



## Asia-Pacific Metrology Programme

**APMP Key Comparison**

**APMP-LK2**

**Calibration of long gauge blocks**

**Final Report: Version B3**

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## 1. Introduction

The Asia Pacific Metrology Programme's Technical Committee for Length (APMP/TCL), held its second meeting at SIRIM Berhad (Malaysia) in August 1998, where it was decided to carry out a regional key comparison on length bar measurements to be coordinated by the Commonwealth Scientific and Industrial Research Organisation – National Measurement Laboratory (CSIRO/NML) with this laboratory acting as the pilot laboratory. The technical protocol was modelled on the protocol for CCL-K2 which was drawn up by Andrew Lewis (CCL Pilot - National Physical Laboratory, UK (NPL)), Jennifer Decker (National Research Council, Canada (NRC)) and Nicholas Brown (APMP Pilot - National Measurement Laboratory, Australia (CSIRO/NML)).

A goal of dimensional metrology key comparisons is to compare routine calibration services offered by NMIs to clients. Uncertainty claims should match those listed in Appendix C of the Mutual Recognition Agreement (MRA) [BIPM, 1999]. To this end, participants in this comparison agree to use the same apparatus and methods as routinely applied to client artefacts. The title for the comparison refers to long gauge blocks because the artefacts are rectangular-section long gauge blocks as specified in ISO 3650 (1998) and not length bars (which are of circular cross-section).

The participant's replies have been collated into an Excel spreadsheet and are shown in Appendix B in an Excel workbook. These results are identified in the text with a B pre-fix.

## 2. Organization

### 2.1 Participants

APMP member laboratories were invited to join the comparison by the pilot laboratory. The final participant list was then circulated within the APMP TCL. The service tested in this comparison is the measurement of central length of gauge blocks covering the range 150 mm to 500 mm to an uncertainty of less than approximately 200 nm for interferometric measurements and 5 µm for comparison measurements (for a coverage factor of  $k = 1$ ).

### 2.2 Participants' details

| INTERFEROMETRIC MEASUREMENTS |   |  |
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Table 1. Participant's details at the start of the comparison

During the comparison some changes were made: CSIR measured all the gauges with an interferometric comparator, KRIS measured the 200 mm and 250 mm gauges by interferometry but the 500 mm with a comparator and SIRIM measuring the 200 mm and 250 mm by interferometry and the 500 mm gauge by comparison. Syria withdrew and Singapore was unable to participate due to instrument failure.

### ***2.3 Comparison Schedule***

The original idea was to have all the interferometric measurements in a first loop, hybrid interferometric/comparison measurements in a second loop and then comparison measurements in a third loop. Compromises had to be made to minimise travel times and to suit participant's requirements. Table B1 shows the original schedule, the actual schedule and the reporting dates of participants. The first delay occurred in customs in India, but this was just a few weeks and SPRING (Singapore) asked for a delay, allowing the schedule to be resumed. Singapore was rescheduled for October 2001 but was still unable to participate. There were further delay in the Philippines and Indonesian customs. NPL (India) asked for a remeasure and it was agreed to fit this in just before the final Pilot measure. However there were very serious delays in the Indian customs resulting in a six month hold up. The final pilot measure confirmed the drift in the length of the standards, however identifying the size of the drift was very dependent on the measurements made by the pilot, so NMIJ agreed to make a final measurement to reduce this dependence.

### ***2.4 Handling and transport***

The standards showed some damage in the form of scratches and low level corrosion, but were still in an acceptable condition for interferometric measurements.

## **3. Reported results**

The Technical Protocol asked the participants to report:

- A1: The central length measured in two orientations and the uncertainty for the average of these measurements, see Table B2 (Measurement Results).
- A2: The observed condition of the measurement surfaces.
- A3: A description of the type of equipment used, the traceability route, the measurement method, platen material (if applicable) and the temperature range of the standards at the time of measurement, see Table B4 (Description of measurement).
- A4: The Uncertainty budget. Most uncertainty budgets did not fit the interferometric model provided because the gauges were measured by comparison. A common factor for all participants should be the uncertainty equation which is part of their CMC claim. Table B4 (Uncertainty of measurement) gives these equations and the claimed uncertainties supplied with the measurements. In some cases these did not agree exactly and the participants have supplied explanations for these discrepancies (attached to Table B4).

## 4. Analysis of the results

### 4.1 Discussion

The aim of this analysis is to find a key reference value which can be used to determine the deviations of the results of each laboratory. Three different approaches are explored, but the final decision on which is most appropriate must be decided by the TCL. Very similar key comparisons have been completed for Gauge blocks (CCL-K1) and for Long Gauge blocks (CCL-K2) and this analysis is based on them and investigates a third approach.

CCL –K1 chose to use a simple average value taken from the participant results after removing measurements that had not complied with the Technical Protocol. This was justified because all participants used the same method of measurement. CCL-K2 chose to use a weighted mean, where the weighting factor was derived from the uncertainties reported by participants. In this case the gauges were much more sensitive to environmental conditions, such as gauge temperature, and the participant's uncertainties had a larger range than was the case for gauge blocks. This comparison has an even larger range of uncertainties than CCL-K2 as it includes the different measurement techniques of interferometry and comparison. A weighted mean of some kind should therefore give a much better results. Over the three year period of the comparison a significant drift in the length of two of the gauge has been observed and it has since been confirmed that this is a characteristic of gauges of this length made by their manufacturer (private communication). A third analysis includes this drift into the reference value.

The statistical background for determining a weighted mean is given below, and is based on the discussion in CCL-K2. This approach requires that the participants have made correct estimates of their uncertainty of measurement, otherwise a too low uncertainty will place undue emphasis on the result of that particular laboratory

### 4.2 Weighting Factors and the Reference Value

Let the measured deviation from nominal size reported by each participant be  $x_i$ , where the number of laboratories is given by  $I$ . Since the three gauge blocks have different lengths, thermal expansion coefficients, material properties *etc*, it is reasonable to expect that the data comes from three separate populations (one per gauge block) and so analysis should be on a gauge-by-gauge basis.

Thus, for a particular gauge block:

Each laboratory reports a measured value,  $x_i$ , and its associated standard uncertainty  $u(x_i)$ .

The normalised weight,  $w_i$ , for the result  $x_i$  is given by:

$$w_i = C \cdot \frac{1}{[u(x_i)]^2} \quad (1)$$

where the normalising factor,  $C$ , is given by:

$$C = \frac{1}{\sum_{i=1}^I \left( \frac{1}{u(x_i)} \right)^2} \quad (2)$$

Then the weighted mean,  $\bar{x}_w$ , is given by:

$$\bar{x}_w = \sum_{i=1}^I w_i \cdot x_i \quad (3)$$

The simple mean uses a weighting factor of one and is given by:

$$\bar{x}_a = \sum_{i=1}^I \frac{x_i}{I} \quad (4)$$

Where there is a drift of the reference value during the comparison, a linear function can be fitted to the reported values using a linear regression. The participant's measurements can be weighted (as for the weighted mean) using the weights given in Equation (1). In this case the reference value becomes:

$$\bar{x}_{Fit}(Date) = A + B \times Date \quad (5)$$

where  $A$  and  $B$  are constants. Each participant, including the pilot, should only contribute once to any determination of a reference value. The comparison reference value  $\bar{x}_{RV}$  can be set equal to the simple mean ( $\bar{x}_a$  - Equation 4), the weighted mean ( $\bar{x}_w$  - Equation 3) or the drifting mean ( $\bar{x}_{Fit}(Date)$  -Equation 5), and these options are discussed below.

### 4.3 Uncertainties

If the artefact uncertainty is ignored, the uncertainty of the reference value can be calculated as either the internal  $u_{int}(\bar{x}_{RV})$  or external  $u_{ext}(\bar{x}_{RV})$  standard deviation. The internal standard deviation is based on the estimated uncertainties  $u(x_i)$  as reported by the participants:

$$u_{\text{int}}(\bar{x}) = \sqrt{\frac{1}{\sum_{i=1}^I \left(\frac{1}{u(x_i)}\right)^2}} = \sqrt{C} \quad (6)$$

The external standard deviation is the standard deviation of the spread of the residuals  $x_i - \bar{x}_{RV}$ , weighted by the uncertainties  $u(x_i)$ :

$$u_{\text{ext}}(\bar{x}) = \sqrt{\frac{1}{(I-1)} \cdot \frac{\sum_{i=1}^I w_i (x_i - \bar{x}_{RV})^2}{\sum_{i=1}^I w_i}} \quad (7)$$

The residuals have an uncertainty which results from the measured value ( $x_i \pm u(x_i)$ ) and the reference value ( $\bar{x}_{RV} \pm u(\bar{x}_{RV})$ ). The uncertainty of the reference value is taken to be the internal uncertainty and the uncertainty of the artefact  $u_{\text{art}}(\bar{x}_{RV})$ . The internal uncertainty can be viewed as setting a limit to the knowable accuracy of any artefact length, given the uncertainty of each measurement. The artefact uncertainty sets a limit on the stability of the artefact during the comparison. The Pilot's measurements provide the best information on artefact changes, given that the same instrument and method were used each time. The uncertainty of the artefact is obtained by repeating the method used to determine the reference value, but only using the Pilot's data. The standard deviation of the mean for just the pilot's measurements,  $J$ , then gives the uncertainty for the artefact. This was obtained for the mean, weighted mean and linear regression reference values.

$$u_{\text{art}}(\bar{x}_{\text{pilot}}) = \sqrt{\frac{\sum_{j=1}^J (x_j - \bar{x}_{\text{pilot}})^2}{J(J-1)}} \quad (8)$$

The uncertainty for each participant's residual is therefore given by:

$$u(x_i - \bar{x}_{RV}) = \sqrt{[u(x_i)]^2 - [u_{\text{int}}(\bar{x}_w)]^2 + [u_{\text{art}}(\bar{x}_{\text{pilot}})]^2} \quad (9)$$

The internal uncertainty is subtracted from the participant's uncertainty because their result has already pulled the reference value in their direction (it has a negative correlation). This could be avoided by excluding them from the reference value they are compared with, but this approach is not used here.

#### 4.4 Analysis using $E_n$ values

A check for statistical consistency of the results with their associated uncertainties can be made by calculating the  $E_n$  value for each laboratory, where  $E_n$  is defined as the ratio of the deviation from the weighted mean, divided by the uncertainty of this deviation, taken for a coverage factor of  $k=2$ :



$$E_n = \frac{x_i - \bar{x}_{RV}}{2 \cdot u(x_i - \bar{x}_{RV})} \quad (10)$$

$E_n$  values should be less than 1, if the participant's result and uncertainty are consistent with the reference value. These values are shown in Tables B6 and Fig B4 for a reference value based on a mean, Table B7 and Fig B5 for a weighted mean and Table B8 and Fig B7 for a reference value determined from a linear regression fitted to the data.

#### 4.5 Birge ratio test

The statistical consistency of a comparison can also be investigated by the Birge ratio  $R_B$ , which compares the observed spread of the results with the spread expected from the individual reported uncertainties.

The application of least squares algorithms and the  $\chi^2$ -test leads to the Birge ratio:

$$R_B = \frac{u_{ext}(\bar{x}_w)}{u_{int}(\bar{x}_w)} \quad (11)$$

The Birge ratio has an expectation value of  $R_B = 1$ , when considering standard uncertainties. For a coverage factor of  $k = 2$ , the expectation value is increased and the data in a comparison are consistent provided that

$$R_B < \sqrt{1 + \sqrt{8/(I-1)}} \quad (12)$$

where  $I$  is the number of laboratories. For the case  $I = 11$ , a value of  $R_B < 1.4$  indicates consistency.

## 5. Comparison with Reference Values

Three reference values are compared in the results, a simple mean (Table 6 & Fig 4), a weighted mean (Table 7 & Fig 5) and a weighted least square fit to include the trend (Table B9 & Fig B6 show the fit, while Table B8 and Fig B7 show the results). Only one measurement from the pilot is used in each case. With the mean and the weighted mean the pilot's value NML-3 is used because this is towards the middle of the comparison. For the linear regression the last pilot measurement (NML-5) is used to spread out the interferometric measurements over as much of the comparison as possible in order to best identify the trend. The second measurement performed by NPL-India (NPL2) is used and the results from ITDI have been excluded.

The interferometric measurements were completed in the first phase of the comparison and establish the reference value at this time with a low uncertainty. This meant that the Pilot's final measurement played a critical role in establishing the artefact drift. To overcome this dependence NMIJ made a second measurement at the end of the comparison and this was included in determining the drift.

Using a simple mean as the Reference Value shows a large scatter of  $E_n$  values (Fig B4) with almost all participants getting a poor result for the 200 mm gauge. The weighted average (Fig B5) gives a much more balanced result, although the interferometric measurements made at the end of the

comparison show large errors. These large errors result from the drift in the length of the standards and removing these from the reference value by fitting a weighted linear regression (Fig B7) takes out this anomaly.

The Birge ratios are shown in Table B10 and summarised below in Table 3 .

| Gauge Length | MEAN | WEIGHTED MEAN | LINEAR FIT |
|--------------|------|---------------|------------|
| 200 mm       | 6.0  | 2.3           | 2.3        |
| 250 mm       | 1.6  | 1.6           | 1.7        |
| 500 mm       | 1.6  | 1.0           | 0.9        |

Table 3. Birge ratios for the three Reference Values.

The Birge ratio should be close to 1.4 and this is roughly the case for the longer gauges. The results for the 200 mm gauge where worse than expected from the participant’s uncertainties. The table shows little difference between the Weighted Mean and the Weighted Linear Fit. The main difference between these two approaches is shown in Table 4 , which shows the artefact uncertainty.

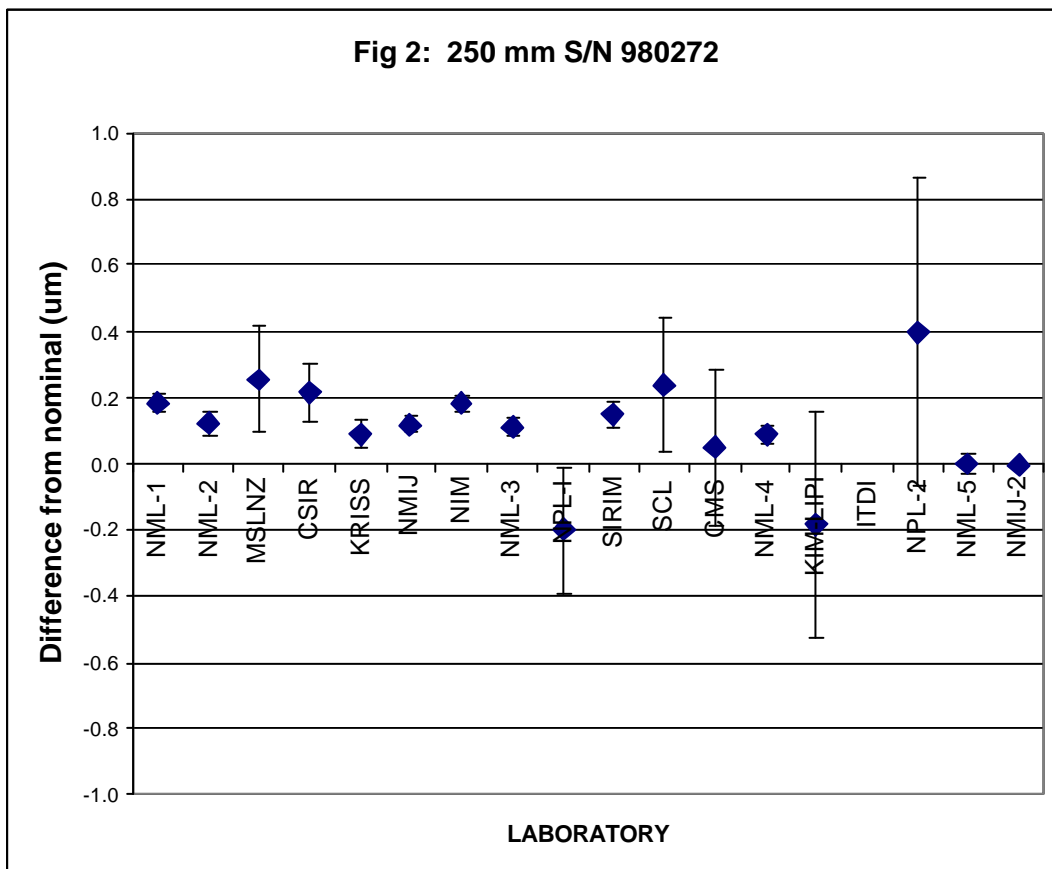
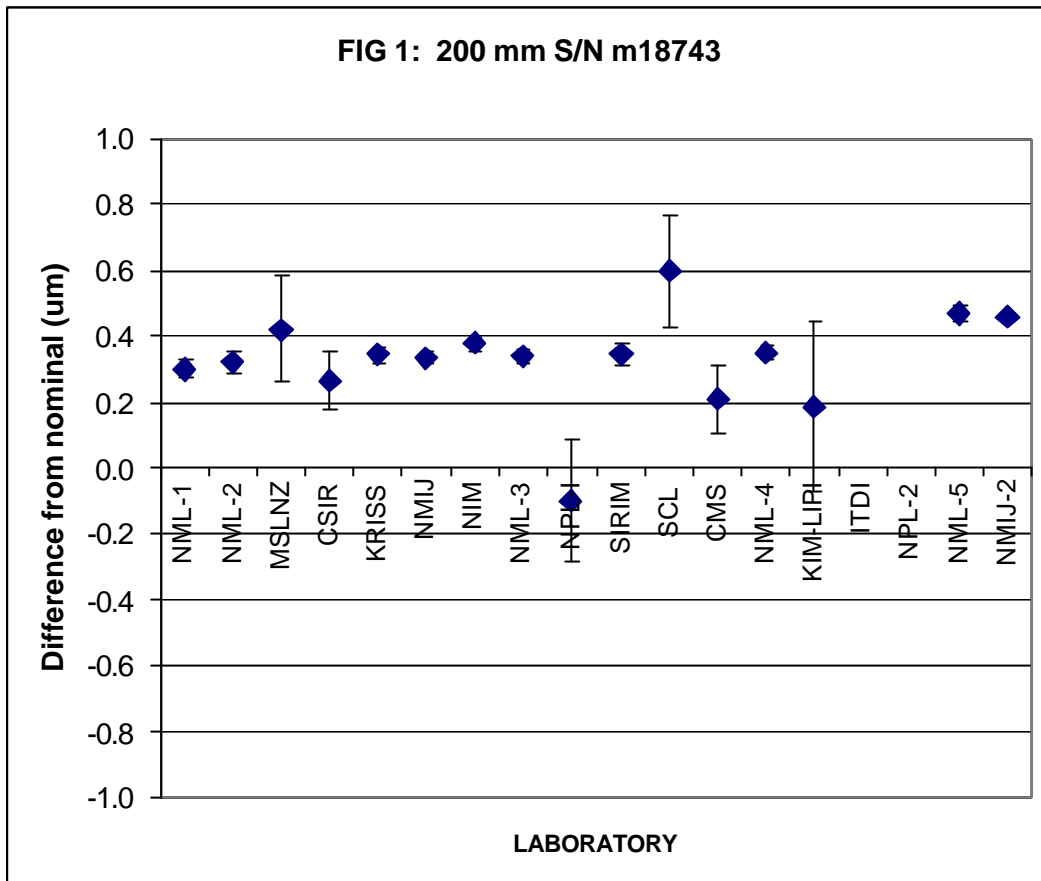
| Gauge Length | MEAN      | WEIGHTED MEAN | LINEAR FIT |
|--------------|-----------|---------------|------------|
| 200 mm       | ±0.029 µm | ±0.029 µm     | ±0.020 µm  |
| 250 mm       | ±0.030 µm | ±0.030 µm     | ±0.015 µm  |
| 500 mm       | ±0.065 µm | ±0.065 µm     | ±0.031 µm  |

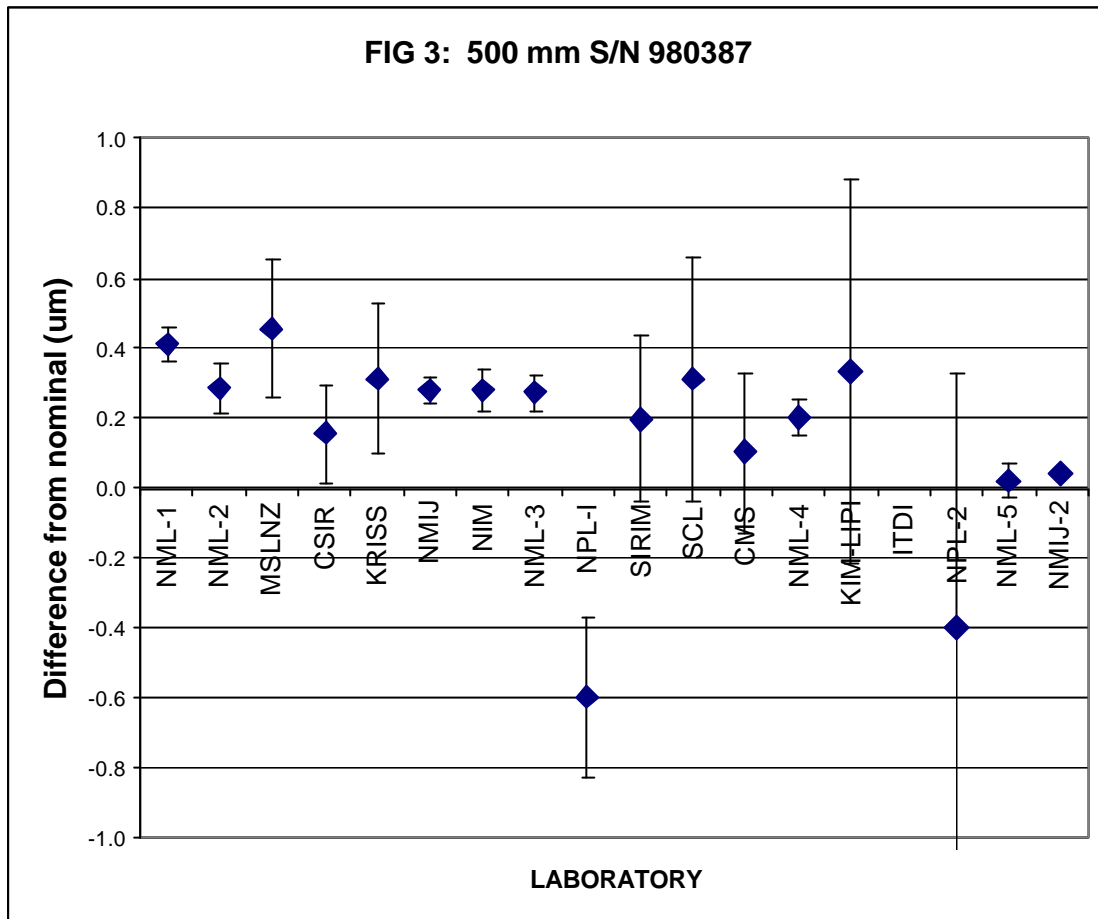
Table 4. Artefact uncertainty  $u(X_{RV})$  ( $K = 1$ )

The Mean and Weighted Mean have virtually the same uncertainty as these are determined from the Pilot’s measurements which had almost the same weighting factor for all the measurements. Only NML 2 had a larger uncertainty as it was a single sided measurement compared to the double sided measurements of the other four. The Linear Fit shows a very significant reduction in artefact uncertainty. The comparison is far more demanding for participants in this case and they are more able to demonstrate their CMC claims with this Reference Value.

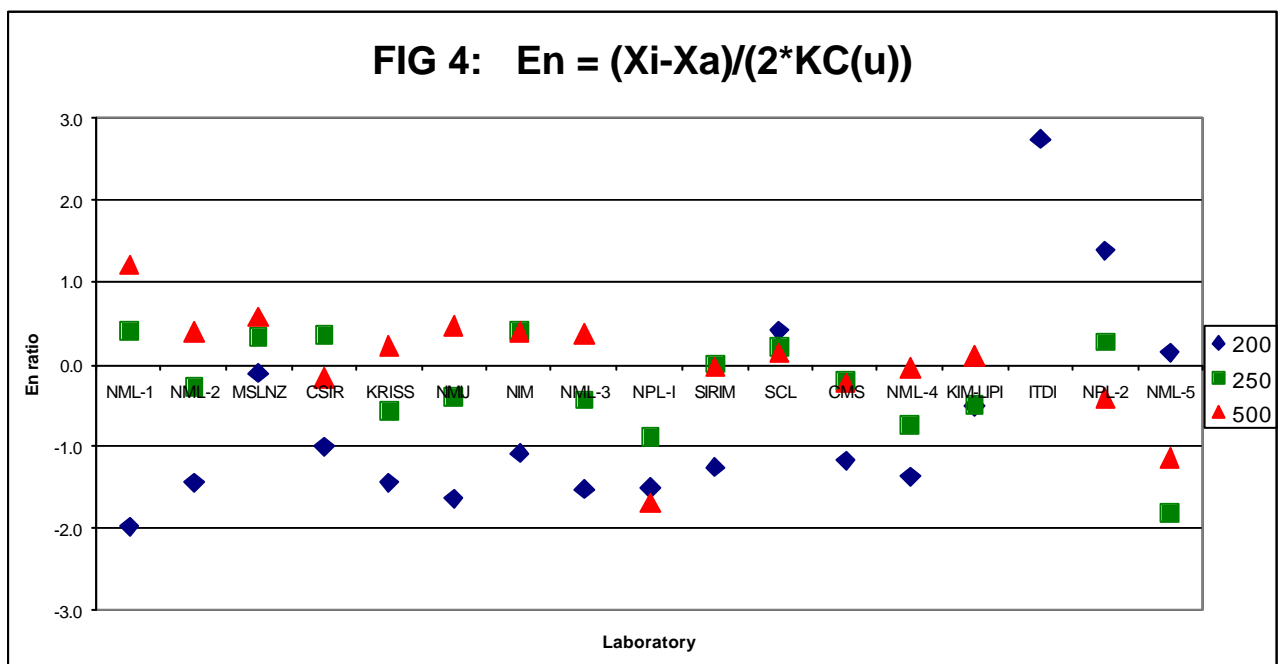
## CRITICAL FIGURES FROM APPENDIX B

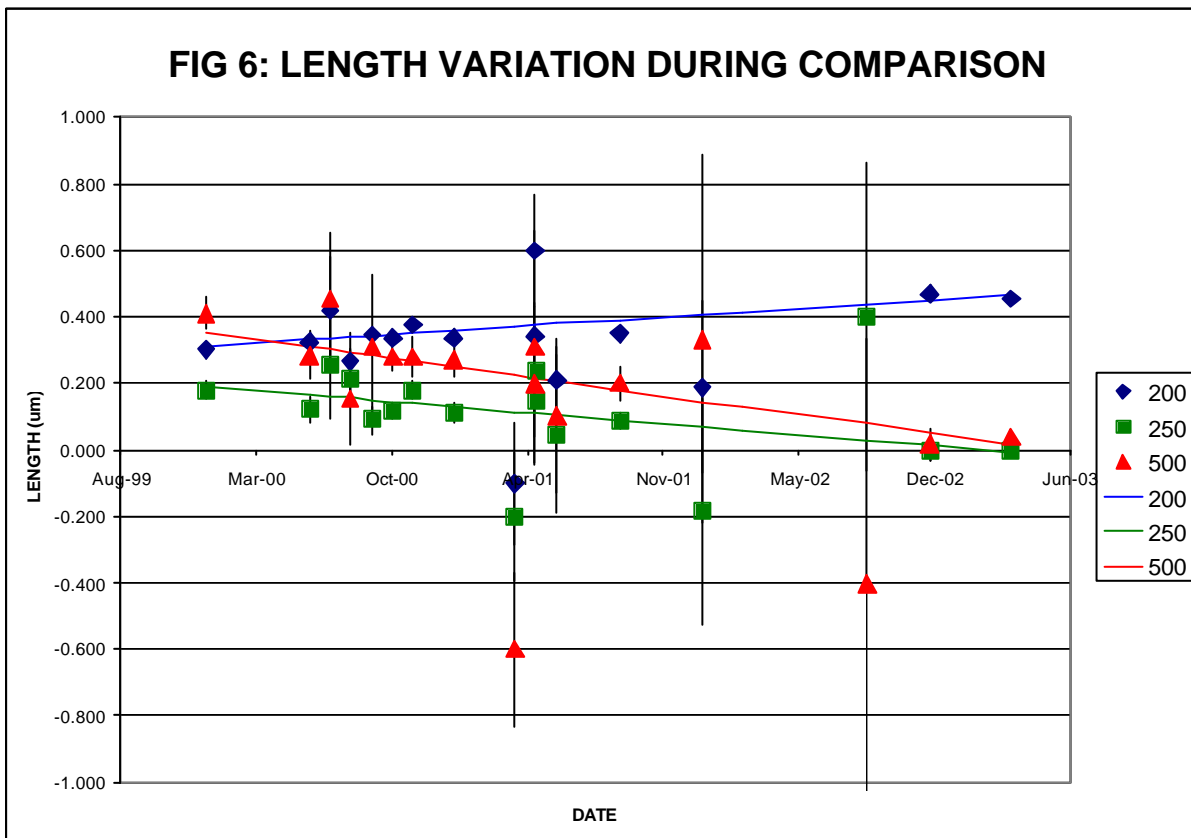
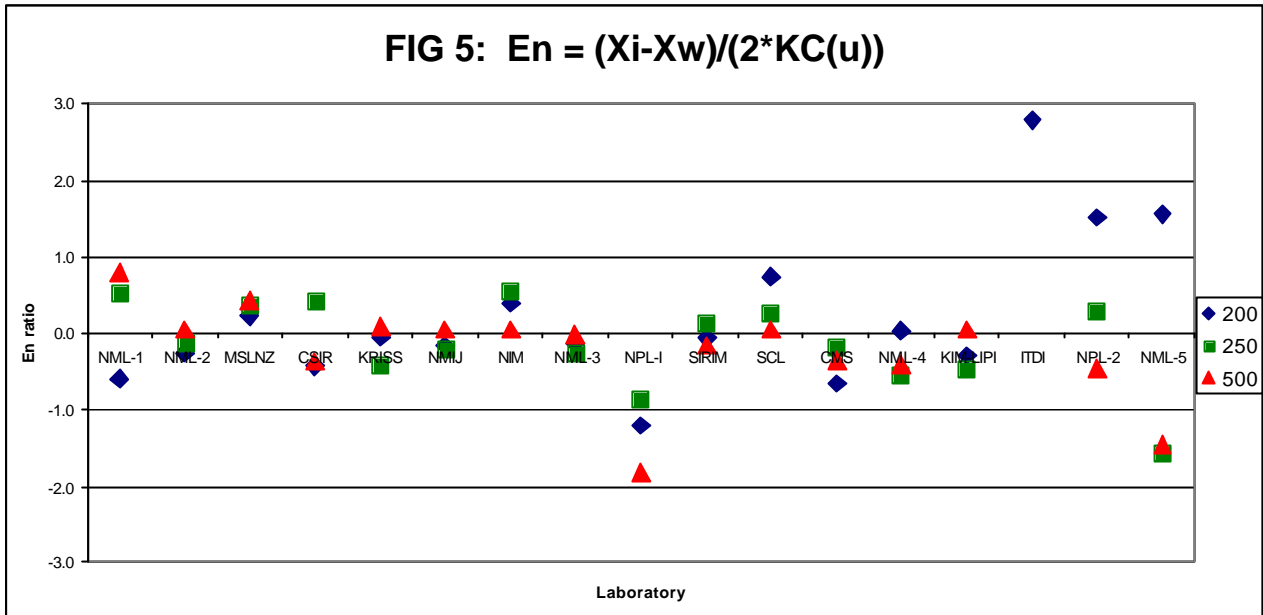
Figure 1 to 3: Measurement data from participants.

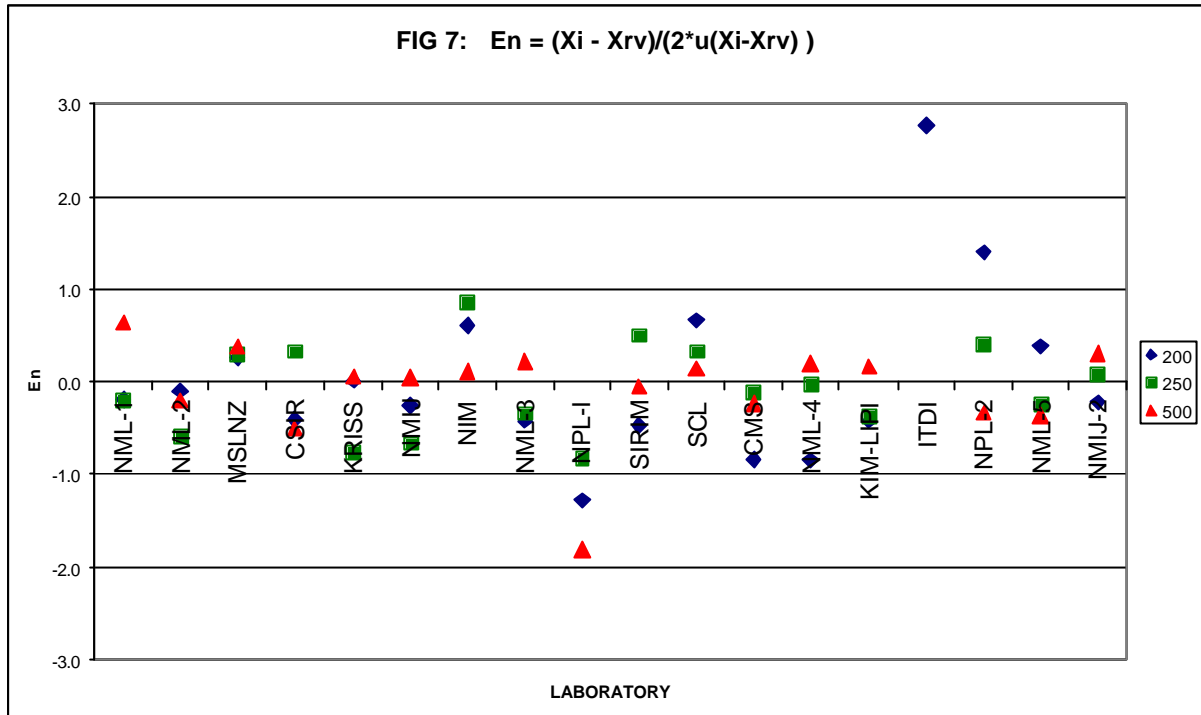


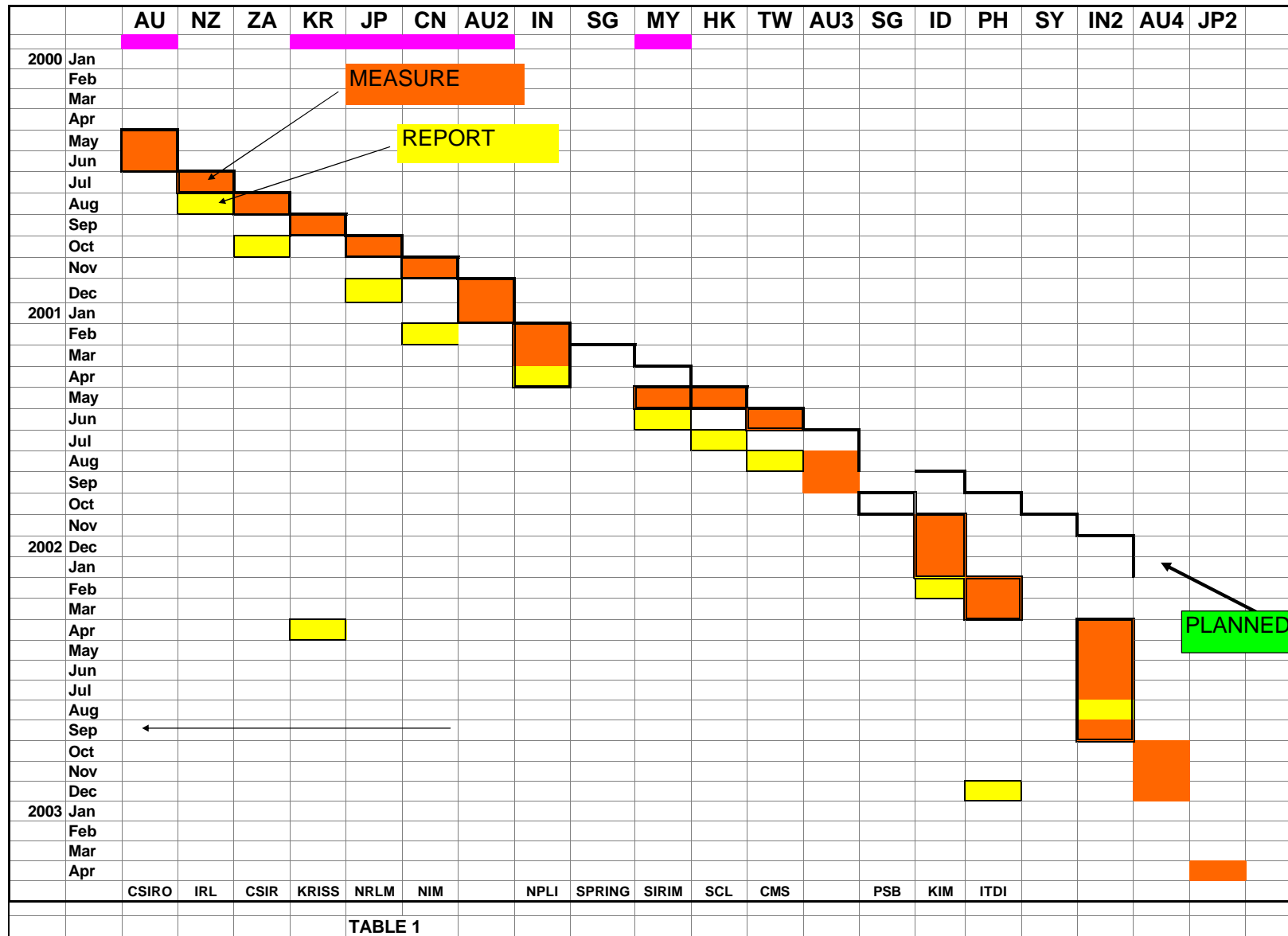


Figures 4 to 7: En values for the three gauges: For simple mean  $X_a$ , weighted mean  $X_w$ , the linear fit and En values for the linear fit  $X_{Fit} (Date)$ .









**TABLE: 2 RAW RESULTS FROM PARTICIPANTS**

|     | Jan-00 |       |              |        | Jun-00 |     |              |       | Jul-00 |       |              |        | Aug-00 |       |              |        | Sep-00 |       |              |        | Oct-00 |       |              |        | Nov-00 |       |              |        |
|-----|--------|-------|--------------|--------|--------|-----|--------------|-------|--------|-------|--------------|--------|--------|-------|--------------|--------|--------|-------|--------------|--------|--------|-------|--------------|--------|--------|-------|--------------|--------|
|     | LHS    | RHS   | Mean         | Uncert | LSU    | RSU | mean         |       | LHS    | RHS   | Mean         | Uncert | LHS    | RHS   | Mean         | Uncert | LHS    | RHS   | Mean         | Uncert | LHS    | RHS   | Mean         | Uncert | LHS    | RHS   | Mean         | Uncert |
| 200 | 0.308  | 0.294 | <b>0.301</b> | 0.024  | 0.324  |     | <b>0.324</b> | 0.034 | 0.390  | 0.440 | <b>0.420</b> | 0.160  | 0.230  | 0.300 | <b>0.265</b> | 0.090  | 0.359  | 0.327 | <b>0.343</b> | 0.024  | 0.328  | 0.344 | <b>0.336</b> | 0.020  | 0.369  | 0.383 | <b>0.376</b> | 0.021  |
| 250 | 0.184  | 0.177 | <b>0.181</b> | 0.028  | 0.121  |     | <b>0.121</b> | 0.039 | 0.260  | 0.250 | <b>0.255</b> | 0.160  | 0.250  | 0.180 | <b>0.215</b> | 0.090  | 0.102  | 0.077 | <b>0.090</b> | 0.041  | 0.126  | 0.110 | <b>0.118</b> | 0.023  | 0.183  | 0.177 | <b>0.181</b> | 0.026  |
| 500 | 0.434  | 0.389 | <b>0.412</b> | 0.050  | 0.283  |     | <b>0.283</b> | 0.070 | 0.500  | 0.410 | <b>0.455</b> | 0.200  | 0.125  | 0.180 | <b>0.155</b> | 0.140  |        |       | <b>0.310</b> | 0.214  | 0.275  | 0.284 | <b>0.280</b> | 0.039  | 0.314  | 0.324 | <b>0.280</b> | 0.060  |

|     | Jan-01 |       |              |        | Apr-01 |     |               |        | May-01 |       |              |        | May-01 |      |             |        | Jun-01 |     |             |        | Sep-01 |       |              |        | Jan-02 |     |               |        |
|-----|--------|-------|--------------|--------|--------|-----|---------------|--------|--------|-------|--------------|--------|--------|------|-------------|--------|--------|-----|-------------|--------|--------|-------|--------------|--------|--------|-----|---------------|--------|
|     | LHS    | RHS   | Mean         | Uncert | LHS    | RHS | Mean          | Uncert | LHS    | RHS   | Mean         | Uncert | LHS    | RHS  | Mean        | Uncert | LHS    | RHS | Mean        | Uncert | LHS    | RHS   | Mean         | Uncert | LHS    | RHS | Mean          | Uncert |
| 200 | 0.336  | 0.340 | <b>0.338</b> | 0.024  |        |     | <b>-0.100</b> | 0.183  | 0.335  | 0.349 | <b>0.342</b> | 0.034  | 0.62   | 0.59 | <b>0.6</b>  | 0.17   |        |     | <b>0.21</b> | 0.1    | 0.353  | 0.344 | <b>0.349</b> | 0.024  |        |     | <b>0.189</b>  | 0.26   |
| 250 | 0.106  | 0.118 | <b>0.112</b> | 0.028  |        |     | <b>-0.200</b> | 0.191  | 0.143  | 0.154 | <b>0.149</b> | 0.041  | 0.21   | 0.27 | <b>0.24</b> | 0.2    |        |     | <b>0.05</b> | 0.12   | 0.100  | 0.075 | <b>0.087</b> | 0.028  |        |     | <b>-0.185</b> | 0.338  |
| 500 | 0.276  | 0.264 | <b>0.27</b>  | 0.050  |        |     | <b>-0.600</b> | 0.228  |        |       | <b>0.195</b> | 0.239  | 0.3    | 0.33 | <b>0.31</b> | 0.35   |        |     | <b>0.1</b>  | 0.23   | 0.201  | 0.197 | <b>0.199</b> | 0.050  |        |     | <b>0.333</b>  | 0.553  |

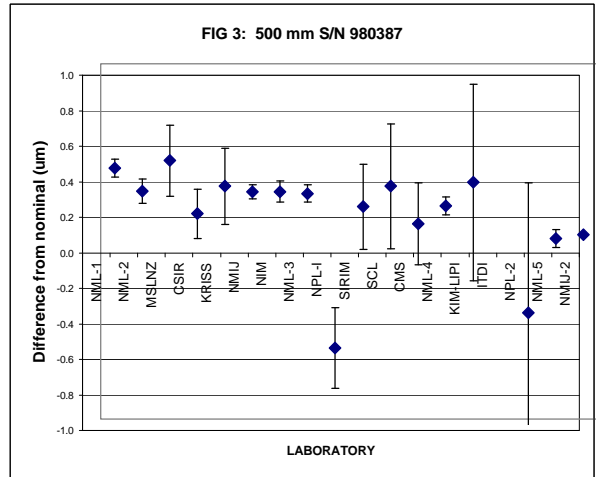
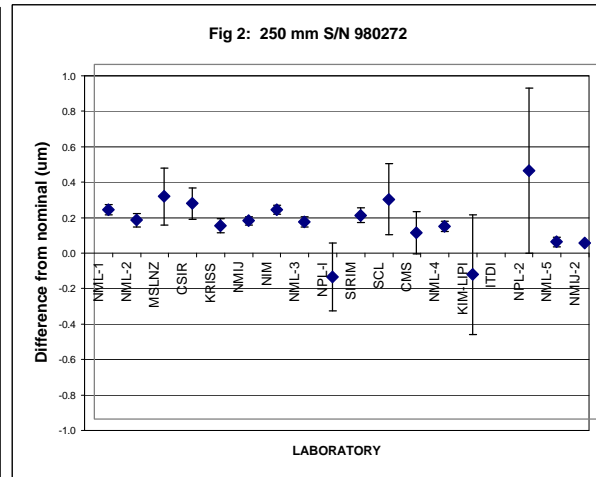
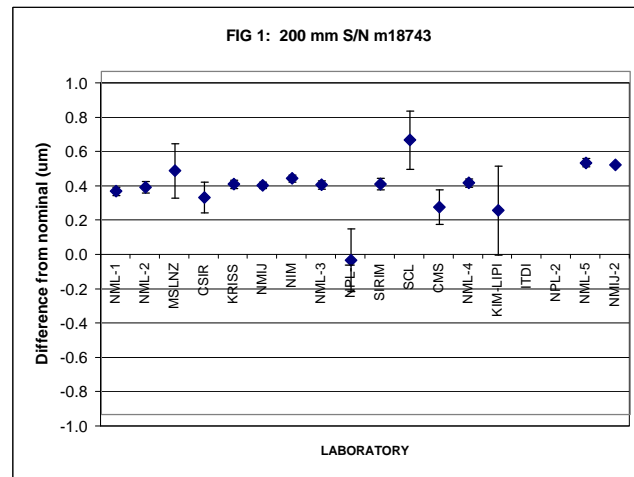
  

|     | Mar-02 |       |             |        | Sep-02 |     |             |        | Dec-02 |        |               |        | Jan-03                                   |     |              |        | Apr-03 |        |               |        |
|-----|--------|-------|-------------|--------|--------|-----|-------------|--------|--------|--------|---------------|--------|--|-----|--------------|--------|--------|--------|---------------|--------|
|     | LHS    | RHS   | Mean        | Uncert | LHS    | RHS | Mean        | Uncert | LHS    | RHS    | Mean          | Uncert | LHS                                      | RHS | Mean         | Uncert | LHS    | RHS    | Mean          | Uncert |
| 200 | 14.2   | 13.2  | <b>13.7</b> | 2.4    |        |     | <b>1.6</b>  | 0.413  | 0.461  | 0.479  | <b>0.468</b>  | 0.024  | Comparison check not included in results |     |              |        | 0.451  | 0.48   | <b>0.455</b>  | 0.023  |
| 250 | 34.6   | 28.7  | <b>31.7</b> | 1.7    |        |     | <b>0.4</b>  | 0.466  | 0.006  | -0.008 | <b>-0.001</b> | 0.028  |  |     | <b>0.528</b> | 0.300  | 0.003  | -0.015 | <b>-0.006</b> | 0.023  |
| 500 | -20.1  | -15.8 | <b>-18</b>  | 1.5    |        |     | <b>-0.4</b> | 0.73   | 0.019  | 0.015  | <b>0.017</b>  | 0.05   |  |     | <b>0.042</b> | 0.400  | 0.051  | 0.026  | <b>0.039</b>  | 0.042  |

**TABLE: 3 RESULTS SUMMARY**

| Laboratory           | NML-1  | NML-2        | MSLNZ        | CSIR         | KRISS        | NMIJ         | NIM          | NML-3        | NPL-1         | SIRIM         | SCL          | CMS          | NML-4        | KIM-LIPI     | ITDI          | NPL-2       | NML-5        | NMIJ-2        |               |
|----------------------|--------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|---------------|---------------|--------------|--------------|--------------|--------------|---------------|-------------|--------------|---------------|---------------|
| Date of measurement  | Jan-00 | Jun-00       | Jul-00       | Aug-00       | Sep-00       | Oct-00       | Nov-00       | Jan-01       | Apr-01        | May-01        | May-01       | Jun-01       | Sep-01       | Jan-02       | Mar-02        | Sep-02      | Dec-02       | Apr-03        |               |
| Length $x_i$         | 200    | <b>0.301</b> | <b>0.324</b> | <b>0.420</b> | <b>0.265</b> | <b>0.343</b> | <b>0.376</b> | <b>0.338</b> | <b>-0.100</b> | <b>0.342</b>  | <b>0.600</b> | <b>0.21</b>  | <b>0.349</b> | <b>0.189</b> | <b>13.7</b>   | <b>1.6</b>  | <b>0.468</b> | <b>0.455</b>  |               |
|                      | 250    | <b>0.181</b> | <b>0.121</b> | <b>0.255</b> | <b>0.215</b> | <b>0.090</b> | <b>0.118</b> | <b>0.181</b> | <b>0.112</b>  | <b>-0.200</b> | <b>0.149</b> | <b>0.240</b> | <b>0.05</b>  | <b>0.087</b> | <b>-0.185</b> | <b>31.7</b> | <b>0.4</b>   | <b>-0.001</b> | <b>-0.006</b> |
|                      | 500    | <b>0.412</b> | <b>0.283</b> | <b>0.455</b> | <b>0.155</b> | <b>0.310</b> | <b>0.280</b> | <b>0.280</b> | <b>0.27</b>   | <b>-0.600</b> | <b>0.195</b> | <b>0.310</b> | <b>0.1</b>   | <b>0.199</b> | <b>0.333</b>  | <b>-18</b>  | <b>-0.4</b>  | <b>0.017</b>  | <b>0.039</b>  |
| Uncertainty $u(x_i)$ | 200    | 0.024        | 0.034        | 0.160        | 0.090        | 0.024        | 0.020        | 0.021        | 0.024         | 0.183         | 0.034        | 0.170        | 0.100        | 0.024        | 0.26          | 2.400       | 0.413        | <b>0.024</b>  | <b>0.023</b>  |
|                      | 250    | 0.028        | 0.039        | 0.160        | 0.090        | 0.041        | 0.023        | 0.026        | 0.028         | 0.191         | 0.041        | 0.200        | 0.120        | 0.028        | 0.338         | 1.700       | 0.466        | <b>0.028</b>  | <b>0.023</b>  |
|                      | 500    | 0.050        | 0.070        | 0.200        | 0.140        | 0.039        | 0.060        | 0.050        | 0.228         | 0.239         | 0.350        | 0.230        | 0.050        | 0.553        | 1.500         | 0.73        | <b>0.050</b> | <b>0.042</b>  |               |

BIPM Appendix B:  
Laboratory individual  
measurements -first page





| Laboratory | Type of equipment   | Traceability route   | Measurement method  | Platen material | Temperature / °C  |
|------------|---|--|---|-----------------|---|
| NML        | CSIRO modified Michelson and Kösters interferometers.   | Directly via use of iodine stabilized lasers at 633 nm, 612 nm, 543 nm.  | Method of excess-fractions. Refractive index determination via air temperature, pressure and humidity measurements. Ciddor equations [1]  | Steel           | 19.90 to 20.10  |
| IR         | PEL Length Bar Comparator, designed and built at PEL, DSIR (now MSL NZ), in 1985. Based on HP5501A laser measurement system.              | HP5501A traceable to NZ Iodine stabilised lasers.  | Comparison against a reference gauge using one fixed headstock and an LVDT/ retroreflector measurement head. Temperature pressure and humidity measured directly and refractive index calculated [2].     |                 | 19.90 to 20.15  |
| CSIR       | The system was built in-house using a HP laser and special design probe.  | HP laser to Iodine stabilised laser at CSIR  | The HP laser measured the displacement of the probe. The probe diameter is calculated on a calibrated gauge block   |                 | 20.028 to 20.065  |
| KRISS      | For 200 mm and 250 mm gauges a Tsugami Twyman-Green laser interferometer. For 500 mm gauge a Federal 8018 mechanical comparator.          | For interferometry a Hg isotope lamp. For comparison, the reference gauge was traceable to PTB.  | For interferometry: Method of excess-fractions. Refractive index determination via air temperature, pressure, CO2 and humidity measurements, using the modified Edlin formula.                            |                 | 19.85 to 20.15  |
| NMIJ       | NRLM, Twyman Green interferometer   | Directly via use of iodine stabilized lasers at 633 nm & 532 nm.   | Method of excess-fractions. Refractive index determination via air temperature, pressure, CO2 and humidity measurements. Ciddor equations   | Steel           | 19.8 to 20.2  |
| NIM        | An improved Kösters interferometer by Carl Zeiss (up to 300 mm). A modified Kösters interferometer (up to 1 m)                            | A Lamb dip frequency stabilised HeNe for the improved Kösters and Kr lamp for the modified Kösters.  | Method of excess-fractions for 200 mm and 250 mm gauges. Optical comparison for 500 mm gauge  |                 | 19.9 to 20.1  |
| NPL-I      | UPMC-850 CARAT, ZEISS Germany   | Step gauge calibrated by PTB   | Comparison with calibrated step gauge   |                 | 1st 20.19 to 20.35<br>2nd 19.55 to 19.70                                      |
| SIRIM      | For 200 mm and 250 mm gauges a NPL-TESA gauge block interferometer. For 500 mm gauge a Single-Axis Universal Measuring Machine. SIP 1002M | For interferometry, two HeNe lasers at 633 nm and 543 nm, calibrated against in-house iodine stabilised lasers. Comparison, reference gauge calibrated at NML /CSIRO | For interferometry, method of excess-fractions. Refractive index determination via air temperature, pressure and humidity measurements. NPL-TESA equations used. Comparison used the substitution method. | Steel           | Interferometry<br>19.814 to 20.176<br>Comparison 19.86<br>to 20.20            |
| SCL        | Compatator: Carl Zeiss, Universal Horizontal Metroscope ULM 01-600C   | Reference gauges calibrated at NPL (UK). Measurement head calibrated by laser interferometer traceable to NPL (UK).  | Comparison by substitution  |                 | 19.88 to 20.10 (200 mm)<br>19.94 to 20.07 (250 mm)<br>19.95 to 20.09 (500 mm) |
| CMS        | SIP 1002 Universal Measurement Machine equipped with Mahr 1302 LVDT sensors.  | Reference gauges calibrated at PTB.  | Comparison by substitution  |                 | Laboratory 19.7 to 20.3<br>difference between<br>gauges < 0.04                |
| KIM-LIPI   | Universal Horizontal Metroscope, ULM Opal 1000  | Reference gauges traceable to NPL (UK)   | Comparison by substitution  |                 | 19.8 to 20.2  |
| ITDI       | Universal Length Measuring machine  | Reference gauges   | Comparison by substitution  |                 | 21.55 to 21.90 (200 mm)<br>21.35 to 22.05 (250 mm)<br>21.55 to 21.90 (500 mm) |

[1] P. E. Ciddor, Refractive index of air: new equations for the visible and near infrared, *Appl. Opt.*, **35**, 1566 –1573 (1996)

[2] R. Muijltwijk, Update of the Edlen Formulae for the Refractive Index of Air, *Metrologia* **25**, 189, (1988)

**TABLE 4: Measurement instruments and conditions reported by the participating laboratories**

$$\text{Standard uncertainty } u(L) = [a, b*L] = \text{SQRT}(a^2+(b*L)^2)$$

| Coeff.    | a     | b     | Length (m) | 0.2   | 0.25  | 0.5   |
|-----------|-------|-------|------------|-------|-------|-------|
| NML       | 0.015 | 0.095 |            | 0.024 | 0.028 | 0.050 |
| IR        | 0.151 | 0.240 |            | 0.158 | 0.162 | 0.193 |
| CSIR      | 0.074 | 0.240 |            | 0.088 | 0.095 | 0.141 |
| KRISS     | 0.015 | 0.089 |            | 0.023 | 0.027 | 0.214 |
| NMIJ      | 0.014 | 0.073 |            | 0.020 | 0.023 | 0.039 |
| NIM       | 0.009 | 0.097 |            | 0.021 | 0.026 | 0.060 |
| NPL-I (1) | 0.180 | 0.260 |            | 0.187 | 0.191 | 0.222 |
| SIRIM     | 0.015 | 0.150 |            | 0.034 | 0.040 | 0.239 |
| SCL       | 0.110 | 0.675 |            | 0.174 | 0.201 | 0.355 |
| CMS       | 0.048 | 0.451 |            | 0.102 | 0.240 | 0.231 |
| KIM-LIPI  | 0.150 | 1.060 |            | 0.260 | 0.305 | 0.551 |
| ITDI      |       |       |            | 2.4   | 1.7   | 1.5   |
| NPL-I (2) | 0.330 | 1.300 |            | 0.420 | 0.463 | 0.729 |

Uncertainty marked yellow is using a different technique

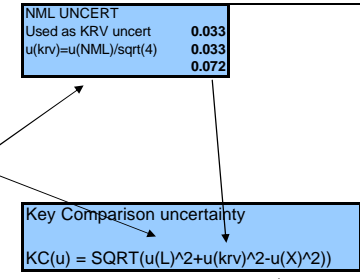
Uncertainty marked blue doesn't fit the formula

**TABLE 5: SUMMARY OF MEASUREMENT UNCERTAINTY**

**TABLE 6: A SIMPLE AVERAGE FOR THE KRV**

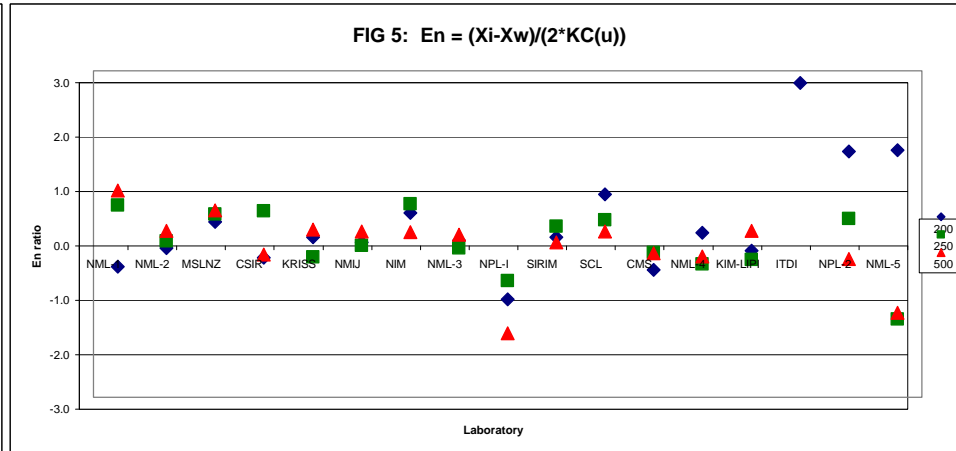
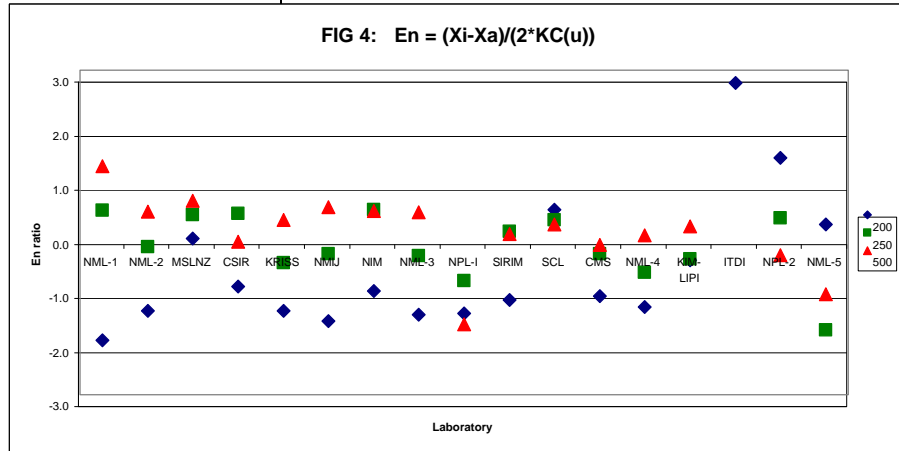
LABORATORY RESULTS FROM "A1 Measurement Results" (Page 2)

| Length L (mm)                   | LABORATORY RESULTS FROM "A1 Measurement Results" (Page 2) |        |        |        |        |        |        |        |        |        |        |       |        |          |        |         |        |        | Average Xa                          |
|---------------------------------|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-------|--------|----------|--------|---------|--------|--------|-------------------------------------|
|                                 | NML-1   | NML-2  | MSLNZ  | CSIR   | KRISS  | NMIJ   | NIM    | NML-3  | NPL-1  | SIRIM  | SCL    | CMS   | NML-4  | KIM-LIPI | ITDI   | NPL-2   | NML-5  | NMIJ-2 |                                     |
| Difference from nominal Xi (um) | 200   | 0.301  | 0.324  | 0.420  | 0.265  | 0.343  | 0.336  | 0.376  | 0.338  | -0.100 | 0.342  | 0.600 | 0.210  | 0.349    | 0.189  | 13.700  | 1.600  | 0.468  | 0.456<br>0.148<br>0.208             |
|                                 | 250   | 0.181  | 0.121  | 0.255  | 0.215  | 0.090  | 0.118  | 0.181  | 0.112  | -0.200 | 0.149  | 0.240 | 0.050  | 0.087    | -0.185 | 31.700  | 0.400  | -0.001 |                                     |
|                                 | 500   | 0.412  | 0.283  | 0.455  | 0.155  | 0.310  | 0.280  | 0.280  | 0.270  | -0.600 | 0.195  | 0.310 | 0.100  | 0.199    | 0.333  | -18.000 | -0.400 | 0.017  |                                     |
| Uncert u(Xi) (um)               | 200   | 0.024  | 0.0336 | 0.16   | 0.09   | 0.024  | 0.02   | 0.021  | 0.024  | 0.183  | 0.034  | 0.17  | 0.1    | 0.024    | 0.26   | 2.4     | 0.413  | 0.024  | 0.065<br>0.066<br>0.145             |
|                                 | 250   | 0.028  | 0.0392 | 0.16   | 0.09   | 0.041  | 0.023  | 0.026  | 0.028  | 0.191  | 0.041  | 0.2   | 0.12   | 0.028    | 0.338  | 1.7     | 0.466  | 0.028  |                                     |
|                                 | 500   | 0.05   | 0.07   | 0.2    | 0.14   | 0.214  | 0.039  | 0.06   | 0.05   | 0.228  | 0.239  | 0.35  | 0.23   | 0.05     | 0.553  | 1.5     | 0.73   | 0.05   |                                     |
| Xi-Xa                           | 200   | -0.155 | -0.132 | -0.036 | -0.191 | -0.113 | -0.120 | -0.080 | -0.118 | -0.556 | -0.114 | 0.144 | -0.246 | -0.107   | -0.267 | 13.244  | 1.144  | 0.012  | 0.065<br>0.066<br>0.145             |
|                                 | 250   | 0.033  | -0.027 | 0.107  | 0.067  | -0.058 | -0.030 | 0.033  | -0.036 | -0.348 | 0.001  | 0.092 | -0.098 | -0.061   | -0.333 | 31.552  | 0.252  | -0.149 |                                     |
|                                 | 500   | 0.204  | 0.075  | 0.247  | -0.053 | 0.102  | 0.072  | 0.072  | 0.062  | -0.808 | -0.013 | 0.102 | -0.108 | -0.009   | 0.125  | -18.208 | -0.608 | -0.191 |                                     |
| En = (Xi-Xa)/(2*KC(u))          | 200   | -1.99  | -1.45  | -0.11  | -1.01  | -1.45  | -1.64  | -1.08  | -1.52  | -1.50  | -1.25  | 0.42  | -1.18  | -1.38    | -0.51  | 2.76    | 1.38   | 0.15   | 0.0001061<br>0.0001638<br>0.0006744 |
|                                 | 250   | 0.40   | -0.27  | 0.33   | 0.35   | -0.57  | -0.39  | 0.42   | -0.43  | -0.90  | 0.01   | 0.23  | -0.39  | -0.74    | -0.49  | 9.28    | 0.27   | -1.81  |                                     |
|                                 | 500   | 1.22   | 0.39   | 0.59   | -0.17  | 0.23   | 0.46   | 0.40   | 0.37   | -1.70  | -0.03  | 0.14  | -0.23  | -0.05    | 0.11   | -6.06   | -0.41  | -1.15  |                                     |



**TABLE 7: A WEIGHTED MEAN FOR THE KRV**

| Length L (mm)          | LABORATORY RESULTS FROM "A1 Measurement Results" (Page 2) |        |        |       |        |        |        |       |        |        |        |       |        |          |        |         |        |        | C = 1/(sum)                         | Wt Average Xw           |
|------------------------|---|--------|--------|-------|--------|--------|--------|-------|--------|--------|--------|-------|--------|----------|--------|---------|--------|--------|-------------------------------------|-------------------------|
|                        | NML-1   | NML-2  | MSLNZ  | CSIR  | KRISS  | NMIJ   | NIM    | NML-3 | NPL-1  | SIRIM  | SCL    | CMS   | NML-4  | KIM-LIPI | ITDI   | NPL-2   | NML-5  | NMIJ-2 |                                     |                         |
| 1/u(Xi)^2              | 200   | 1736   | 886    | 39    | 123    | 1736   | 2500   | 2268  | 1736   | 30     | 865    | 35    | 100    | 1736     | 15     | 0       | 6      | 1736   | 0.0001061<br>0.0001638<br>0.0006744 | 0.347<br>0.135<br>0.272 |
|                        | 250   | 1276   | 651    | 39    | 123    | 595    | 1890   | 1479  | 1276   | 27     | 595    | 25    | 69     | 1276     | 9      | 0       | 5      | 1276   |                                     |                         |
|                        | 500   | 400    | 204    | 25    | 51     | 22     | 657    | 278   | 400    | 19     | 18     | 8     | 19     | 400      | 3      | 0       | 2      | 400    |                                     |                         |
| Xi*(1/u(Xi)^2)*C       | 200   | 0.055  | 0.030  | 0.002 | 0.003  | 0.063  | 0.089  | 0.090 | 0.062  | 0.000  | 0.031  | 0.002 | 0.002  | 0.064    | 0.000  | 0.000   | 0.001  | 0.086  | 0.347<br>0.135<br>0.272             | 0.347<br>0.135<br>0.272 |
|                        | 250   | 0.038  | 0.013  | 0.002 | 0.004  | 0.009  | 0.037  | 0.044 | 0.023  | -0.001 | 0.015  | 0.001 | 0.001  | 0.018    | 0.000  | 0.002   | 0.000  | 0.000  |                                     |                         |
|                        | 500   | 0.111  | 0.039  | 0.008 | 0.005  | 0.005  | 0.124  | 0.052 | 0.073  | -0.008 | 0.002  | 0.002 | 0.001  | 0.054    | 0.001  | -0.005  | -0.001 | 0.005  |                                     |                         |
| (Xi-Xw)                | 200   | -0.046 | -0.023 | 0.073 | -0.082 | -0.004 | -0.011 | 0.029 | -0.009 | -0.447 | -0.005 | 0.253 | -0.137 | 0.002    | -0.158 | 13.353  | 1.253  | 0.121  | 0.347<br>0.135<br>0.272             | 0.347<br>0.135<br>0.272 |
|                        | 250   | 0.046  | -0.014 | 0.120 | 0.080  | -0.045 | -0.017 | 0.046 | -0.023 | -0.335 | 0.014  | 0.105 | -0.085 | -0.048   | -0.320 | 31.565  | 0.265  | -0.136 |                                     |                         |
|                        | 500   | 0.140  | 0.011  | 0.183 | -0.117 | 0.038  | 0.007  | 0.008 | -0.002 | -0.872 | -0.077 | 0.038 | -0.172 | -0.073   | 0.061  | -18.272 | -0.672 | -0.255 |                                     |                         |
| En = (Xi-Xw)/(2*KC(u)) | 200   | -0.60  | -0.26  | 0.22  | -0.43  | -0.06  | -0.16  | 0.38  | -0.12  | -1.21  | -0.06  | 0.73  | -0.66  | 0.02     | -0.30  | 2.78    | 1.51   | 1.55   | 0.347<br>0.135<br>0.272             | 0.347<br>0.135<br>0.272 |
|                        | 250   | 0.53   | -0.13  | 0.37  | 0.42   | -0.42  | -0.21  | 0.55  | -0.26  | -0.86  | 0.14   | 0.26  | -0.34  | -0.55    | -0.47  | 9.28    | 0.28   | -1.56  |                                     |                         |
|                        | 500   | 0.79   | 0.05   | 0.43  | -0.37  | 0.08   | 0.04   | 0.04  | -0.01  | -1.82  | -0.15  | 0.05  | -0.36  | -0.42    | 0.05   | -6.08   | -0.46  | -1.45  |                                     |                         |



**TABLE 8: A LINEAR FIT TO THE INTERFEROMETRIC DATA FOR THE KRV:**

Data used for the linear regression is identified as a darker yellow. Data used to determine the artefact uncertainty is identified as a darker green.

| Length (mm)                      | LABORATORY RESULTS FROM sheet "A1 Measurement Results" (Page 2) |        |        |       |        |        |        |       |        |        |        |       |        |          |        |         |        |        |        |  |
|----------------------------------|---|--------|--------|-------|--------|--------|--------|-------|--------|--------|--------|-------|--------|----------|--------|---------|--------|--------|--------|--|
|                                  | NML-1   | NML-2  | MSLNZ  | CSIR  | KRISS  | NMIJ   | NIM    | NML-3 | NPL-1  | SIRIM  | SCL    | CMS   | NML-4  | KIM-LIPI | ITDI   | NPL-2   | NML-5  | NMIJ-2 |        |  |
| Difference from nominal (x (um)) | 200   | 0.301  | 0.324  | 0.420 | 0.265  | 0.343  | 0.336  | 0.376 | 0.338  | -0.100 | 0.342  | 0.600 | 0.210  | 0.349    | 0.189  | 13.700  | 1.600  | 0.468  | 0.455  |  |
|                                  | 250   | 0.181  | 0.121  | 0.255 | 0.215  | 0.090  | 0.118  | 0.181 | 0.112  | -0.200 | 0.149  | 0.240 | 0.050  | 0.087    | -0.185 | 31.700  | 0.400  | -0.001 | -0.006 |  |
|                                  | 500   | 0.412  | 0.283  | 0.455 | 0.155  | 0.310  | 0.280  | 0.280 | 0.270  | -0.600 | 0.195  | 0.310 | 0.100  | 0.199    | 0.333  | -18.000 | -0.400 | 0.017  | 0.039  |  |
| u(Xi) (um)                       | 200   | 0.024  | 0.0336 | 0.16  | 0.09   | 0.024  | 0.02   | 0.021 | 0.024  | 0.183  | 0.034  | 0.17  | 0.1    | 0.024    | 0.26   | 2.4     | 0.413  | 0.024  | 0.023  |  |
|                                  | 250   | 0.028  | 0.0392 | 0.16  | 0.09   | 0.041  | 0.023  | 0.026 | 0.028  | 0.191  | 0.041  | 0.2   | 0.12   | 0.028    | 0.338  | 1.7     | 0.466  | 0.028  | 0.023  |  |
|                                  | 500   | 0.050  | 0.070  | 0.200 | 0.140  | 0.214  | 0.039  | 0.060 | 0.050  | 0.228  | 0.239  | 0.350 | 0.230  | 0.050    | 0.553  | 1.500   | 0.730  | 0.050  | 0.042  |  |
| Xi - Xfi                         | 200   | -0.010 | -0.006 | 0.086 | -0.073 | 0.001  | -0.010 | 0.026 | -0.020 | -0.470 | -0.032 | 0.226 | -0.168 | -0.041   | -0.217 | 13.286  | 1.162  | 0.018  | -0.010 |  |
|                                  | 250   | -0.010 | -0.045 | 0.094 | 0.060  | -0.060 | -0.027 | 0.041 | -0.017 | -0.314 | 0.040  | 0.131 | -0.054 | -0.001   | -0.253 | 31.642  | 0.373  | -0.013 | 0.003  |  |
|                                  | 500   | 0.058  | -0.028 | 0.153 | -0.138 | 0.025  | 0.003  | 0.013 | 0.020  | -0.824 | -0.021 | 0.094 | -0.107 | 0.018    | 0.187  | -18.129 | -0.477 | -0.034 | 0.022  |  |

NML UNCERT  
Used as Xrv uncert  
 $u(krv)=u(NML)/\sqrt{4}$

u(Xrv) 0.011  
0.008  
0.019

NML u(Xnml-Xrv)

0.022  
0.016  
0.038

**BIPM Appendix B: Graph uncertainties**

|           | NML-1 | NML-2 | MSLNZ | CSIR  | KRISS | NMIJ  | NIM   | NML-3 | NPL-1 | SIRIM | SCL   | CMS   | NML-4 | KIM-LIPI | ITDI  | NPL-2 | NML-5 | NMIJ-2 |       |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|-------|-------|-------|--------|-------|
| u(Xi-Xrv) | 200   | 0.024 | 0.034 | 0.160 | 0.090 | 0.024 | 0.020 | 0.021 | 0.024 | 0.183 | 0.034 | 0.170 | 0.100 | 0.024    | 0.260 | 2.400 | 0.413 | 0.024  | 0.023 |
|           | 250   | 0.026 | 0.038 | 0.160 | 0.089 | 0.040 | 0.020 | 0.024 | 0.026 | 0.191 | 0.040 | 0.200 | 0.120 | 0.026    | 0.338 | 1.700 | 0.466 | 0.026  | 0.020 |
|           | 500   | 0.046 | 0.067 | 0.199 | 0.139 | 0.213 | 0.033 | 0.056 | 0.046 | 0.227 | 0.238 | 0.349 | 0.229 | 0.046    | 0.553 | 1.500 | 0.730 | 0.046  | 0.037 |

Key Comparison uncertainty  
 $u(Xi-Xrv) = \sqrt{u(Xi)^2 + u(Xint)^2 + u(Xrv)^2}$

|                         | NML-1 | NML-2 | MSLNZ | CSIR | KRISS | NMIJ  | NIM   | NML-3 | NPL-1 | SIRIM | SCL   | CMS  | NML-4 | KIM-LIPI | ITDI  | NPL-2 | NML-5 | NMIJ-2 |       |
|-------------------------|-------|-------|-------|------|-------|-------|-------|-------|-------|-------|-------|------|-------|----------|-------|-------|-------|--------|-------|
| En = (Xi-Xrv)/(2*KC(u)) | 200   | -0.20 | -0.10 | 0.27 | -0.41 | 0.01  | -0.26 | 0.61  | -0.42 | -1.28 | -0.47 | 0.66 | -0.84 | -0.85    | -0.42 | 2.77  | 1.41  | 0.38   | -0.22 |
|                         | 250   | -0.20 | -0.59 | 0.30 | 0.33  | -0.76 | -0.66 | 0.87  | -0.34 | -0.82 | 0.50  | 0.33 | -0.23 | -0.03    | -0.37 | 9.31  | 0.40  | -0.24  | 0.07  |
|                         | 500   | 0.64  | -0.21 | 0.38 | -0.50 | 0.06  | 0.05  | 0.11  | 0.22  | -1.82 | -0.04 | 0.13 | -0.23 | 0.20     | 0.17  | -6.04 | -0.33 | -0.38  | 0.30  |

**BIPM Appendix B: Page 2 - x<sub>F1</sub>**

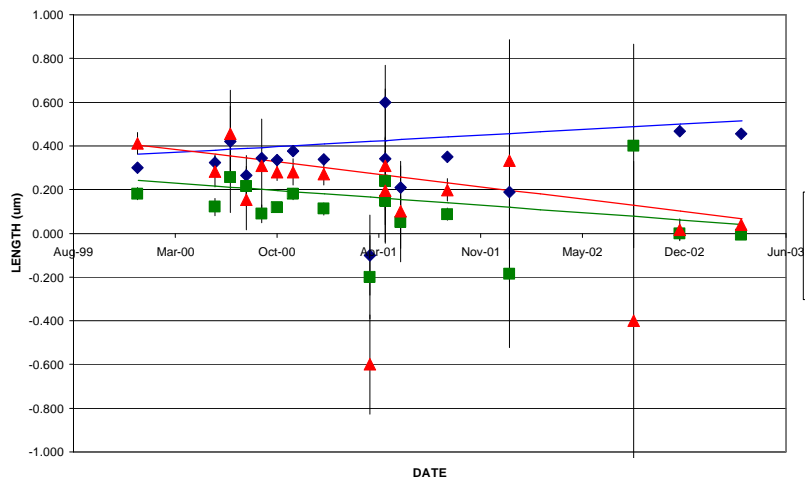
**TABLE 9: LINEAR FIT TO INTERFEROMETRIC DATA**

| Reference value (Xrv) | Xrv=A+B*Date |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |  |
|-----------------------|--------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--|
|                       | Jan-00       | Jun-00 | Jul-00 | Aug-00 | Sep-00 | Oct-00 | Nov-00 | Jan-01 | Apr-01 | May-01 | May-01 | Jun-01 | Sep-01 | Jan-02 | Mar-02 | Sep-02 | Dec-02 | Apr-03 |  |
| 200                   | 0.311        | 0.330  | 0.334  | 0.338  | 0.342  | 0.346  | 0.350  | 0.358  | 0.370  | 0.374  | 0.374  | 0.378  | 0.390  | 0.406  | 0.414  | 0.438  | 0.450  | 0.465  |  |
| 250                   | 0.191        | 0.166  | 0.161  | 0.155  | 0.150  | 0.145  | 0.140  | 0.129  | 0.114  | 0.109  | 0.109  | 0.104  | 0.088  | 0.068  | 0.058  | 0.027  | 0.012  | -0.009 |  |
| 500                   | 0.354        | 0.311  | 0.302  | 0.293  | 0.285  | 0.276  | 0.267  | 0.250  | 0.224  | 0.216  | 0.216  | 0.207  | 0.181  | 0.146  | 0.129  | 0.077  | 0.051  | 0.017  |  |

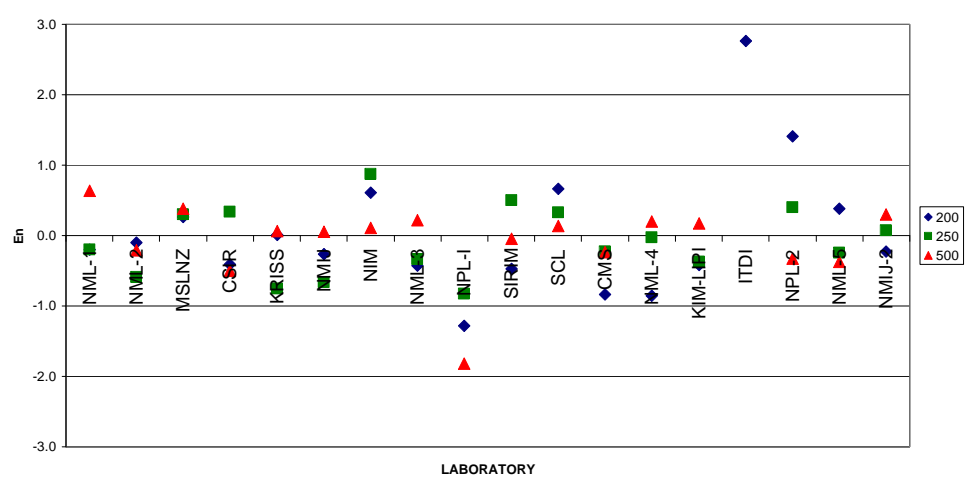
Fit values from ORIGIN  
B 0.00013038 A -4.45171  
-0.0001687 6.35194  
-0.0002842 10.73433

From "Birge Ratio"  
Internal uncert u(Xint)  
0.011  
0.014  
0.028

**FIG 6: LENGTH VARIATION DURING COMPARISON**



**FIG 7: En = (Xi - Xrv)/(2\*u(Xi-Xrv))**



**TABLE 10: BIRGE RATIO FOR THE THREE KEY REFERENCE VALUES**

| Gauge Length<br>(mm)               | LABORATORY RESULTS FROM "A1 Measurement Results" (Page 2) |        |        |       |        |        |        |        |        |        |        |       |        |          |        |         |        |        |       |
|------------------------------------|---|--------|--------|-------|--------|--------|--------|--------|--------|--------|--------|-------|--------|----------|--------|---------|--------|--------|-------|
|                                    | NML-1   | NML-2  | MSLNZ  | CSIR  | KRISS  | NMIJ   | NIM    | NML-3  | NPL-1  | SIRIM  | SCL    | CMS   | NML-4  | KIM-LIPI | ITDI   | NPL-2   | NML-5  |        |       |
| Difference from nominal<br>Xi (um) | 200   | 0.301  | 0.324  | 0.420 | 0.265  | 0.343  | 0.336  | 0.376  | 0.338  | -0.100 | 0.342  | 0.600 | 0.210  | 0.349    | 0.189  | 13.700  | 1.600  | 0.468  |       |
|                                    | 250   | 0.181  | 0.121  | 0.255 | 0.215  | 0.090  | 0.118  | 0.181  | 0.112  | -0.200 | 0.149  | 0.240 | 0.050  | 0.087    | -0.185 | 31.700  | 0.400  | -0.001 |       |
|                                    | 500   | 0.412  | 0.283  | 0.455 | 0.155  | 0.310  | 0.280  | 0.280  | 0.270  | -0.600 | 0.195  | 0.310 | 0.100  | 0.199    | 0.333  | -18.000 | -0.400 | 0.017  |       |
| Uncert u(Xi)<br>(um)               | 200   | 0.024  | 0.0336 | 0.16  | 0.09   | 0.024  | 0.02   | 0.021  | 0.024  | 0.183  | 0.034  | 0.17  | 0.1    | 0.024    | 0.26   | 2.4     | 0.413  | 0.024  |       |
|                                    | 250   | 0.028  | 0.0392 | 0.16  | 0.09   | 0.041  | 0.023  | 0.026  | 0.028  | 0.191  | 0.041  | 0.2   | 0.12   | 0.028    | 0.338  | 1.7     | 0.466  | 0.028  |       |
|                                    | 500   | 0.05   | 0.07   | 0.2   | 0.14   | 0.214  | 0.039  | 0.06   | 0.05   | 0.228  | 0.239  | 0.35  | 0.23   | 0.05     | 0.553  | 1.5     | 0.73   | 0.05   |       |
| 1/u(Xi)^2                          | 200   | 1736   | 886    | 39    | 123    | 1736   | 2500   | 2268   | 1736   | 30     | 865    | 35    | 100    | 1736     | 15     | 0       | 6      | 1736   |       |
|                                    | 250   | 1276   | 651    | 39    | 123    | 595    | 1890   | 1479   | 1276   | 27     | 595    | 25    | 69     | 1276     | 9      | 0       | 5      | 1276   |       |
|                                    | 500   | 400    | 204    | 25    | 51     | 22     | 657    | 278    | 400    | 19     | 18     | 8     | 19     | 400      | 3      | 0       | 2      | 400    |       |
| C = 1/sum                          |   |        |        |       |        |        |        |        |        |        |        |       |        |          |        |         |        |        |       |
|                                    |   |        |        |       |        |        |        |        |        |        |        |       |        |          |        |         |        |        |       |
|                                    |   |        |        |       |        |        |        |        |        |        |        |       |        |          |        |         |        |        |       |
| AVERAGE                            |   |        |        |       |        |        |        |        |        |        |        |       |        |          |        |         |        |        |       |
| Xi-Xa                              | 200   | -0.16  | -0.13  | -0.04 | -0.19  | -0.11  | -0.12  | -0.08  | -0.12  | -0.56  | -0.11  | 0.14  | -0.25  | -0.11    | -0.27  | 13.24   | 1.14   | 0.01   | 0.13  |
|                                    | 250   | 0.03   | -0.03  | 0.11  | 0.07   | -0.06  | -0.03  | 0.03   | -0.04  | -0.35  | 0.00   | 0.09  | -0.10  | -0.06    | -0.33  | 31.55   | 0.25   | -0.15  | 0.05  |
|                                    | 500   | 0.20   | 0.08   | 0.25  | -0.05  | 0.10   | 0.07   | 0.07   | 0.06   | -0.81  | -0.01  | 0.10  | -0.11  | -0.01    | 0.13   | -18.21  | -0.61  | -0.19  | 0.07  |
| External uncert u(Xa)              |   |        |        |       |        |        |        |        |        |        |        |       |        |          |        |         |        |        |       |
| (Xi-Xa)^2/u(Xi)^2                  | 200   | 41.857 | 15.498 | 0.051 | 4.517  | 22.276 | 36.164 | 14.612 | 24.285 | 9.240  | 11.296 | 0.715 | 6.065  | 19.978   | 1.057  | 30.451  | 7.669  | 0.239  | 0.065 |
|                                    | 250   | 1.412  | 0.465  | 0.450 | 0.559  | 1.982  | 1.671  | 1.638  | 1.628  | 3.314  | 0.001  | 0.213 | 0.663  | 4.704    | 0.969  | 344.480 | 0.293  | 28.214 | 0.022 |
|                                    | 500   | 16.654 | 1.149  | 1.526 | 0.143  | 0.227  | 3.365  | 1.442  | 1.540  | 12.558 | 0.003  | 0.085 | 0.220  | 0.032    | 0.051  | 147.346 | 0.694  | 14.585 | 0.046 |
| WEIGHTED AVERAGE                   |   |        |        |       |        |        |        |        |        |        |        |       |        |          |        |         |        |        |       |
| (Xi-Xw)                            | 200   | -0.046 | -0.023 | 0.073 | -0.082 | -0.004 | -0.011 | 0.029  | -0.009 | -0.447 | -0.005 | 0.253 | -0.137 | 0.002    | -0.158 | 13.353  | 1.253  | 0.121  | 0.121 |
|                                    | 250   | 0.046  | -0.014 | 0.120 | 0.080  | -0.045 | -0.017 | 0.046  | -0.023 | -0.335 | 0.014  | 0.105 | -0.085 | -0.048   | -0.320 | 31.565  | 0.265  | -0.136 | 0.05  |
|                                    | 500   | 0.140  | 0.011  | 0.183 | -0.117 | 0.038  | 0.007  | 0.008  | -0.002 | -0.872 | -0.077 | 0.038 | -0.172 | -0.073   | 0.061  | -18.272 | -0.672 | -0.255 | 0.07  |
| External uncert (Wt Ave)           |   |        |        |       |        |        |        |        |        |        |        |       |        |          |        |         |        |        |       |
| (Xi-Xw)^2/u(Xi)^2                  | 200   | 3.744  | 0.487  | 0.206 | 0.839  | 0.034  | 0.327  | 1.849  | 0.155  | 5.978  | 0.026  | 2.207 | 1.889  | 0.004    | 0.371  | 30.953  | 9.198  | 25.233 | 0.025 |
|                                    | 250   | 2.741  | 0.121  | 0.566 | 0.797  | 1.186  | 0.524  | 3.179  | 0.654  | 3.070  | 0.123  | 0.277 | 0.498  | 2.896    | 0.894  | 344.765 | 0.324  | 23.469 | 0.022 |
|                                    | 500   | 7.807  | 0.023  | 0.835 | 0.702  | 0.031  | 0.034  | 0.016  | 0.002  | 14.637 | 0.105  | 0.012 | 0.561  | 2.149    | 0.012  | 148.390 | 0.848  | 26.070 | 0.027 |
| WEIGHTED LINEAR FIT                |   |        |        |       |        |        |        |        |        |        |        |       |        |          |        |         |        |        |       |
| Xi - Xfi                           | 200   | -0.010 | -0.006 | 0.086 | -0.073 | 0.001  | -0.010 | 0.026  | -0.020 | -0.470 | -0.032 | 0.226 | -0.168 | -0.041   | -0.217 | 13.286  | 1.162  | 0.018  | 0.018 |
|                                    | 250   | -0.010 | -0.045 | 0.094 | 0.060  | -0.060 | -0.027 | 0.041  | -0.017 | -0.314 | 0.040  | 0.131 | -0.054 | -0.001   | -0.253 | 31.642  | 0.373  | -0.013 | 0.05  |
|                                    | 500   | 0.058  | -0.028 | 0.153 | -0.138 | 0.025  | 0.003  | 0.013  | 0.020  | -0.824 | -0.021 | 0.094 | -0.107 | 0.018    | 0.187  | -18.129 | -0.477 | -0.034 | 0.07  |
| External uncert Lin Fit            |   |        |        |       |        |        |        |        |        |        |        |       |        |          |        |         |        |        |       |
| (Xi-Xfi)^2/u(Xi)^2                 | 200   | 0.162  | 0.037  | 0.286 | 0.666  | 0.000  | 0.270  | 1.483  | 0.721  | 6.599  | 0.887  | 1.767 | 2.825  | 2.927    | 0.696  | 30.647  | 7.921  | 0.593  | 0.025 |
|                                    | 250   | 0.133  | 1.293  | 0.349 | 0.440  | 2.145  | 1.378  | 2.515  | 0.390  | 2.708  | 0.941  | 0.427 | 0.203  | 0.003    | 0.560  | 346.442 | 0.641  | 0.202  | 0.023 |
|                                    | 500   | 1.347  | 0.157  | 0.583 | 0.978  | 0.014  | 0.008  | 0.045  | 0.161  | 13.073 | 0.008  | 0.072 | 0.217  | 0.131    | 0.114  | 146.079 | 0.427  | 0.471  | 0.026 |

BIRGE RATIO  
number 11  
Rb 1.4

C = 1/sum  
0.0001167  
0.0001825  
0.0007771

Internal uncert u(Xint)  
0.011  
0.014  
0.028

BIRGE RATIO  
11.6  
3.4  
2.5  
6.0  
1.6  
1.6  
2.3  
1.6  
1.0

BIPM Appendix  
B: Page 2  
uncertainties  $u_{R2}$

**TABLE: 3**

| Laboratory          | NML-1  | NML-2  | MSLNZ  | CSIR   | KRISS  | NMIJ   | NIM    | NML-3  | NPL-1  | SIRIM    | SCL       | CMS    | NML-4  | KIM-LIPI | ITDI      | NPL-2  | NML-5    |        |
|---------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----------|-----------|--------|--------|----------|-----------|--------|----------|--------|
| Date of measurement | Jan-00 | Jun-00 | Jul-00 | Aug-00 | Sep-00 | Oct-00 | Nov-00 | Jan-01 | Apr-01 | May-01   | May-01    | Jun-01 | Sep-01 | Jan-02   | Mar-02    | Sep-02 | Dec-02   |        |
| Length $x_i$        | 200    | 0.301  | 0.324  | 0.420  | 0.265  | 0.343  | 0.336  | 0.376  | 0.338  | -0.100   | 0.342     | 0.600  | 0.21   | 0.349    | 0.189     | 13.7   | 1.6      | 0.468  |
|                     | 250    | 0.181  | 0.121  | 0.255  | 0.215  | 0.090  | 0.118  | 0.181  | 0.112  | -0.200   | 0.149     | 0.240  | 0.05   | 0.087    | -0.185    | 31.7   | 0.4      | -0.001 |
|                     | 500    | 0.412  | 0.283  | 0.455  | 0.155  | 0.310  | 0.280  | 0.280  | 0.27   | -0.600   | 0.195     | 0.310  | 0.1    | 0.199    | 0.333     | -18    | -0.4     | 0.017  |
| $u(X_i-X_{rv})$     | 200    | 0.024  | 0.034  | 0.160  | 0.090  | 0.024  | 0.020  | 0.021  | 0.024  | 0.183002 | 0.0340123 | 0.170  | 0.100  | 0.024    | 0.2600016 | 2.400  | 0.413001 | 0.024  |
|                     | 250    | 0.026  | 0.038  | 0.160  | 0.089  | 0.040  | 0.020  | 0.024  | 0.026  | 0.190697 | 0.0395623 | 0.200  | 0.120  | 0.026    | 0.3378286 | 1.700  | 0.465876 | 0.026  |
|                     | 500    | 0.046  | 0.067  | 0.199  | 0.139  | 0.213  | 0.033  | 0.056  | 0.046  | 0.227091 | 0.2381328 | 0.349  | 0.229  | 0.046    | 0.5526258 | 1.500  | 0.729717 | 0.046  |

From "measurement results" page 2

From "Drift removed" page 6

9: LINEAR FIT TO INTERFEROMETRIC DATA

| Reference value ( $X_{rv}$ ) | $X_{rv}=A+B*Date$ |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |
|------------------------------|-------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|                              | Jan-00            | Jun-00 | Jul-00 | Aug-00 | Sep-00 | Oct-00 | Nov-00 | Jan-01 | Apr-01 | May-01 | May-01 | Jun-01 | Sep-01 | Jan-02 | Mar-02 | Sep-02 | Dec-02 |
| 200                          | 0.311             | 0.330  | 0.334  | 0.338  | 0.342  | 0.346  | 0.350  | 0.358  | 0.370  | 0.374  | 0.374  | 0.378  | 0.390  | 0.406  | 0.414  | 0.438  | 0.450  |
| 250                          | 0.191             | 0.166  | 0.161  | 0.155  | 0.150  | 0.145  | 0.140  | 0.129  | 0.114  | 0.109  | 0.109  | 0.104  | 0.088  | 0.068  | 0.058  | 0.027  | 0.012  |
| 500                          | 0.354             | 0.311  | 0.302  | 0.293  | 0.285  | 0.276  | 0.267  | 0.250  | 0.224  | 0.216  | 0.216  | 0.207  | 0.181  | 0.146  | 0.129  | 0.077  | 0.051  |

$X_i - X_{rv}$

x 2

BIPM Appendix B: Degrees of equivalence - page 3 and graphs

**TABLE: 3**

| Laboratory          | NML/CSIRO-1 | NML/CSIRO-2 | MSLIRL | NML/CSIR | KRISS  | NMIJ   | NIM    | ML/CSIRONPL/CSIR- | SIRIM  | SCL    | CMS    | ML/CSIRO | KIMLIPI | ITDI   | NPL/CSIR-ML/CSIRO-5 |         |        |        |
|---------------------|-------------|-------------|--------|----------|--------|--------|--------|-------------------|--------|--------|--------|----------|---------|--------|---------------------|---------|--------|--------|
| Date of measurement | Jan-00      | Jun-00      | Jul-00 | Aug-00   | Sep-00 | Oct-00 | Nov-00 | Jan-01            | Apr-01 | May-01 | May-01 | Jun-01   | Sep-01  | Jan-02 | Mar-02              | Sep-02  | Dec-02 |        |
| Length $x_i$        | 200         | -0.010      | -0.006 | 0.086    | -0.073 | 0.001  | -0.010 | 0.026             | -0.020 | -0.470 | -0.032 | 0.226    | -0.168  | -0.041 | -0.217              | 13.286  | 1.162  | 0.018  |
|                     | 250         | -0.010      | -0.045 | 0.094    | 0.060  | -0.060 | -0.027 | 0.041             | -0.017 | -0.314 | 0.040  | 0.131    | -0.054  | -0.001 | -0.253              | 31.642  | 0.373  | -0.013 |
|                     | 500         | 0.058       | -0.028 | 0.153    | -0.138 | 0.025  | 0.003  | 0.013             | 0.020  | -0.824 | -0.021 | 0.094    | -0.107  | 0.018  | 0.187               | -18.129 | -0.477 | -0.034 |
| Uncertainty         | 200         | 0.048       | 0.067  | 0.320    | 0.180  | 0.048  | 0.040  | 0.042             | 0.048  | 0.366  | 0.068  | 0.340    | 0.200   | 0.048  | 0.520               | 4.800   | 0.826  | 0.048  |
| $2u(X_i-X_{rv})$    | 250         | 0.052       | 0.075  | 0.319    | 0.179  | 0.079  | 0.041  | 0.047             | 0.052  | 0.381  | 0.079  | 0.399    | 0.239   | 0.052  | 0.676               | 3.400   | 0.932  | 0.052  |
|                     | 500         | 0.091       | 0.134  | 0.398    | 0.277  | 0.426  | 0.067  | 0.113             | 0.091  | 0.454  | 0.476  | 0.699    | 0.458   | 0.091  | 1.105               | 3.000   | 1.459  | 0.091  |

