	6	2,6	20	1	7,5	20	3,3	10
t_G	9	7	5,8	30	2,8	5	11	9	17	8	17	6	8	10	20	3,1	23,2
α	2	1	0,5	0,1	0,1	1	5	14	11,6	0	0,6	0,6	0,3	3,5	2	3,3	0,2
δ_{Ω}	0	0,6	0,8	0,6	1	1	5	0,6	1	1	0,2	0,01	0,2	0,6	16	5,4	2
$S (\delta\Delta l_s)$	0	0,5	0,2	0,9	0,7	1		0,9	0,2	10	0,2	0,007	0,3	0,2	0,63	0,2	5
δ_A	3	3,7	2,5	5,2	4	3	2,5	3,5	6,3	4	2	0,5	3,2	3	12	3	5
δ_G	5	1,5	2,5	3,5		2,5	3	1,5	2,1	2	2	3,6	2	2,5	10	2	10
δ_w	5	3,5	4,3	5	5,1	4	5	2,9	6	6	6	6,5	6	3	20	4	6
Δl_{ϕ}	3,4	4,2	1,5	5	7,3	4	6	6,3	6	7	6	5,8	7	5	10	5	15
Other contributions not reported																	
Combined standard uncertainty	16	12	11	33	15	15	17	20	24	20	19	13	18	17	41	15	37

table 9 . Standard uncertainties (nm) quoted by the different laboratories for the major uncertainty contributions for a 100 mm steel gauge block.

8 ANALYSIS OF THE RESULTS

8.1 WITHDRAWAL

A primary descriptive analysis of the results shows that some laboratories either didn't fulfill all the comparison requirements and/or are out of the rest of the population of results for a large part of a set of gauge blocks.

GUM indicated that they didn't measure phase correction due to the lack of time that was the consequence of a shift of the schedule.

After the publication of the first draft, two other laboratories were contacted to explain the possible reasons for these shifts.

CMI reported that during steel gauge blocks measurements they got some problems with their dew point sensor, but the main error seems to come from the phase correction. The laboratory found unusual values that can be attributed to the quality of the platen and of the gauge blocks. This hypothesis was confirmed by the measurements made on a new platen.

OMH explained the systematic error by the measurement procedure. The deviation comes from the measuring method. As OMH used only one light source (laser at 633 nm), the laboratory had to measure the gauge blocks by mechanical comparison first. In this case the initial value was wrong (probably due to temperature correction) and lead to look for the length in a wrong half wavelength. Consequently OMH got, in 2004, a green stabilized laser which is traceable to BEV (Austria). This new laser source is now installed on the interferometer and OMH is ready to participate in a further comparison.

During the EUROMET meeting in 2002, all participants agreed with the withdrawal of GUM for the two sets of gauge blocks, the withdrawal of CMI only for steel gauge blocks and the withdrawal of OMH only for TC gauge blocks.

8.2 OUTLIERS

Some reported measurement results seem to be inconsistent with other results, and may change the estimate and specially the reference values.

That's why a numerical outliers test was performed according to the ISO 5725-2: 1994¹. The Grubbs' test was applied for one outlying observation. Details of this test are given in *appendix 2*.

Results were declared outliers in the cases where the test gave a *Grubbs' value* greater than its 1 % critical value.

For Steel gauge blocks :

Two results (1,01 and 1,1 mm from OMH) gave a Grubbs' value (respectively 3,452 and 3,151) greater than the 1% critical value of 2.852.

These results are excluded from all the following computations.

For TC gauges blocks :

None results are outliers.

¹ ISO 5725-2 : Accuracy (trueness and precision) of measurement methods and results – part 2 : Basic method for the determination of repeatability and reproducibility of a standard measurement method.

8.3 STABILITY OF THE GAUGE BLOCKS

The examination of the results shows, in some cases, tendency that could be interpreted as possible drift of the gauge blocks.

At this stage, one can make the hypothesis that this tendency comes from either a physical effect (material or wringing) and / or is due to the sequence of the laboratories.

8.3.1 Statistical analysis

The following table gives for each gauge blocks the estimated slope b_1 of the regression line on the results versus the time of measurement.

The *t student* test is applied for testing the signification of the slope in the hypothesis of b_1 equal to 0. The tables below give the statistic $T = \frac{b_1}{\sqrt{\text{var}(b_1)}}$.

For this test the *T value* is compared to $t_{1-\frac{\alpha}{2}}(\nu)$, which is equal to 2,160 for $\nu = 13$ and 2,145 for $\nu = 14$ for $\alpha = 5\%$ and respectively 3,012 and 2,977 for $\alpha = 1\%$.

A value equal or greater than the $t_{1-\frac{\alpha}{2}}(\nu)$ means that the slope is significantly different from 0.

Nominal value (mm)	0,5	1,01	1,1	6	7	8	15	80	90	100
<i>T</i>	0,925	2,044	1,054	3,400	0,814	2,137	0,050	1,750	0,111	0,166
<i>n</i> ($\nu=n-2$)	16 (14)	15 (13)	15 (13)	16 (14)	16 (14)	16 (14)	16 (14)	16 (14)	16 (14)	16 (14)
Slope (nm per month)	-0,329	-0,456	-0,365	-0,655	-0,186	-1,006	0,014	-1,311	0,084	-0,160
<i>u</i> (slope)	0,4	0,2	0,3	0,2	0,2	0,5	0,3	0,7	0,8	1,0

table 10 a : Steel gauge blocks ; value and *t student* test the slope of regression line

Nominal value (mm)	0,5	1	1,01	1,1	5	6	8	80	90	100
<i>T</i>	2,286	2,330	2,713	2,150	1,814	2,733	2,853	4,692	4,164	3,352
<i>n</i> ($\nu=n-2$)	16 (14)	16 (14)	16 (14)	16 (14)	16 (14)	16 (14)	16 (14)	16 (14)	16 (14)	16 (14)
Slope (nm per month)	-0,84	-0,92	-1,01	-0,86	-0,52	-0,68	-0,69	-1,67	-1,73	-1,77
<i>u</i> (slope)	0,37	0,40	0,37	0,40	0,29	0,25	0,24	0,36	0,42	0,53

table 10 b : TC gauge blocks ; value and *t student* test the slope of regression line

8.3.2 Complementary analysis

The statistical analysis is dependent on the leverage effects of those laboratories which are rather far from the regression line at the beginning or at the end of the loop. This analysis requires a complementary analysis based on measurements made by the pilot laboratory.

Monitoring of the pilot laboratory

The pilot laboratory made 3 calibrations, in July 1998, July 1999 and October 2000. An additional measurement was made on May 2001 which confirms the tendency of the first three measurements (this last measurement was made only on one face, and is not reported in this report)

The following figures show the drift for the 6 mm, 8 mm and 80 mm steel gauges blocks.

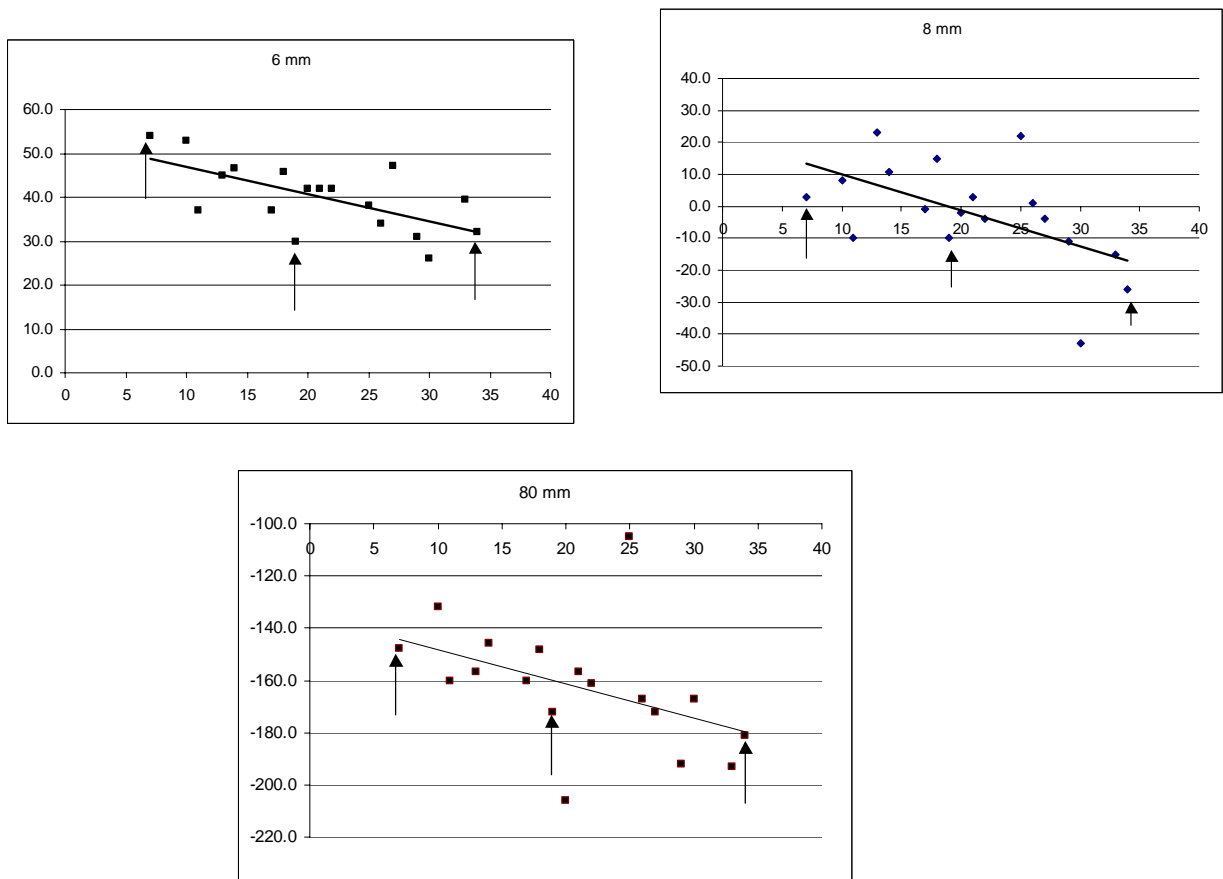


Figure 5 : Deviation from nominal length (nm) versus time (month) for steel gauge blocks : 6 mm, 8 mm, 80 mm ; results of the pilot laboratory are identified by arrows.

and for the 80 mm, 90 mm, 100 mm tungsten carbide gauge blocks

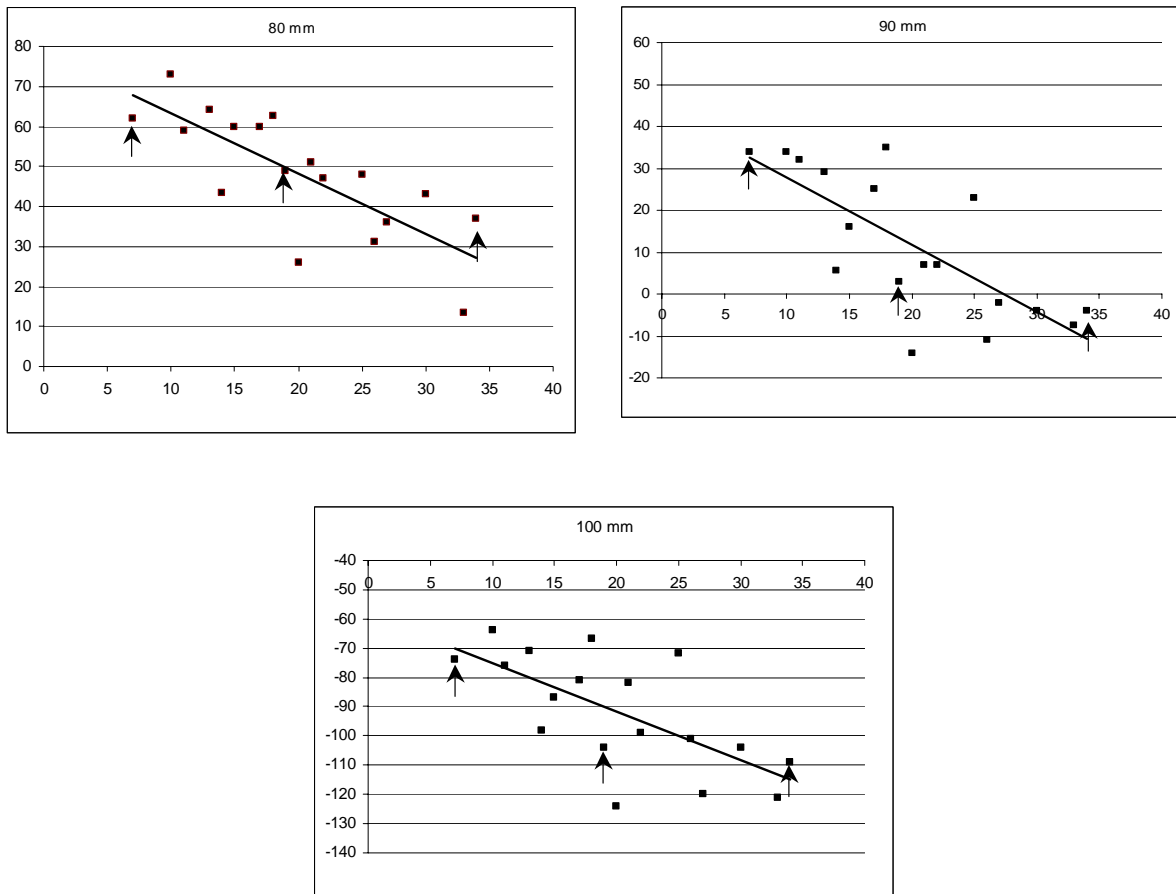


Figure 6 : Deviation from nominal length (nm) versus time (month) for tungsten carbide gauge blocks : 80 mm, 90 mm, 100 mm ; results of the pilot laboratory are identified by arrows.

Steel gb				TC gb			
Nom. value (mm)	1999/1998 (nm)	2000/1998 (nm)	u_c (nm)	Nom. value (mm)	1999/1998 (nm)	2000/1998 (nm)	u_c (nm)
0.5	-9	-33	14	0.5	-3	-11	14
1.01	-8	-1	14	1	2	-12	14
1.1	-11	-13	14	1.01	-12	-12	14
6	-24	-22	14	1.1	-9	-17	14
7	2	4	14	5	-23	-25	14
8	-13	-29	14	6	-17	-19	14
15	3	-6	14	8	-18	-20	14
80	-24	-33	20	80	-13	-25	14
90	4	5	21	90	-31	-38	16
100	-30	-38	23	100	-30	-35	16

table 11 : Differences of the pilot laboratory measurements made in 1999 and 2000 with respect to the first measurements of 1998.

8.3.3 Conclusion

For the tungsten carbide gauge blocks, there was a significant drift during the comparison for all the set. For steel gauge blocks, the drift is significant only for a part of the set. These drifts were confirmed through the pilot laboratory measurements and don't come from a leverage effect.

At the EUROMET length meeting in 2002, it was decided to apply the same treatment for all gauges blocks, steel and tungsten carbide.

A linear fit was removed from the data to take this drift into account . This means that all results were corrected with a *drift correction*, depending on the row of the laboratory.

9 CORRECTED RESULTS

9.1 DRIFT CORRECTION

The correction is based on the straight regression line according to the following equation:

$$X = b_0 + b_1T , \quad (2)$$

where X is the modeled value of the measurand at time T .

b_1 values are those obtained from the regression of all the results and are given in tables 10.

Each result is corrected by

$$-b_1(T - T_0) \quad (3)$$

where T and T_0 are respectively the period of measurement for the concerned laboratory and the first laboratory.

9.2 DEVIATION FROM NOMINAL LENGTH AFTER DRIFT CORRECTION

laboratory	Steel gauge blocks : Nominal length in mm									
	0,5	1,01	1,1	6	7	8	15	80	90	100
BNM-LNE	-15	88	69	54	-22	3	38	-148	39	-127
METAS	-12	104	79	55	-13	11	56	-128	79	-115
PTB	-23	91	63	40	-30	-6	39	-155	54	-144
NPL	-21	93	75	49	-11	29	41	-149	63	-119
SMD	-12	92	86	53	-16	17	47	-137	69	-130
MIKES	-18	99	64	44	-17	9	47	-147	53	-139
DFM	-14	100	79	53	-17	26	54	-134	80	-120
JV	-12	104	73	51	-20	11	38	-189	6	-189
SP	-10	95	63	51	-18	17	42	-139	60	-141
IMGC	-17	88	68	52	-27	11	55	-141	48	-147
IPQ	-8	95	72	50	-14	40	61	-81	102	-64
BEV	-12	97	78	46	-17	20	48	-142	51	-141
CEM	-13	102	90	60	-13	16	56	-146	34	-151
OMH	10	34	17	45	-15	11	39	-163	78	-85
NMI-VSL	-41	83	53	41	-35	-20	42	-137	47	-145
SMU	-22	96	78	57	-14	11	36	-159	57	-145

table 12a : Steel gauge blocks ; deviation from nominal value (in nm) with drift correction

laboratory	TC gauge blocks : Nominal length in mm									
	0,5	1	1,01	1,1	5	6	8	80	90	100
BNM-LNE	-5	31	101	78	26	30	2	62	34	-74
METAS	-6	25	95	65	9	16	-10	78	39	-59
PTB	-16	14	86	60	-4	8	-24	66	39	-69
NPL	3	33	108	76	19	25	-3	74	39	-60
SMD	-11	18	91	58	2	10	-14	55	18	-86
CMI	7	31	101	77	15	22	-6	73	30	-73
MIKES	0	24	99	69	6	14	-13	77	42	-63
DFM	9	42	105	73	13	13	-2	81	54	-47
JV	-3	24	104	74	2	9	-13	48	8	-101
SP	0	18	95	71	10	15	-11	74	31	-57
IMGC	-2	13	97	65	7	11	-10	72	33	-72
IPQ	-28	1	76	43	-2	11	-12	78	54	-40
BEV	14	45	119	93	16	26	-1	63	22	-67
CEM	6	31	103	75	9	16	-9	69	33	-85
NMI-VSL	-5	30	107	77	11	15	-1	81	36	-63
SMU	-12	24	84	60	20	27	-15	57	37	-75

table 12b : TC gauge blocks ; deviation from nominal value (in nm) with drift correction

10 STATISTICAL ANALYSIS

10.1 AVERAGE DEVIATION AND STANDARD DEVIATION ; DEFINITIONS

For each laboratory, the average $\langle \Delta l \rangle$ of the deviations from the mean of all gauge blocks is calculated as follows :

$$\langle \Delta l \rangle = \frac{1}{N} \sum_{j=1}^N (x_j - \bar{x}_j) \quad (4)$$

where x_j is the result for the individual gauge block j and \bar{x}_j the mean of all laboratories for that gauge block. $N = 10$ is the number of gauge blocks for each material.

By the same way, the standard deviations s of the differences $x_j - \bar{x}_j$ are calculated:

$$s^2 = \frac{1}{N-1} \sum_{j=1}^N \left((x_j - \bar{x}_j) - \langle \Delta l \rangle \right)^2 \quad (5)$$

The normalised standard deviation s_n is the standard deviation divided by the standard uncertainty u_j given by the laboratory for the individual gauge block j

$$s_n^2 = \frac{1}{N-1} \sum_{j=1}^N \left(\frac{(x_j - \bar{x}_j - \langle \Delta l \rangle)}{u_j} \right)^2 \quad (6)$$

10.2 RESULTS

The following tables and figures give the result of the computation of $\langle \Delta l \rangle$, s and s_n for each laboratory and gauge block material.

Results are given for the simple mean value calculated on the original data and for the mean value in the case of drift correction.

	Steel gauge blocks (original data)			Steel gauge blocks with drift correction		
	Δl (nm)	s (nm)	s_n	Δl (nm)	s (nm)	s_n
BNM-LNE	1,0	9,7	0,8	-4,7	6,8	0,5
METAS	13,4	7,1	0,7	9,0	7,0	0,7
PTB	-5,8	4,0	0,4	-9,7	4,4	0,5
NPL	5,5	8,3	0,6	2,4	7,9	0,5
SMD	7,0	5,5	0,5	4,3	4,8	0,5
MIKES	-1,9	4,2	0,4	-3,2	4,1	0,4
DFM	9,1	6,4	0,5	8,2	6,4	0,5
JV	-15,2	25,5	1,5	-15,3	25,6	1,5
SP	-1,0	5,3	0,4	-0,6	5,4	0,4
IMGC	-4,5	6,7	0,6	-3,6	6,9	0,6
IPQ	20,5	26,2	2,0	22,7	26,4	2,1
BEV	-2,4	4,7	0,4	0,2	5,2	0,4
CEM	-2,1	12,5	1,1	0,9	12,9	1,1
OMH	-0,9	23,5	1,1	-5,2	33,1	1,0
NMI-VSL	-14,5	13,2	1,1	-13,7	11,1	0,9
SMU	-10,5	11,2	0,4	-3,1	8,1	0,3

table 13a : Steel gauge blocks ; Average deviation, standard deviation and normalised standard deviation from mean and linear regression line for all laboratories

	Tungsten carbide gauge blocks (original data)			Tungsten carbide gauge blocks with drift correction		
	ΔI (nm)	s (nm)	s_n	ΔI (nm)	s (nm)	s_n
BNM-LNE	17,3	4,3	0,4	4,2	8,0	0,8
METAS	10,7	9,2	0,9	0,8	5,1	0,5
PTB	0,5	10,1	1,2	-8,3	6,4	0,7
NPL	13,8	2,7	0,2	7,1	1,7	0,1
SMD	-4,5	2,5	0,3	-10,2	4,5	0,5
MIKES	8,1	3,5	0,3	3,5	4,7	0,4
DFM	3,6	5,5	0,5	1,2	4,5	0,4
JV	11,1	8,4	0,7	9,7	7,9	0,7
SP	-9,9	13,5	0,8	-9,1	13,2	0,8
IMGC	-1,6	4,8	0,3	0,2	5,2	0,3
IPQ	-5,9	4,0	0,4	-3,0	4,2	0,4
BEV	-12,2	17,6	1,4	-6,1	19,6	1,5
CEM	1,5	14,5	1,3	8,7	12,1	1,1
OMH	-8,0	9,3	0,9	0,6	7,0	0,7
NMI-VSL	-9,7	8,4	0,7	4,4	4,7	0,5
SMU	-17,6	13,4	0,5	-3,7	8,7	0,4

table 13b: TC gauge blocks ; Average deviation, standard and normalised standard deviation from mean and linear regression line for all laboratories

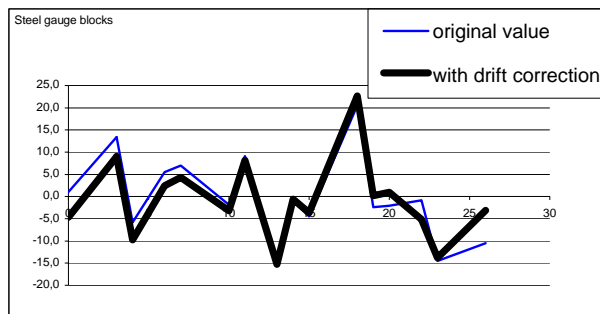


Figure 7a: Steel gauge blocks. : Average deviation from mean and regression line according to the row in the schedule

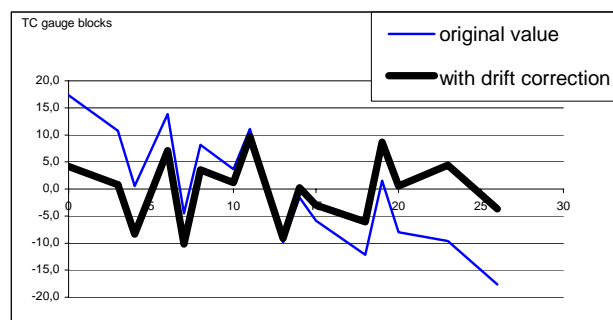


Figure 7b : TC gauge blocks : Average deviation from mean and regression line according to the row in the schedule

10.3 STATISTICAL DISTRIBUTION

Figures 8 to 11 show the histograms of the normalized deviations from the means of all measurement results.

Normalized deviations are obtained by the ratio of the difference of each result with the mean value (calculated without and with drift correction) divided by the standard uncertainty of each individual result.

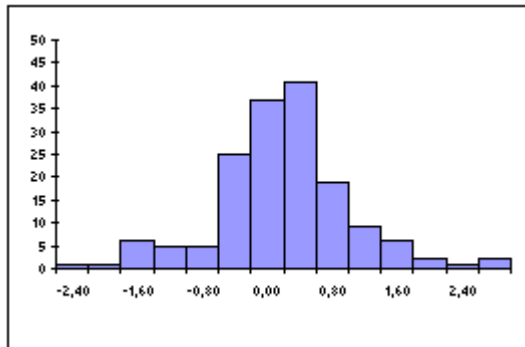


Figure 8 : Steel gauge blocks ; deviation from mean for raw data ($s = 0.86$)

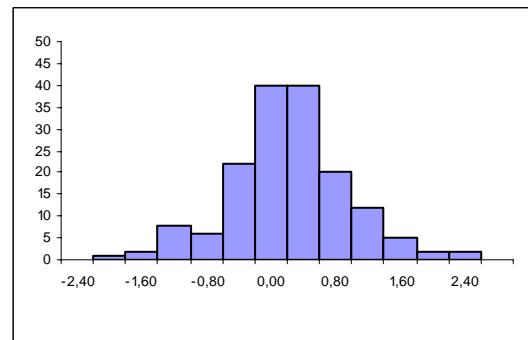


Figure 10 : Tungsten carbide gauge blocks ; deviation from mean for raw data ($s = 0,73$)

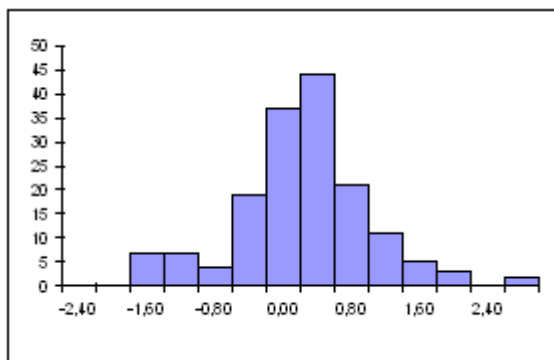


Figure 9 : Steel gauge blocks ; deviation from mean for corrected values ($s = 0.82$)

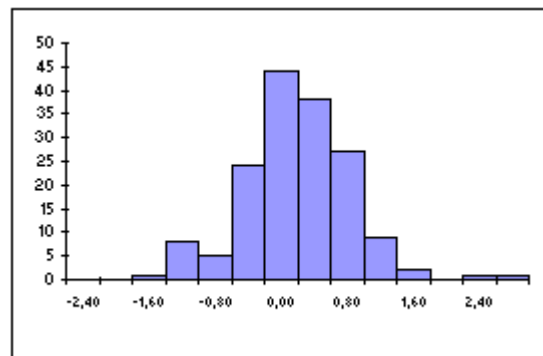


Figure 11 : Tungsten carbide gauge blocks ; deviation from mean for corrected values ($s = 0.66$)

10.4 STATISTICAL CONSISTENCY

The statistical consistency of a comparison can be investigated by the so-called Birge ratio R_B [3], which compares the observed spread of the results with the spread expected from the individual reported uncertainties.

The application of least squares algorithms and the χ^2 -test leads to the Birge ratio

$$R_B = \frac{u_{ext}}{u_{in}}, \tag{7}$$

where u_{in} , the internal standard deviation, is given by the reported uncertainties

$$u_{in} = \left(\sum_{i=1}^n u^{-2}(x_i) \right)^{-1/2}, \tag{8}$$

and the external standard deviation u_{ext} is the standard deviation of the spread of the results x_i , weighted by the associated uncertainties $u(x_i)$:

$$u_{ext} = \left(\frac{\sum u^{-2}(x_i)(x_i - \bar{x}_w)^2}{(n-1)\sum u^{-2}(x_i)} \right)^{1/2}. \tag{9}$$

\bar{x}_w is the weighted mean given by

$$\bar{x}_w = \frac{\sum_{i=1}^n u^{-2}(x_i) \cdot x_i}{\sum_{i=1}^n u^{-2}(x_i)}. \tag{10}$$

The Birge ratio has an expectation value of $R_B = 1$. For a coverage factor of $k = 2$, the data in a comparison are consistent provided that

$$R_B < \sqrt{1 + \sqrt{8/(n-1)}}. \tag{11}$$

For $n = 16$ participating laboratories, a value $R_B > 1,32$ (*1,33 for $n=15$) may be interpreted such that the laboratories have underestimated their uncertainties.

Steel	0,5 mm	1,01 mm	1,1 mm	6 mm	7 mm	8 mm	15 mm	80 mm	90 mm	100 mm
u_{in}	2,69	2,69	2,64	2,64	2,65	2,72	3,74	3,95	4,22	2,69
u_{ext}	2,17	2,94	1,55	1,74	3,35	1,93	5,44	5,42	7,07	2,17
R_B	0,81	1,09	0,59	0,66	1,26	0,71	1,45	1,37	1,67	0,81
$R_{B <}$	1,32	1,33*	1,33*	1,32	1,32	1,32	1,32	1,32	1,32	1,32

TC	0,5 mm	1 mm	1,01 mm	1,1 mm	5 mm	6 mm	8 mm	80 mm	90 mm	100 mm
u_{in}	2,57	2,53	2,57	2,49	2,50	2,50	2,50	3,07	3,21	3,33
u_{ext}	2,41	2,75	2,22	2,61	1,95	1,64	1,80	2,22	2,55	3,31
R_B	0,94	1,09	0,86	1,05	0,78	0,66	0,72	0,72	0,80	0,99
$R_{B <}$	1,32	1,32	1,32	1,32	1,32	1,32	1,32	1,32	1,32	1,32

table 14 : Birge ratio for corrected results

11 CONCLUSIONS

- This comparison involves 17 laboratories in all, 16 laboratories for each type of gauge blocks. Considering the number of laboratories two facts can be noticed :

- the duration of the comparison, two years only for the measurement period,
- the damage on the gauge blocks by repeated wringing.

The last point is important in such international comparison, leading to CMC, which should provide all laboratories with the same quality of standards.

In this comparison, specially for steel gauge blocks, it appears that surfaces rapidly suffered from the repeated wringings so that some laboratories, at the end of the loop, could not measure both faces and had some difficulties in determining the phase correction.

Consequently, in some cases, laboratories have increased their uncertainties of measurement compared to their best measurement capability.

It points out that the organisation of future comparison shall deal with the number of participants, which are now more than 20 within Euromet for this type of measurement, and take into account the relative fragility of some materials to support a great number of wringings.

- During this comparison a drift was observed on some gauges blocks.

It is obvious that some of the options retained in this report are still open to discussion and improvement for future comparison.

The first problem to be solved was to state on the significance of the drift. In this report analyses were made both on pilot laboratory monitoring and general trend given by the results of all participants. Analyses were made on the visual trend of graphs, completed by a statistical analysis.

The second problem was how to deal with the drift ; there are two possibilities :

- the first one consists in estimating the reference value with the original set of data and then completing the uncertainty budget on this value with an additional uncertainty based on the drift. If there is evidence on the drift phenomenon, it could lead to increase the uncertainty on the reference value and overestimate the deviation of the results of some laboratories from the reference value.

- the second one, which has been applied in this report, consists in applying drift correction based on a linear trend. In this case the reference value is given by the intercept of the regression line. This solution leads to reduce, in most cases, the difference with the reference value but the consequence is that the uncertainty on this difference is dependent on the quality of the estimation of the trend.

At the 14th TCL meeting it was decided to apply a drift correction on all gauge blocks of the two sets. The reason was mainly to apply homogeneous treatment to all gauge blocks. Once again, this could be discussed.

However, for future comparison the question is how to avoid or manage such situations, either by special manufacturing (?), or by selecting in advance a stabilized sets of gauge blocks or by having more than one laboratory monitor the measurand during the comparison.

- With the agreement of all other participants, two laboratories withdrew for some results after analysis of the results of the first draft and one for all gauge blocks due to the lack of time for determining phase correction.

Following these first conclusions, an additional comparison (EUROMET project # 643, now EUROMET **L-K1a**) was organized for those laboratories which have withdrawn and for those laboratories which have found explanations for any significant offset of their results and improved their process since the publication of the draft A of this report.

- The measurement results show a good agreement between laboratories. The deviation from the average is, except very few results, smaller than the stated standard uncertainties. Except for a few laboratories, the uncertainties given by all participants are rather coherent both in the components and in the quantification. This can be explained by the fact that many laboratories have the same type of interferometer.

12 REFERENCES

[1] - R. Thalmann, CCL key comparison CCL-K1 : *Calibration of gauge blocks by interferometry*, *Metrologia*, 2002, **39**, 165-177 (available on BIPM Web site, <http://www.bipm.org>)

[2] ISO 5725-2 : *Accuracy (trueness and precision) of measurement methods and results – part 2 : Basic method for the determination of repeatability and reproducibility of a standard measurement method*.

[3] *Statistical Analysis of Interlaboratory Comparisons*, Report on EUROMET workshop held at the National Physical Laboratory (UK) 11-12 November 1999

13 ACKNOWLEDGEMENTS

The pilot would like to thank all the members of the Euromet Technical Committee Length group and specially the participants of this comparison for their help and useful comments on this report.

APPENDIX 1: MEASUREMENT CONDITIONS

Table A1. Measurement instruments and conditions reported by the participating laboratories.

Lab.	Make and type of interferometer	Light sources, Wave lengths	Method of fringe fraction determination	Refractive index determination	Temperature range / °C
LNE	NPL-TESA Automatic gauge block interferometer, Twyman Green design	He-Ne Laser $\lambda = 633$ nm He-Ne Laser $\lambda = 543$ nm	Horizontal scan through interference pattern digitised through video-interface. The signal (intensity profile) is filtered. The fringe fraction is obtained from the phase shift between the profiles at the middle height of the sin curves fitted on the intensity profile signal.	Edlèn's equation as modified by Birch and Downs (<i>Metrologia</i> 31, 1994, 315-316)	19.8 °C- 20.2 °C
METAS	NPL-TESA Automatic gauge block interferometer, Twyman Green design	He-Ne Laser $\lambda = 633$ nm He-Ne Laser $\lambda = 543$ nm	Video-electronic fringe fractioning (extraction of a TV-image column through the center of the gauge block [transversal direction] and fitting a sinusoidal function through the intensity profile sections on the gauge block and the base plate; the fringe fraction is obtained from the phase shift between the profile sections).	Edlèn's equation	19,95 °C 20,03 °C
NPL	NPL-TESA Automatic gauge block interferometer, Twyman Green design	Zeeman stabilized He-Ne lasers at 633 nm and 543 nm. Sources are calibrated by beat frequency comparison against iodine-stabilised reference lasers	Vertical scan through interference pattern digitised via electronic interface into a computer. Computer determines location of fringe minima on platen and gauge surfaces within the scanned data. Difference between actual minima on gauge and minima predicted by interpolation of fringes on platen is taken as the fringe fraction (suitably scaled). Operates separately at each wavelength. Only red (633 nm) wavelength fringe fraction is used in length determination - green fringe fraction verifies correct fringe order number only.	Edlèn's equation as modified by Birch and Downs (<i>Metrologia</i> 31, 1994, 315-316)	19,950 °C 20,018 °C

Lab.	Make and type of interferometer	Light sources, Wave lengths	Method of fringe fraction determination	Refractive index determination	Temperature range / °C
PTB	Michelson interferometer, modified TSUGAMI	Wavelength stabilized He-Ne lasers at 633 nm and 543 nm, Rb-stabilized diode-laser at 780 nm	Automatic interference evaluation by phase stepping-interferometry	Edlèn's equation, <i>Metrologia</i> 35, 133-139 (1998)	Steel : 19,948°C to 20,002 °C TC : 19,950 °C to 19,982 °C
SMD	NPL-TESA Automatic gauge block interferometer, Twyman Green design	He-Ne Laser $\lambda = 633$ nm He-Ne Laser $\lambda = 543$ nm	According to the standard method of TESA automatic gauge block interferometer	Edlèn's equation as modified by Birch and Downs (<i>Metrologia</i> 31, 1994, 315-316)	Steel : 19,979°C to 20,077 °C TC : 19,999 °C to 20,068 °C
CMI	NPL-TESA Automatic gauge block interferometer, Twyman Green design	He-Ne Laser $\lambda = 633$ nm He-Ne Laser $\lambda = 543$ nm	According to the standard method of TESA automatic gauge block interferometer	According to NPL-TESA software	19,779°C to 19,965 °C
MIKES	NPL-TESA Automatic gauge block interferometer, Twyman Green design	He-Ne Laser $\lambda = 633$ nm He-Ne Laser $\lambda = 543$ nm	According to the standard method of TESA automatic gauge block interferometer	Edlèn's equation	20,08 °C to 20,19 °C
DFM	NPL-TESA Automatic gauge block interferometer, Twyman Green design	Zeeman stabilized He-Ne lasers at 633 nm and 543 nm.	Linear interpolation between fringes inside and outside the gauge block area. Green laser only used for establishing integer fringe order and not for the gauge block length.	Edlèn's equation as modified by Birch and Downs (<i>Metrologia</i> 31, 1994, 315-316)	19,7 °C to 20,2 °C

Lab.	Make and type of interferometer	Light sources, Wave lengths	Method of fringe fraction determination	Refractive index determination	Temperature range / °C
JV	NPL-TESA Automatic gauge block interferometer, Twyman Green design	Zeeman stabilized He-Ne lasers at 633 nm and 543 nm	According to the standard method of TESA automatic gauge block interferometer	Edlèn's equation as modified by Birch and Downs (<i>Metrologia</i> 30, 1993, 155-162) ⁽¹⁾	L < 10 mm $\Delta T < 0,283$ °C L < 25mm $\Delta T < 0,212$ °C L > 25 mm $\Delta T < 0,141$ °C $\Delta T = T_{\text{gauge}} - 20$ °C
SP	Carl Zeiss Jena interferometer, Köster type	Kr-lamp, 4 wavelengths (red, yellow, green, violet)	Visual	Edlèn's equation	20 °C to 20,2 °C
IMGC	Hilger & Watts T 190 NPL-H&W (Fizeau interferometer)	Zeeman stabilized He-Ne lasers at 633 nm Kr ⁸⁶ spectral lamp $\lambda_2 = 645,8$ nm $\lambda_3 = 605,7$ nm $\lambda_4 = 450,3$ nm	Video signal acquisition, digital signal filtering	Edlèn's equation (<i>Metrologia</i> v.2, 1966, 71-80) modified by Birch and Downs (<i>Metrologia</i> 31, 1994, 315-316)	19,73 °C to 20,00 °C
IPQ	NPL-TESA Automatic gauge block interferometer, Twyman Green design	He-Ne Laser $\lambda = 633$ nm He-Ne Laser $\lambda = 543$ nm	First find the position of fringe minima in relation to the TV line number. Second find a more precise position using a least squares fitting routine. Next determine the phase displacement at the gauge centre by fitting the fringe number on the reference flat to a quadratic form. The exact fringe number at the gauge centre can be calculated for the fringes on the gauges. The difference in two values of fringe number provides the fringe fraction.	Edlèn's equation (<i>Metrologia</i> v.2, 1966, 71-80)	(20,5 ± 0,3) °C
BEV	Carl Zeiss Jena interferometer, Köster type	Cd 114 spectral lamp 644,02480 nm, 508,72379 nm, 480,12251 nm, 467,94581 nm	Peltier cooled CCD camera for the recording of interferograms. Dedicated interferometric fringe processing software developed for BEV (Appl. Opt. 199, 38, pp 101 ff.)	Edlèn's equation as modified by Birch and Downs (<i>Metrologia</i> 30, 1993, 155-162 and <i>Metrologia</i> 31, 1994, 315-316)	19,952 °C to 20,162 °C

Lab.	Make and type of interferometer	Light sources, Wave lengths	Method of fringe fraction determination	Refractive index determination	Temperature range / °C
CEM	NPL-TESA Automatic gauge block interferometer, Twyman Green design	He-Ne Laser $\lambda = 633$ nm He-Ne Laser $\lambda = 543$ nm	According to the standard method of TESA automatic gauge block interferometer	Edlèn's equation	$(20 \pm 0,05)$ °C
OMH	Carl Zeiss Jena interferometer, Köster type	Frequency stabilized He-Ne Laser $\lambda = 633$ nm; stabilized by temperature; made by a Hungarian manufacturer	The gauge blocks were measured by comparative method first in order to have the integer part of number of half wavelength. Then the gauge blocks were wrung onto platen made from quartz. To observe the fringe fraction a CCD camera was used and the images of the individual blocks were stored in a computer. To evaluate the image and to determine the fractional part, graphical method was used. In order to have the length in the centre of the gauges mathematical linear polynomial regression method was used.	Edlèn's equation	19,7 °C to 20,1 °C
NMi	Carl Zeiss Jena interferometer, Köster type	He-Ne Laser $\lambda = 633$ nm He-Ne Laser $\lambda = 543$ nm He-Ne Laser $\lambda = 612$ nm	By moving the gauge block up/down (using piezo-actuator) the interference pattern is scanned over a hairline. The piezo's voltage is measured when an interference line is aligned with the airline. The exact fractions are calculated from the voltage values of the interference lines of the gauge block and the optical flat.	Edlèn's equation	19,95 °C to 20,05 °C
SMU	Carl Zeiss Jena interferometer, Köster type	Kr ⁸⁶ spectral lamp $\lambda = 645,8$ nm $\lambda = 587,2$ nm $\lambda = 565,1$ nm $\lambda = 450,3$ nm	The method is based on the determination of the difference between the fractions of wavelength corresponding to the nominal length and that one corresponding to the real length of the gauge block, for each of four wavelengths.	Edlèn's equation	19,920 °C to 20,112 °C

	Steel gauge blocks			TC gauge blocks			Wringing fluid
	material	mean	range	material	mean	range	
BNM-LNE	steel	-21		steel	-15		yes
METAS	steel	-12		TC	-12		
PTB	steel		-5 ; +2	TC		-10 ; -2	no
NPL	steel	-45,5	-44,1 ; -46,7	TC	+2,3	+0,3 ; +4,3	
SMD	steel	-26,3		TC	-43,7		
CMI	--	--	--	steel		-40 ; -36	yes
MIKES	steel		-43 ; -34	steel		-50,5 ; -40, 2	yes
DFM	steel	-8,4		steel	-36,7		
JV	steel		-6,8 ; -16	TC		-50,1 ; -53,3	Yes (paraffin oil)
SP	steel	0		TC	-10		
IMGC	steel	-28		steel	-30		
IPQ	steel	-24,22		steel	-71,68		yes
BEV	single crystalline quartz	+32		single crystalline quartz	+7		no
CEM	steel	-20		TC	+1		
OMH	quartz	+30		--	--	--	
NMI-VSL	glass (low expansion)	44,1		glass (low expansion)	18,8		
SMU	steel	-7		steel	-48		

Table A2 : Material of the reference flats and values of phase correction in nanometer applied to the measurement results.

APPENDIX 2 : DETERMINATION OF THE REFERENCE VALUES

CONSISTENCY OF RESULTS AND OUTLIERS

Some reported results seem to be inconsistent with other results, and may change the estimate and specially reference values.

GRUBBS' TEST

The Grubbs' test according to the ISO 5725-2 : 1994² was applied for one outlying observation.

The test is applied on the smallest and largest values of the set of data, x_i for $i = 1, 2, \dots, p$. The Grubbs' statistic is calculated as

$$G_p = \frac{(x_p - \bar{x})}{s} \quad \text{or} \quad G_p = \frac{(\bar{x} - x_1)}{s} \quad \text{where}$$

$$\bar{x} = \frac{1}{p} \sum_{i=1}^p x_i \quad \text{and} \quad s = \sqrt{\frac{1}{p-1} \sum_{i=1}^p (x_i - \bar{x})^2}$$

If the test statistic is greater than or equal to its 5% critical value and less than or equal to its 1% critical value, the item tested is called a straggler. If the test is greater than its 1% critical value, the item is called a statistical **outlier**.

Outliers do not participate in the calculation of the means.

² ISO 5725-2 : Accuracy (trueness and precision) of measurement methods and results – part 2 : Basic method for the determination of repeatability and reproducibility of a standard measurement method.

OUTLIERS

The Grubbs' test gives the same results for both cases of corrected and original values.

Steel	0,5 mm	1,01 mm	1,1 mm	6 mm	7 mm	8 mm	15 mm	80 mm	90 mm	100 mm
max	10,2	104,4	90,3	60,1	-10,9	40,1	60,8	-81,5	102,5	-64,1
min	-41,4	34,0	17,0	39,6	-34,7	-19,9	35,6	-189,0	5,9	-188,9
G max	2,436	0,795	1,256	1,792	1,176	1,976	1,836	2,832	2,024	2,387
G min	2,550	3,492	3,121	1,849	2,393	2,390	1,297	2,081	2,326	2,040
GRUBBS 95%	2,585	2,585	2,585	2,585	2,585	2,585	2,585	2,585	2,585	2,585
GRUBBS 99%	2,852	2,852	2,852	2,852	2,852	2,852	2,852	2,852	2,852	2,852

table A3a. Corrected values; Results of the Grubb's test for steel gauge blocks: Bold values are outliers

TC	0,5 mm	1 mm	1,01 mm	1,1 mm	5 mm	6 mm	8 mm	80 mm	90 mm	100 mm
max	13,9	45,5	119,2	93,3	26,0	30,0	2,0	81,3	54,1	-40,1
min	-27,9	0,6	76,2	43,4	-3,9	7,7	-24,3	47,7	8,5	-101,0
G max	1,650	1,823	1,992	2,110	1,970	1,889	1,615	1,210	1,690	1,890
G min	2,403	2,213	2,099	2,339	1,706	1,280	2,268	2,157	2,212	2,197
GRUBBS 95%	2,585	2,585	2,585	2,585	2,585	2,585	2,585	2,585	2,585	2,585
GRUBBS 99%	2,852	2,852	2,852	2,852	2,852	2,852	2,852	2,852	2,852	2,852

table A3b. Corrected values; Results of the Grubb's test for TC gauge blocks.

REFERENCE VALUES

ARITHMETIC MEAN

The arithmetic mean reference value x_{ref} is calculated by the average of all measurement values x_i :

$$x_{ref} = \frac{1}{n} \sum_{i=1}^n x_i \quad (A1)$$

The arithmetic mean does not take into account the uncertainty of the individual results contributing to the reference value.

The standard uncertainty $u(x_{ref})$ of the arithmetic mean can either be determined by application of the error propagation law, i.e. by taking into account the uncertainties $u(x_i)$ of the individual results [Eq. (A2a)], or by the spread of the results, i.e. by the standard deviation divided by the square root of the number n of results contributing to the mean [Eq. (A2b)].

$$u(x_{ref}) = \frac{1}{n} \sqrt{\sum_{i=1}^n u^2(x_i)} = \frac{u_{rms}}{\sqrt{n}} \quad (A2a)$$

or

$$u(x_{ref}) = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^n (x_i - x_{ref})^2} . \quad (A2b)$$

ARITHMETIC MEAN IN CASE OF DRIFT CORRECTION

The drift of the gauge block is estimated by a linear regression defined by the straight regression line according to the following equation:

$$X = b_0 + b_1 T \quad (A3)$$

where the time $T = 0$ is corresponding to the first measurement and

$$b_0 = \bar{x} - b_1 \bar{t} \quad (A4)$$

$$b_1 = \frac{\sum (t_i - \bar{t})(x_i - \bar{x})}{\sum (t_i - \bar{t})^2} \quad (A5)$$

If we calculate the arithmetic mean x_{refc} for all the corrected results $(x_i - b_1 t_i)$, we obtain a

formula $x_{refc} = \frac{1}{n} \sum_{i=1}^n (x_i - b_1 t_i)$, which is equivalent to

$$x_{refc} = \bar{x} - b_1 \bar{t} = b_0 \quad (A6)$$

In this case the reference value is estimated by the intercept of the regression line.

Thus the standard uncertainty $u(x_{refc})$ is given by

$$u(x_{refc}) = \sqrt{\frac{\sigma^2 \sum_{i=1}^n t_i^2}{n \sum_{i=1}^n (t_i - \bar{t})^2}} \quad (A7)$$

$$\text{with } \sigma^2 = \frac{\sum (x_i - \hat{x}_i)^2}{n-2}$$

In this case the uncertainty on the reference value depends on the deviation of all the results from the model, i.e. the regression line and σ^2 is similar to an external deviation (A2b).

UNCERTAINTY OF THE DIFFERENCE BETWEEN CORRECTED RESULTS AND THE REFERENCE VALUES

For the arithmetic mean approach, the expanded uncertainty $U(\Delta x)$ is given by

$$U(\Delta x) = 2 \sqrt{\left(1 - \frac{2}{n}\right) u^2(x_i) + \frac{1}{n^2} \sum_{j=1}^n u^2(x_j)}. \quad (A8)$$

In the case of a drift correction, the difference between the results and the reference value becomes :

$$\Delta x_i = [x_i - b_1 t_i] - (b_0) \quad (A9)$$

This formula is equivalent to $\Delta x_i = x_i - \hat{x}_{(ref)_i}$, where $\hat{x}_{(ref)_i}$ is the modeled value, given by the linear regression.

In this case, the expanded uncertainty $U(\Delta x)$ is given by

$$U(\Delta x) = 2 \sqrt{\sigma^2 \left(1 - \frac{1}{n} - \frac{(t_i - \bar{t})^2}{\sum (t_i - \bar{t})^2}\right)} \quad (A10)$$

This formula shows that $U(\Delta x)$ depends on the uncertainty of the slope of the fitted line but doesn't take into account the proper uncertainty of each laboratory.

One idea was to complete the formulas above with the uncertainty of each results, but in addition to the fact that this is not statistically correct, these formulas lead to overestimate the uncertainty on $U(\Delta x)$

It seems to the TCL group that one acceptable solution is to combined *simply* $u(x_{refc})$ and $u(x_i)$ as follows

$$U(\Delta x) = 2 \sqrt{\frac{\sigma^2 \sum_{i=1}^n t_i^2}{n \sum_{i=1}^n (t_i - \bar{t})^2} + u^2(x_i)} \quad (\text{A11})$$

which is not fully correct as the value of X_{refc} and x_i are correlated, but leads to values that seem to be reasonable.

Further comments

In this report it was decided to apply drift correction and then calculate the uncertainty on the reference value as given by (A7) formula.

An other possibility was to estimate the reference value with the simple mean and then estimate the uncertainty on this value by a quadratic combination of $u(x_{ref})$ (formula A2) and a component dependent on the drift.

This component could be evaluated via the change in length calculated with the slope of the trend and assuming a rectangular distribution.

$$u_{drift} = \sqrt{\frac{b_1 \times duration}{12}} \quad (\text{A12})$$

The result of this combination is given in tables 6.

In these tables the different calculations are compared. It shows that for steel gauge blocks the uncertainty on the reference value (line $u(x_{ref}c)$) and line *Combination of A2a and A12* are not really different except for 90 mm and 100 mm where the uncertainty on X_{refc} is affected by the bad adjustment of the data to the regression line as the slope is not really important.

For TC gauge blocks the uncertainty on $u(x_{ref}c)$ is always lower than the *Combination of A2a and A12* , which confirms the necessary correction of drift, which improves both the estimation of the reference value and its uncertainty.

Steel gauge blocks	Nominal value (mm)	0,5	1,01	1,1	6	7	8	15	80	90	100
Number of lab.		16	15	15	16	16	16	16	16	16	16
Ref. value corrected from drift x_{ref_c}	Formula A4	-15,0	95,2	72,8	50,0	-18,7	13,0	46,0	-143,4	57,5	-131,4
$u(x_{ref_c})$	Formula A7	5,4	3,3	5,1	2,9	3,5	7,2	4,2	11,4	11,6	14,7
Arithmetic mean	Formula A1	-19,3	89,5	68,2	41,4	-21,2	-0,3	46,2	-160,7	58,7	-133,5
Standard uncertainty	Formula A2a	2,7	1,8	2,6	1,9	1,7	4,0	2,0	6,0	5,6	7,1
Standard uncertainty	Formula A2b	3,5	3,1	3,1	3,3	3,4	3,3	3,5	4,8	5,2	5,5
Standard uncertainty derive from drift	Formulas A12	2,5	4,1	4,1	4,9	1,4	7,5	0,1	9,8	0,6	1,2
Combination of A2a and A12		4,3	5,1	5,1	5,9	3,6	8,2	3,5	11,0	5,2	5,7

Table A6a

TC gauge blocks	Nominal value (mm)	0,5	1	1,01	1,1	5	6	8	80	90	100
Number of lab.		16	16	16	16	16	16	16	16	16	16
Ref. value corrected from drift x_{ref_c}	Formula A4	-3,1	25,2	98,3	69,6	10,0	16,7	-8,9	69,2	34,4	-68,2
$u(x_{ref_c})$	Formula A7	5,2	5,7	5,3	5,7	4,1	3,6	3,4	5,1	5,9	7,6
Arithmetic mean	Formula A1	-13,4	13,9	85,8	59,1	3,5	8,3	-17,4	48,7	13,1	-90,1
Standard uncertainty	Formula A2a	3,0	3,3	3,2	3,2	2,3	2,2	2,1	4,0	4,4	5,0
Standard uncertainty	Formula A2b	3,0	2,9	3,0	2,9	2,9	2,9	2,9	3,5	3,7	3,9
Standard uncertainty derive from drift	Formulas A12	6,3	6,9	7,6	6,4	3,9	5,1	5,1	12,5	13,0	13,3
Combination of A2a and A12		7,0	7,5	8,2	7,0	4,9	5,9	5,9	13,0	13,5	13,9

Table A6b

Table A6a/ A6b . Reference values and associated standard uncertainties (in nm) estimated with the intercept of the regression line ; comparison with the mean of raw data.

**APPENDIX 3 : COMPARISON OF RESULTS WITH THE REFERENCE VALUES
AND E_n VALUES**

	0,5 mm	1,01 mm	1,1 mm	6 mm	7 mm	8 mm	15 mm	80 mm	90 mm	100 mm
BNM-LNE	0 ± 23	-7 ± 23	-4 ± 23	4 ± 21	-3 ± 21	-10 ± 25	-8 ± 22	-5 ± 36	-19 ± 38	4 ± 43
METAS	3 ± 22	9 ± 22	6 ± 23	5 ± 17	5 ± 18	-2 ± 22	10 ± 20	15 ± 32	21 ± 33	17 ± 38
PTB	-8 ± 19	-4 ± 20	-9 ± 20	-10 ± 17	-12 ± 17	-19 ± 21	-7 ± 18	-11 ± 30	-4 ± 31	-13 ± 37
NPL	-6 ± 30	-2 ± 30	2 ± 30	-1 ± 29	8 ± 29	16 ± 31	-5 ± 31	-6 ± 60	6 ± 66	12 ± 72
SMD	3 ± 22	-3 ± 23	13 ± 23	3 ± 20	3 ± 21	4 ± 24	1 ± 21	6 ± 35	12 ± 36	1 ± 41
MIKES	-3 ± 23	3 ± 24	-9 ± 24	-6 ± 21	2 ± 22	-4 ± 25	1 ± 22	-3 ± 38	-4 ± 40	-8 ± 45
DFM	1 ± 21	5 ± 22	7 ± 22	3 ± 20	2 ± 20	13 ± 23	8 ± 21	10 ± 41	22 ± 44	11 ± 50
JV	3 ± 28	9 ± 29	0 ± 29	0 ± 27	-1 ± 27	-2 ± 30	-8 ± 28	-46 ± 47	-52 ± 50	-58 ± 55
SP	5 ± 27	0 ± 27	-10 ± 27	1 ± 25	0 ± 26	4 ± 29	-4 ± 27	5 ± 41	2 ± 43	-9 ± 49
IMGC	-2 ± 21	-7 ± 22	-4 ± 22	2 ± 19	-8 ± 19	-2 ± 23	9 ± 21	2 ± 40	-10 ± 42	-15 ± 48
IPQ	7 ± 28	0 ± 28	-1 ± 28	0 ± 26	5 ± 26	27 ± 29	15 ± 27	62 ± 34	45 ± 35	67 ± 39
BEV	3 ± 32	1 ± 23	5 ± 23	-4 ± 21	1 ± 21	7 ± 25	2 ± 24	1 ± 35	-6 ± 36	-10 ± 46
CEM	2 ± 20	7 ± 20	18 ± 21	10 ± 18	5 ± 18	3 ± 22	10 ± 19	-2 ± 36	-23 ± 39	-19 ± 44
OMH	25 ± 63	-59 ± 41	-54 ± 42	-5 ± 54	4 ± 56	-2 ± 54	-7 ± 63	-20 ± 77	21 ± 81	46 ± 87
NMI-VSL	-26 ± 25	-12 ± 25	-19 ± 25	-9 ± 23	-16 ± 24	-33 ± 27	-4 ± 24	7 ± 36	-10 ± 38	-14 ± 43
SMU	-7 ± 47	1 ± 47	6 ± 47	7 ± 47	5 ± 47	-2 ± 48	-10 ± 47	-15 ± 68	-1 ± 72	-13 ± 79

Table 12a. Differences of measured lengths of steel gauge blocks with respect to the reference values and expanded uncertainties of these differences according to formula **A9** and **A11**. All results are in nm.

	0,5 mm	1 mm	1,01 mm	1,1 mm	5 mm	6 mm	8 mm	80 mm	90 mm	100 mm
BNM-LNE	-2 ± 23	6 ± 23	3 ± 23	8 ± 23	16 ± 22	13 ± 21	11 ± 21	-7 ± 22	0 ± 25	-6 ± 27
OFMET	-3 ± 22	0 ± 22	-3 ± 22	-5 ± 20	-1 ± 18	-1 ± 18	-1 ± 18	9 ± 22	5 ± 23	10 ± 27
PTB	-13 ± 19	-12 ± 20	-12 ± 19	-9 ± 20	-14 ± 18	-9 ± 18	-15 ± 17	-4 ± 21	5 ± 22	-1 ± 24
NPL	6 ± 30	7 ± 30	10 ± 30	7 ± 30	9 ± 29	8 ± 29	6 ± 29	5 ± 35	5 ± 36	8 ± 41
MD	-8 ± 22	-7 ± 22	-7 ± 22	-12 ± 22	-8 ± 21	-7 ± 21	-5 ± 21	-14 ± 22	-17 ± 23	-18 ± 25
CMI	10 ± 19	6 ± 20	3 ± 19	7 ± 20	5 ± 18	6 ± 18	3 ± 18	4 ± 28	-5 ± 31	-5 ± 34
MIKES	3 ± 23	-1 ± 23	1 ± 23	-1 ± 23	-4 ± 22	-3 ± 22	-4 ± 22	7 ± 29	8 ± 31	5 ± 34
DFM	12 ± 21	16 ± 22	7 ± 21	3 ± 22	3 ± 20	-3 ± 20	7 ± 20	12 ± 36	20 ± 39	21 ± 43
JV	0 ± 30	-1 ± 30	6 ± 30	5 ± 30	-8 ± 29	-8 ± 29	-4 ± 29	-22 ± 35	-26 ± 36	-33 ± 39
SP	3 ± 27	-7 ± 27	-3 ± 27	1 ± 27	0 ± 26	-2 ± 26	-2 ± 26	5 ± 34	-3 ± 36	11 ± 39
IMGC	1 ± 21	-12 ± 21	-1 ± 21	-5 ± 21	-3 ± 20	-5 ± 20	-1 ± 19	3 ± 23	-1 ± 25	-4 ± 27
IPQ	-25 ± 27	-25 ± 28	-22 ± 28	-26 ± 28	-12 ± 27	-5 ± 26	-3 ± 26	9 ± 28	20 ± 28	28 ± 30
BEV	17 ± 32	20 ± 23	21 ± 32	24 ± 23	6 ± 22	9 ± 21	8 ± 21	-7 ± 24	-12 ± 27	1 ± 28
CEM	9 ± 20	6 ± 20	5 ± 20	6 ± 20	0 ± 19	-1 ± 18	0 ± 18	0 ± 24	-2 ± 26	-16 ± 29
NMI-VSL	-2 ± 21	4 ± 22	9 ± 21	7 ± 22	1 ± 20	-2 ± 20	8 ± 20	12 ± 24	1 ± 26	5 ± 28
SMU	-9 ± 47	-1 ± 47	-14 ± 47	-9 ± 47	10 ± 46	10 ± 47	-6 ± 46	-12 ± 55	3 ± 58	-7 ± 60

Table 12b. Differences of measured lengths of tungsten carbide gauge blocks with respect to the reference values and expanded uncertainties of these differences according to formula **A9** and **A11**. All results are in nm.

	0,5 mm	1,01 mm	1,1 mm	6 mm	7 mm	8 mm	15 mm	80 mm	90 mm	100 mm
BNM-LNE	0,0	-0,3	-0,2	0,2	-0,2	-0,4	-0,4	-0,1	-0,5	0,1
METAS	0,1	0,5	0,3	0,3	0,3	-0,1	0,5	0,5	0,6	0,4
PTB	-0,4	-0,3	-0,5	-0,6	-0,7	-0,9	-0,4	-0,4	-0,1	-0,4
NPL	-0,2	-0,1	0,1	0,0	0,3	0,5	-0,2	-0,1	0,1	0,2
SMD	0,1	-0,2	0,6	0,2	0,2	0,2	0,1	0,2	0,3	0,0
MIKES	-0,1	0,2	-0,4	-0,3	0,1	-0,2	0,0	-0,1	-0,1	-0,2
DFM	0,1	0,3	0,3	0,2	0,1	0,6	0,4	0,2	0,5	0,2
JV	0,1	0,3	0,0	0,0	0,0	-0,1	-0,3	-0,97	-1,04	-1,04
SP	0,2	0,0	-0,4	0,0	0,0	0,1	-0,2	0,1	0,1	-0,2
IMGC	-0,1	-0,4	-0,2	0,1	-0,4	-0,1	0,4	0,1	-0,2	-0,3
IPQ	0,3	0,0	0,0	0,0	0,2	0,9	0,5	1,8	1,3	1,7
BEV	0,1	0,1	0,2	-0,2	0,1	0,3	0,1	0,0	-0,2	-0,2
CEM	0,1	0,4	0,9	0,6	0,3	0,1	0,5	-0,1	-0,6	-0,4
OMH	0,4	-1,4	-1,3	-0,1	0,1	0,0	-0,1	-0,3	0,3	0,5
NMI-VSL	-1,06	-0,5	-0,8	-0,4	-0,7	-1,2	-0,2	0,2	-0,3	-0,3
SMU	-0,2	0,0	0,1	0,1	0,1	0,0	-0,2	-0,2	0,0	-0,2

Table 13a : E_n values for steel gauge blocks calculated from deviation Δx given in table 12a : $E_n = |\Delta x| / U(\Delta x)$

	0,5 mm	1 mm	1,01 mm	1,1 mm	5 mm	6 mm	8 mm	80 mm	90v	100 mm
BNM-LNE	-0,1	0,3	0,1	0,4	0,7	0,6	0,5	-0,3	0,0	-0,2
OFMET	-0,2	0,0	-0,1	-0,3	-0,1	0,0	-0,1	0,4	0,2	0,4
PTB	-0,7	-0,6	-0,6	-0,5	-0,8	-0,5	-0,9	-0,2	0,2	0,0
NPL	0,2	0,2	0,3	0,2	0,3	0,3	0,2	0,1	0,1	0,2
MD	-0,3	-0,3	-0,3	-0,5	-0,4	-0,3	-0,3	-0,7	-0,7	-0,7
CMI	0,5	0,3	0,1	0,4	0,3	0,3	0,2	0,1	-0,1	-0,1
MIKES	0,2	0,0	0,0	0,0	-0,2	-0,1	-0,2	0,3	0,3	0,1
DFM	0,6	0,8	0,3	0,1	0,1	-0,2	0,3	0,3	0,5	0,5
JV	0,0	0,0	0,2	0,1	-0,3	-0,3	-0,1	-0,6	-0,7	-0,8
SP	0,1	-0,3	-0,1	0,1	0,0	-0,1	-0,1	0,2	-0,1	0,3
IMGC	0,0	-0,6	-0,1	-0,2	-0,2	-0,3	0,0	0,1	-0,1	-0,2
IPQ	-0,9	-0,9	-0,8	-0,9	-0,4	-0,2	-0,1	0,3	0,7	0,9
BEV	0,5	0,9	0,7	1,03	0,3	0,4	0,4	-0,3	-0,5	0,0
CEM	0,4	0,3	0,2	0,3	0,0	-0,1	0,0	0,0	-0,1	-0,6
NMI-VSL	-0,1	0,2	0,4	0,3	0,1	-0,1	0,4	0,5	0,1	0,2
SMU	-0,2	0,0	-0,3	-0,2	0,2	0,2	-0,1	-0,2	0,1	-0,1

Table 13b : E_n values for tungsten carbide gauge blocks calculated from deviation Δx given in table 12b :

$$E_n = |\Delta x| / U(\Delta x)$$