

Key Comparison CCEM-K7 : AC Voltage Ratio Draft B Report - Version I - Part 1

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This report provides the results of a key comparison, CCEM-K7, organised under the auspices of the Comité Consultatif d'Électricité et Magnétisme (CCEM) and carried out by a group of National Metrology Institutes (NMIs). The measured parameters were a set of ratios of a type of voltage transformer called an Inductive Voltage Divider (IVD). A single custom-made IVD - the *Travelling Standard* - was designed and used specifically for this comparison.

Participating Laboratories: ¹NPL[†] ²CEM ³IEN ⁴KRISS ⁵LNE ⁶METAS ⁷NIM ⁸NIST
⁹NMIA ¹⁰NMIJ ¹¹VSL ¹²NPLI ¹³NRC ¹⁴PTB ¹⁵SP ¹⁶UME ¹⁷VNIIM

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1 Introduction

Many nations have signed up to a Mutual Recognition Arrangement (MRA) to improve and simplify trade amongst themselves. The MRA process is controlled by the Comité International des Poids et Mesures (CIPM) formed from delegates to the Conférence Général des Poids et Mesures (CGPM) which is itself comprised of delegates from countries that are signatories to the Convention du Mètre.

One of the key points of the MRA is mutual recognition of measurement capabilities between nations. In order to facilitate this process, the CIPM and the Bureau International des Poids et Mesures (BIPM) have organised a series of key comparisons between the National Metrology Institutes (NMIs) with electrical measurements organised under the auspices of the Comité consultatif d'Électricité et Magnétisme (CCEM).

2 Organisation

2.1 Organisation

The comparison was started as proposal No. CCE/97-31, put to the 21st session of the CCE¹ in June 1997 by Ian Robinson (NPL) and through discussion between interested parties lead to a informal meeting at CPEM '98², held in Washington D.C. , USA

The Pilot Laboratory for key comparison CCEM-K7 was the National Physical Laboratory (NPL) UK : NPL's role was to

- Prepare and disseminate the *Technical Protocol* document to participants;
- Coordinate the testing schedule ;
- Organise the despatch and return of the *Travelling Standard* between participant and pilot laboratory, or between participants as required;
- Collect, archive and analyse the reported data;
- Prepare and disseminate the Draft A and Draft B Reports

¹Consultative Committee on Electricity - renamed CCEM in 1997

²Conference on Precision Electromagnetic Measurements

2.2 Participants

Table 1 lists all participants in CCEM-K7 (arranged in alphabetical order of Identifier)

Table 1: CCEM-K7 Participants

<i>Identifier</i>	<i>Name</i>	<i>Country</i>	<i>(RMO) ^a</i>
<i>CEM ^b</i>	<i>Centro Español de Metrologia</i>	<i>Spain</i>	<i>EUROMET</i>
<i>IEN</i>	<i>Istituto Elettrotecnico Nazionale Galileo Ferraris</i>	<i>Italy</i>	<i>EUROMET</i>
<i>KRISS</i>	<i>Korea Research Institute of Standards and Science</i>	<i>Korea, Republic of</i>	<i>APMP</i>
<i>LCIE</i>	<i>Laboratoire Central des Industries Electriques</i>	<i>France</i>	<i>EUROMET</i>
<i>METAS ^c</i>	<i>Metrology and Accreditation Switzerland</i>	<i>Switzerland</i>	<i>EUROMET</i>
<i>NIM</i>	<i>National Institute of Metrology</i>	<i>China</i>	<i>APMP</i>
<i>NIST ^d</i>	<i>National Institute of Standards and Technology</i>	<i>USA</i>	
<i>NMIA ^e</i>	<i>National Measurement Institute</i>	<i>Australia</i>	<i>APMP</i>
<i>NMIJ ^f</i>	<i>National Metrology Institute of Japan</i>	<i>Japan</i>	<i>APMP</i>
<i>NMi-VSL</i>	<i>Nederlands Meetinstituut - Van Swinden Laboratorium</i>	<i>The Netherlands</i>	<i>EUROMET</i>
<i>NPL</i>	<i>National Physical Laboratory</i>	<i>United Kingdom</i>	<i>EUROMET</i>
<i>NPLI</i>	<i>National Physical Laboratory of India</i>	<i>India</i>	<i>APMP</i>
<i>NRC</i>	<i>National Research Council</i>	<i>Canada</i>	<i>SIM</i>
<i>PTB</i>	<i>Physikalisch-Technische Bundesanstalt</i>	<i>Germany</i>	<i>EUROMET</i>
<i>SP</i>	<i>SP Technical Research Institute of Sweden</i>	<i>Sweden</i>	<i>EUROMET</i>
<i>UME ^g</i>	<i>Tubitak Ulusal Metrologi Enstitüsü</i>	<i>Turkey</i>	<i>MENAMET</i>
<i>VNIIM ^h</i>	<i>D.I. Mendeleyev Institute for Metrology</i>	<i>Russian Federation</i>	<i>COOMET</i>

^a*Regional Metrology Organisation*

^b*Joined 2001-01*

^c*formerly OFMET*

^d*Withdrew in 2000-11; re-joined in 2002-08*

^e*formerly CSIRO-NML*

^f*formerly ETL*

^g*Joined 2001-10*

^h*Joined 2001-01*

2.3 Testing Schedule

The initial plan called for all testing to be completed within a 2 year period using an 8-week cycle of 2 weeks for despatch between laboratories , 4 weeks for testing at the participant laboratory and another 2 weeks for despatch back to the pilot laboratory. At the commencement of the comparison the schedule was heavily loaded towards the latter half of the project, leaving large gaps in the 1st year and no spare capacity the 2nd year.

At the beginning of the 2nd year, the program was extended to be over 3 years duration. This decision was made because of numerous delays caused by shipment problems, 3 new laboratories joined the comparison and several participants requested an opportunity to re-measure.

The pilot laboratory was responsible for the arrangement and cost of the outward journey to participant laboratories - the participant laboratory was responsible for the arrangement and cost of the return or onward journey.

Table 2: Calibration Schedule

<i>Identifier</i>	<i>Country</i>	Year	Weeks	Month (approx)
<i>NMi-VSL</i>	<i>The Netherlands</i>	1999	44-47	November
<i>PTB</i>	<i>Germany</i>	1999- 2000	50-07	December-January
<i>NMIA</i>	<i>Australia</i>	2000 2001	30-35 30-36	July-August July-August
<i>NMIJ</i>	<i>Japan</i>	2000	34-38	September
<i>METAS</i>	<i>Switzerland</i>	2000	44-49	November
<i>NRC</i>	<i>Canada</i>	2000- 2001 2003	51-05 08-11	December-January February-March
<i>NPLI</i>	<i>India</i>	2001	10-17	March-April
<i>SP</i>	<i>Sweden</i>	2001	24-27	June
<i>NIM</i>	<i>China</i>	2001	40-51	October-December
<i>LCIE</i>	<i>France</i>	2002	05-08	February
<i>KRISS</i>	<i>Korea, Republic of</i>	2002	12-15	March-April
<i>IEN</i>	<i>Italy</i>	2002	19-22	May
<i>CEM</i>	<i>Spain</i>	2002 2003	23-26 02-03	June January
<i>VNIIM</i>	<i>Russian Federation</i>	2002	29-41	July-September
<i>UME</i>	<i>Turkey</i>	2002	44-47	November
<i>NPL</i>	<i>United Kingdom</i>	2003	01-05	January
<i>NIST</i>	<i>USA</i>	2003	14-24	April-June

3 Calibration Artefact & Shipping

3.1 The Travelling Standard

A custom-made IVD was designed and constructed specifically for the comparison by Greig Small and John Fiander of NMI, Australia [1] and delivered to NPL in June 1999. The IVD is a 2-stage design with fixed output taps for 10-section (i.e. decade), 11-section and 0.01 ratio windings. Figure 1 is a schematic of the IVD. All connections are made via BPO (MUSA) coaxial sockets on the top panel of the divider. The outer terminals of all the coaxial connectors except the EXCITE terminal are connected in parallel, by direct contact to the copper slab that forms the top panel. The IVD dimensions are approximately (length x width x height) 175 x 125 x 145 mm (165 mm including MUSA sockets) and net weight is 4.90 kg. It fits snugly within a red wooden box that comes in two parts, so the lid can be removed for access to the terminals and the IVD can be left within the lower box. The box parts join up via 8 long screws through the edges of the lid. The box dimensions are 195 x 155 x 195 mm. A tamper-proof seal was applied to one of the torx screws holding the transformer to the metal case at the beginning of the programme. It remained intact until intentionally broken at the pilot laboratory in January 2002. It was re-sealed and remained intact until the end of the programme.

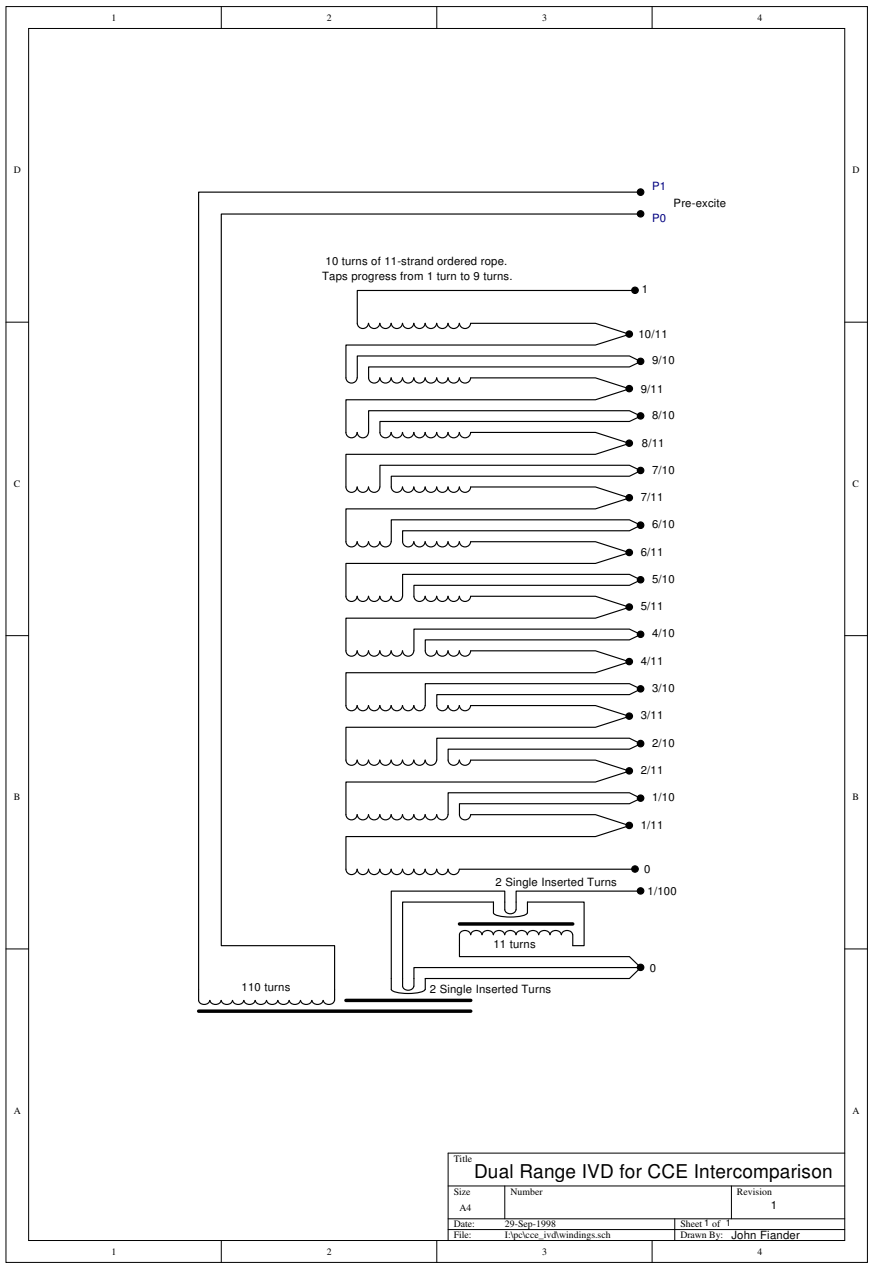


Figure 1

In January 2002, on return from NIM, China there was evidence of growth of some organic compounds (“mould”) within the BPO (MUSA) sockets. The sockets were carefully cleaned using paper soaked in propan-2-ol. The standard was dismantled and checked inside for further evidence of mould - none was found. Routine checks at the pilot laboratory did not show any anomalous or significant changes to the measured parameters.

3.2 Accessories

The set of accessories accompanied the travelling standard were :-

- A 3 mm hex ball driver, for the 8 bolts that secure the red wooden box top to bottom
- 4 off BNC - dual 4 mm adaptors
- 4 off BPO (MUSA) - BNC adaptors
- 2 off BNC - single 4 mm adaptors

3.3 Transit cases

Special care was applied to the way in which the travelling standard was packed and transported between laboratories. As the schedules were tight, air-freight was the only viable option.

During transit, the red wooden box was secured in a large ABS plastic case with padded foam inserts. This plastic case and the accessories case were in turn secured within a large aluminum transit case with padded foam inserts.

The transit system was tested at NPL sometime prior to the first scheduled visit : The aluminum transit case was held approximately 1 m above a concrete pavement and dropped, with one corner of the case pointing towards the ground. Before-and-after measurements did not reveal any significant change to the measured parameters of the travelling standard.

Shock detectors were mounted on the red wooden box, on the ABS case and the other transit case; only one of these detectors was activated throughout the entire programme. Minor damage was observed during the comparison to the outer transit case and repairs and additions were made to the securing latches and hasps. No visible damage was seen or reported to the intermediate ABS case, the red wooden box or the travelling standard itself.

4 Measurements

Details of the tests, methods, defining conditions and related information were given in the Technical Procedure document in Appendix R.

5 Method of measurement used by each participant

5.1 Method of measurement used by CEM, Spain

The ratios 0.1 to 0.9 and 1/11 to 10/11 have been measured by the known "straddling" method with direct and reverse connections using a calibration transformer in taps 0.1 to 0.9 and 1/11 to 10/11.

The ratio 0.01 was measured using an auxiliary voltage ratio transformer whose deviation from its nominal value at ratio 0.1 is previously measured, with an input voltage of 1V. This previous measurement is carried out by the same method as above. The most outstanding elements that the measuring system incorporates are: triaxial connections between the IVD traveling standard and the calibration transformer; specially designed networks to detect current, to give potential to the intermediate screen, to adjust cross-capacitances in the lines and to provide injections in phase and quadrature to produce the main balance.

An RC network adjusted, jointly with the rest of the components in the line of quadrature injection, produces a precise 90° in advance at 1 kHz.

5.2 Method of measurement used by IEN, Italy

The measurements have been carried out with a new system for ac voltage ratio absolute calibration. The system is based on the simplest possible application of the bootstrap method [2–5]. The system is built around a new two-stage divider, having k: 10, k: 11 and 1: 100 ratios; a two-stage bootstrap transformer (1:10 and 1:11 ratios) with a guarded one-turn secondary 8 stage with adjustable cross-capacitances; two-secondaries (k:10 and k:11) transformer for easy guarding. The system does not involve injection/detection transformers, but is based on a direct measurement of the voltage differences with a purpose-built vector voltmeter [6]. During the step-up the voltage differences are read directly with the vector voltmeter which has an extremely high common-mode rejection ratio, exceeding 180 dB at 1 kHz. The vector voltmeter has the following properties:

1. completely guarded structure to achieve very high apparent input impedance;
2. an extremely high common-mode rejection ratio (CMRR), greater than 180 dB at 1 kHz
3. a programmable-gain amplification stage (full-scale dynamics are in the range 500 ~V to 500 n V)
4. a dual-channel synchronous demodulating stage with high harmonic rejection;
5. 8 battery operation
6. computer-controlled operation, via an optical fibre interface [7].

The method is insensitive to several possible systematic errors of the vector voltmeter or those coming from the bootstrap operation

5.3 Method of measurement used by KRISS, Korea

The AC voltage ratio calibration system of KRISS [8] has the following functions: - Ratio terminals: 0.01, 0.1 to 1.0, 0.01 to 0.1, and 1/11 to 10/11, - Resolution for α and β : 10^{-10} , - Self-calibration by comparison with the reference winding using tri-axial cables, - Adjustment function to minimize internal loading caused by distributed shunt impedances and leakage impedances in both the divider winding and the calibration system.

A modified method for consecutive comparison of a voltage difference on the taps of each section with a stable isolated reference voltage was used for calibration of the ratio transformer. The reference voltage is formed by special tri-axial winding of the same transformer, so the used method is rather self-calibration than calibration. The creating of EMF in both ratio and reference winding by the same magnetic flux eliminates any nonlinear distortions between them which are fundamental restriction of accuracy in this method. The potentials of internal screens of tri-axial cables connecting to both the reference winding and a measured section of the ratio winding, are driven by auxiliary sources. Any offset in the detector is eliminated by injection of a current in an additional winding of the detector transformer instead of balancing of currents in gaps of tri-axial cable.

5.4 Method of measurement used by LCIE, France

The divider CCEM-ACVRS01 was calibrated by comparison with a reference divider ESI type DT72A previously calibrated.

The comparison was carried out directly for all ratios by means of a General Radio zero detector and a voltage injection device adjusted in phase and quadrature. The voltage injection device is introduced in the detection arm of the bridge.

The determination of the ESI type DT72A divider linearity errors was performed as the following:

1. Direct comparison between the unknown divider ratio and the same ratio of a tare divider for all positions of the 1st, 2nd and 3rd decades.
2. Insertion of a voltage transformer (1) in the detection arm and comparison between the unknown divider ratio and the tare divider ratio for the just inferior position for all positions of the 1st, 2nd and 3rd decades.

By combining all values of voltage injected (in phase and quadrature) to obtain the balance, it is possible to determine the linearity error of the unknown divider.

(1) Ratio 1:10 for the 1st decade, 1:100 for the 2nd decade and 1:1000 for the 3rd decade.

5.5 Method of measurement used by METAS, Switzerland

The $n/10$ and $n/11$ ratios of the CCEM-ACVRS01 Inductive Voltage Divider (IVD) were calibrated using the so-called "boot-strap" method [9, 10] either with or without the use of an auxiliary IVD.

When an auxiliary IVD(AUX) is used the measurement procedure is as follows. First, both dividers are compared with their outputs set at the same nominal value. An injection system ΔE_n and a null detector is used to measure the voltage difference. Then, the IVD is set one step higher and a supplementary step-injection system was introduced in the circuit. The injected voltage $\Delta E'_n$ needed to null the detector is measured. Finally, the AUX is set one step higher. Both IVD outputs are nominally equal again, so they can be compared

without the step- injection system. This procedure of increasing first the UUT and then the AUX can be repeated from the lowest to the highest ratio output of the IVD. The set of equations involving the measured values of ΔE_n and $\Delta E'_n$ can be solved and the in-phase and quadrature deviations from the nominal output ratios can be calculated for both dividers. In the case when all the ratio taps of the UUT are separately available, the auxiliary IVD is not needed any more and the ratio $\{n\}$ of the UUT can be directly compared to the tap $\{n+1\}$. The number of steps needed to calibrate UUT is smaller than for the method using an auxiliary IVD.

The 1/100 ratio of the CCEM-ACVRS01 IVD was calibrated by comparison with our reference 8-Dial IVD (PR1). In this case, the error of the output ratio of the UUT is directly determined by comparison to the same nominal output ratio of the reference IVD which has a known error.

5.6 Method of measurement used by NIM, China

The linear correction $\Delta K_{\Sigma L}(K_N)$ of transfer ratio for the core of shielding plug of all indications i/N ($i=0\sim N$, $N=10$ or 11) have determined by the Method "Comparison Method with Bidirectional (positive/negative) Reference Winding" [11], Design principle of the measurement method are following:

1. The reference voltage divider will provide the supporting voltage for the reference winding. Then the comparison between reference winding voltage and the sectional voltage of calibrated voltage divider could be processed under the practical working conditions of the calibrated voltage divider.
2. The Reference voltage of Reference Winding is defined as an increment of this winding voltage when connect of its original winding is changed without short to input voltage of the calibrated voltage divider. Based on cumulate principle this Reference voltage is irrelevant without the potential of low point by this Reference Winding, if its circuit parameters are linear.
3. For the linear correction $\Delta K_{\Sigma L}(K_N)$ of transfer ratio two independent measurements will be taken by commutate the original winding. The average of two independent measurement results will be used as final result. The practical design of measurement process calculate that, most errors of the measurement equipment can be eliminated by the subtracting of the data pairs. Most of the remaining error factors will produce opposite errors during the bidirection (positive/negative) process. So, besides providing the measurement result of linear corrections $\Delta K_{\Sigma L}(K_N)$, this method can also provide the impersonal experimental base for evaluating the limit error (extended uncertainty) of the measurement result.

5.7 Method of measurement used by NIST, USA

Two methods were used to determine the errors of the IVD. With the exception of the 0.01 tap for which a “bootstrap” bridge was used, the bridge was based upon the step-up or “straddling” method. The IVDs were of a two-stage design with additional circuitry to perform triaxial guarding, coaxial equalization, Wagner isolation, auxiliary signal injection and phase sensitive detection. With this straddling method, a critical factor influencing bridge accuracy is the common mode rejection of the auxiliary IVD. The 1:1 ratio errors of the auxiliary IVD must remain constant irrespective of the common mode level to which it is driven. This requirement is addressed by vary careful design of guarded shields that enclose the auxiliary IVD.

5.8 Method of measurement used by NMI, Australia

AC voltage ratio is measured at NMIA using a build-up technique based on the method described by Thompson [12]. The signal measurement system is described by Small and Leslie [13]. Further to Thompson’s original work, the build-up measurement system at NMIA has undergone considerable refinement and development. The main changes are: 1. Use of a bootstrap transformer separate from the transformer being calibrated to provide the isolated reference voltage. The shields of the purpose built bootstrap transformer are designed in such a way that the variation of output voltage with offset voltage is of the order of 1 nV/V and may be determined with small uncertainty (linear regression on multiple measurements sets was used to further reduce uncertainty). 2. Use of pre-excited (rather than single excitation) transformers, ensuring that output voltages are well defined and stable with respect to each other. 3. Core sense windings on the coaxial chokes and manually-adjusted injectors have been added where required. Note that each injection voltage is adjusted for every measurement. The uncertainty due to residual current imbalance has been determined experimentally to be negligible by testing the sensitivity of the measurement to the adjustment of the injectors. 4. The on/off switching of the calibrate signal is arranged so that the detector circuit impedance and the loading of the source remain unchanged. 5. A single wire connects the grounds of the bootstrap transformer and the ac voltage ratio standard. By coaxial injection into its supply lead, the ground potential of the bootstrap transformer is adjusted for zero current in this wire, so that the grounds of the two transformers are equalised without causing unbalanced current flow in the coaxial conductors of the voltage comparator. The single wire is wrapped around the comparator circuit to minimise the area of the loop represented by the single wire and the outer conductors of the voltage comparator, thereby ensuring that the measurement is insensitive to coupling into this loop. The ratio of the 0.01 output voltage to the 0.1 output voltage was measured by comparison with a 10:1 inductive voltage divider previously calibrated by the build-up technique. From the build-up calibration of the decade output voltage, the 0.01 ratio was then calculated.

5.9 Method of measurement used by NMI (ETL), Japan

Measurement method The traveling IVD was calibrated using a build-up method, the principle of which is based on the techniques developed by A. M. Thompson [12]. So-called "special connectors" were used in the measurements, by which output voltage of the IVD can be measured as the open circuit voltage at its port. The measurement equipment including the special connectors was fabricated and evaluated by ourselves [14, 15].

5.10 Method of measurement used by NMI - Van Swinden Laboratorium, The Netherlands

A bootstrapping method [9] is used for calibrations of inductive voltage dividers (IVD) and voltage ratio transformers. In this comparison, the bootstrapping method was applied to measure the $n/10$ and $n/11$ ratio taps of the (traveling) inductive voltage divider (IVD). The 0.01 output was measured by comparison with a calibrated IVD. For the bootstrapping method, the uncertainty in the ratio measurements is mainly determined by the contributions of the ratio difference measurements between the divider under test (traveling standard) and the auxiliary divider. The measurements are performed by null-detection on a phase sensitive detector. An injection system in series with the output of the auxiliary divider is used to balance the bridge.

5.11 Method of measurement used by NPL, UK

The NPL AC voltage ratio measurement system based on a bootstrapping method [2, 5] was used for the intercomparison measurements. The traveling inductive voltage divider (IVD) was compared against the NPL reference IVD, with both IVDs energised from a common AC voltage, with the voltage difference between their outputs being measured. The ratio errors of the traveling IVD were calculated from these measured differences and the known errors of the NPL reference IVD. This NPL reference IVD is maintained by measuring the difference voltage between adjacent taps.

5.12 Method of Measurement used by NPL, India

LF Impedance group of NPLI is having two standard IVDs. Each section of these IVDs is approachable through BPO connectors. Calibration of Standard IVD is done by absolute method using injection voltage technique. This setup was established in 1987 and reported in CPEM 88 [16]. This setup was improved to achieve an uncertainty of 1 to 2 parts in 10^9 . Calibration of 8 decade IVDs is done by 1:1 comparison against standard IVD using injection voltage technique. The traveling standard was calibrated in this setup.

8 decade IVDs are used as reference standards for calibration of 6 and 7 decade IVDs received for calibration from industries and calibration laboratories [17–20].

5.13 Method of measurement used by NRC, Canada

The measurements of the CCEM-K7 comparison were made at 10 volts and 1 kHz using a modified straddle comparison bridge somewhat similar in basic design to that described in [5] (page 140). The major design differences include:

1. A coaxial straddle transformer with an internal 2-stage injection transformer on the central lead Triaxial connections between the straddle transformer, the triaxial detector and measured transformer.
2. Triaxial connections between the straddle transformer, the triaxial detector and measured transformer.
3. Triaxial definition up to the BPO connectors, which attach to the measured transformer.
4. A single detector (triaxial) on the central lead.
5. Measurements are only made with a single loop of the straddle transformer at one time, the other straddle transformer lead being left open.
6. Sensitivity of each single loop measurement is determined in situ.

Detector offsets were determined by reversal of the triaxial detector and measurement of the detector signal versus guard voltage of each single loop using a BPO Tee and with the straddle transformer driven with zero voltage. The successive ratios of the CCEM-K7 transformer's decade and eleventh taps were measured using the straddle transformer bridge. The linearized tap ratios were subsequently determined.

The 0.01 tap was measured by comparing a 1, 0.1, 0 transformer to the CCEM-K7 transformer and then reducing the drive of the 1, 0.1, 0 transformer by 10 and comparing to the CCEM-K7 0.1, 0.01, and 0 taps. An estimate of the voltage coefficient of the transformer is included in the uncertainty budget.

A single loop measurement consists of 25 measurements, each in a lockin amplifier time constant of 300 mS and at a range corresponding to $\sim 1.5 \times 10^{-6}$ of full scale (10V).

An additional 25 measurements with 1×10^{-6} injected into the central lead was used to determine the detector sensitivity which varies slightly for each loop. These are repeated for each of the two leads.

The reported results are the average of several complete straddle measurement sets.

5.14 Method of measurement used by PTB, Germany

The principle of the calibration is the well-known "bootstrap"-method. The constant output voltage of a reference transformer is compared in i steps (e.g. $i = 10$ for decade dividers with 10 winding groups) with the unknown output voltages of each winding group of the inductive voltage divider (device under test). The complex voltage differences, relative to the input voltage, are measured using a lock-in amplifier. From these readings the in-phase and quadrature voltage ratios as well as the uncertainties are then calculated using the software "GUM Workbench" (Version 1.2.11.56 Win32, see www.metrodata.de).

In Appendix N the calculation of the in-phase voltage ratio and the corresponding uncertainty at the nominal value 0.6 is shown as an example. The 10 in-phase readings α_1 to α_{10} of the difference voltages are added and divided by (-10) to receive the in-phase correction K_{wR} of the reference transformer. The corrections K_{wt1} to K_{wt6} of each winding group can then be calculated by adding the in-phase correction K_{wR} of the reference transformer to the in-phase readings α_1 to α_6 . Summing up the in-phase corrections of every winding group K_{wt1} to K_{wt6} will result in the in-phase correction K_{w6} of the voltage ratio 0.6. The final result ipvr6 is the sum of the nominal value of Port 6 (= 0.6) and the in-phase correction K_{w6} .

5.15 Method of measurement used by SP, Sweden

The travelling Inductive Voltage Divider CCEM-ACVRS01 was calibrated by comparison with SPs primary inductive voltage divider in an inductive voltage divider comparison bridge. The comparison bridge consists of a two-staged primary inductive voltage divider and a R-C circuit for balancing the quadrature voltage difference between the primary standard and the inductive voltage divider under test. The in-phase voltage ratios and the quadrature voltage ratios for the primary inductive voltage divider were calibrated by comparison with a two-staged inductive voltage divider of the same construction with the in-phase voltage ratios and the quadrature voltage ratios determined at SP by a capacitance permutation method in a capacitance bridge. The permutation method is based on five 100 pF and two 50 pF capacitance standards of very high short term stability. The capacitance standards are compared with 1:1 ratio measurements. Then, by measurements with different capacitance combinations, are the ratio corrections determined for the capacitance bridge inductive voltage divider. One 1000 pF and one 10 pF capacitance standard which are measured in a 1:10 ratio against the 100 pF reference is used for the determination of the 1:100 ratio corrections.

5.16 Method of measurement used by UME, Turkey

An AC voltage ratio measurement system was used for the intercomparison measurements. This system is a homemade and primary measurement system based on Bootstrapping method [9]. n/10 and n/11 reference two-stage transformers were used to compare two adjacent outputs of n/10 and n/11 at travelling standard. In order to calibrate 1/100 ratio, a reference two-stage 1/100 transformer and a two-decade reference IVD were used. Second decade of the reference IVD was calibrated with 1/100 two-stage transformer based on Bootstrapping method. Real and imaginer errors of 1/100 two-stage transformer were calculated by using the calibration values of the reference IVD then 1/100 ratios of the travelling standard and 1/100 two-stage transformer were compared. Phase calibration of injection network was performed by using a 100 pF Fused-Silica standard capacitor and a 100 Ω Vishay resistor [5]. Injection transformers were calibrated with a reference 7-decade IVD [21].

5.17 Method of measurement used by VNIIM, Russia

Self-calibration method using a calibration transformer with a single output voltage ("bootstrap" method) is applied. It is analogous to the method [9] but has the following improvements:

1. additional winding of self-calibrated two-stage transformer is used instead of the external calibration transformer;
2. this additional winding is provided with double shielding;
3. conductor-pair coaxial design is used for the whole circuit;
4. special injection circuit is used for "zeroing" of an detector, to eliminate the currents (due to nonbalanced cross capacitance) that flow through detector and give incorrect balances;
5. self-calibrating transformer is supplied with two inputs - for 100 and 110 turns - for both magnetizing and ratio windings.

6 Results

6.1 Handling & treatment

Every participating laboratory sent a report to the Pilot Laboratory detailing the test method used to obtain the results and uncertainty budgets, in accordance with ISO “Guide to Uncertainty in Measurements” guidelines, referred to as the GUM model. [22]

6.2 Statistical analysis

Details of the statistical analysis can be found in Part 2 of the report.

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7 Appendix

Uncertainty Budgets and Technical Protocol Document

Appendix A

Appendix - A - Uncertainty Budget for CEM, Spain

Ratios 0.1 to 0.9

The real ratio of the output n of the IVD is : $(n/10) + E(n)$, for $n = 1$ to 9.

We calculate $E(n)$ by the expression:

$$E(n) = (n/10) [0.9 \bar{a}_1 + 0.8 \bar{a}_2 + \dots + 0.1 \bar{a}_9 - ((n-1)/n) \bar{a}_1 - ((n-2)/n) \bar{a}_2 - ((n-3)/n) \bar{a}_3 - \dots - (1/n) \bar{a}_{n-1}]$$

in which \bar{a}_n is obtained according to Kibble and Rayner (Coaxial AC Bridges)

In CEM system, the \bar{a}_n values are obtained from the following expressions:

$$\bar{a}_{n \text{ (phase)}} = 10 r_0 (r_{1p} - r_{2p} + r'_{1p} - r'_{2p}) / R$$
$$\bar{a}_{n \text{ (quad)}} = 10 r_0 (r_{1q} - r_{2q} + r'_{1q} - r'_{2q}) / R \cdot R_{RC}$$

being r , R y R_{RC} parameters that result of auxiliary IVD readings use to balance and transformer injections.

Table 1 and Table 2 include, as an example, the uncertainty budget of $\bar{a}_{n \text{ (phase)}}$ and $\bar{a}_{n \text{ (quad)}}$ respectively, for positions of measurement $n = 1$ and $n = 2$.

Ratios 1/11 to 10/11

The real ratio of the output n of the IVD is : $(n/11) + E(n)$, for $n = 1$ to 10.

We calculate $E(n)$ by the expression:

$$E(n) = (n/11) [(10/11) \bar{a}_1 + (9/11) \bar{a}_2 + \dots + (1/11) \bar{a}_1 - ((n-1)/n) \bar{a}_1 - ((n-2)/n) \bar{a}_2 - ((n-3)/n) \bar{a}_3 - \dots - (1/n) \bar{a}_{n-1}]$$

In this case the \bar{a}_n values are obtained in a similar way as before.

Ratios 0.01

The real ratio of the output of the IVD is : $0,01 + E_{(0.01)}$. This $E_{(0.01)}$ deviation is obtained from the deviation of 0.1 ratio of an auxiliary transformer (input 1 V), and of the deviation of 0.1 ratio of the IVD travelling standard (input 10 V) previously measured.

Table 1:
 \bar{a}_1 and \bar{a}_2 phase values uncertainty budget

Position: 0 - 0,1 - 0,2 $\bar{a}_1 = -460.10^{-9}$

Quantity X_i	Standard uncertainty	Probability distribution	Sensitivity coefficient c_i	Standard uncertainty contribution($\times 10^{-9}$)
r_0	10^{-5}	Rectangular/B	$4,9.10^{-8}$	0
r_1	1.10^{-6}	Rectangular/B	10^{-2}	10
r_2	1.10^{-6}	Rectangular/B	10^{-2}	10
r_1	1.10^{-6}	Rectangular/B	10^{-2}	10
r_2	1.10^{-6}	Rectangular/B	10^{-2}	10
R	10^{-1}	Rectangular/B	$4,9.10^{-10}$	0
\bar{a}_1	$3,7.10^{-8}/\sqrt{5}$	Gauss/A	1	17
				$u(\bar{a}_1) = 26.10^{-9}$

$$v_{\text{eff}} = 4 \frac{26^4}{16,6^4} = 24$$

Position: 0,1 - 0,2 - 0,3 $\bar{a}_2 = -54.10^{-9}$

Quantity X_i	Standard uncertainty	Probability distribution	Sensitivity coefficient c_i	Standard uncertainty contribution($\times 10^{-9}$)
r_0	10^{-5}	Rectangular/B	$0,9.10^{-8}$	0
r_1	1.10^{-6}	Rectangular/B	10^{-2}	10
r_2	1.10^{-6}	Rectangular/B	10^{-2}	10
r_1	1.10^{-6}	Rectangular/B	10^{-2}	10
r_2	1.10^{-6}	Rectangular/B	10^{-2}	10
R	10^{-1}	Rectangular/B	9.10^{-10}	0
\bar{a}_2	$2,7.10^{-8}/\sqrt{5}$	Gauss/A	1	12
				$u(\bar{a}_2) = 23.10^{-9}$

$$v_{\text{eff}} = 4 \frac{23,4^4}{12,1^4} = 55$$

Table 2:
 \bar{a}_1 and \bar{a}_2 quadrature values uncertainty budget

Position: 0 - 0,1 - 0,2 $\bar{a}_1 = + 460.10^{-9}$

Quantity X_i	Standard uncertainty	Probability distribution	Sensitivity coefficient c_i	Standard uncertainty contribution($\times 10^{-9}$)
r_0	10^{-5}	Rectangular/B	$4,7.10^{-6}$	0
r_1	5.10^{-5}	Rectangular/B	$6,8.10^{-5}$	3
r_2	5.10^{-5}	Rectangular/B	$6,8.10^{-5}$	3
r_1	5.10^{-5}	Rectangular/B	$6,8.10^{-5}$	3
r_2	5.10^{-5}	Rectangular/B	$6,8.10^{-5}$	3
R	10^{-1}	Rectangular/B	$4,7.10^{-10}$	0
R_{RC}	5	Rectangular/B	$3,1.10^{-9}$	16
a_1	$3,1.10^{-8}/\sqrt{5}$	Gauss/A	1	14
				$u(\bar{a}_1) = 22.10^{-9}$

$$v_{\text{eff}} = 4 \frac{22^4}{14^4} = 24$$

Position: 0,1 - 0,2 - 0,3 $\bar{a}_2 = + 136.10^{-9}$

Quantity X_i	Standard uncertainty	Probability distribution	Sensitivity coefficient c_i	Standard uncertainty contribution($\times 10^{-9}$)
r_0	10^{-5}	Rectangular/B	18.10^{-7}	0
r_1	5.10^{-5}	Rectangular/B	$6,8.10^{-5}$	3
r_2	5.10^{-5}	Rectangular/B	$6,8.10^{-5}$	3
r_1	5.10^{-5}	Rectangular/B	$6,8.10^{-5}$	3
r_2	5.10^{-5}	Rectangular/B	$6,8.10^{-5}$	3
R	10^{-1}	Rectangular/B	$1,4.10^{-9}$	0
	5	Rectangular/B	$1,2.10^{-9}$	6
\bar{a}_2	$1,3.10^{-8}/\sqrt{5}$	Gauss/A	1	6
				$u(\bar{a}_2) = 11.10^{-9}$

$$v_{\text{eff}} = 4 \frac{11^4}{6^4} = 45$$

Appendix B

Appendix - B - Uncertainty Budget for NMI, Australia (formally CSIRO)

The following sources of uncertainty were considered:

Type A:

Each ratio was measured on at least three separate occasions. The mean value of the measured ratios is reported, and a Type A uncertainty calculated.

Type B:

Reference uncertainty. AC voltage ratio is realised directly at NML and, as such, does not depend on references calibrated at NML or by another laboratory.

Linearity of detector system. The linearity of the detector system (including the lock-in amplifier) was measured and found to be better than 1 in 10^5 of the measured signal (no deviations from linearity could be detected to this level). The measured signal is, in all cases, at least 1×10^{-6} of the input voltage, V . We are confident therefore that the uncertainty from this source is less than $1 \times 10^{-11} V$. Assuming a square law distribution with a semi-range of $0.5 \times 10^{-11} V$, we estimate the standard uncertainty as $0.29 \times 10^{-10} V$ with 50 degrees of freedom.

Definition of calibrate signal. The calibrate signal of $1 \times 10^{-6} V$ is derived from two cascaded 1000:1 separately-excited ratio transformers whose ratio errors are not expected to be greater than 1 in 10^5 . We assume that the ratio error for each 1000:1 transformer is zero, and take the uncertainty in the error as 1×10^{-5} of the applied voltage, treated as a square law distribution. We therefore take the uncertainty in the calibrate signal as $1.15 \times 10^{-11} V$. The uncertainty in the measured errors of the IVD under test depend on the uncertainty in the calibrate signal in a complex way, but provided the measured errors are no greater than the calibrate signal then the uncertainty in the measured errors of the IVD under test will be no greater than the uncertainty in the calibrate signal. We expect therefore that the uncertainty from this source is less than 1.15×10^{-11} of input voltage with infinite degrees of freedom.

Temperature. Variations in the ratio errors with variations in temperature are accounted for in the Type A assessment. Any uncertainty in the ratio error associated with the uncertainty in the temperature measurement is assumed to be negligible.

Magnetization of artefact. The IVD was demagnetized at the outset and again at the conclusion of the calibration. There was no evidence of acquired residual magnetization on either occasion. We assume therefore that any uncertainty in the ratio errors due to magnetization of the artefact is negligible.

Frequency. Ratio errors at 55 Hz were found to be strongly frequency dependent. Ratio errors measured at 55.0 Hz differed from those measured at

55.1 Hz by typically 2 in 10^9 of input voltage. Since our realisation of frequency is many orders of magnitude better than 0.1 Hz, we believe it is unlikely that this source of uncertainty is significant. However, no attempt has been made to accurately determine the frequency dependence of the ratio errors or to include frequency dependence as a source of uncertainty.

Excitation level. The input voltage was measured with a calibrated, 5½ digit multimeter. No attempt has been made to accurately determine the dependence of the ratio errors on excitation level or to include excitation level as a source of uncertainty.

Resolution. The resolution of the system is $\pm 1 \times 10^{-11} V$. Treated as a square law distribution, the resolution uncertainty is therefore $0.58 \times 10^{-11} V$ with infinite degrees of freedom.

Summary of uncertainties:

Source	Assessed as:	Standard Uncertainty	Degrees of Freedom
Repeatability	Type A	$(1 - 44) \times 10^{-10}$	2-7
Linearity of detector system	Type B	0.03×10^{-10}	50
Definition of calibrate signal	Type B	0.12×10^{-10}	Infinite
Resolution	Type B	0.06×10^{-10}	Infinite

Appendix C

IV. MEASUREMENTS

The travelling standard [10] has a tap configuration different from Fig. 2, since the standard is constructed to provide *at the same time* k/10, k/11 and 1/100 ratios. In particular,

- all connectors are BPO MUSA instead of triax;
- the magnetizing 0, 100 and 110 taps are a single BPO connector;
- the feeding 0, 100 and 110 taps are a single connector called input;
- the output 0, 0.1, 0.2, ... 0.9, 1.0 taps are splitted in two sets of taps called 0, 1/10, ... 9/10 and 1/11, 2/11, ... 10/11;
- the output 0.01 is called 1/100;
- the output 0 is not shorted, since the voltage output is defined in a non-coaxial way as the voltage between tap k and tap 0.

Calibration of k/10 and k/11 taps

Apart from these differences, the circuit employed is the same of Fig. 2 and the procedure for calibration of k/10 and k/11 taps is similarly carried out.

Calibration of 1/100 tap

The calibration of 1/100 tap is carried out by comparison of the taps 1/10, 1/100 and 0 of the travelling standard against the taps 1, 1/10 and 0 of an auxiliary IVD1, in turn calibrated with the circuit of Fig. 2 and energized with an auxiliary 10:1 IVD2. IVD1 is calibrated using the bootstrap technique; IVD2 ratio errors don't enter the calibration of 1/100 tap.

V. UNCERTAINTY EVALUATION

The uncertainty in the estimation of the ratio errors $\{r_k\}$ can be computed from the uncertainty in the estimation of the set $\{e_k\}$ using Eqn. 3.

Assuming that measurement errors given by :

- limited resolution of the measurement device (voltmeter reading);
 - uncertainty of V_{in} ;
 - effects of temperature coefficients of the system within the temperature range considered (23 ± 0.5 °C);
 - effects of humidity coefficients of the system within the humidity range considered (50 ± 10 RH)
 - effects of frequency coefficient of the standard measured within the frequency range considered (997-1003 Hz);
- are negligible,

we can model the errors during the measurements of $\{e_k\}$ as follows:

$$e_k = (1 + g_k) \cdot (e_k^* + c_k + \Delta E_k) + o_k \quad (4)$$

where

- e_k^* is the true value of e_k ;
- e_k is the voltage reading;
- g_k is the complex gain error of the measurement. Takes into account the residual gain error of the voltmeter after the calibration procedure, residual loadings on the voltage taps;
- c_k is the complex common-mode error. Takes into account the limited common-mode rejection ratio of the voltmeter
- ΔE_k is the variation of E because of imperfect or unequalized guarding, causing loading of MIVD, in the measurement arm. Takes also into account guard currents in BT;
- o_k is the measurement complex offset. Takes into account the electronic voltage offset of the voltmeter, and interferences due to stray magnetic fields from the magnetic devices.

Reasonable assumptions:

- $g_k = g$. Short-term constancy of the gain error during the measurement cycle;
- $o_k = o$. Short-term constancy of the offset during the measurement cycle. It assumes that the electronics is stable during the measurement period, and that possible interference effects due to stray magnetic fields from the magnetic devices (each one carrying constant flux during the entire measurement) are constant;
- $c_k = c \cdot (k/n)$. The common-mode error is considered proportional to the corresponding common-mode voltage, in turn proportional to the tap index. This is the standard assumption for CM error in electronic amplifiers.
- $\Delta E_k = \Delta E \cdot (k/n)$. E deviation proportionality to unequalized guard currents or loading effects, in turn proportional to the corresponding common-mode voltage.

and consequently

$$e_k = (1 + g) \cdot \left(e_k^* + \frac{k}{n} (c + \Delta E) \right) + o \quad (5)$$

Recalling Eqn. 3:

$$r_k = \frac{\sum_{i=1}^k e_i - \frac{k}{n} \sum_{i=1}^n e_i}{V_{in}} \quad (6)$$

by direct substitution,

$$r_k = (1 + g) \frac{\sum_{i=1}^k e_i^* - \frac{k}{n} \sum_{i=1}^n e_i^*}{V_{in}} + \frac{(1 + g)}{V_{in}} \left[(c + \Delta E) \cdot \left(\sum_{i=1}^k \frac{i}{n} - \frac{k}{n} \sum_{i=1}^n \frac{i}{n} \right) \right] + \frac{1}{V_{in}} \left[o \cdot \left(\sum_{i=1}^k i - \frac{k}{n} \sum_{i=1}^n i \right) \right]; \quad (7)$$

but

$$\frac{\sum_{i=1}^k e_i^* - \frac{k}{n} \sum_{i=1}^n e_i^*}{V_{in}} = r_k^*; \quad (8)$$

$$\sum_{i=1}^k \frac{i}{n} = \frac{k(k+1)}{2n} \text{ and similarly } \sum_{i=1}^n \frac{i}{n} = \frac{(n+1)}{2} \quad (9)$$

in conclusion,

$$r_k = (1 + g) \cdot r_k^* + \frac{(1 + g)}{V_{in}} \left[\frac{k(k-n)}{2n} \cdot (c + \Delta E) \right], \quad (10)$$

The factor $p(k) = \frac{k(k-n)}{2n}$ has the shape given in Fig. 3.

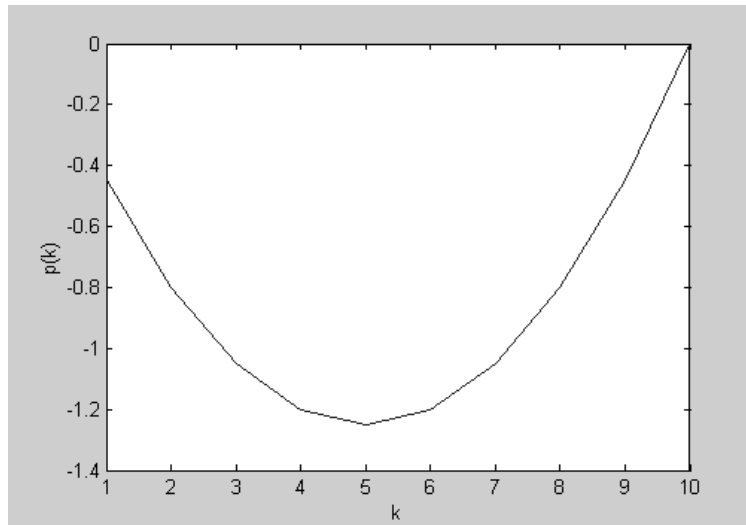


Fig. 3. Shape of $p(k)$ factor.

and its maximum absolute value is $|p(k/2)| = n/8$.

In conclusion, a constant offset error has no effect on r_k ; both the gain error g and the relative CM errors c/V_{in} and $\Delta E/V_{in}$ have, for $n=10$ or $n=11$, sensitivity coefficients for the propagation of uncertainty of e_k to r_k , close to unity.

Estimation of the gain error g

The gain error results from calibration errors of the voltmeter and drifts after calibration. We estimate a gain error

$$g = 0 + j0; u(g) = 0.01, \text{ Gaussian}$$

Estimation of the common-mode error c

Testing the voltmeter for common-mode rejection showed no appreciable common-mode effects within the noise of the voltmeter. Thus we put the uncertainty for c equal to the rms noise of the voltmeter

$$c = 0 + j0 \text{ V}; u(c) = 10 \text{ nV}, \text{ Gaussian}$$

Estimation of the voltage error ΔE

This is the most difficult error to ascertain. We put

$$\Delta E = 0 + j0; u(\text{real}(\Delta E)) = 10^{-9} V_{in}, \text{ Gaussian}; u(\text{imag}(\Delta E)) = 4 \cdot 10^{-9} V_{in}, \text{ Gaussian}$$

Evaluation of sensitivity coefficients

By differentiation, and computing at point $[g = 0, c = 0, \Delta E = 0]$:

$$s(r_k) = r_k^* \quad (11)$$

$$s(c) = \frac{1}{V_{in}} \left[\frac{k(k-n)}{2n} \right] \quad (12)$$

$$s(\Delta E) = \frac{1}{V_{in}} \left[\frac{k(k-n)}{2n} \right] \quad (13)$$

Standard uncertainty

The estimation of the standard uncertainty of ratio error is considered as the quadratic sum of type B uncertainty (estimated as above) and type A uncertainty, estimated as the experimental standard deviation of several measurement cycles (more than 10). The expanded uncertainty has been evaluated considering infinite degrees of freedom for type B uncertainty, and for a 95% confidence interval, by calculating the equivalent degrees of freedom with the Welch-Satterthwaite formula.

1/100 tap

A straightforward evaluation of uncertainty for 1/100 tap require a complex analysis since many passages are involved. For simplicity, we make a simplified evaluation with the same expressions of k/10 and k/11 taps, but expanding the instrumental uncertainty.

VI. RESULTS

The calibration results, and standard and expanded uncertainties, are reported in Tab. 1.

VII. CONCLUSIONS

We developed at IEN an implementation of the well-known bootstrap technique for the calibration of ac voltage ratio standards. The implementation is simple, and quick to operate, thanks to the use of a special vector voltmeter, to permit direct reading of voltage differences. A guarded network comprising a bootstrap transformer and a guard divider, permits to calibrate k/10, k/11 and 1/100 taps of a two-stage divider, which in turn will be used (with a similar network using the same components) to calibrate customers' dividers. Construction details, examples of calibration and notes about uncertainty are given.

Appendix D

Appendix - D - Uncertainty Budget for KRISS, Korea
UNCERTAINTY BUDGET FOR RATIO OF 0.01 (KRISS)

No	Cause of uncertainty	Type	In-phase			Quadrature		
			Standard uncertainty	Accuracy of uncer. estimate (%)	Calculated degree of freedom	Standard uncertainty	Accuracy of uncer. estimate (%)	Calculated degree of freedom
1	Adjustment of injection circuits	B	4×10^{-9}	30	6	5×10^{-9}	30	6
2	Zero balance of detector transformer	B	4×10^{-9}	30	6	5×10^{-9}	30	6
3	Potential difference among outer conductor of BPO connectors	B	4×10^{-9}	30	6	5×10^{-9}	30	6
4	Magnetic field effect on reference voltage circuit	B	4×10^{-9}	30	6	5×10^{-9}	30	6
5	Leakage capacitance through screen of coaxial cable	B	4×10^{-9}	30	6	5×10^{-9}	30	6
6	Repeatability of measurements	A	4×10^{-9}		14	10×10^{-9}		14
Combined standard uncertainty			9.80×10^{-9}			1.5×10^{-8}		
Expanded uncertainty			2.0×10^{-8}			3.0×10^{-8}		
Effective degrees of freedom			37			40		
Coverage factor			2.03			2.02		

UNCERTAINTY BUDGET FOR RATIO FROM 0.1 TO 0.9 (KRISS)

No	Cause of uncertainty	Type	In-phase			Quadrature		
			Standard uncertainty	Accuracy of uncer. estimate (%)	Calculated degree of freedom	Standard uncertainty	Accuracy of uncer. estimate (%)	Calculated degree of freedom
1	Adjustment of injection circuits	B	4×10^{-9}	20	13	5×10^{-9}	20	13
2	Zero balance of detector transformer	B	4×10^{-9}	20	13	6×10^{-9}	25	8
3	Potential difference among outer conductor of BPO connectors	B	2×10^{-9}	20	13	5×10^{-9}	25	8
4	Magnetic field effect on reference voltage circuit	B	3×10^{-9}	30	6	5×10^{-9}	25	8
5	Leakage capacitance through screen of coaxial cable	B	4×10^{-9}	25	8	5×10^{-9}	25	8
6	Repeatability of measurements	A	4×10^{-10}		29	5×10^{-10}		29
Combined standard uncertainty			7.82×10^{-9}			1.17×10^{-8}		
Expanded uncertainty			1.6×10^{-8}			2.4×10^{-8}		
Effective degrees of freedom			42			42		
Coverage factor			2.02			2.02		

UNCERTAINTY BUDGET FOR RATIO FROM 1/11 TO 10/11 (KRISS)

No	Cause of uncertainty	Type	In-phase			Quadrature		
			Standard uncertainty	Accuracy of uncer. estimate (%)	Calculated degree of freedom	Standard uncertainty	Accuracy of uncer. estimate (%)	Calculated degree of freedom
1	Adjustment of injection circuits	B	5×10^{-9}	20	13	5×10^{-9}	20	13
2	Zero balance of detector transformer	B	5×10^{-9}	25	8	6×10^{-9}	25	8
3	Potential difference among outer conductor of BPO connectors	B	2×10^{-9}	20	13	5×10^{-9}	25	8
4	Magnetic field effect on reference voltage circuit	B	3×10^{-9}	25	8	5×10^{-9}	25	8
5	Leakage capacitance through screen of coaxial cable	B	4×10^{-9}	25	8	5×10^{-9}	25	8
6	Repeatability of measurements	A	5×10^{-10}		19	5×10^{-10}		19
Combined standard uncertainty			8.90×10^{-9}			1.17×10^{-8}		
Expanded uncertainty			1.8×10^{-8}			2.4×10^{-8}		
Effective degrees of freedom			37			42		
Coverage factor			2.03			2.02		

Appendix E

Appendix - E - Uncertainty Budget for LCIE, France

For ratios 1/10, 2/10, 3/10, 4/10, 5/10, 6/10, 7/10, 8/10 and 9/10 :

From the equation 1 ,

$$u(N_x) = \sqrt{2.(1 - N_n + N_n^2).u^2(K) + u^2(\varepsilon_n^1)}$$

With $u(K) = u(K_{inj}) = u(K_{inj0}) = u(K_{inj1})$ and u absolute uncertainties.

For ratios 1/100 and 1/11 :

From the equation 2 ,

$$u(N_x) = \sqrt{(10.N_n)^2 .u^2(\varepsilon_1^1) + u^2(\varepsilon_n^2) + 2.(1 - N_n + N_n^2 + (10.N_n)^2).u^2(K)}$$

With $u(K) = u(K_{inj}) = u(K_{inj0}) = u(K_{inj1})$ and u absolute uncertainties.

For ratios 2/11, 3/11, 4/11, 5/11, 6/11, 7/11, 8/11, 9/11 and 10/11 :

From the equation 3 ,

$$u(N_x) = \sqrt{2.(1 - N_n + N_n^2).u^2(K) + u^2(\varepsilon)}$$

With $u(K) = u(K_{inj}) = u(K_{inj0}) = u(K_{inj1})$ and u absolute uncertainties.

Uncertainty components (phase)

Component	Ratio	Standard uncertainty value	Probability distribution/ method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Reference divider (linearity error)	1/100	$1,1 \cdot 10^{-8}$	Normal/B	1	$1,1 \cdot 10^{-8}$	500
	1/10	$1,1 \cdot 10^{-8}$			$1,1 \cdot 10^{-8}$	500
	2/10	$1,9 \cdot 10^{-8}$			$1,9 \cdot 10^{-8}$	500
	3/10	$2,5 \cdot 10^{-8}$			$2,5 \cdot 10^{-8}$	500
	4/10	$2,8 \cdot 10^{-8}$			$2,8 \cdot 10^{-8}$	500
	5/10	$2,9 \cdot 10^{-8}$			$2,9 \cdot 10^{-8}$	500
	6/10	$2,8 \cdot 10^{-8}$			$2,8 \cdot 10^{-8}$	500
	7/10	$2,5 \cdot 10^{-8}$			$2,5 \cdot 10^{-8}$	500
	8/10	$1,9 \cdot 10^{-8}$			$1,9 \cdot 10^{-8}$	500
	9/10	$1,1 \cdot 10^{-8}$			$1,1 \cdot 10^{-8}$	500
	1/11	$1,1 \cdot 10^{-8}$			$1,1 \cdot 10^{-8}$	500
	2/11	$1,9 \cdot 10^{-8}$			$1,9 \cdot 10^{-8}$	500
	3/11	$2,5 \cdot 10^{-8}$			$2,5 \cdot 10^{-8}$	500
	4/11	$2,8 \cdot 10^{-8}$			$2,8 \cdot 10^{-8}$	500
	5/11	$2,9 \cdot 10^{-8}$			$2,9 \cdot 10^{-8}$	500
	6/11	$2,9 \cdot 10^{-8}$			$2,9 \cdot 10^{-8}$	500
	7/11	$2,8 \cdot 10^{-8}$			$2,8 \cdot 10^{-8}$	500
8/11	$2,5 \cdot 10^{-8}$	$2,5 \cdot 10^{-8}$	500			
9/11	$1,9 \cdot 10^{-8}$	$1,9 \cdot 10^{-8}$	500			
10/11	$1,1 \cdot 10^{-8}$	$1,1 \cdot 10^{-8}$	500			
Temperature effect	All	$0,2 \cdot 10^{-8}$	ArcSinus/B	1	$0,2 \cdot 10^{-8}$	500
Compensation voltage effect	All	$1,8 \cdot 10^{-8}$	Rectangular/B	$\sqrt{2 \cdot (1 - N_n + N_n^2)}$	$\sqrt{2 \cdot (1 - N_n + N_n^2)} \cdot 1,8 \cdot 10^{-8}$	500
Detector stability	All	$0,2 \cdot 10^{-8}$	Rectangular/B	$\sqrt{2 \cdot (1 - N_n + N_n^2)}$	$\sqrt{2 \cdot (1 - N_n + N_n^2)} \cdot 0,2 \cdot 10^{-8}$	500
Upstream decades effect	1/100	0	Rectangular/B	1	0	500
	1/10	0			0	500
	2/10	0			0	500
	3/10	0			0	500
	4/10	0			0	500
	5/10	0			0	500
	6/10	0			0	500
	7/10	0			0	500
	8/10	0			0	500
	9/10	0			0	500
	1/11	0			0	500
	2/11	$6,0 \cdot 10^{-8}$			$1,8 \cdot 10^{-8}$	500
	3/11	$6,0 \cdot 10^{-8}$			$1,8 \cdot 10^{-8}$	500
	4/11	$6,0 \cdot 10^{-8}$			$1,8 \cdot 10^{-8}$	500
	5/11	$6,0 \cdot 10^{-8}$			$1,8 \cdot 10^{-8}$	500
	6/11	$6,0 \cdot 10^{-8}$			$1,8 \cdot 10^{-8}$	500
	7/11	$6,0 \cdot 10^{-8}$			$1,8 \cdot 10^{-8}$	500
8/11	$6,0 \cdot 10^{-8}$	$1,8 \cdot 10^{-8}$	500			
9/11	$6,0 \cdot 10^{-8}$	$1,8 \cdot 10^{-8}$	500			
10/11	$6,0 \cdot 10^{-8}$	$1,8 \cdot 10^{-8}$	500			

Ratio	Combined uncertainty (2σ)
1/100	$1,1 \cdot 10^{-8}$
1/10	$2,6 \cdot 10^{-8}$
2/10	$2,9 \cdot 10^{-8}$
3/10	$3,3 \cdot 10^{-8}$
4/10	$3,5 \cdot 10^{-8}$
5/10	$3,6 \cdot 10^{-8}$
6/10	$3,5 \cdot 10^{-8}$
7/10	$3,3 \cdot 10^{-8}$
8/10	$2,9 \cdot 10^{-8}$
9/10	$2,6 \cdot 10^{-8}$
1/11	$3,5 \cdot 10^{-8}$
2/11	$3,4 \cdot 10^{-8}$
3/11	$3,7 \cdot 10^{-8}$
4/11	$3,9 \cdot 10^{-8}$
5/11	$4,0 \cdot 10^{-8}$
6/11	$4,0 \cdot 10^{-8}$
7/11	$3,9 \cdot 10^{-8}$
8/11	$3,7 \cdot 10^{-8}$
9/11	$3,4 \cdot 10^{-8}$
10/11	$3,1 \cdot 10^{-8}$

Uncertainties components (quadrature)

Component	Ratio	Standard uncertainty value	Probability distribution/ method of evaluation	Sensitivity coefficient	Uncertainty contribution	Degrees of freedom
Reference divider (linearity error)	1/100	$1,1 \cdot 10^{-8}$	Normal/B	1	$1,1 \cdot 10^{-8}$	500
	1/10	$1,1 \cdot 10^{-8}$			$1,1 \cdot 10^{-8}$	500
	2/10	$1,9 \cdot 10^{-8}$			$1,9 \cdot 10^{-8}$	500
	3/10	$2,5 \cdot 10^{-8}$			$2,5 \cdot 10^{-8}$	500
	4/10	$2,8 \cdot 10^{-8}$			$2,8 \cdot 10^{-8}$	500
	5/10	$2,9 \cdot 10^{-8}$			$2,9 \cdot 10^{-8}$	500
	6/10	$2,8 \cdot 10^{-8}$			$2,8 \cdot 10^{-8}$	500
	7/10	$2,5 \cdot 10^{-8}$			$2,5 \cdot 10^{-8}$	500
	8/10	$1,9 \cdot 10^{-8}$			$1,9 \cdot 10^{-8}$	500
	9/10	$1,1 \cdot 10^{-8}$			$1,1 \cdot 10^{-8}$	500
	1/11	$1,1 \cdot 10^{-8}$			$1,1 \cdot 10^{-8}$	500
	2/11	$1,9 \cdot 10^{-8}$			$1,9 \cdot 10^{-8}$	500
	3/11	$2,5 \cdot 10^{-8}$			$2,5 \cdot 10^{-8}$	500
	4/11	$2,8 \cdot 10^{-8}$			$2,8 \cdot 10^{-8}$	500
	5/11	$2,9 \cdot 10^{-8}$			$2,9 \cdot 10^{-8}$	500
	6/11	$2,9 \cdot 10^{-8}$			$2,9 \cdot 10^{-8}$	500
7/11	$2,8 \cdot 10^{-8}$	$2,8 \cdot 10^{-8}$	500			
8/11	$2,5 \cdot 10^{-8}$	$2,5 \cdot 10^{-8}$	500			
9/11	$1,9 \cdot 10^{-8}$	$1,9 \cdot 10^{-8}$	500			
10/11	$1,1 \cdot 10^{-8}$	$1,1 \cdot 10^{-8}$	500			
Temperature effect	All	$0,2 \cdot 10^{-8}$	ArcSinus/B	1	$0,2 \cdot 10^{-8}$	500
Compensation voltage effect	All	$1,8 \cdot 10^{-8}$	Rectangular/B	$\sqrt{2 \cdot (1 - N_n + N_n^2)}$	$\sqrt{2 \cdot (1 - N_n + N_n^2)} \cdot 1,8 \cdot 10^{-8}$	500
Detector stability	All	$0,2 \cdot 10^{-8}$	Rectangular/B	$\sqrt{2 \cdot (1 - N_n + N_n^2)}$	$\sqrt{2 \cdot (1 - N_n + N_n^2)} \cdot 0,2 \cdot 10^{-8}$	500
Upstream decades effect	1/100	0	Rectangular/B	1	0	500
	1/10	0			0	500
	2/10	0			0	500
	3/10	0			0	500
	4/10	0			0	500
	5/10	0			0	500
	6/10	0			0	500
	7/10	0			0	500
	8/10	0			0	500
	9/10	0			0	500
	1/11	0			0	500
	2/11	$240 \cdot 10^{-8}$			$69 \cdot 10^{-8}$	500
	3/11	$240 \cdot 10^{-8}$			$69 \cdot 10^{-8}$	500
	4/11	$240 \cdot 10^{-8}$			$69 \cdot 10^{-8}$	500
	5/11	$240 \cdot 10^{-8}$			$69 \cdot 10^{-8}$	500
	6/11	$240 \cdot 10^{-8}$			$69 \cdot 10^{-8}$	500
7/11	$240 \cdot 10^{-8}$	$69 \cdot 10^{-8}$	500			
8/11	$240 \cdot 10^{-8}$	$69 \cdot 10^{-8}$	500			
9/11	$240 \cdot 10^{-8}$	$69 \cdot 10^{-8}$	500			
10/11	$240 \cdot 10^{-8}$	$69 \cdot 10^{-8}$	500			

Ratio	Combined uncertainty (2σ)
1/100	$1,1 \cdot 10^{-8}$
1/10	$2,6 \cdot 10^{-8}$
2/10	$2,9 \cdot 10^{-8}$
3/10	$3,3 \cdot 10^{-8}$
4/10	$3,5 \cdot 10^{-8}$
5/10	$3,6 \cdot 10^{-8}$
6/10	$3,5 \cdot 10^{-8}$
7/10	$3,3 \cdot 10^{-8}$
8/10	$2,9 \cdot 10^{-8}$
9/10	$2,6 \cdot 10^{-8}$
1/11	$3,5 \cdot 10^{-8}$
2/11	$69 \cdot 10^{-8}$
3/11	$69 \cdot 10^{-8}$
4/11	$69 \cdot 10^{-8}$
5/11	$69 \cdot 10^{-8}$
6/11	$69 \cdot 10^{-8}$
7/11	$69 \cdot 10^{-8}$
8/11	$69 \cdot 10^{-8}$
9/11	$69 \cdot 10^{-8}$
10/11	$69 \cdot 10^{-8}$

Appendix F

Appendix - F - Uncertainty Budget for METAS, Switzerland

Tables below give the detailed uncertainty budget for the ratios 1/100 and n/10 measured at 10 V, 1000 Hz.

in-phase		ratio										n
Uncertainty components (10^{-8})	type	1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10		
reproducibility of ratio measurements	A	0.2	0.3	0.3	0.4	0.5	0.9	1.2	1.3	1.1	12	
injection accuracy and linearity	B	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	8	
offset of the comparator bridge	B	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	50	
offset of the step-transformer	B	0.4	0.7	1.0	1.1	1.2	1.1	1.0	0.7	0.4	8	
load of the IVD	B	0.3	0.4	0.4	0.3	0.2	0.3	0.4	0.4	0.3	50	
combined standard uncertainty	1σ	0.8	1.1	1.2	1.3	1.4	1.6	1.7	1.6	1.3		
effective degrees of freedom		64	32	21	17	17	25	29	27	26		
coverage factor		2.1	2.1	2.1	2.2	2.2	2.1	2.1	2.1	2.1		
expanded uncertainty (95.45%)		1.6	2.2	2.6	2.9	3.0	3.3	3.5	3.5	2.7		

quadrature		ratio										n
Uncertainty components (10^{-8})	type	1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10		
reproducibility of ratio measurements	A	0.4	0.6	0.8	0.8	0.7	0.6	0.5	0.3	0.2	12	
injection accuracy and linearity	B	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	8	
offset of the comparator bridge	B	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	50	
offset of the step-transformer	B	2.1	3.7	4.8	5.5	5.8	5.5	4.8	3.7	2.1	8	
load of the IVD	B	2.6	3.8	3.9	3.2	2.0	3.2	3.9	3.8	2.6	50	
combined standard uncertainty	1σ	3.5	5.4	6.3	6.5	6.2	6.5	6.3	5.4	3.5		
effective degrees of freedom		44	31	22	15	11	15	21	31	43		
coverage factor		2.1	2.1	2.1	2.2	2.3	2.2	2.1	2.1	2.1		
expanded uncertainty (95.45%)		7.1	11.3	13.4	14.2	14.1	14.2	13.4	11.3	7.1		

1/100 ratio		type	in-phase	quadrature	n
Uncertainty components (10^{-8})					
reproducibility of ratio measurements		A	1.3	2.2	3
injection accuracy and linearity		B	0.3	0.6	8
offset of the comparator bridge		B	0.5	0.5	50
uncertainty on the reference		B	3.0	3.5	50
combined standard uncertainty		1σ	3.3	4.2	
effective degrees of freedom			47	29	
coverage factor			2.1	2.1	
expanded uncertainty (95.45%)			6.8	8.9	

Tables below give the detailed uncertainty budget for the ratios 1/100 and n/10 measured at 3 V, 55 Hz.

in-phase		ratio									
Uncertainty components (10^{-8})	type	1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10	n
reproducibility of ratio measurements	A	1.2	1.9	2.4	2.8	3.0	2.3	2.1	3.9	2.5	12
injection accuracy and linearity	B	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	8
offset of the comparator bridge	B	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	50
offset of the step-transformer	B	0.2	0.4	0.5	0.6	0.6	0.6	0.5	0.4	0.2	8
load of the IVD	B	0.1	0.2	0.2	0.2	0.1	0.2	0.2	0.2	0.1	50
combined standard uncertainty	1σ	2.4	2.9	3.2	3.5	3.7	3.1	3.0	4.5	3.3	
effective degrees of freedom		15	18	17	16	15	18	18	12	16	
coverage factor		2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	
expanded uncertainty (95.45%)		5.3	6.2	6.9	7.7	8.0	6.8	6.4	9.9	7.1	

quadrature		ratio									
Uncertainty components (10^{-8})	type	1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10	n
reproducibility of ratio measurements	A	1.3	1.7	2.1	2.6	3.1	1.9	1.8	2.3	1.4	12
injection accuracy and linearity	B	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	8
offset of the comparator bridge	B	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	50
offset of the step-transformer	B	1.0	1.8	2.4	2.8	2.9	2.8	2.4	1.8	1.0	8
load of the IVD	B	1.3	1.9	2.0	1.6	1.0	1.6	2.0	1.9	1.3	50
combined standard uncertainty	1σ	4.6	5.1	5.5	5.7	5.9	5.5	5.4	5.3	4.6	
effective degrees of freedom		13	20	24	24	24	22	22	22	14	
coverage factor		2.2	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.2	
expanded uncertainty (95.45%)		10.1	11.0	11.8	12.2	12.6	11.7	11.5	11.4	10.1	

1/100 ratio				
Uncertainty components (10^{-8})	type	in-phase	quadrature	n
reproducibility of ratio measurements	A	3.0	1.1	3
injection accuracy and linearity	B	2.0	4.0	8
offset of the comparator bridge	B	0.5	0.5	50
uncertainty on the reference	B	8.0	8.5	8
combined standard uncertainty	1σ	8.8	9.5	
effective degrees of freedom		11	12	
coverage factor		2.3	2.3	
expanded uncertainty (95.45%)		19.8	21.3	

Appendix G

Appendix G - Uncertainty Budget for NIM, China

In our tests the evaluation values of the type A may be neglected. The half of difference of the two correction values in positive direction and negative direction measurement respectively at a ratio point is taken as the extended uncertainty of the average value of these two values mentioned in our table of uncertainties with $k \approx 2.5$, where the actual distribution for error is identified as negative kurtosis and its effective degree of freedom can be thought as any great number.

NIM Uncertainty Tables 1000Hz (x E-9)										
Ratio	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
In-Phase	10	5	15	10	10	5	10	10	10	5
Quadrature	10	10	10	10	15	20	25	25	20	10
Ratio	1/11	2/11	3/11	4/11	5/11	6/11	7/11	8/11	9/11	10/11
In-Phase	5	5	5	10	10	10	10	10	10	10
Quadrature	10	15	15	25	25	15	15	10	20	15



In “*comparison protocol*” the expression of accuracy keep to document GUM95, but same prescribes in GUM95 are improper with practice of Inductive Voltage Divider. Following main problem can be demonstrated:

- 1) GUM95 negates term “*true value*” of *measurand* for this reason that “*true value*” of *measurand* could not be confirmed accurately. The essential of the “*true value*” of *measurand* is its definition, which is what *value* wanted measure with what condition. Section 2 of this paper shown that, for same *measurand* its different work state correspond with different definitions of the “*true value*”. In accuracy valuation the “*true value*” of *measurand* is need been completely accurate never. . In accuracy valuation instead of the “*true value*” is used its estimated value— “*conventional true value*” usually, difference between which and “*true value*” is negligible than the estimated error. The matter of Section 2 of this paper is to confirm the definitions of the “*true value*” in this comparison.

- 2) The errors of Inductive Voltage Divider possess following characteristics: The dispersion of these errors in changeless connection is wee, i.e. most of the error is system error. For measurement with such error single measurement is enough because the repetitious measurement has the same result as single measurement. Such error would be changed markedly if the connection of Inductive Voltage Divider has been changed. The changed value of such error is direct proportional to the changed parameter of the

connection. Repetitious measurement without control of its error reasons will have result with undefined meaning and will lack reproducibility. The correction method for such error valuation is ascertaining the limit area of such error through ascertaining the limit area of its error reason and its error coefficient. For example, limit of error ($\Delta\dot{Z}_1\Delta\dot{Y}_1+\Delta\dot{Z}_0\Delta\dot{Y}_0$) in the equation (2-3) can be valuated through stipulating allowable value of impedance $\Delta\dot{Z}_1$ and $\Delta\dot{Z}_0$ and measuring the input conductance $\Delta\dot{Y}_1$ and $\Delta\dot{Y}_0$ of Inductive Voltage Divider. In accuracy valuation for Inductive Voltage Divider Type A evaluation of GUM95 is inept usually, and diffusely used is Type B evaluation of GUM95 with limit control of error reason. Under limit control of error reason the error possesses realizing limit and random probability distribution, thereout the *standard deviations* and *coverage factors* of such errors are random also, which are required by GUM95.

Concept of “*Degree Free*” is right and realizing for Type A evaluation of GUM95, but is inexistent for Type B evaluation of GUM95. *Effective “Degree Free”* for Type B evaluation of GUM95 is a mimic with theoretical mistake. Under limit control the variable possesses negative kurtosis. With uniform reliability the coverage factor for any variable with negative kurtosis will be less than the coverage factor for normal distribution variable, accordingly *Effective “Degree Free”* for any variable with negative kurtosis ought be more than ∞ .

Appendix H

Appendix - H - Uncertainty Budget for NIST, USA

The estimated uncertainties of the straddling bridge measurements at 1 kHz are based on an evaluation of the errors associated with: bridge and IVD under test reversals, imperfect current equalization, injection IVD errors, and common-mode rejection of the triaxial injection/detection networks. These sources of error are summarized in Table I, where the individual components are expressed in terms of their contributions to in-phase uncertainties (U_I) and quadrature uncertainties (U_Q). In addition to the dominant sources of type B measurement uncertainties listed in Table I, estimates of the system's type A uncertainties are based upon several full sets of straddling bridge measurement runs. It should be noted that the error associated with imperfect current equalization is the dominant source of error in this bridge. In addition, this error is somewhat dependent upon the input impedance of the IVD under test. The total uncertainties of the intercomparison results are based upon the values given in Table I, with additional type A uncertainties associated with the actual IVD measurements added in a root sum of squares (RSS) fashion.

Table I. Uncertainty estimates for the NIST straddling bridge at 1 kHz.

Type B Uncertainty Components (k=1)	U_I (parts in 10^6)	U_Q (parts in 10^6)
Imperfect current equalization (choke errors)	0.02	0.07
Detection network offsets / and common mode errors	0.02	0.04
Injection IVD ratio errors	0.01	0.03
Auxiliary IVD common mode ratio dependencies	0.01	0.02
Magnitude and phase errors of Network B injection signals	0.01	0.01
Type A Uncertainty Components (k=1)		
Connection repeatability during bridge/DUT reversals	0.01	0.01
Bridge balance algorithm induced noise	0.005	0.01
Totals (k=2)	0.07	0.18

Appendix I

Appendix - I - Uncertainty Budget for NMIJ, Japan (Formerly ETL)

IN-PHASE

Label	Source of uncertainty					Combined standard uncertainty
	Repeated observations	Special connector	Detector linearity	Detector phase angle	Resolution	
0.9	0.21E-08	0.05E-08	0.07E-08	0.34E-08	0.02E-08	0.41E-08
0.8	0.20E-08	0.05E-08	0.06E-08	0.31E-08	0.02E-08	0.38E-08
0.7	0.31E-08	0.04E-08	0.06E-08	0.28E-08	0.01E-08	0.43E-08
0.6	0.22E-08	0.04E-08	0.05E-08	0.25E-08	0.01E-08	0.34E-08
0.5	0.13E-08	0.04E-08	0.04E-08	0.22E-08	0.01E-08	0.26E-08
0.4	0.08E-08	0.03E-08	0.04E-08	0.19E-08	0.01E-08	0.22E-08
0.3	0.09E-08	0.03E-08	0.03E-08	0.16E-08	0.01E-08	0.19E-08
0.2	0.08E-08	0.02E-08	0.03E-08	0.13E-08	0.01E-08	0.15E-08
0.1	0.05E-08	0.01E-08	0.02E-08	0.09E-08	0.00E-08	0.10E-08
0.01	4.00E-08	0.01E-08	0.12E-08	0.12E-08	0.00E-08	4.00E-08
10/11	0.12E-08	0.06E-08	0.07E-08	0.36E-08	0.02E-08	0.39E-08
9/11	0.11E-08	0.05E-08	0.07E-08	0.33E-08	0.02E-08	0.36E-08
8/11	0.12E-08	0.05E-08	0.06E-08	0.30E-08	0.02E-08	0.33E-08
7/11	0.11E-08	0.04E-08	0.06E-08	0.28E-08	0.01E-08	0.31E-08
6/11	0.11E-08	0.04E-08	0.05E-08	0.25E-08	0.01E-08	0.28E-08
5/11	0.09E-08	0.04E-08	0.04E-08	0.22E-08	0.01E-08	0.25E-08
4/11	0.08E-08	0.03E-08	0.04E-08	0.19E-08	0.01E-08	0.21E-08
3/11	0.07E-08	0.03E-08	0.03E-08	0.16E-08	0.01E-08	0.18E-08
2/11	0.07E-08	0.02E-08	0.03E-08	0.13E-08	0.01E-08	0.15E-08
1/11	0.05E-08	0.01E-08	0.02E-08	0.09E-08	0.00E-08	0.10E-08

QUADRATURE

Label	Source of uncertainty					Combined standard uncertainty
	Repeated observations	Special connector	Detector linearity	Detector phase angle	Resolution	
0.9	0.42E-08	0.54E-08	0.34E-08	1.35E-08	0.02E-08	1.55E-08
0.8	0.44E-08	0.49E-08	0.31E-08	1.24E-08	0.02E-08	1.44E-08
0.7	0.48E-08	0.45E-08	0.28E-08	1.13E-08	0.01E-08	1.34E-08
0.6	0.47E-08	0.40E-08	0.25E-08	1.01E-08	0.01E-08	1.21E-08
0.5	0.35E-08	0.36E-08	0.22E-08	0.89E-08	0.01E-08	1.05E-08
0.4	0.34E-08	0.31E-08	0.19E-08	0.77E-08	0.01E-08	0.92E-08
0.3	0.31E-08	0.26E-08	0.16E-08	0.65E-08	0.01E-08	0.78E-08
0.2	0.24E-08	0.20E-08	0.13E-08	0.51E-08	0.01E-08	0.61E-08
0.1	0.14E-08	0.14E-08	0.09E-08	0.34E-08	0.00E-08	0.40E-08
0.01	7.50E-08	0.09E-08	0.12E-08	2.31E-08	0.00E-08	7.85E-08
10/11	0.21E-08	0.57E-08	0.36E-08	1.43E-08	0.02E-08	1.59E-08
9/11	0.23E-08	0.53E-08	0.33E-08	1.32E-08	0.02E-08	1.48E-08
8/11	0.33E-08	0.48E-08	0.30E-08	1.21E-08	0.02E-08	1.38E-08
7/11	0.34E-08	0.44E-08	0.28E-08	1.11E-08	0.01E-08	1.27E-08
6/11	0.38E-08	0.40E-08	0.25E-08	0.99E-08	0.01E-08	1.16E-08
5/11	0.35E-08	0.35E-08	0.22E-08	0.88E-08	0.01E-08	1.04E-08
4/11	0.28E-08	0.30E-08	0.19E-08	0.76E-08	0.01E-08	0.89E-08
3/11	0.23E-08	0.25E-08	0.16E-08	0.64E-08	0.01E-08	0.74E-08
2/11	0.19E-08	0.20E-08	0.13E-08	0.50E-08	0.01E-08	0.59E-08
1/11	0.13E-08	0.14E-08	0.09E-08	0.34E-08	0.00E-08	0.40E-08

Appendix J

APPENDIX - J - Uncertainty budget for NMI-VSL, THE NETHERLANDS

The measurements are performed by null-detection on a phase sensitive detector. An injection system in series with the output of the auxiliary divider is used to balance the bridge.

The injection voltage is found from the following equation:

$$V_{inj} = (\alpha + j\beta) \cdot V$$

where the in-phase (α) and quadrature (β) ratio differences are given by:

$$\alpha = \frac{r}{N_i \cdot N_R \cdot R} \quad (1) \quad \text{and} \quad \beta = \frac{\omega r C}{N_i \cdot N_C} \quad (2)$$

where: r is a fixed resistor, R is an adjustable resistor, C is an adjustable capacitor, N_i is a fixed transformer ratio and N_R and N_C are selectable transformer ratios.

The uncertainty budgets for the nominal ratios $U_{0.1n}$ where $n = (0, 1, 2, \dots, 10)$ are determined as follows. The measured ratios are given by $U_{0.1n} + a_{0.1n} + j b_{0.1n}$, where $a_{0.1n}$ and $b_{0.1n}$ are respectively the in-phase and quadrature deviations from the nominal ratios.

By definition, $a_{0.1n} = 0$ and $b_{0.1n} = 0$ for $n = 0$ and $n = 10$. Therefore, the sum of the 20 measured ratio differences and ten times the step ratio ($U_s + a_s + j b_s$) equals 1:

$$10 \cdot (U_s + a_s + j b_s) + \sum_{m=1}^{10} (\alpha_{2m} + j\beta_{2m} - \alpha_{2m-1} - j\beta_{2m-1}) = 1 \quad (3)$$

In this case, the nominal step ratio $U_s = 0.1$ and a_s and b_s are the in-phase and quadrature deviation from the nominal step ratio. From equation (3) the values of a_s and b_s can be determined:

$$a_s = \frac{1}{10} \cdot \sum_{m=1}^{10} \alpha_{2m-1} - \alpha_{2m} \quad (4) \quad \text{and} \quad b_s = \frac{1}{10} \cdot \sum_{m=1}^{10} \beta_{2m-1} - \beta_{2m} \quad (5)$$

For each of the steps ($0.1 \cdot n$) where $n = (1, 2, 3, \dots, 9)$:

$$a_{0.1n} = n \cdot a_s + \sum_{m=1}^n \alpha_{2m} - \alpha_{2m-1} \quad (6) \quad \text{and} \quad b_{0.1n} = n \cdot b_s + \sum_{m=1}^n \beta_{2m} - \beta_{2m-1} \quad (7)$$

The equations (1), (2), (4) and (5) can be substituted in (6) and (7). This yields (8) and (9), which will be used as the model equations to calculate the uncertainty budget.

For all of the ratio difference measurements, r , N_i and ω stay the same, so they are fully correlated for all difference measurements. The values of R and C change from one measurement to another and therefore can be considered to be independent (uncorrelated). Within a series of measurements, often only 2 or 3 different settings of N_R are N_C used. So for both N_R as well as for N_C there is a strong correlation between the difference measurements. For reasons of simplification of the uncertainty calculations, it is assumed that both N_R and N_C are fully correlated in all measurements by taking $N_R = 10$ and $N_C = 10$ for all measurements.

This assumption is reasonable if it is kept in mind that the values of R in the equations can be smaller or larger by a factor of 10 with respect to the actual resistance values, and both R and C can be either positive or negative in the equations.

$$a_{0.1n} = \frac{r}{N_i N_R} \left(\sum_{m=1}^n \left(\frac{1}{R_{2m}} - \frac{1}{R_{2m-1}} \right) + \frac{n}{10} \sum_{m=1}^{10} \left(\frac{1}{R_{2m-1}} - \frac{1}{R_{2m}} \right) \right) + n \cdot \delta a_s \quad (8)$$

$$b_{0.1n} = \frac{\omega r}{N_i N_C} \left(\sum_{m=1}^n (C_{2m} - C_{2m-1}) + \frac{n}{10} \sum_{m=1}^{10} (C_{2m-1} - C_{2m}) \right) + n \cdot \delta b_s + 2n \cdot \delta b_g \quad (9)$$

From equations (8) and (9) it can be seen that extra terms have been included to take into account small changes in the step ratio (δa_s and δb_s) during the series of measurements. Some problems have been observed with the guarding of the injection and detection transformers. Since, this influences mainly the quadrature reading, δb_g has been introduced to take these effects into account.

By calculating the partial derivatives from equation (8), the individual uncertainty contributions for each of the parameters are found for the in-phase measurements. Similarly, the uncertainty contributions for the quadrature measurements can be derived from equation (9).

On the (0.1·*n*) ratio taps, 9 series of measurements have been performed. The type A standard uncertainty is found from the standard deviation divided by the root of the number of series of measurements.

The total uncertainty for the in-phase deviation $a_{0.1n}$ from nominal is found from the root sum square of all contributions. Similarly, the total uncertainty for the quadrature deviation $b_{0.1n}$ is found from the root sum square of all contributions.

The expanded uncertainties are found from the total uncertainties multiplied by a coverage factor $k = 2$, which corresponds to a coverage probability of about 95 %.

The equations as given above have been numerically worked out in the tables on the following pages.

It can be shown that by good approximation:

$$\begin{aligned} u(a_{0.1}) &= u(a_{0.9}) & u(b_{0.1}) &= u(b_{0.9}) \\ u(a_{0.2}) &= u(a_{0.8}) & u(b_{0.2}) &= u(b_{0.8}) \\ u(a_{0.3}) &= u(a_{0.7}) & u(b_{0.3}) &= u(b_{0.7}) \\ u(a_{0.4}) &= u(a_{0.6}) & u(b_{0.4}) &= u(b_{0.6}) \end{aligned}$$

(Only the standard uncertainties in the terms may be slightly different, but in practice this effect is negligible.)

The method as described above was also applied for the (*n*/11) steps, where $n = (0, 1, \dots, 11)$. In this case there are 11 steps and a total of 22 ratio difference measurements.

The 0.01 ratio output tap of the traveling standard was compared with a reference inductive voltage divider set to a nominal output ratio of 0.01. The uncertainty in the reference divider output ratio is determined in a similar way as explained above. For the comparison with the traveling standard, three extra difference measurements are required: 0, 1, and 0.01.

Parameter values and uncertainties:

	Value		Uncertainty	Type / distribution / comments
r	0.2000 Ω	$u(r)$	0.001 Ω	A, normal, the same for all measurements of α and β
N_i	10 V/V	$u(N_i)$	1.00E-05	B, rectangular, the same for all measurements of α and β
N_R	10 V/V	$u(N_R)$	1.00E-05	B, rectangular, typical value
R	10 k Ω	$u(R)$	0.5 k Ω	B, rectangular, R worst case, uncertainty limited by detector resolution
ω	6283 rad/s	$u(\omega)$	5.00E-05	A, normal, $\omega = 2\pi f$, the same for all measurements of β
N_C	10 V/V	$u(N_C)$	1.00E-05	B, rectangular, typical value
C	100 nF	$u(C)$	0.5 nF	B, rectangular, C worst case, uncertainty limited by detector resolution
δa_s	0 V/V	$u(\delta a_s)$	1.00E-09 V/V	B, rectangular
δb_s	0 V/V	$u(\delta b_s)$	5.00E-08 V/V	B, rectangular
δb_g	0 V/V	$u(\delta b_g)$	5.00E-08 V/V	B, rectangular
$(1/R_{2m-1} - 1/R_{2m})$			1.00E-04 Ω^{-1}	typical value
$(C_{2m-1} - C_{2m})$			-9.0E-08 F ⁻¹	typical value
$1/10 \cdot \Sigma(1/R_{2m} - 1/R_{2m-1})$			1.00E-04 Ω^{-1}	typical value
$1/10 \cdot \Sigma(C_{2m} - C_{2m-1})$			1.59E-07 F ⁻¹	typical value

In-phase:

Step 0.1						
Quantity	Estimate	Uncertainty	Distribution	Standard uncertainty	Sensitivity	Contribution
X_i	x_i			$u(x_i)$	c_i	$u_i(y)$
r	0.2 Ω	0.001 Ω	normal	0.001 Ω	2.00E-06 Ω^{-1}	2.00E-09 V/V
N_R	10 V/V	1.00E-05	rectangular	5.77E-06	-4.00E-07	-2.31E-12 V/V
N_i	10 V/V	1.00E-05	rectangular	5.77E-06	-4.00E-07	-2.31E-12 V/V
R_1 to R_{2n}	10 k Ω	0.5 k Ω	rectangular	0.289 k Ω	1.8E-11 Ω^{-1}	7.35E-09 V/V
R_{2n+1} to R_{20}	10 k Ω	0.5 k Ω	rectangular	0.289 k Ω	-2E-12 Ω^{-1}	2.45E-09 V/V
δa_s	0 V/V	1.00E-09 V/V	rectangular	5.77E-10 V/V	1	5.77E-10 V/V
$s(a)$	0 V/V	3.00E-09 V/V	normal	3.00E-09 V/V	1	3.00E-09 V/V
					$k = 1$	8.56E-09 V/V
					$k = 2$	1.71E-08 V/V

Step 0.2						
Quantity	Estimate	Uncertainty	Distribution	Standard uncertainty	Sensitivity	Contribution
X_i	x_i			$u(x_i)$	c_i	$u_i(y)$
r	0.2 Ω	0.001 Ω	normal	0.001 Ω	2.00E-06 Ω^{-1}	4.00E-09 V/V
N_R	10 V/V	1.00E-05	rectangular	5.77E-06	-4.00E-07	-4.62E-12 V/V
N_i	10 V/V	1.00E-05	rectangular	5.77E-06	-4.00E-07	-4.62E-12 V/V
R_1 to R_{2n}	10 k Ω	0.5 k Ω	rectangular	0.289 k Ω	1.8E-11 Ω^{-1}	1.04E-08 V/V
R_{2n+1} to R_{20}	10 k Ω	0.5 k Ω	rectangular	0.289 k Ω	-2E-12 Ω^{-1}	2.31E-09 V/V
δa_s	0 V/V	1.00E-09 V/V	rectangular	5.77E-10 V/V	1	8.16E-10 V/V
$s(a)$	0 V/V	4.00E-09 V/V	normal	4.00E-09 V/V	1	4.00E-09 V/V
					$k = 1$	1.21E-08 V/V
					$k = 2$	2.42E-08 V/V

Step 0.3						
Quantity	Estimate	Uncertainty	Distribution	Standard uncertainty	Sensitivity	Contribution
X_i	x_i			$u(x_i)$	c_i	$u_i(y)$
r	0.2 Ω	0.001 Ω	normal	0.001 Ω	2.00E-06 Ω^{-1}	6.00E-09 V/V
N_R	10 V/V	1.00E-05	rectangular	5.77E-06	-4.00E-07	-6.92E-12 V/V
N_i	10 V/V	1.00E-05	rectangular	5.77E-06	-4.00E-07	-6.92E-12 V/V
R_1 to R_{2n}	10 k Ω	0.5 k Ω	rectangular	0.289 k Ω	1.8E-11 Ω^{-1}	1.27E-08 V/V
R_{2n+1} to R_{20}	10 k Ω	0.5 k Ω	rectangular	0.289 k Ω	-2E-12 Ω^{-1}	2.16E-09 V/V
δa_s	0 V/V	1.00E-09 V/V	rectangular	5.77E-10 V/V	1	1.00E-09 V/V
$s(a)$	0 V/V	4.00E-09 V/V	normal	4.00E-09 V/V	1	4.00E-09 V/V
					$k = 1$	1.48E-08 V/V
					$k = 2$	2.96E-08 V/V

Step 0.4						
Quantity	Estimate	Uncertainty	Distribution	Standard uncertainty	Sensitivity	Contribution
X_i	x_i			$u(x_i)$	c_i	$u_i(y)$
r	0.2 Ω	0.001 Ω	normal	0.001 Ω	2.00E-06 Ω^{-1}	8.00E-09 V/V
N_R	10 V/V	1.00E-05	rectangular	5.77E-06	-4.00E-07	-9.23E-12 V/V
N_i	10 V/V	1.00E-05	rectangular	5.77E-06	-4.00E-07	-9.23E-12 V/V
R_1 to R_{2n}	10 k Ω	0.5 k Ω	rectangular	0.289 k Ω	1.8E-11 Ω^{-1}	1.47E-08 V/V
R_{2n+1} to R_{20}	10 k Ω	0.5 k Ω	rectangular	0.289 k Ω	-2E-12 Ω^{-1}	2.00E-09 V/V
δa_s	0 V/V	1.00E-09 V/V	rectangular	5.77E-10 V/V	1	1.15E-09 V/V
$s(a)$	0 V/V	5.00E-09 V/V	normal	5.00E-09 V/V	1	5.00E-09 V/V
					$k = 1$	1.76E-08 V/V
					$k = 2$	3.52E-08 V/V

Step 0.5						
Quantity	Estimate	Uncertainty	Distribution	Standard uncertainty	Sensitivity	Contribution
X_i	x_i			$u(x_i)$	c_i	$u_i(y)$
r	0.2 Ω	0.001 Ω	normal	0.001 Ω	2.00E-06 Ω^{-1}	9.99E-09 V/V
N_R	10 V/V	1.00E-05	rectangular	5.77E-06	-4.00E-07	-1.15E-11 V/V
N_i	10 V/V	1.00E-05	rectangular	5.77E-06	-4.00E-07	-1.15E-11 V/V
R_1 to R_{2n}	10 k Ω	0.5 k Ω	rectangular	0.289 k Ω	1.8E-11 Ω^{-1}	1.64E-08 V/V
R_{2n+1} to R_{20}	10 k Ω	0.5 k Ω	rectangular	0.289 k Ω	-2E-12 Ω^{-1}	1.83E-09 V/V
δa_s	0 V/V	1.00E-09 V/V	rectangular	5.77E-10 V/V	1	1.29E-09 V/V
$s(a)$	0 V/V	6.00E-09 V/V	normal	6.00E-09 V/V	1	6.00E-09 V/V
					$k = 1$	2.03E-08 V/V
					$k = 2$	4.05E-08 V/V

Quadrature:

Step 0.1						
Quantity	Estimate	Uncertainty	Distribution	Standard uncertainty	Sensitivity	Contribution
X_i	x_i			$u(x_i)$	c_i	$u_i(y)$
r	0.2 Ω	0.001 Ω	normal	0.001 Ω	4.35E-06 Ω^{-1}	4.35E-09 V/V
ω	6283.185 rad/s	5.00E-05	normal	0.00005	8.69E-07	4.35E-11 V/V
N_C	10 V/V	1.00E-05	rectangular	5.77E-06	-8.69E-07	-5.02E-12 V/V
N_i	10 V/V	1.00E-05	rectangular	5.77E-06	-8.69E-07	-5.02E-12 V/V
C_1 to C_{2n}	100 nF	0.5 nF	rectangular	0.289 nF	11.31 F^{-1}	4.62E-09 V/V
C_{2n+1} to C_{20}	100 nF	0.5 nF	rectangular	0.289 nF	-1.26 F^{-1}	1.54E-09 V/V
δb_s	0 V/V	5.00E-08 V/V	rectangular	2.89E-08 V/V	1	2.89E-08 V/V
δb_g	0 V/V	5.00E-08 V/V	rectangular	2.89E-08 V/V	1	4.08E-08 V/V
$s(b)$	0 V/V	1.50E-08 V/V	normal	1.50E-08 V/V	1	1.50E-08 V/V
					$k = 1$	5.26E-08 V/V
					$k = 2$	1.05E-07 V/V

Step 0.2						
Quantity	Estimate	Uncertainty	Distribution	Standard uncertainty	Sensitivity	Contribution
X_i	x_i			$u(x_i)$	c_i	$u_i(y)$
r	0.2 Ω	0.001 Ω	normal	0.001 Ω	4.35E-06 Ω^{-1}	8.69E-09 V/V
ω	6283.185 rad/s	5.00E-05	normal	0.00005	8.69E-07	8.69E-11 V/V
N_C	10 V/V	1.00E-05	rectangular	5.77E-06	-8.69E-07	-1.00E-11 V/V
N_i	10 V/V	1.00E-05	rectangular	5.77E-06	-8.69E-07	-1.00E-11 V/V
C_1 to C_{2n}	100 nF	0.5 nF	rectangular	0.289 nF	11.31 F^{-1}	6.53E-09 V/V
C_{2n+1} to C_{20}	100 nF	0.5 nF	rectangular	0.289 nF	-1.26 F^{-1}	1.45E-09 V/V
δb_s	0 V/V	5.00E-08 V/V	rectangular	2.89E-08 V/V	1	4.08E-08 V/V
δb_g	0 V/V	5.00E-08 V/V	rectangular	2.89E-08 V/V	1	5.77E-08 V/V
$s(b)$	0 V/V	1.80E-08 V/V	normal	1.80E-08 V/V	1	1.80E-08 V/V
					$k = 1$	7.38E-08 V/V
					$k = 2$	1.48E-07 V/V

Step 0.3						
Quantity	Estimate	Uncertainty	Distribution	Standard uncertainty	Sensitivity	Contribution
X_i	x_i			$u(x_i)$	c_i	$u_i(y)$
r	0.2 Ω	0.001 Ω	normal	0.001 Ω	4.35E-06 Ω^{-1}	1.30E-08 V/V
ω	6283.185 rad/s	5.00E-05	normal	0.00005	8.69E-07	1.30E-10 V/V
N_C	10 V/V	1.00E-05	rectangular	5.77E-06	-8.69E-07	-1.51E-11 V/V
N_i	10 V/V	1.00E-05	rectangular	5.77E-06	-8.69E-07	-1.51E-11 V/V
C_1 to C_{2n}	100 nF	0.5 nF	rectangular	0.289 nF	11.31 F^{-1}	8.00E-09 V/V
C_{2n+1} to C_{20}	100 nF	0.5 nF	rectangular	0.289 nF	-1.26 F^{-1}	1.36E-09 V/V
δb_s	0 V/V	5.00E-08 V/V	rectangular	2.89E-08 V/V	1	5.00E-08 V/V
δb_g	0 V/V	5.00E-08 V/V	rectangular	2.89E-08 V/V	1	7.07E-08 V/V
$s(b)$	0 V/V	2.10E-08 V/V	normal	2.10E-08 V/V	1	2.10E-08 V/V
					$k = 1$	9.04E-08 V/V
					$k = 2$	1.81E-07 V/V

Step 0.4						
Quantity	Estimate	Uncertainty	Distribution	Standard uncertainty	Sensitivity	Contribution
X_i	x_i			$u(x_i)$	c_i	$u_i(y)$
r	0.2 Ω	0.001 Ω	normal	0.001 Ω	4.35E-06 Ω^{-1}	1.74E-08 V/V
ω	6283.185 rad/s	5.00E-05	normal	0.00005	8.69E-07	1.74E-10 V/V
N_C	10 V/V	1.00E-05	rectangular	5.77E-06	-8.69E-07	-2.01E-11 V/V
N_i	10 V/V	1.00E-05	rectangular	5.77E-06	-8.69E-07	-2.01E-11 V/V
C_1 to C_{2n}	100 nF	0.5 nF	rectangular	0.289 nF	11.31 F^{-1}	9.23E-09 V/V
C_{2n+1} to C_{20}	100 nF	0.5 nF	rectangular	0.289 nF	-1.26 F^{-1}	1.26E-09 V/V
δb_s	0 V/V	5.00E-08 V/V	rectangular	2.89E-08 V/V	1	5.77E-08 V/V
δb_g	0 V/V	5.00E-08 V/V	rectangular	2.89E-08 V/V	1	8.16E-08 V/V
$s(b)$	0 V/V	2.20E-08 V/V	normal	2.20E-08 V/V	1	2.20E-08 V/V
					$k = 1$	1.04E-07 V/V
					$k = 2$	2.09E-07 V/V

Step 0.5						
Quantity	Estimate	Uncertainty	Distribution	Standard uncertainty	Sensitivity	Contribution
X_i	x_i			$u(x_i)$	c_i	$u_i(y)$
r	0.2 Ω	0.001 Ω	normal	0.001 Ω	4.35E-06 Ω^{-1}	2.17E-08 V/V
ω	6283.185 rad/s	5.00E-05	normal	0.00005	8.69E-07	2.17E-10 V/V
N_C	10 V/V	1.00E-05	rectangular	5.77E-06	-8.69E-07	-2.51E-11 V/V
N_i	10 V/V	1.00E-05	rectangular	5.77E-06	-8.69E-07	-2.51E-11 V/V
C_1 to C_{2n}	100 nF	0.5 nF	rectangular	0.289 nF	11.31 F^{-1}	1.03E-08 V/V
C_{2n+1} to C_{20}	100 nF	0.5 nF	rectangular	0.289 nF	-1.26 F^{-1}	1.15E-09 V/V
δb_s	0 V/V	5.00E-08 V/V	rectangular	2.89E-08 V/V	1	6.45E-08 V/V
δb_g	0 V/V	5.00E-08 V/V	rectangular	2.89E-08 V/V	1	9.13E-08 V/V
$s(b)$	0 V/V	2.20E-08 V/V	normal	2.20E-08 V/V	1	2.20E-08 V/V
					$k = 1$	1.16E-07 V/V
					$k = 2$	2.33E-07 V/V

Appendix K

Appendix - K - Uncertainty Budget for NPL, UK							
Uncertainty budget for In-Phase measurements of decade and 11-section taps at 1 kHz							
Source	Description of uncertainty	type	Value x 10E-9	distribution	divisor	factor	u(y) 10E-9
Injection system	accuracy, linearity, offset	B	4	normal	2.0	1	2.0
Reference transformer	Linearity, offset	B	5	normal	2.0	1	2.5
Comparator	Linearity, offset	B	4	normal	2.0	1	2.0
Reference transformer	loading caused by Test IVD	B	5	normal	2.0	1	2.5
Injection system	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Reference transformer	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Comparator	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Test IVD system	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
	measurement repeatability	A	11	normal	1.0	1	11.0
RSS	combined (U=1)			normal			12.4
RSS	Expanded (U=2) : k(U=1)			normal (k=2)			25
	coverage factor, k						2.03
	$u^4c(y)$						23562.25
	$\text{sum}(u^4(y)/V(\text{eff}))$						300.3841
	OVERALL v(eff)						78.44

Uncertainty budget for Quadrature measurements of decade and 11-section taps at 1 kHz							
Source	Description of uncertainty	type	Value x 10E-9	distribution	divisor	factor	u(y) 10E-9
Injection system	accuracy, linearity, offset	B	4	normal	2.0	1	2.0
Reference transformer	Linearity, offset	B	5	normal	2.0	1	2.5
Comparator	Linearity, offset	B	4	normal	2.0	1	2.0
Reference transformer	loading caused by Test IVD	B	5	normal	2.0	1	2.5
Quadrature network	attenuation value, linearity	B	3	normal	2.0	1	1.5
Injection system	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Reference transformer	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Comparator	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Test IVD system	temperature coefficient, short-term stability measurement repeatability	B A	3 11	rect 100% normal	1.732 1.0	1 1	1.7 11.0
RSS	combined (U=1)			normal			12.5
RSS	Expanded (U=2) : k(U=1)			normal (k=2)			25
	coverage factor, k						2.03
	$u^4c(y)$						24258.06
	$\text{sum}(u^4(y)/V(\text{eff}))$						300.485
	OVERALL $v(\text{eff})$						80.73

Uncertainty budget for In-Phase measurements of 0.01 tap at 1 kHz							
Source	Description of uncertainty	type	Value x 10E-9	distribution	divisor	factor	u(y) 10E-9
Injection system	accuracy, linearity, offset	B	4	normal	2.0	1	2.0
Standard IVD	calibration uncertainty at 0.01 setting	B	43	normal	2.0	1	21.5
Comparator	Linearity, offset	B	4	normal	2.0	1	2.0
Injection system	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Standard IVD	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Comparator	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
system	measurement repeatability	A	11	normal	1.0	1	11.0
RSS	combined (U=1)			normal			24.5
RSS	Expanded (U=2) : k(U=1)			normal (k=2)			50
	coverage factor, k						2.04
	$u^4c(y)$						360300
	sum($u^4(y)/V(\text{eff})$						5445.96
	OVERALL v(eff)						66.16

Uncertainty budget for Quadrature measurements of 0.01 tap at 1 kHz							
Source	Description of uncertainty	type	Value x 10E-9	distribution	divisor	factor	u(y) 10E-9
Injection system	accuracy, linearity, offset	B	4	normal	2.0	1	2.0
Standard IVD	calibration uncertainty at 0.01 setting	B	43	normal	2.0	1	21.5
Comparator	Linearity, offset	B	4	normal	2.0	1	2.0
Quadrature network	attenuation value, linearity	B	3	normal	2.0	1	1.5
Injection system	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Standard IVD	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Comparator	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
system	measurement repeatability	A	11	normal	1.0	1	11.0
RSS	combined (U=1)			normal			24.5
RSS	Expanded (U=2) : k(U=1)			normal (k=2)			50
	coverage factor, k						2.04
	$u^4c(y)$						363006
	$\text{sum}(u^4(y)/V(\text{eff}))$						5446.06
	OVERALL $v(\text{eff})$						66.65

Uncertainty budget for In-Phase measurements of decade and 11-section taps at 55 Hz							
Source	Description of uncertainty	type	Value x 10E-9	distribution	divisor	factor	u(y) 10E-9
Injection system	accuracy, linearity, offset	B	20	normal	2.0	1	10.0
Reference transformer	Linearity, offset	B	20	normal	2.0	1	10.0
Comparator	Linearity, offset	B	20	normal	2.0	1	10.0
Reference transformer	loading caused by Test IVD	B	40	normal	2.0	1	20.0
Injection system	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Reference transformer	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Comparator	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Test IVD system	temperature coefficient, short-term stability measurement repeatability	B A	3 20	rect 100% normal	1.732 1.0	1 1	1.7 20.0
RSS	combined (U=1)			normal			33.3
RSS	Expanded (U=2) : k(U=1)			normal (k=2)			69
	coverage factor, k						2.06
	$u^4c(y)$						1236544
	$\text{sum}(u^4(y)/V(\text{eff}))$						28551.4
	OVERALL $v(\text{eff})$						43.31

Uncertainty budget for Quadrature measurements of decade and 11-section taps at 55Hz							
Source	Description of uncertainty	type	Value x 10E-9	distribution	divisor	factor	u(y) 10E-9
Injection system	accuracy, linearity, offset	B	20	normal	2.0	1	10.0
Reference transformer	Linearity, offset	B	20	normal	2.0	1	10.0
Comparator	Linearity, offset	B	20	normal	2.0	1	10.0
Reference transformer	loading caused by Test IVD	B	40	normal	2.0	1	20.0
Quadrature network	attenuation value, linearity	B	15	normal	2.0	1	7.5
Injection system	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Reference transformer	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Comparator	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Test IVD system	temperature coefficient, short-term stability measurement repeatability	B A	3 20	rect 100% normal	1.732 1.0	1 1	1.7 20.0
RSS	combined (U=1)			normal			34.2
RSS	Expanded (U=2) : k(U=1)			normal (k=2)			70
	coverage factor, k						2.05
	$u^4c(y)$						1364808
	$\text{sum}(u^4(y)/V(\text{eff}))$						28694.00
	OVERALL $v(\text{eff})$						47.56

Uncertainty budget for In-Phase measurements of 0.01 tap at 55 Hz							
Source	Description of uncertainty	type	Value x 10E-9	distribution	divisor	factor	u(y) 10E-9
Injection system	accuracy, linearity, offset	B	20	normal	2.0	1	10.0
Standard IVD	calibration uncertainty at 0.01 setting	B	133	normal	2.0	1	66.5
Comparator	Linearity, offset	B	20	normal	2.0	1	10.0
Injection system	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Standard IVD	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Comparator	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
system	measurement repeatability	A	20	normal	1.0	1	20.0
RSS	combined (U=1)			normal			70.9
RSS	Expanded (U=2) : k(U=1)			normal (k=2)			148
	coverage factor, k						2.09
	$u^4c(y)$						25313477
	$\text{sum}(u^4(y)/V(\text{eff}))$						893734.5
	OVERALL v(eff)						28.32

Uncertainty budget for Quadrature measurements of 0.01 tap at 55 Hz							
Source	Description of uncertainty	type	Value x 10E-9	distribution	divisor	factor	u(y) 10E-9
Injection system	accuracy, linearity, offset	B	20	normal	2.0	1	10.0
Standard IVD	calibration uncertainty at 0.01 setting	B	133	normal	2.0	1	66.5
Comparator	Linearity, offset	B	20	normal	2.0	1	10.0
Quadrature network	attenuation value, linearity	B	20	normal	2.0	1	10.0
Injection system	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Standard IVD	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
Comparator	temperature coefficient, short-term stability	B	3	rect 100%	1.732	1	1.7
system	measurement repeatability	A	20	normal	1.0	1	20.0
RSS	combined (U=1)			normal			71.6
RSS	Expanded (U=2) : k(U=1)			normal (k=2)			150
	coverage factor, k						2.09
	$u^4c(y)$						26329727
	$\text{sum}(u^4(y)/V(\text{eff}))$						894184.5
	OVERALL v(eff)						29.45

Appendix L

Appendix - L - Uncertainty Budget for NPLI, India

In Phase

Uncertainty Budget for n/10 Ratio (n=1,2,9) at 1 kHz

Source of Uncertainty X_i	Estimates x_i $\times 10^{-9}$	Limits $\pm x_i$ $\times 10^{-9}$	Probability Distribution Type A or B factor	Standards Uncertainty $u(x_i) \times 10^{-9}$	Sensitivity Coefficient c_i	Uncertainty Contribution $u_i(y) \times 10^{-9}$	Degree of Freedom ν_i
Std.IVD	4	2	Normal Type B 2	2	1	2.0	∞
Injection	2	1	Normal Type B 2	1	1	1.0	∞
Repeatability			Normal Type A $\sqrt{20}$	0.3	1	0.3	20
U_c						2.5	∞
Expanded Uncertainty			k=2			5.0	∞

Uncertainty Budget for n/11 Ratio (n=1,2,10) at 1 kHz

Source of Uncertainty X_i	Estimates x_i $\times 10^{-9}$	Limits $\pm x_i$ $\times 10^{-9}$	Probability Distribution Type A or B factor	Standards Uncertainty $u(x_i) \times 10^{-9}$	Sensitivity Coefficient c_i	Uncertainty Contribution $u_i(y) \times 10^{-9}$	Degree of Freedom ν_i
Std.IVD	7.4	3.7	Normal Type B 2	3.7	1	3.7	∞
Injection	2.0	1.0	Normal Type B 2	1.0	1	1.0	∞
Repeatability			Normal Type A $\sqrt{15}$	2.4	1	2.4	15
U_c						4.5	∞
Expanded Uncertainty			k=2			9.0	∞

Uncertainty Budget for n/10 Ratio (n=1,2,9) at 55 Hz

Source of Uncertainty X_i	Estimates x_i $\times 10^{-8}$	Limits $\pm x_i$ $\times 10^{-8}$	Probability Distribution Type A or B factor	Standards Uncertainty $u(x_i) \times 10^{-8}$	Sensitivity Coefficient c_i	Uncertainty Contribution $u_i(y) \times 10^{-8}$	Degree of Freedom ν_i
Std.IVD	2	1	Normal Type B 2	1	1	1.0	∞
Injection	2	1	Normal Type B 2	1	1	1.0	∞
Repeatability			Normal Type A $\sqrt{20}$	0.9	1	0.9	20
U_c						1.5	∞
Expanded Uncertainty			k=2			3.0	∞

Uncertainty Budget for n/11 Ratio (n=1,2,10) at 55 kHz

Source of Uncertainty X_i	Estimates x_i $\times 10^{-9}$	Limits $\pm x_i$ $\times 10^{-9}$	Probability Distribution Type A or B factor	Standards Uncertainty $u(x_i) \times 10^{-9}$	Sensitivity Coefficient c_i	Uncertainty Contribution $u_i(y) \times 10^{-9}$	Degree of Freedom ν_i
Std.IVD	4	2	Normal Type B 2	2	1	2.0	∞
Injection	2	1	Normal Type B 2	1	1	1.0	∞
Repeatability			Normal Type A $\sqrt{15}$	1.5	1	1.5	15
U_c						2.5	∞
Expanded Uncertainty			k=2			5.0	∞

Quadrature

Uncertainty Budget for n/10 Ratio (n=1,2,9) at 1 kHz

Source of Uncertainty X_i	Estimates x_i $\times 10^{-7}$	Limits $\pm x_i$ $\times 10^{-7}$	Probability Distribution Type A or B factor	Standards Uncertainty $u(x_i) \times 10^{-7}$	Sensitivity Coefficient c_i	Uncertainty Contribution $u_i(y) \times 10^{-7}$	Degree of Freedom ν_i
Std.IVD	2	1	Normal Type B 2	1	1	1.0	∞
Injection	1	0.5	Normal Type B 2	0.5	1	0.5	∞
Repeatability			Normal Type A $\sqrt{20}$	0.89	1	0.9	20
U_c						1.5	∞
Expanded Uncertainty			k=2			3.0	∞

Uncertainty Budget for n/11 Ratio (n=1,2,10) at 1 kHz

Source of Uncertainty X_i	Estimates x_i $\times 10^{-7}$	Limits $\pm x_i$ $\times 10^{-7}$	Probability Distribution Type A or B factor	Standards Uncertainty $u(x_i) \times 10^{-7}$	Sensitivity Coefficient c_i	Uncertainty Contribution $u_i(y) \times 10^{-7}$	Degree of Freedom ν_i
Std.IVD	4	2	Normal Type B 2	2.0	1	2.0	∞
Injection	1	0.5	Normal Type B 2	1	1	0.5	∞
Repeatability			Normal Type A $\sqrt{15}$	1.3	1	1.3	15
U_c						2.5	∞
Expanded Uncertainty			k=2			5.0	∞

Uncertainty Budget for n/10 Ratio (n=1,2,9) at 55 Hz

Source of Uncertainty X_i	Estimates x_i $\times 10^{-7}$	Limits $\pm x_i$ $\times 10^{-7}$	Probability Distribution Type A or B factor	Standards Uncertainty $u(x_i) \times 10^{-7}$	Sensitivity Coefficient c_i	Uncertainty Contribution $u_i(y) \times 10^{-7}$	Degree of Freedom ν_i
Std.IVD	2	1	Normal Type B 2	2	1	1	∞
Injection	1	0.5	Normal Type B 2	1	1	0.5	∞
Repeatability			Normal Type A $\sqrt{20}$	0.98	1	1.0	20
U_c						1.5	∞
Expanded Uncertainty			k=2			3.0	∞

Uncertainty Budget for n/11 Ratio (n=1,2,10) at 55 kHz

Source of Uncertainty X_i	Estimates x_i $\times 10^{-7}$	Limits $\pm x_i$ $\times 10^{-7}$	Probability Distribution Type A or B factor	Standards Uncertainty $u(x_i) \times 10^{-7}$	Sensitivity Coefficient c_i	Uncertainty Contribution $u_i(y) \times 10^{-7}$	Degree of Freedom ν_i
Std.IVD	4	2	Normal Type B 2	2	1	2.0	∞
Injection	1	0.5	Normal Type B 2	1	1	0.5	∞
Repeatability			Normal Type A $\sqrt{15}$	1.3	1	1.3	15
U_c						2.5	∞
Expanded Uncertainty			k=2			5.0	∞

Appendix M

Appendix - M - Uncertainty Budget for NRC, Canada

The direct straddle determinations of the 10 and 11 taps of the CCEM K7 transformer (DUT) were performed by measuring the K and DS data sets and then determining the sequential straddle ratios (S_n and their uncertainties) of the CCEM K7 transformer. There were 8 or 9 such sets of these determinations. Although we could (and did) determine the 8 or 9 sets of DUT tap ratios from this information and assess their uncertainties, I decided that it was slightly better to analyse the results as a single set. Thus each S_n was averaged and an uncertainty was assigned from its scatter the standard deviation (std) of the results. Then the DUT tap ratios were determined from this data.

The sequential straddle ratios, S_n , were then converted to the DUT tap ratios as described at the end of the Straddle Equations document. The uncertainties of these final tap ratios can be determined by combining the S_n uncertainties as prescribed by the straddle equations.

However there is an additional problem with the Type A uncertainty calculation. We have to assign the value of two adjacent taps to their nominal values before we can assign the rest of the tap ratios and re-normalize the entire set. Normally this is done by setting the value of the 0 and 1 taps to exactly 0 and 1, determining the values of the taps 2 to 10 and then renormalizing so that tap 0 = 0 and tap 10 = 1. However any adjacent pair of taps could have been chosen as nominal for the process. The final tap values are exactly the same but the uncertainties are not (assuming that each S_n uncertainty is different). Instead we have calculated the DUT ratios and their uncertainties of all 10 possible adjacent pairs of taps and averaged the results and the uncertainties. This distributes the uncertainties more properly among the DUT taps (otherwise tap 1 would always have 0 uncertainty).

The CCEM K7 uncertainty budgets are divided into each tap set 10, 11, and 0.01 and into inphase and out-of-phase (quadrature) sheets. The following explains the components of the 10 and 11 tap sheets.

Component 1

This is the variation (std) of the 8 or 9 sets of complete straddle determinations of the decade or eleventh's taps. It could have been determined exactly as stated but instead was determined by quoting the maximum tap uncertainty as described two paragraphs above.

Component 2

This is the uncertainty from the individual data sets, 25 data points per measurement and 4 K and 4 DS measurements forming a single determination of an S_n (it is about $0.0003 \cdot 10^{-6}$). But S_n is not a DUT tap ratio. On average, each S_n is multiplied by 9 to become a tap ratio.

Component 3

The offset of the main (triaxial) detector. This is measured and in the K measurements and removed from each S_n determination. Both the static and V_g dependent offset are corrected for in this fashion. However, there is still the stability of this effect, which is estimated from the repeatability of these measurements and

includes the effect of capacitive current flows from the guards to the DUT outer conductor.

Component 4

The guard accuracy is limited by the accuracy of the main guard IVD (an NL IVD box $\sim(3, j5) 10^{-6}$) and the accuracy of the DUT drive setting to VG ($<1 10^{-6}$). The sensitivity is detailed in the Type B uncertainty spreadsheet.

Component 5

The red and blue guard accuracy is also limited by the accuracy of the red or blue guard IVD (another NL IVD box $\sim(3, j5) 10^{-6}$). The sensitivity is also detailed in the Type B uncertainty spreadsheet.

Component 6

The dependence of the straddle transformer ratio to Vg. This is assessed in each DUT tap ratio determination as the std of the straddle transformer's ratio determined at each Vg.

Component 7 & 8

There are different winding impedances associated with each loop measurement and the contact resistances of the BPO connectors can vary from insertion to insertion. This is corrected for by determining the sensitivity of every loop measurement. The general effect is also reduced and made more constant by the addition of the 7 ohm resistor in the central inner loop path. However, the stability of the $1 10^{-6}$ calibration pulse size is an indication of the stability of the uncompensated or varying impedances.

Component 9

The outer conductor of the BPO taps may be at different potentials. This is supposed to be transformed into compensating potentials on the inner conductor loop but the equalizer (only three turns) is not perfect or even optimal. A worse case estimate of the possible potential was determined by considering the drive current for the DUT and estimating the voltage developed over 12 cm of copper 5mm thick. An estimate of the conversion efficiency of the equalizer was determined from three turns of that core and the outer conductor impedances

Component 10

A simple estimate of the detector linearity (1/5000) multiplied by the range of the lockin amplifier.

Component 11


Intermodulation distortion. The inphase and quadrature values of the 2nd to 8th harmonic were measured. The worse case is the 2nd and 3rd harmonics combining to create a fundamental signal. A conversion factor of 1/1000 was assumed which should be quite reasonable.


The 0.01 Tap Uncertainty Budgets are somewhat simpler and include only three components

- The variation of the comparison determinations of the 0.01 tap
- The uncertainty of the 0.1 tap of the CCEM K7 as previously determined. This is actually applied three times, the DUT 0.1 to NL 0.1, the DUT 0.1 to NL 1.0 at reduced drive and the DUT 0.01 against the NL 0.1.
- The voltage coefficient correction for the 0.1 tap of the second comparison IVD from 10 V FS to 1 V FS.

Appendix N

Appendix - N - Uncertainty Budget for PTB, Germany

	PTB Uncertainty Budget																																																																
<p>PTB Uncertainty Budget</p> <p>Calculation of the In-Phase Voltage Ratio and Uncertainty at Nominal Value 0.6 (Input Voltage 10 V, Frequency 995 Hz)</p> <p>Model Equation:</p> $K_{WR} = -1/10 * (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10});$ $K_{wt1} = \alpha_1 + K_{WR}; K_{wt2} = \alpha_2 + K_{WR};$ $K_{wt3} = \alpha_3 + K_{WR}; K_{wt4} = \alpha_4 + K_{WR};$ $K_{wt5} = \alpha_5 + K_{WR}; K_{wt6} = \alpha_6 + K_{WR};$ $K_{w6} = K_{wt1} + K_{wt2} + K_{wt3} + K_{wt4} + K_{wt5} + K_{wt6};$ <p>Label6=0.6;</p> $ipvr6 = \text{Label6} + (K_{w6}) * (1E-9)$ <p>List of Quantities:</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 15%;">Quantity</th> <th style="width: 15%;">Unit</th> <th style="width: 70%;">Definition</th> </tr> </thead> <tbody> <tr><td>α_1</td><td>nV/V</td><td>Reading of the In-Phase Voltage Difference of Winding Group 1</td></tr> <tr><td>α_2</td><td>nV/V</td><td>Reading of the In-Phase Voltage Difference of Winding Group 2</td></tr> <tr><td>α_3</td><td>nV/V</td><td>Reading of the In-Phase Voltage Difference of Winding Group 3</td></tr> <tr><td>α_4</td><td>nV/V</td><td>Reading of the In-Phase Voltage Difference of Winding Group 4</td></tr> <tr><td>α_5</td><td>nV/V</td><td>Reading of the In-Phase Voltage Difference of Winding Group 5</td></tr> <tr><td>α_6</td><td>nV/V</td><td>Reading of the In-Phase Voltage Difference of Winding Group 6</td></tr> <tr><td>α_7</td><td>nV/V</td><td>Reading of the In-Phase Voltage Difference of Winding Group 7</td></tr> <tr><td>α_8</td><td>nV/V</td><td>Reading of the In-Phase Voltage Difference of Winding Group 8</td></tr> <tr><td>α_9</td><td>nV/V</td><td>Reading of the In-Phase Voltage Difference of Winding Group 9</td></tr> <tr><td>α_{10}</td><td>nV/V</td><td>Reading of the In-Phase Voltage Difference of Winding Group 10</td></tr> <tr><td>K_{WR}</td><td>nV/V</td><td>In-Phase Correction of the Reference Transformer</td></tr> <tr><td>K_{wt1}</td><td>nV/V</td><td>In-Phase Correction of Winding Group 1</td></tr> <tr><td>K_{wt2}</td><td>nV/V</td><td>In-Phase Correction of Winding Group 2</td></tr> <tr><td>K_{wt3}</td><td>nV/V</td><td>In-Phase Correction of Winding Group 3</td></tr> <tr><td>K_{wt4}</td><td>nV/V</td><td>In-Phase Correction of Winding Group 4</td></tr> <tr><td>K_{wt5}</td><td>nV/V</td><td>In-Phase Correction of Winding Group 5</td></tr> <tr><td>K_{wt6}</td><td>nV/V</td><td>In-Phase Correction of Winding Group 6</td></tr> <tr><td>K_{w6}</td><td>nV/V</td><td>In-Phase Correction of Output 6 (Port 6)</td></tr> <tr><td>Label6</td><td>V/V</td><td>Nominal Value 0.6 (Port 6)</td></tr> <tr><td>ipvr6</td><td>V/V</td><td>In-Phase Voltage Ratio at Nominal Value 0.6</td></tr> </tbody> </table>			Quantity	Unit	Definition	α_1	nV/V	Reading of the In-Phase Voltage Difference of Winding Group 1	α_2	nV/V	Reading of the In-Phase Voltage Difference of Winding Group 2	α_3	nV/V	Reading of the In-Phase Voltage Difference of Winding Group 3	α_4	nV/V	Reading of the In-Phase Voltage Difference of Winding Group 4	α_5	nV/V	Reading of the In-Phase Voltage Difference of Winding Group 5	α_6	nV/V	Reading of the In-Phase Voltage Difference of Winding Group 6	α_7	nV/V	Reading of the In-Phase Voltage Difference of Winding Group 7	α_8	nV/V	Reading of the In-Phase Voltage Difference of Winding Group 8	α_9	nV/V	Reading of the In-Phase Voltage Difference of Winding Group 9	α_{10}	nV/V	Reading of the In-Phase Voltage Difference of Winding Group 10	K_{WR}	nV/V	In-Phase Correction of the Reference Transformer	K_{wt1}	nV/V	In-Phase Correction of Winding Group 1	K_{wt2}	nV/V	In-Phase Correction of Winding Group 2	K_{wt3}	nV/V	In-Phase Correction of Winding Group 3	K_{wt4}	nV/V	In-Phase Correction of Winding Group 4	K_{wt5}	nV/V	In-Phase Correction of Winding Group 5	K_{wt6}	nV/V	In-Phase Correction of Winding Group 6	K_{w6}	nV/V	In-Phase Correction of Output 6 (Port 6)	Label6	V/V	Nominal Value 0.6 (Port 6)	ipvr6	V/V	In-Phase Voltage Ratio at Nominal Value 0.6
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ipvr6	V/V	In-Phase Voltage Ratio at Nominal Value 0.6																																																															
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PTB Uncertainty Budget		
α_1 :	Type B rectangular distribution Value: 12 nV/V Halfwidth of Limits: 10 nV/V	
α_2 :	Type B rectangular distribution Value: 56 nV/V Halfwidth of Limits: 10 nV/V	
α_3 :	Type B rectangular distribution Value: 63 nV/V Halfwidth of Limits: 10 nV/V	
α_4 :	Type B rectangular distribution Value: 76 nV/V Halfwidth of Limits: 10 nV/V	
α_5 :	Type B rectangular distribution Value: 87 nV/V Halfwidth of Limits: 10 nV/V	
α_6 :	Type B rectangular distribution Value: 88 nV/V Halfwidth of Limits: 10 nV/V	
α_7 :	Type B rectangular distribution Value: 87 nV/V Halfwidth of Limits: 10 nV/V	
α_8 :	Type B rectangular distribution Value: 68 nV/V Halfwidth of Limits: 10 nV/V	
α_9 :	Type B rectangular distribution Value: 53 nV/V Halfwidth of Limits: 10 nV/V	
α_{10} :	Type B rectangular distribution Value: 13 nV/V Halfwidth of Limits: 10 nV/V	
K_{WR} :	Temporary	
K_{Wt1} :	Temporary	
K_{Wt2} :	Temporary	
K_{Wt3} :	Temporary	
K_{Wt4} :	Temporary	
Date: 02/14/2003	File: CCEMK7-2	Page 2 of 3

K_{wt5} : Temporary

K_{wt6} : Temporary

K_{w6} : Temporary

Label6: Temporary

ipvr6: Result

Uncertainty Budget:

Quantity	Value	Standard Uncertainty	Degrees of Freedom	Sensitivity Coefficient	Uncertainty Contribution	Index
α_1	12.00 nV/V	5.77 nV/V	infinity	$400 \cdot 10^{-12}$	$2.3 \cdot 10^{-9}$ V/V	6.7 %
α_2	56.00 nV/V	5.77 nV/V	infinity	$400 \cdot 10^{-12}$	$2.3 \cdot 10^{-9}$ V/V	6.7 %
α_3	63.00 nV/V	5.77 nV/V	infinity	$400 \cdot 10^{-12}$	$2.3 \cdot 10^{-9}$ V/V	6.7 %
α_4	76.00 nV/V	5.77 nV/V	infinity	$400 \cdot 10^{-12}$	$2.3 \cdot 10^{-9}$ V/V	6.7 %
α_5	87.00 nV/V	5.77 nV/V	infinity	$400 \cdot 10^{-12}$	$2.3 \cdot 10^{-9}$ V/V	6.7 %
α_6	88.00 nV/V	5.77 nV/V	infinity	$400 \cdot 10^{-12}$	$2.3 \cdot 10^{-9}$ V/V	6.7 %
α_7	87.00 nV/V	5.77 nV/V	infinity	$-600 \cdot 10^{-12}$	$-3.5 \cdot 10^{-9}$ V/V	15.0 %
α_8	68.00 nV/V	5.77 nV/V	infinity	$-600 \cdot 10^{-12}$	$-3.5 \cdot 10^{-9}$ V/V	15.0 %
α_9	53.00 nV/V	5.77 nV/V	infinity	$-600 \cdot 10^{-12}$	$-3.5 \cdot 10^{-9}$ V/V	15.0 %
α_{10}	13.00 nV/V	5.77 nV/V	infinity	$-600 \cdot 10^{-12}$	$-3.5 \cdot 10^{-9}$ V/V	15.0 %
K_{wR}	-60.30 nV/V	1.83 nV/V				
K_{wt1}	-48.30 nV/V	5.48 nV/V				
K_{wt2}	-4.30 nV/V	5.48 nV/V				
K_{wt3}	2.70 nV/V	5.48 nV/V				
K_{wt4}	15.70 nV/V	5.48 nV/V				
K_{wt5}	26.70 nV/V	5.48 nV/V				
K_{wt6}	27.70 nV/V	5.48 nV/V				
K_{w6}	20.20 nV/V	8.94 nV/V				
Label6	0.6 V/V	0.0 V/V				
ipvr6	0.6000000202 V/V	$8.94 \cdot 10^{-9}$ V/V	infinity			

Result: Quantity: ipvr6
 Value: 0.600000020 V/V
 Expanded Uncertainty: $\pm 18 \cdot 10^{-9}$ V/V
 Coverage Factor: 2.0
 Coverage: t-table 95%

Appendix O

Appendix - O - Uncertainty budget for SP, Sweden

Uncertainty budget for in phase voltage ratio of CCEM-ACVRS01

Quantity X_i	Standard uncertainty $u(x_i)$, (10^{-9} of input)	Probability distribution / method of evaluation	Sensitivity coefficient c_i	Uncertainty contribution u_i (10^{-9} of input)	Degree of freedom ν_i
In phase voltage ratio determination with capacitance permutation	18	Normal/ A,B	1	18	50
Comparison bridge meas. of primary standard (A)	10	Normal/ B	1	10	∞
*Experimental standard deviation of the mean for the measurement A	4	Normal/ A	1	4	2
Comparison bridge meas. of ACVRS01 (B)	10	Normal/ B	1	10	∞
*Experimental standard deviation of the mean for the measurement B	6	Normal/ A	1	6	2
Ratio of ACVRS01				24	115
Expanded uncertainty (k=2):				$48 \cdot 10^{-9}$	

Uncertainty budget for quadrature voltage ratio of CCEM-ACVRS01

Quantity X_i	Standard uncertainty $u(x_i)$, (10^{-9} of input)	Probability distribution / method of evaluation	Sensitivity coefficient c_i	Uncertainty contribution u_i (10^{-9} of input)	Degree of freedom ν_i
Quadrature voltage ratio determination with capacitance permutation	20	Normal/A, B	1	20	50
Comparison bridge meas. of primary standard (A)	20	Normal/B	1	20	∞
*Experimental standard deviation of the mean for the measurement A	3	Normal/A	1	3	2
Comparison bridge meas. of ACVRS01 (B)	20	Normal/B	1	20	∞
*Experimental standard deviation of the mean for the measurement B	7	Normal/A	1	7	2
Ratio of ACVRS01				35.5	5722
Expanded uncertainty (k=2):				$71 \cdot 10^{-9}$	

*The experimental standard deviation of the mean is a typical value for all ratios and the largest experimentally found in the calibration of the primary standard or the CCEM-ACVRS01 standard.

The standard uncertainty has been determined in accordance with the Guide to the expression of Uncertainty in Measurement (GUM), ISO, 1995.

The long term stability of the calibrated object is not included in the reported expanded uncertainty of measurement.

Appendix P

Appendix - P - Uncertainty budget for UME, Turkey

Uncertainty Budget of X measurements of 1/10 ratio

Uncertainty components	Standard uncertainty	Degrees of Freedom
From real and imaginary injection circuit	1.00E-08	12
From guard arrangement	1.50E-08	10
Stability of 1/10 reference divider	5.00E-09	5
Asymmetry of cables and shields of 1/10 reference divider	5.00E-09	20
Asymmetry of the connection cables used between 1/10 two-stage transformer and travelling standard	1.00E-08	20
Errors from cables and connection leads between inputs and outputs (1.0 and 1.0, 0.0 and 0.0)	1.00E-08	5
Standard deviation of the measurements	1.00E-09	13
RSS total uncertainty (1σ)	2.40E-08	38.8 (k=2.07)
Expanded uncertainty (95.45%)	5.0E-08	

Uncertainty Budget of Y measurements of 1/10 ratio

Uncertainty components	Standard uncertainty	Degrees of Freedom
From real and imaginary injection circuit	1.50E-08	12
From guard arrangement	2.00E-08	10
Stability of 1/10 reference divider	5.00E-09	5
Asymmetry of cables and shields of 1/10 reference divider	5.00E-09	20
Asymmetry of the connection cables used between 1/10 two-stage transformer and travelling standard	1.00E-08	20
Errors from cables and connection leads between inputs and outputs (1.0 and 1.0, 0.0 and 0.0)	1.00E-08	5
Standard deviation of the measurements	1.00E-09	16
RSS total uncertainty (1σ)	2.96E-08	33.5 (k=2.09)
Expanded uncertainty (95.45%)	6.2E-08	

Uncertainty Budget of X measurements of 1/11 ratio

Uncertainty components	Standard uncertainty	Degrees of Freedom
From real and imaginary injection circuit	1.00E-08	12
From guard arrangement	1.50E-08	10
Stability of 1/11 reference divider	5.00E-09	5
Asymmetry of cables and shields of 1/11 reference divider	5.00E-09	20
Asymmetry of the connection cables used between 1/11 two-stage transformer and travelling standard	1.00E-08	20
Errors from cables and connection leads between inputs and outputs (1.0 and 1.0, 0.0 and 0.0)	1.00E-08	5
Standard deviation of the measurements	1.00E-09	18
RSS total uncertainty (1σ)	2.40E-08	38.8 (k=2.07)
Expanded uncertainty (95.45%)	5.0E-08	

Uncertainty Budget of Y measurements of 1/11 ratio

Uncertainty components	Standard uncertainty	Degrees of Freedom
From real and imaginary injection circuit	1.50E-08	12
From guard arrangement	2.00E-08	10
Stability of 1/11 reference divider	5.00E-09	5
Asymmetry of cables and shields of 1/11 reference divider	5.00E-09	20
Asymmetry of the connection cables used between 1/11 two-stage transformer and travelling standard	1.00E-08	20
Errors from cables and connection leads between inputs and outputs (1.0 and 1.0, 0.0 and 0.0)	1.00E-08	5
Standard deviation of the measurements	2.00E-09	16
RSS total uncertainty (1σ)	2.96E-08	33.5 (k=2.09)
Expanded uncertainty (95.45%)	6.2E-08	

Uncertainty Budget of X measurements of 1/100 ratio

Uncertainty components	Standard uncertainty	Degrees of Freedom
From real and imaginary injection circuit	2.00E-08	12
From guard arrangement	2.50E-08	10
Stability of 1/100 reference divider	5.00E-09	5
Stability of two-decade reference Inductive Voltage Divider	5.00E-09	5
Asymmetry of cables and shields of 1/100 reference divider	2.00E-08	20
Asymmetry of the connection cables used between 1/100 (or 11/1 or 100/1) two-stage transformer and travelling standard	1.50E-08	20
Errors from cables between inputs and outputs (1.0 and 1.0, 0.00 and 0.00)	1.50E-08	5
Standard deviation of the measurement of reference inductive divider and 1/100 reference divider	3.00E-09	13
Standard deviation of the measurement of 1/100 reference divider and 1/100 ratio of the transfer standard	1.00E-09	13
RSS total uncertainty (1σ)	4.4E-08	51 (k=2.05)
Expanded uncertainty (95.45%)	9.0E-08	

Uncertainty Budget of Y measurements of 1/100 ratio

Uncertainty components	Standard uncertainty	Degrees of Freedom
From real and imaginary injection circuit	2.00E-08	12
From guard arrangement	2.50E-08	10
Stability of 1/100 reference divider	5.00E-09	5
Stability of two-decade reference IVD	5.00E-09	5
Asymmetry of cables and shields of 1/100 reference divider	2.00E-08	20
Asymmetry of the connection cables used between 10/1 (or 11/1 or 100/1) two-stage transformer and travelling standard	1.50E-08	20
Errors from cables between inputs and outputs (1.0 and 1.0, 0.00 and 0.00)	1.50E-08	5
Standard deviation of the measurement of reference inductive divider and 1/100 reference divider	3.00E-09	13
Standard deviation of the measurement of 1/100 reference divider and 1/100 ratio of the transfer standard	1.00E-09	13
RSS total uncertainty (1σ)	4.4E-08	51 (k=2.05)
Expanded uncertainty (95.45%)	9.0E-08	

Appendix Q

Appendix - Q- Uncertainty budget of VNIIM, Russia

Measurements at 1000 Hz

Symbol	Type	In phase (α)			Quadrature (β)		
		Standard uncertainty	Accuracy of U estimate (%)	Calculated degrees of freedom	Standard Uncertainty	Accuracy of U estimate (%)	Calculated degrees of freedom
U_1	B	5×10^{-9}	25	8	8×10^{-9}	25	8
U_2	B	5×10^{-9}	25	8	8×10^{-9}	25	8
U_3	B	2×10^{-9}	25	8	4×10^{-9}	25	8
U_4	B	4×10^{-9}	25	8	5×10^{-9}	25	8
U_5	B	2×10^{-9}	25	8	4×10^{-9}	25	8
U_6	A	2×10^{-9}	–	14	1×10^{-9}	–	14
u_c		8.83×10^{-9}			1.36×10^{-8}		
U		1.8×10^{-8}			2.8×10^{-8}		
$\nu(\text{eff})$		31			30		
k		2.04			2.04		

NOTE 1.

u_1 : Instability of cross-currents balance in the detector transformer

u_2 : Inner loading in the reference voltage source

u_3 : Insufficient shielding in coaxial cables

u_4 : Insufficient effectiveness of coaxial chock

u_5 : Non-linearity of the balancing circuits

u_6 : Experimental standard deviation

u_c : Combined standard uncertainty

U : Expanded uncertainty

ν_{eff} : Effective degrees of freedom

k : Coverage factor

NOTE 2.

All uncertainties are expressed in part of input voltage.

Appendix R

CCEM Key Comparison

Comparison of Alternating Voltage Ratio using an Inductive Voltage Divider

Technical Protocol

(Draft 0.29)

18th October 1999

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Introduction

This comparison is designated as a key comparison of the Consultative Committee for Electricity and Magnetism, CCEM. The comparison will start in November 1999 and last 30 months, with the National Physical Laboratory, UK (NPLUK) acting as the pilot laboratory. Contacts details for NPLUK are listed in **Appendix A**.

This document supplements the document 'Guidelines for the organisation of CCEM key comparisons' dated March 1999 hereafter called 'The Guidelines'. In case of a conflict between the Guidelines and this document the Guidelines will take precedence unless there is an explicit statement to the contrary.

Participation

The comparison is open to laboratories meeting both the technical requirements for the comparison and the requirements in Appendix 1 of the Guidelines.

Technical Requirements for the Comparison

Participants should be able to measure both orthogonal components of the alternating voltage ratio of the travelling standard with an expanded uncertainty ($k=2$) of less than 1×10^{-7} of the input .

Where participants are comparing the travelling standard against a reference calibrated by another laboratory, this must be declared in the results.

Participants must be able to conduct the measurements in equilibrium conditions at a temperature of either 20 ± 1 °C or 23 ± 1 °C.

Form of Comparison

By default, each stage of the comparison will consist of a measurement by a participating laboratory followed by a measurement by the pilot laboratory. By arrangement with the pilot laboratory the participating laboratory can forward the instrument to the next participant provided that there is minimal risk to the travelling standard.

Pre Comparison Activities

The stability of the travelling standard will be assessed in two ways: Firstly by comparison of measurements made by CSIRO, its manufacturer, with those made by NPLUK, the pilot

laboratory after shipment from CSIRO to NPLUK. Secondly by repeated measurement interleaved by local travel at the pilot laboratory prior to the start of the comparison.

Handling of the Travelling Standard

The travelling standard should be examined immediately upon receipt. Its condition should be noted and communicated to the pilot laboratory. **Appendix E** contains a form to aid this procedure. Please fax or E-mail the completed form to Stephen Bryant at NPLUK.

The travelling standard should only be handled by authorised persons and stored in such a way as to prevent damage.

The travelling standard should be examined before despatch and any change in condition during the measurement at each laboratory should be communicated to the pilot laboratory.

Please inform the pilot laboratory and the next laboratory via fax or e-mail when the travelling standard is about to be sent to the next recipient.

After the measurements please ensure that all items are packed before shipment, particularly the adapters. Always use the original packaging.

Transportation of the Travelling Standard

It is of utmost importance that the travelling standard be transported in a manner in which it will not be lost or damaged.

The travelling standard should be dispatched in the original packing case and marked as 'Fragile'. A list of the contents and accessories is given in **Appendix D**.

The travelling standard will be accompanied by a customs carnet where appropriate or documentation identifying the items uniquely.

Transportation is each laboratory's responsibility and cost. Each participating laboratory covers the costs for its own measurements, transportation and any customs charges as well as for any damages that may have occurred within its country. The overall costs for the organisation and for the devices are covered by the organising pilot laboratory. The pilot laboratory has no insurance for any loss or damage of the standards during transportation.

The packing details are contained within the packing crate.

Allotment of Time

Each laboratory will receive the travelling standard according to an agreed timetable. A final set of measurements will be made at the end of the comparison by the pilot laboratory.

The comparison will be scheduled on an eight week cycle, within this cycle each laboratory has **3 weeks** for measurement and 1 week for return of the standard. For the comparison to be completed on schedule it is important that participating laboratories abide by this schedule.

If for any reason, a participating laboratory finds that it cannot measure the travelling standard and ship it on schedule the pilot laboratory should be contacted without delay for instructions.

It may be necessary either to return the standard to the pilot laboratory or send it directly to another, designated, laboratory. Efforts will be made to reschedule incomplete measurements later in the comparison.

Comparisons will operate on 8-week cycles are detailed below:-

Week(s)	Operation
1	Shipment from pilot laboratory to participant laboratory
2-4	Measurement at participant laboratory
5	Shipment back to the pilot laboratory from participant laboratory
6-8	Measurement at the pilot laboratory

Wherever possible, allowance will be made for extended national holidays e.g. Easter, Christmas and other periods of reduced activity e.g. summer recess.

Participants are requested to use the yyyy-mm-dd format for dates. Reference : ISO 8601:1998

Description of the Travelling Standard

The travelling standard is an auto-transformer, having a fixed set of taps. It has been designed and built specifically for this comparison. The device has a fixed ratio magnetising winding and multi-tapped main or ratio winding. All connections are made to BPO type coaxial sockets, also known as MUSA sockets.

The preferred method of connection is direct to the BPO sockets. However, suitable adapters will be provided to allow connections via binding post, 4mm (banana) terminal and BNC.

Key

BPO - BPO socket.

BNC - BPO socket to BNC.

BPA - BPO socket to BNC Binding Post Adapter.

Defining Conditions

The voltage at a coaxial port is defined as the open circuit voltage existing between the inner and outer conductors of the coaxial connector at that port.

The INPUT VOLTAGE of the divider is defined as the voltage at the port marked “1” of the divider minus the voltage at the port marked “0”.

The OUTPUT VOLTAGE of the divider at a given port is defined as the voltage at the port minus the voltage at the port marked “0”.

The VOLTAGE RATIO of the divider at a given port is the complex quantity equal to the OUTPUT VOLTAGE divided by the INPUT VOLTAGE.

The NOMINAL VOLTAGE RATIO of a given port of the divider is a real number equal to the VOLTAGE RATIO of the corresponding port on an equivalent ideal divider.

The IN-PHASE VOLTAGE RATIO is the real part of the VOLTAGE RATIO.

The QUADRATURE VOLTAGE RATIO is the imaginary part of the VOLTAGE RATIO.

The IN-PHASE RATIO ERROR is the IN-PHASE RATIO minus the NOMINAL VOLTAGE RATIO.

Measurement Conditions

The divider should be energised with identical sinusoidal alternating voltages applied to the ports marked “EXCITE” and “INPUT” see Figure 1, Appendix C. These voltages must be sufficiently stable in both frequency and amplitude for a single value, obtained from traceable calibrated instruments, to be given for each set of results. Amplitude values will be expressed in RMS [root-mean-square] Volts, frequencies in Hz.

It is recommended that the travelling standard be energised and given a minimum 2 days (48 hours) to reach equilibrium with the laboratory environment before measurements are made.

The temperature of the case of the travelling standard must be measured using traceable calibrated instruments with an uncertainty of ± 0.5 °C or better.

The travelling standard is intended to be calibrated in an upright and level condition.

Mandatory Measurements

Frequency Hz	Input Volts V (rms)	Ratios
1000	10	0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
1000	10	$\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}, \frac{5}{11}, \frac{6}{11}, \frac{7}{11}, \frac{8}{11}, \frac{9}{11}, \frac{10}{11}$

Optional Measurements

Frequency Hz	Input Volts V (rms)	Ratios
55	3	0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
55	3	$\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}, \frac{5}{11}, \frac{6}{11}, \frac{7}{11}, \frac{8}{11}, \frac{9}{11}, \frac{10}{11}$

Measurement Uncertainty

The uncertainty of measurement shall be estimated according to the *ISO Guide to the Expression of Uncertainty in Measurement*.

Participants will provide a value for the expanded uncertainty together with a value of the coverage factor, k and a value for the actual or estimated degrees of freedom. The expanded uncertainty is expected to be a normal distribution, assessed at the 2σ or 95.45% confidence level.

Reporting of Results

The results should be communicated to the pilot laboratory within 6 weeks of the completion of measurements.

An example of a report giving the measurement results appears in **Appendix F**. It should be noted that these sample results bear no resemblance to the results expected for the travelling standard. The report is designed for the automated processing of the results of the comparison and should be completed using an ASCII text editor or equivalent and returned in electronic form. The first column of the report, denoted as “**LABEL**”, is to be used in the automatic data processing system and therefore should NOT under any circumstances be modified. The text in this column is the nominal measured value at each port and corresponds to the port configuration notation as shown in Figure 1.

For the purposes of result processing each participant in the comparison has been designated a unique alpha numeric identifier. This identifier **MUST** appear in column two of the report. A list of these identifiers is given in **Appendix B**.

A template for the report is shown in **Appendix G** with an * denoting where text or results should be entered. This template will be available as an ASCII file which can be downloaded from the dedicated Web site the address of which is given in Appendix A.

Participating laboratories are encouraged to sign the report electronically using one of the systems commonly available.

Following receipt of all measurement reports from the participating laboratories, the pilot laboratory will analyse the results and prepare the reports on the comparison as required by the guidance document.

Appendix A: Pilot Laboratory Details

Dedicated World Wide Web / Internet sitecceaivr.npl.co.uk
??? address to be confirmed ???

Dedicated E-Mail addressccem.acivr@npl.co.uk

Dedicated Fax number+44 (0)20 8943 6341

Laboratory contacts

Name & E-Mail Address :	Direct Phone +44 (0)20 8943 xxxx.	Location
Dr Ian A Robinson ian.robinson@npl.co.uk	7139 6129	Room 31, Building 1 (Bushy House)
Janet H Belliss janet.belliss@npl.co.uk	6294 6129	Room 11, Building 1 (Bushy House)
Stephen Bryant stephen.bryant@npl.co.uk	6172 6294	Room 12, Building 1 (Bushy House)

Address for correspondence

Mr Stephen Bryant, Room 11, Bushy House
 Centre for Electromagnetic & Time Metrology
 National Physical Laboratory
 Queens Road
 Teddington
 TW11 0LW
 UK

Address for Goods / shipments

Goods In
 Building 2000
 National Physical Laboratory
 Queens Road
 Teddington
 TW11 0LW
 UK

Appendix B: Designated Participant Identifiers.

Each participant has been given a unique alpha numeric identifier up to six characters in length and this must be used in the report file submitted by each participant. A second identifier is also given which will be used in the final report of the comparison for all table and graphical representations of results.

Participant Identifier for Report Processing	Identifier for Table & Graphical Representation.	Participating Laboratory
LCIE	BNM-LCIE	Bureau International de Metrologie, Laboratoire Central des Industries Electriques
CSIRO	CSIRO	Commonwealth Scientific & Industrial Research Organisation
ETL	ETL	Electrotechnical Laboratory
IEN	IEN	Istituto Elettrotecnico Nazionale
KRISS	KRISS	Korea Research Institute of Standards & Science
NIM	NIM	National Institute of Metrology
NIST	NIST	National Institute of Standards & Technology
VSL	NMi-VSL	NMi Van Swinden Laboratorium
NPLI	NPL(India)	National Physical Laboratory, India
NPLUK	NPLUK	National Physical Laboratory, UK
NRC	NRC	Institute for National Measurement Standards
OFMET	OFMET	Swiss Federal Office of Metrology
PTB	PTB	Physikalisch-Technische Bundesanstalt
SP	SP	SP - Swedish National Testing & Research Institute

Appendix C: Characteristics of the Travelling Standard



Figure 1: Travelling ac voltage ratio standard

Diagram of the port configuration as shown in Figure 1.

INPUT	0.2	0.4	0.6	0.8	1
EXCITE	0.1	0.3	0.5	0.7	0.9
0	1/11	3/11	5/11	7/11	9/11
0.01	2/11	4/11	6/11	8/11	10/11

Appendix D: Contents List

1) Travelling ac voltage ratio standard Serial Number CCEM-ACVRS01.

The following adapters are housed in Accessory Case (Figures 2A and 2B)

- 2) 4 off BPO to BNC Adapters
- 3) 4 off BNC to twin binding post Adapters
- 4) 2 off BPO to single binding post Adapters
- 5) 1 off 3mm Hex driver



Figure 2A: Accessories



Figure 2B: Accessories & Case

The travelling standard is housed the Wooden Box (Figures 3 & 4) which in turn is housed in the Polycarbonate Case (Figure 5) and then placed in the Packing Crate (Figure 6) which is foam lined.



Figure 3: Standard in wooden box



Figure 4: Wooden box & 150mm rule



Figure 5: Polycarbonate case

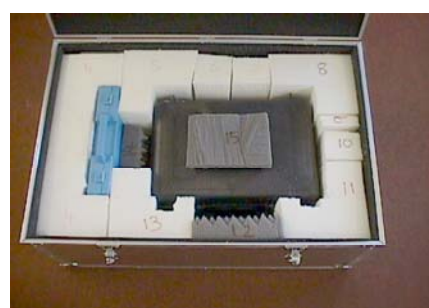


Figure 6: Outer Packing Crate

Appendix E: Receipt Notification Form.

Fax/Email Receipt Notification Fax/Email Receipt Notification Fax/Email Receipt
Notification/Email Receipt Notification

CCE Key Comparison
Intercomparison of Alternating Voltage Ratio

TO Stephen Bryant Room 12 Building 1
Centre for Electromagnetic & Time Metrology ... National Physical Laboratory
Queens Road Teddington TW11 0LW UK

FAX +44 (0)20 8943 6341

FROM.....

Please tick appropriate box and supply details where required.

I confirm receipt of the travelling standard on
(Date)

After a visual inspection, there is no damage to the travelling standard

OR

Damage has been found : (details)

A complete set of adapters has been received in good order.

OR

The following adapters were missing / damaged (details)

Other comments

Signature..... Date.....

Name(BLOCK CAPS)

Appendix F: Example of Report.

Label	Lab	Date	Conn	Volt	Freq	Temp	Tmpu	ipvr	ipvd	ipvu	ipk	qvr	qdf	quv	qk
0.9	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.900000021	200	5.0E-08	2.0	0.000000088	199	7.0E-08	2.0
0.8	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.800000032	200	5.0E-08	2.0	0.000000080	199	7.0E-08	2.0
0.7	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.700000043	200	5.0E-08	2.0	0.000000070	199	7.0E-08	2.0
0.6	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.600000054	200	5.0E-08	2.0	0.000000050	199	7.0E-08	2.0
0.5	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.500000065	200	5.0E-08	2.0	0.000000050	199	7.0E-08	2.0
0.4	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.400000076	200	5.0E-08	2.0	0.000000040	199	7.0E-08	2.0
0.3	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.300000087	200	5.0E-08	2.0	0.000000020	199	7.0E-08	2.0
0.2	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.200000098	200	5.0E-08	2.0	0.000000020	199	7.0E-08	2.0
0.1	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.100000109	200	5.0E-08	2.0	0.000000010	199	7.0E-08	2.0
0.01	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.010000111	200	5.0E-08	2.0	0.000000020	199	7.0E-08	2.0
10/11	NPL	1999-10-28	BPO	10	1000.00	20.0	0.5	0.909090955	200	5.0E-08	2.0	-0.000000010	199	7.0E-08	2.0
9/11	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.818181859	200	5.0E-08	2.0	-0.000000020	199	7.0E-08	2.0
8/11	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.727272763	200	5.0E-08	2.0	-0.000000030	199	7.0E-08	2.0
7/11	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.636363667	200	5.0E-08	2.0	-0.000000030	199	7.0E-08	2.0
6/11	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.545454571	200	5.0E-08	2.0	-0.000000050	199	7.0E-08	2.0
5/11	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.454545475	200	5.0E-08	2.0	-0.000000060	199	7.0E-08	2.0
4/11	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.363636379	200	5.0E-08	2.0	-0.000000070	199	7.0E-08	2.0
3/11	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.272727283	200	5.0E-08	2.0	-0.000000070	199	7.0E-08	2.0
2/11	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.181818187	200	5.0E-08	2.0	-0.000000090	199	7.0E-08	2.0
1/11	NPL	1999-10-27	BPO	10	1000.00	20.0	0.5	0.090909091	200	5.0E-08	2.0	-0.000000070	199	7.0E-08	2.0
0.9	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.900000212	170	16.0E-08	2.1	0.000000650	150	19.0E-08	2.1
0.8	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.800000281	170	16.0E-08	2.1	0.000000580	150	19.0E-08	2.1
0.7	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.700000356	170	16.0E-08	2.1	0.000000510	150	19.0E-08	2.1
0.6	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.600000419	170	16.0E-08	2.1	0.000000440	150	19.0E-08	2.1
0.5	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.500000328	170	16.0E-08	2.1	0.000000370	150	19.0E-08	2.1
0.4	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.400000557	170	16.0E-08	2.1	0.000000300	150	19.0E-08	2.1
0.3	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.300000626	170	16.0E-08	2.1	0.000000230	150	19.0E-08	2.1
0.2	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.200000695	170	16.0E-08	2.1	0.000000160	150	19.0E-08	2.1
0.1	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.100000564	170	16.0E-08	2.1	0.000000120	150	19.0E-08	2.1
0.01	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.010000125	170	16.0E-08	2.1	0.000000120	150	19.0E-08	2.1
10/11	NPL	1999-10-28	BPO	3	55.00	20.0	0.5	0.909090984	170	16.0E-08	2.1	-0.000000050	150	19.0E-08	2.1
9/11	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.818181875	170	16.0E-08	2.1	-0.000000120	150	19.0E-08	2.1
8/11	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.727272766	170	16.0E-08	2.1	-0.000000190	150	19.0E-08	2.1
7/11	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.636363657	170	16.0E-08	2.1	-0.000000260	150	19.0E-08	2.1
6/11	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.545454548	170	16.0E-08	2.1	-0.000000330	150	19.0E-08	2.1
5/11	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.454545439	170	16.0E-08	2.1	-0.000000400	150	19.0E-08	2.1
4/11	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.363636331	170	16.0E-08	2.1	-0.000000470	150	19.0E-08	2.1
3/11	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.272727221	170	16.0E-08	2.1	-0.000000540	150	19.0E-08	2.1
2/11	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.181818112	170	16.0E-08	2.1	-0.000000570	150	19.0E-08	2.1
1/11	NPL	1999-10-27	BPO	3	55.00	20.0	0.5	0.090909003	170	16.0E-08	2.1	-0.000000580	150	19.0E-08	2.1

KEY

Identifier	Description
Label	Nominal Value at each port.
Lab	Unique Designated Identifier
Date	Mean Date of Measurement. Format yyyy-mm-dd
Conn	Connection to standard. See page 5 for key.
Volt	Input Voltage (rms)
Freq	Frequency (Hz)
Temp	Temperature of standard
Tmpu	Temperature Uncertainty
ipvr	In Phase Voltage Ratio
ipvd	In Phase Degrees of Freedom
ipvu	In Phase Voltage Ratio Uncertainty
ipk	In Phase Coverage Factor k
qvr	Quadrature Voltage Ratio
qdf	Quadrature Degrees of Freedom
quv	Quadrature Uncertainty
qk	Quadrature Coverage Factor k

