EURAMET Key Comparison

Luminous Intensity EURAMET.PR-K3.a
and
Luminous Flux EURAMET.PR-K4

Technical Protocol

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1 Introduction

This report describes two international key comparisons of the values of luminous intensity and luminous flux, which are transferred by batches of incandescent lamps from the participants to the pilot laboratory. The Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig (Germany) was asked to act as pilot laboratory for both key comparisons. When it was decided to carry out the EURAMET Key Comparison, the Institute National de Métrologie (BNM-INM / CNAM, France) and the Instituto Nazionale di Ricerca Metrologica (INRIM, Italy) agreed to act as link laboratories for both units.

These comparisons are carried out under the auspices of the European Association of National Metrology Institutes (EURAMET) as the Regional Metrology Organisation (RMO). They are denoted EURAMET-K3a and EURAMET-K4 key comparison for luminous intensity and luminous flux, respectively. Key comparisons are intended to determine the Degrees of Equivalence (DOE) for each participant and the associated expanded uncertainty. The DOE for a quantity states for a participant the relative difference of his value with the related Key Comparison Reference Value (KCRV).

The KCRVs for luminous intensity and luminous flux were determined in former key-comparisons, initialized by the Comité Consultative de Photométrie et Radiométrie (CCPR), denoted as CCPR-K3a and CCPR-K4, respectively, and piloted by the PTB, too. At that time the resulting CCPR-KCRVs were calculated as weighted average from the values of the accepted participants and a minimum cutoff was applied. All results were published [1] in 1999 and the DOEs are listed in the data base [2] of the Bureau Internationale des Poids et Mesures (BIPM). Copies relevant for these comparisons are shown in Annex A.

Meanwhile, the CCPR-KCRVs are maintained over nearly one decade by the participants of the CCPR comparisons and three of the former participants are now link partners for the EURAMET key comparisons. These partners transfer their maintained values by batches of lamps to the pilot laboratory, which measures the lamps and evaluates a weighted average for each quantity as the EURAMET-KCRVs. The average reduces the uncertainty contributions from maintenance, transfer and from the measurements performed at the pilot and link laboratories and furthermore. It ensures that the EURAMET-KCRVs are as close as possible to the original CCPR-KCRVs.

The luminous intensity and luminous flux values transferred from a participant are compared with the related EURAMET-KCRVs and for the two quantities the DOEs with associated uncertainties are evaluated independently of each other. Following the guidelines within the Mutual Recognition Arrangement (MRA) [3] the first results are focused on the consistency of the data sets and relative numbers are collected in a "Draft A" document for the distribution to all participants and link partners. Later, the detailed results will be summarised in a final draft for discussion and an abstract will be published in a metrological journal after the acceptance. Finally, the DOEs of all participants will be added to the BIPM data base.

All participating laboratories are from National Metrological Institutes (NMI) and thus, accredited according to the ISO 17025 [4]. This ensures that the calibration procedures are certified, the units of all references are traceable to national or international standards and the stated uncertainties are evaluated und reported according to the "Guide to the Expression of Uncertainty in Measurement" (GUM) [5]. The pilot laboratory needs these information and especially the complete uncertainty budgets with all correlated contributions for a correct averaging of the measurement results.

The Technical Protocol is divided into two parts, where Part I is dealing with organisational and general information regarding the comparison.

In Part II, the principle models for an evaluation of the measurement results and uncertainties are summarised, to prevent the community in these key comparisons from misunderstanding of calculations and from the need for future explanations about reported data, their meaning, content and presentation. The models are intended to harmonise the use of symbols and terms, they may also guide the participants, how to describe their measurements. They definitely shall not restrict or direct a participant to modify or even change his calibration procedures.

Based on principle methods for the measurement of the two photometric quantities, which are valid for the participants, the link partners and the pilot laboratory, the most general models are presented using either detector based or source based procedures for the calibration of the transfer standards. Examples, how to report the data are given additionally. The calibration methods at the pilot laboratory are explained and the evaluation and reporting of the raw-data in relative presentation are shown. Finally, the determination of the DOE with associated uncertainty is explained. These principle steps in the main chapters are supplemented with more details in several annexes.

2 Part I:

2.1 Organization

2.1.1 Participants

The list of participants which was submitted to the CCPR for approval, was drafted by the pilot laboratory taking account of the RMOs of EURAMET. All participants must be able to demonstrate independent traceability to the realization of the quantity, or make clear the route of traceability to the quantity via another named laboratory. By their declared intention to participate in this key comparison, the laboratories accept the general instructions and the technical protocols written down in this document and commit themselves to follow the procedures strictly. Once the protocol and list of participants has been agreed, no change to the protocol or list of participants may be made without prior agreement of all participants.

2.1.2 Participants' details

Contact	Institute	Contact Details	Shortcut
Nikolay Alexandrov	Bulgarian Institute for Metrology 52-B, G.M. Dimitrov Blvd. 1125 Sofia Bulgaria	Tel. +359 2 974 31 61 Fax +359 2 974 08 96 Email: nikal_alex@abv.bg	BIM
Anne Andersson	Swedish National Testing and Research Institute P.O. Box 857 SE-501 15 Borås Sweden	Tel. +46 105 165403 Fax +46 33 16 56 20 Email:anne.andersson@sp.se	SP
Pasi Manninen	Helsinki University of Technology and Centre for Metrology Metrology Research Institute P.O. Box 3000, FI-02015 TKK, Finland	Tel. +358 9 451 2308 Fax +358 9 451 2222 Email: pasi.manninen@tkk.fi	MIKES
Dorota Soboto	Central Office of Measures Optical Radiation Division Elektrralna 2; 00-950 Warsaw Poland	Tel. +48 22 581 9295 Fax +48 22 581 9388 Email: radiation@gum.gov.pl	GUM

Roman Dubnicka	Slovak Institute of Metrology Karloveská 63 SK-842 55 Bratislava Slovakia	Tel: +421 2 602 94 247 Fax: +421 2 602 94 521 Email: rdubnicka@smu.gov.sk	SMU
Olivier Pellegrino	Instituto Português da Qualidade Laboratório Central de Metrologia Rua António Gião PT-2829-513 Caparica Portugal	Tel: +351 21 294 81 79 Fax: +351 21 294 81 88 Email: opellegrino@mail.ipq.pt	IPQ
Peter Rosenkranz	Bundesamt für Eich- und Vermessungswesen Gruppe Eichwesen (Metrology Service) Arltgasse 35 AT-1160 Wien Austria	Tel: +43 1 21110 6415 Fax: +43 1 21110 6340 Email: p.rosenkranz@metrologie.at	BEV
Mihai Simionescu	National Institute of Metrology Şos. Vitan-Bârzeşti 11, Sector 4 Bucureşti 042122 Romania	Tel: +40 21 334 50 60 Fax: +40 21 334 53 45 Email: mihai.simionescu@inm.ro	INM
Marek Smid	Czech Metrology Institute V Botanice 4 CZ-150 72 Praha 5 Czech Republic	Tel: +420 602 751 168 Fax: +420 257 328 077 Email: msmid@cmi.cz	CMI
Kamuran Türkoğlu	Ulusal Metroloji Enstitüsü TÜBITAK-UME P.O. Box 54, TR-Gebze Kocaeli 41470, Turkey	Tel: +90 262 679 50 00 Ext: 3353 Fax: +90 262 679 50 01 Email: akt@ume.tubitak.gov.tr	UME
H.C.D. (Daniel) Bos	NMi Van Swinden Laboratorium B.V. Thijsseweg 11 2629 JA Delft Netherlands	Tel: +31 15 269 15 96 Fax: +31 15 261 29 71 Email: Dbos@Nmi.nl	NMi VSL
Pedrag Vukadin	Bureau of Measures and Precious Metals, Mike Alasa 14 YU - 11 000 Beograd Serbia	Tel: +381 11 3282 736 Fax: +381 11 21 81 668 Email: vukadin@szmdm.sv.gov.yu	BMPM
Olga B. Tarasova	Belarussian State, Institute of Metrology 93, Starovilensky trakt Minsk, 220053 Belarus	Tel: +375 17 23 4 98 20 Fax: +375 17 28 80 938 Email: khairova@belgim.by	BelGIM

Maria Luisa Rastello	Istituto Nazionale di Ricerca Metrologica, Strada delle Cacce 73 IT-10135 Torino Italy	Tel: +39 011 3919 219 Fax: +39 011 3463 84 Email: rastello@inrim.it	INRIM
Gael Obein	Institut National de Métrologie, 61 rue du Landy 93210 La Plaine Saint-Denis, France	Tel: + 33 1 58 80 87 88 Fax: + 33 1 58 80 89 00 Email: gael.obein@cnam.fr	CNAM
Armin Sperling	Physikalisch-Technische Bundesanstalt 4.12 Photometry Bundesallee 100 D 38116 Braunschweig Germany	Tel: +49 531 592 4120 Fax: + 49 531 592 4170 Email: Armin.Sperling@ptb.de	PTB

2.2 Form of comparison

The comparison will principally be carried out by the calibration of a group of transfer standard lamps. The used type of lamps have to show a reasonable stability and robustness. If used with care, they should be capable to transfer the luminous intensity and luminous flux quantities, which is maintained at a participating laboratory, to the pilot laboratory and vice versa. Each participant will make his own set of lamps available to minimise the effects of ageing and transfer. It is also necessary that these sets of lamps will reside at the participant to allow the participant directly to maintain the compared quantity.

A full description of the transfer standard lamps selected for use in this comparison was enquired in advance by a questionnaire and is given in Chapter 2.21 of this protocol. The minimum set of any transfer standards used for this comparison should be a group of four lamps. This minimises the risk of unknown drift and damage and improves the ascertainment of the participants Degree Of Equivalence (DOE) (see Technical Protocol, Part II).

All transfer standard lamps provided by the participants and used in this comparison, have to be seasoned. They have to be sent together with detailed operation conditions used at the participant laboratory (including pictures). If expedient, a suitable holder may be supplied, too.

The comparison will take the form of a star-type comparison, carried out in 1 phase for each, luminous intensity and luminous flux calibration. The artefacts (lamps) will initially be calibrated by the participant laboratory. They will then be sent to the pilot laboratory to perform calibration according to the KCRV. The standard lamps will then be returned to the participant laboratory to carry out a repeated calibration to monitor drift. For the details regarding the analysis of the lamp data see Technical Protocol Part II.

PTB will act as the pilot laboratory. INRIM and CNAM will act as link laboratories. All results are to be communicated directly to the pilot laboratory as soon as possible and certainly within 6 weeks of the completion of all the measurements by a laboratory.

The participating laboratories are asked to specify a preferred timetable slot for their own measurements of the transfer standard lamps - the timetable given below has been drawn up taking the last preferences into account if possible (may be readapt according until start of comparison).

Within the time schedule, each laboratory has 8 weeks for calibration and transportation. With its confirmation to participate, each laboratory has confirmed that it is capable to perform the measurements in the time allocated to it. It ensures that the relatively short timetable to complete the comparison is met.

If for some reasons, the measurement facility is not ready or customs clearance takes too much time in a country, the participating laboratory must contact the pilot laboratory immediately to discuss further details and changes of the measurement timetable. It may be possible for the participant to continue to take part by returning the calibrated lamps back to the pilot laboratory at an agreed later date. However, in view of the large amount of work for the pilot laboratory and the need for a strict timetable to allow the comparison to take place, this may not be possible. If this is the case the participant and their results may have to be excluded from the final report. Exclusion may also occur if the results are not available in time to prepare the draft report.

2.2.1 Timetable

Intensity	Time Schedule (Draft)	Flux
Action to at the latest at the end of:	**Action**	Action to at the latest at the end of:
cleared	receipt of the "Announcement" and „Reply form“ by the participants	**cleared**
cleared	„Reply form“ sent to PTB	**cleared**
cleared	coordination between link laboratories regarding the technical protocol	**cleared**
cleared	receipt of the „Technical Protocol“ by the participants	**cleared**
cleared	comments sent to PTB	**cleared**
4/2008	1st calibration of transfer standards (participant)	7/2008
5-6/2008	transportation of transfer standards to PTB	9/2008
7/2008	measurements at PTB	11/2008
9/2008	transportation of transfer standards to participants	1/2009
11/2008	2nd calibration of transfer standards (participant)	3/2009
2/2009	final data from participants arrive at PTB	5/2009
	…….	
End of 2009	draft A of report	End of 2009
2010	final report	2010

2.3 Handling of artefacts

If ever possible, standard lamps should be transported by hand-carriage from the participant to the pilot laboratory and back again to the participant. The standard lamps should only be handled by authorized persons and stored and packed in such a way as to prevent damage.

The standard lamps should be examined immediately upon receipt at final destination. However, care should be taken to ensure that the lamps and packaging have sufficient time to acclimatise to the actual environment thus preventing any condensation etc. The condition of the lamps and associated packaging should be noted and communicated to the pilot laboratory. Please use the fax form in Annex I.

After the very first calibration at the participant no cleaning of any lamp windows, apertures or envelopes should be attempted. No parts other than noted within operating conditions belonging to specific lamps should be removed from or connected to this lamp. If a standard lamp appears damaged a replacement if possible will be only available from the participant laboratory. However, appropriate insurance should be taken out by participating laboratories to cover the cost of such a replacement if the damage occurred in transit.

During operation of the standard lamps any unusual occurrence, e.g. change of voltage, change in output etc. should be notified immediately to the pilot laboratory before proceeding.

Please inform the pilot laboratory via fax or e-mail when the measurement on the standard lamps are completed to arrange a suitable date for transportation or dispatch.

After the measurements, the lamps should be re-packaged in their original transit cases. Ensure that the content of the package is complete before shipment. Always use the original packaging.

2.4 Transport of artefacts

It is of utmost importance that the artefacts be transported in a manner in which they will not be lost, damaged or handled by un-authorised persons.

Packaging for the artefacts should be made which is suitably robust to protect the artefacts from being deformed or damaged during transit.

Artefacts (luminous intensity and luminous flux lamps) should as a preference be carried by hand between participating laboratories, either by personal road transport, sea, or in an aircraft cabin. However, recognising that this may result in high financial costs to some participants and recognising that the lamp systems are fragile and may be subject to change in their characteristics from transportation, it should be noted that there is as yet no conclusive evidence indicating whether lamps are less likely to change when hand carried as opposed to a careful postal service. They should under all circumstances be marked as ‘Fragile’.

The artefacts should be accompanied by a suitable customs carnet (where appropriate) or documentation identifying the items uniquely. The participants have to pay attention to the import/export regulations during transport, which may be different for every country. The packaging should be lockable e.g. by clasp, so that it will be easy to open with minimum delay to allow customs inspections to take place.

The participating laboratories are responsible for the transport of their own lamps and the costs involved. Each participating laboratory covers the costs for its own measurements, transportation and any customs charges as well as for any damages that may have occurred within its country. The overall costs for the organisation of the comparison are covered by the pilot laboratory. The pilot laboratory has no insurance for any loss or damage of the standards during transportation.

2.5 Description of the standards

2.5.1 Transfer Standards used within the comparison

The measurement artefacts are specially developed transfer standard lamps for luminous intensity and luminous flux. The use of these lamps was decided and determined by the participants on request of the pilot laboratory.

Luminous intensity transfer standards	Luminous flux transfer standards
 I1: “OSRAM WI41/G”	 F1: “POLARON LF200W”
 I2: “OSRAM WI40/G”	 F2: “POLARON LF200W-ZZ”

 I3: “POLARON (similar to LIS-T75)”	 F3: “OSRAM WI5”
 I4: “POLARON LIS/IF/IC/S”	 F4: “OSRAM WI40/Globe”
 I5: “SIS 40-100”	 F5: “OSRAM WI40/G”

No picture available I6: “FEL”	 F6: “GEC-Hirst 200W”
	 F7: “MAZDA 115V, 200W”
	No picture available F8: “FEL”

Figure 1: Different types of standard luminous intensity lamps (left column) and standard luminous flux lamps (right column) delivered by the participants and used within this comparison.

Expected types and number of lamps use by the participants within the comparison

Institute	Luminous intensity lamp types	Luminous flux lamp types
BIM	4 pieces I1	4 pieces F4
SP	3 pieces I1	3 pieces F6
MIKES	4 pieces I1	4 pieces F4
GUM	5 pieces I1	4 pieces F1
SMU	4 pieces I4	
IPQ	4 pieces I1	
BEV	4 pieces I1	4 pieces F7
INM	4 pieces I1 & I2	6 pieces F3
CMI	4 pieces I1	4 pieces F3
UME	6 pieces I1	5 pieces F1
NMi VSL	3 pieces I1 & 3 pieces I6	3 pieces F5 & 3 pieces F8
BMPM	4 pieces I1 & 4 pieces I3 (GEC T75-375)	6 pieces F5
BelGIM	4 pieces I5	
INRIM	6 pieces I1	4 (6) pieces F1
CNAM	4 pieces I1 & 4 pieces I3	6 pieces F1
PTB	6 pieces I1	6 pieces F1 & F2

2.6 Measurement Conditions

2.6.1 Traceability

Length measurements should be independently traceable to the latest realisation of the meter.

Temperature measurements should be made using the International Temperature Scale of 1990 (ITS-90).

Electrical measurements should be independently traceable to the latest realisations of the amp and volt.

2.6.2 Measurands

The measurand is the luminous intensity or luminous flux of a lamp. These photometric quantities should be measured for the defined operating conditions of each lamp, where the operating current (or in some special cases the operating voltage) acts as the setting parameter. The measurements should be performed in suitable laboratory accommodation maintained at a temperature of 20 to 25 °C. The temperature of the laboratory during the time of the measurements should be reported.

The luminous intensity of the appropriate lamp should be measured independently at least 2 times; the luminous flux should be measured independently at least 2 times. Each independent measurement should consist of the lamp being realigned in the measurement facility and being switched off and on after a break of at least 1 h for each lamp. Each independent measurement set should be reported. It should be noted that each independent measurement may consist of more than one set of measurements, the exact number should be that normally used by the participating laboratory to obtain the appropriate accuracy as limited by the noise characteristics of their specific measurement facility. The exact number of measurements used should be stated in the measurement report but only the mean or final declared value of the set is required to be included. Participants are reminded that the luminous intensity as well as the luminous flux of the transfer standard lamps will change as a function of the operational burning time and so it is recommended that this is kept to a minimum.

The measurements should be taken at a sufficient large distance. At PTB, measurement will also be carried out at that distance which was used by the participant to correct for the influence of different solid angles used.

2.6.3 Geometrical conditions

All participants have to describe their geometrical measurement conditions using Annex G. The basic conditions used within this comparison are as follows:

Luminous intensity standard lamps:

- The optical axis of the bench is horizontal and central to the filament.
- The distance is measured from the centre of the filament.
- The plane, containing optical axis and lamp axis (cap down) is vertical.
- For lamp types I1, I3, and I4, only the light passing through the square opening

on the front mask is measured (light from outer bulb is shielded).

- For all other types of lamp, light including from the entire bulb should be measured. Details especially for the WI40/G have to be stated by the participants laboratory using Annex G.
- For lamp I6 (FEL), the light from the base part of the lamp (below quartz bulb) must be shielded. If an alignment jig is used, the optical axis of the lamp is perpendicular from the alignment jig surface. The alignment jig used by the participant must be shipped with the lamp. Details have to be stated by the participants laboratory using Annex G
- For each type of luminous intensity lamp, a recommended measurement distance will be agreed in advance to measurements among the pilot and the participants. (This does not mean every lab has to use that distance. Each participant, and pilot also, will need to evaluate related uncertainty if their measurement distance is significantly different from that).
- OSRAM: optical axis is rectangular to the filament plane.
- Polaron: optical axis is rectangular to the flat window. Special lamp holders should be shipped with the lamps.
- FEL lamps: the beginning of the light path and the measurement direction has to be determined by the participant.

Luminous flux standard lamps:

- Lamp axis (cap up) is vertical.
- All light emitted by the lamp will be measured.

2.6.4 Electrical conditions

All transfer standard lamps have to be operated with DC power where the lamp current (or lamp voltage) is stabilized. If not stated elsewhere, lamp voltages should be measured using 4-pole technique directly at the cap. Some lamps mounted in specific holders do have separate contacts for current supply and voltage measurement.

The Polaron lamps used for luminous intensity should be measured in true 4-pole technique (see Fig. 2), using temporary contacts and ensuring low uncertainty of lamp voltage values. Due to this technique, the readings for this lamps voltage have a high repeatability.

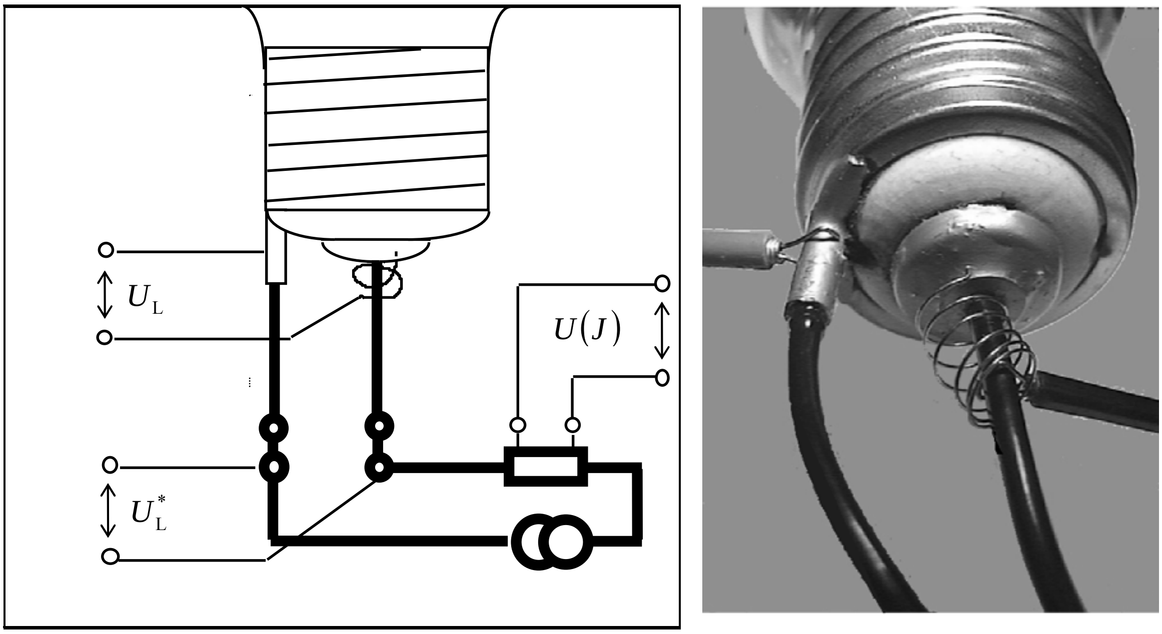

Figure 2: (*left)* schematic diagram showing the circuitry normally used for Polaron luminous intensity lamps. Instead of using the Voltage drop U^*_L at the high current clamps, which is influenced by the current flow over the contact resistance, the lamp voltage U_L at the soldered thread should be used.

(right) photo of the cap with clamps as additional contacts for voltage measurement

2.6.5 Measurement instructions

Before connecting to any electrical power supply, the standard lamps should be inspected for damage or contamination of either the window of the lamp, the cap or its supporting mount. Any damage should be documented by photos and a drawing using the appropriate form in Annex F and the pilot laboratory should be informed immediately.

Before switching on the current for any lamp, an appropriate time recording device and notebook should be established to allow the operation time for each lamp to be recorded. An example form which could be used has been attached as Annex H. A summary sheet such as this should be completed for and kept with each lamp and returned to the pilot laboratory as part of the final report.

After connecting the electrical power to the lamps, the prescribed warm-up procedure for each lamp should be followed. Operational parameters for each lamp (specified in the lamp operating procedure) should be recorded and compared to those supplied with the lamp.

The operational conditions and alignment procedure for each lamp should be noted and followed according to the details described in the notes supplied with each lamp. A photograph should be taken from the lamp installed and kept by the participants for documentation and quality insurance.

The photometric quantity (luminous intensity or luminous flux depending on the lamp and the part of the comparison) of the transfer standards should then be measured together (at the same time if possible) with the electrical values.

The signed results of the measurements together with the operating condition (e.g. lamp number, current, voltage, distribution temperature or corr. colour temperature) and the uncertainty (k=1) have to be posted to the pilot laboratory by FAX or regular mail. Electronic mail may be used additionally for convenience and rapid response.

The value of the distribution temperature or corr. colour temperature should be within the range of 2600 K to 3200 K. This value has to be determined separately to correct for the colour mismatch of the used photometer if necessary.

No other measurements are to be attempted by the participants nor any modification to the operating conditions during the course of this comparison. The transfer standards used in this comparison should not be used for any purpose other than described in this document nor given to any party other than the predetermined participants in the comparison.

Any information obtained relating to the use or any results obtained by a participant during the course of the comparison shall be sent only to the pilot laboratory who will be responsible for co-ordinating how the information should be disseminated to other participants. No communication whatsoever regarding any details of the comparison other than the general conditions described in this protocol shall occur between any of the participants or any party external to the comparison without the written consent of the pilot laboratory. The pilot laboratory will in turn seek permission of all the participants. This is to ensure that no bias from whatever accidental means can occur.

3 Part II:

Models for the Evaluation of Values, Uncertainties and Degrees of Equivalence

G. Sauter 2008

3.1 Principles

3.2 Measurement of Photometric Quantities

3.2.1 Fundamental Models for the Evaluation of Photometric Quantities

The transfer standards in the EURAMET key comparisons of photometric quantities are incandescent lamps owned by the participants. Their relative spectral distributions $S(\lambda)$ are similar to a Planckian radiator and characterised by distribution temperatures T.

For the measurement of luminous intensity $I(T)$ the transfer standard and a photometer head are aligned to the optical axis of a photometer bench and located in a distance d, which is sufficiently large for a valid inverse squared law. The transfer standard illuminates the entrance aperture of a photometer head with the illuminance $E(T)$.

The different directions of emittance of a transfer standard are stated by spherical coordinates $\{r,\vartheta,\varphi\}$. Usually, lamps selected as transfer standards for luminous flux have a distribution temperature T not much depending on the direction. Their illuminances $E(\vartheta,\varphi,T)$ on a fictitious surface totally enclosing the luminous flux standard can be described by a product of two factors: The illuminance $E_{\mathrm{G}}(T)=E(\vartheta_0,\varphi_0,T)$ in a fixed direction ϑ_0,φ_0 with a constant value characterised by a distribution temperature T, and a relative function $g(\vartheta,\varphi)=E(\vartheta,\varphi,T)/E(\vartheta_0,\varphi_0,T)$ of the angular distribution, which is normalised to unity in the specified direction.

The (total) luminous flux of a transfer standard is measured with a goniophotometer or an integrating sphere. A goniophotometer (index "G") with radius r_{G} measures the angular variation of the illuminance $E(\vartheta,\varphi,T)=E_{\mathrm{G}}(T)\cdot g(\vartheta,\varphi)$. The integral G (see equ. [1]) of the relative angular distribution $g(\vartheta,\varphi)$ has the dimension of a solid angle. It is taken over the full solid angle and determined by numerical evaluation.

An integrating sphere (index "S") with inner surface area $4\,\pi\cdot r_{\mathrm{S}}^2$ (r_{S} is radius) and internal coating with reflectance ρ_{S} is characterised by the throughput τ_{S}. The installed photometer measures the indirect illuminance $E_{\mathrm{S}}(T)$ generated by the luminous flux

$\Phi(T)$ of the transfer standard inside the sphere. These definitions lead to the fundamental measurement equations.

$$\begin{aligned}
&I(T) = d^2 \cdot E(T) \cdot \Omega_0^{-1} \\
&\Phi(T) = r_G^2 \cdot E_G(T) \cdot G; \qquad G = \int_0^{\pi} \left[\int_0^{2\pi} g(\vartheta, \varphi)\, \mathrm{d}\varphi \right] \sin\vartheta \, \mathrm{d}\vartheta \\
&\Phi(T) = \tau_S \cdot E_S(T); \qquad \tau_S = 4\,\pi \cdot r_S^2 \cdot \rho_S / (1 - \rho_S)
\end{aligned} \tag{1}$$

Note: The unit of solid angle $\Omega_0 = 1\,\mathrm{sr} = 1$ is added for consistency of the dimensions in the first equation. This will not be repeated in this document to get shorter writings of the equations.

3.2.2 Detector-based Calibration of Photometric Quantities

The illuminances in these equations are measured as output signals y of photometers with luminous responsivities s_v and current-to-voltage converters with a gain setting resistor R_g. The photometer heads are matched to $V(\lambda)$ with mismatch corrections $F(S(\lambda))$ for any relative spectral distribution $S(\lambda)$ of the source.

Due to the (nearly) Planckian distributions $P(\lambda,T)$ of incandescent lamps used as transfer standards, an approximation $F(P(\lambda,T)) \approx (T/T_A)^m$ is valid with an exponent m denoted as mismatch index, and the distribution temperature $T_A = 2856\,\mathrm{K}$ referring to CIE illuminant A (see Annex B).

$$E(T) = \frac{y}{R_g \cdot s_v} \left(\frac{T}{T_A} \right)^m \tag{2}$$

Principle models for an evaluation of the values of transfer standards and valid for <u>detector based calibrations</u> are received from the combination of equ. (1) with equ. (2).

In goniophotometers the output voltage of the photometer may be converted by an additional conversion factor w in a frequency, which allows for an arbitrary timing of the readings.

The reflectance of the sphere coating depends marginally on wavelengths, which acts as a modification of the $V(\lambda)$-match of the photometer head and so the mismatch index of the photometer head is replaced by a mismatch index m_S of the complete sphere photometer. The equation for the integrating sphere is in brackets to remind, that it cannot be used directly, because the throughput τ_S has to be determined separately by substitution method.

$$I(T) = d^2 \cdot \frac{y}{R_g \cdot s_v} \left(\frac{T}{T_A} \right)^m; \qquad \begin{aligned} &\Phi(T) = r_G^2 \cdot \frac{y_G}{w \cdot R_G \cdot s_v} \left(\frac{T}{T_A} \right)^m \cdot G \\ &\left[\Phi(T) = \tau_S \cdot \frac{y_S}{R_S \cdot s_v} \left(\frac{T}{T_A} \right)^{m_S} \right] \end{aligned} \tag{3}$$

3.2.3 Source-based Calibration of Photometric Quantities

For a source based calibration the responsivity s_v in the equations is eliminated by substitution method using reference standards with properties similar to those of the transfer standards and referred by symbols marked with index "R". Participants using different types of references for the calibration of their goniophotometer or integrating sphere will find models for the evaluation in Annex C.

More effects from the operational conditions for the standards are summarised in correction factors $corr, corr_\mathrm{G}, corr_\mathrm{S}$ with values close to unity, but different for the various measurement methods. These are explained in Annex D.

$$I(T) = I_\mathrm{R}(T_\mathrm{R}) \cdot \frac{y \cdot R_\mathrm{g,\,R}}{y_\mathrm{R} \cdot R_\mathrm{g}} \cdot \left(\frac{T}{T_\mathrm{R}}\right)^m \cdot \left(\frac{d}{d_\mathrm{R}}\right)^2 \cdot corr$$

$$\Phi(T) = \Phi_\mathrm{R}(T_\mathrm{R}) \cdot \frac{y_\mathrm{G} \cdot R_\mathrm{G,\,R}}{y_\mathrm{G,\,R} \cdot R_\mathrm{G}} \left(\frac{T}{T_\mathrm{R}}\right)^m \cdot \frac{G}{G_\mathrm{R}} \cdot corr_\mathrm{G} \tag{4}$$

$$\Phi(T) = \Phi_\mathrm{R}(T_\mathrm{R}) \cdot \frac{y_\mathrm{S} \cdot R_\mathrm{S,\,R}}{y_\mathrm{S,\,R} \cdot R_\mathrm{S}} \left(\frac{T}{T_\mathrm{R}}\right)^{m_S} \cdot \frac{\tau_\mathrm{S}}{\tau_\mathrm{S,\,R}} \cdot corr_\mathrm{S}$$

Note: In most cases, the radius r_G of a goniophotometer in equ. [1] is constant and its uncertainty contribution is negligible. Therefore it may be cancelled out during substitution. Nevertheless, for special goniophotometer with adjustable radius an additional term $\left(\frac{r_\mathrm{G}}{r_\mathrm{G,R}}\right)^2$ may be necessary.

3.3 Degrees of Equivalence

3.3.1 Equivalence with KCRV

There are two statements on the Degree Of Equivalence (DOE): One defines the equivalence D_i of the values of one participant with the KCRV, the "reference value of a key comparison" and the other statement deals with the equivalence D_{ij} with the "value of a second participant".

The DOE D_i for the i-th participant with the (mean) value x_i of a quantity X determined in a CCPR key comparison with reference value x_R denoted as CCPR-KCRV is defined as the ratio of the two values subtracted by unity. The associated uncertainty $u(D_i)$ is presented using the approximation $x_i / x_\mathrm{R} \approx 1$.

$$D_i = \frac{x_i}{x_\mathrm{R}} - 1; \qquad u(D_i) \approx \sqrt{u_\mathrm{rel}^2(x_i) + u_\mathrm{rel}^2(x_\mathrm{R})} \tag{5}$$

The relative uncertainties $u_{\mathrm{rel}}(x_{\mathrm{R}})$ associated to CCPR-KCRVs of the key comparison of luminous intensity $u_{\mathrm{rel}}(x_{\mathrm{R}}) = 0.09\ \%$ and luminous flux $u_{\mathrm{rel}}(x_{\mathrm{R}}) = 0.1\ \%$ were found in Annex A to be negligible, if compared with the relative uncertainties $u_{\mathrm{rel}}(x_i)$ associated to the values of all participants in these comparisons.

The values D_i with associated expanded uncertainties $U(D_i) = 2\,u(D_i)$ are given in the BIPM-database and those of the three link partners in the EURAMET key comparisons described in this report are shown in Table 1 using a percentage presentation.

Table 1 DOE values D_i and associated expanded $k = 2$ uncertainties $U(D_i)$ taken from BIPM database [2] for the two photometric quantities and the three link partners for the EURAMET key comparisons of the same quantities.

	PTB	IEN (INRIM)	BNM (LNE-INM/CNAM)
luminous Intensity	(-0.31 ± 0.40)%	(-0.43 ± 0.90)%	(0.89 ± 0.60)%
luminous flux	(-0.42 ± 0.56)%	(-0.06 ± 0.96)%	(0.69 ± 0.58)%

For RMO key comparisons (superscript "(m)"), the related reference values $x_{\mathrm{R}}^{(\mathrm{m})}$ denoted as RMO-KCRV are evaluated by the pilot laboratory as weighted mean of the values transferred from at minimum two link partners. The relative difference between this reference value and that of the CCPR-KCRV x_{R} should be negligible. For an analysis of the uncertainty, the relative difference is accounted as a factor $\varepsilon^{(\mathrm{m})}$ with a value $\left|\varepsilon^{(\mathrm{m})}\right| << 1$ and an associated uncertainty $u\left(\varepsilon^{(\mathrm{m})}\right)$. The DOE $D_i^{(\mathrm{m})}$ with uncertainty $u\left(D_i^{(\mathrm{m})}\right)$ is evaluated similar to the definition in equ. (5) and the uncertainty is presented using the approximation $\left|D_i^{(\mathrm{m})}\right| << 1$.

$$D_i^{(\mathrm{m})} = \frac{x_i}{x_{\mathrm{R}}\left(1 + \varepsilon^{(\mathrm{m})}\right)} - 1; \qquad u\left(D_i^{(\mathrm{m})}\right) \approx \sqrt{u_{\mathrm{rel}}^2(x_i) + \left[u_{\mathrm{rel}}^2(x_{\mathrm{R}}) + \frac{1}{x_{\mathrm{R}}^2} u_{\mathrm{rel}}^2\left(\varepsilon^{(\mathrm{m})}\right)\right]} \tag{6}$$

The uncertainties associated to the DOEs of different participants are correlated because of the identical uncertainty contributions stated in brackets in equ. (6). Provided, the key comparison would be performed with negligible uncertainty contributions associated to the reference value and to the relative difference, then the uncertainty of the DOE can be approximated by $u\left(D_i^{(\mathrm{m})}\right) \approx u_{\mathrm{rel}}(x_i)$ with negligible correlation.

3.3.2 Equivalence with the Value of a Participant

The DOE D_{ij} for the values of two participants (indexes "i" and "j") is defined as the difference $D_{ij} = D_i - D_j$ and determined from the values x_i and x_j of participants in the same key comparison with DOEs D_i and D_j.

In general, the participants may have joined two different RMO key comparisons marked by superscripts "(1)" and "(2)". Then the DOE D_{ij} is the difference of twice the equ. (6) with an associated uncertainty $u(D_{ij})$, which is presented using the approximations $x_i / x_\mathrm{R} \approx 1$, $x_j / x_\mathrm{R} \approx 1$, $\left|\varepsilon^{(1)}\right| << 1$, $\left|\varepsilon^{(2)}\right| << 1$.

$$D_{ij} = \frac{x_i}{x_\mathrm{R} \cdot \left(1 + \varepsilon^{(1)}\right)} - \frac{x_j}{x_\mathrm{R} \cdot \left(1 + \varepsilon^{(2)}\right)}; \qquad u(D_{ij}) \approx \sqrt{u_\mathrm{rel}^2(x_i) + u_\mathrm{rel}^2(x_\mathrm{j}) + u^2\left(\varepsilon^{(1)}\right) + u^2\left(\varepsilon^{(2)}\right)} \tag{7}$$

It should be noted, that the uncertainty $u(D_{ij})$ is

- independent of the uncertainties associated to the KCRVs for all participants, who joined the same comparison (CCPR or RMO).
- increased by contribution $u^2\left(\varepsilon^{(m)}\right)$, if one participant joined an RMO comparison.
- enlarged by variances $u^2\left(\varepsilon^{(1)}\right), u^2\left(\varepsilon^{(2)}\right)$, if different RMO comparisons are involved.

3.4 Transfer Standards and Measurements

3.4.1 Elimination of a Correlation by Factorisation

The $1 \le i \le ni$ participants transfer their units realised for a photometric quantity X with batches of $1 \le k \le nk_i'$ lamps. The photometric values $x_{i\,k}$ within one batch are measured by the same procedure and they are traceable to the maintained national or international standard. In the uncertainty budgets for these values most of the contributions dealing with mechanical, electrical, thermal, temporal and optical properties are constant for all transfer standards and mainly the contributions from stability and alignment show variations for individual lamps. Therefore, all the uncertainties $u(x_{i\,k})$ associated to the values $x_{i\,k}$ are correlated, because of the constant part.

This correlation is eliminated, if the value $x_{i\,k}$ is presented as a product of two factors C_i and $y_{i\,k}$. The first factor $C_i \equiv 1$ is unity with an associated relative uncertainty $u_\mathrm{rel}(C_i)$ combined from all constant contributions. The second factor $y_{i\,k}$ with just the value $x_{i\,k}$ and an associated relative uncertainty $u_\mathrm{rel}(y_{i\,k})$, which combines only contributions individual for each lamp (see Annex E). This factorisation is valid for all values transferred with batches of transfer standards from participants and from link partners. It is also valid for the EURAMET-KCRV, realised at the pilot laboratory from the values transferred by the link partners.

$$x_{i\,k} = C_i \cdot y_{i\,k}; \qquad u_{\text{rel}}\left(x_{i\,k}\right) = \sqrt{u_{\text{rel}}^2\left(C_i\right) + u_{\text{rel}}^2\left(y_{i\,k}\right)} \tag{8}$$

A participant of a comparison takes part for each quantity with a batch of up to six transfer standards of the same type of lamps (or up to two times four standards of two different types). He reports two sets of data marked as "initial" and "return" for one quantity. These data sets are measured before travelling the standards to the pilot laboratory and after their return. A change of a transfer standard due to the shipment is unlikely, if the batch was hand-carried or transported with a professional packing.

Damage by travelling or outliers due to any other reason should not affect the result of a comparison for a participant. Therefore, <u>on his request</u> one of the six transfer standards (or two of the eight) can be withdrawn. The values of the remaining transfer standards in the batch will be used for the comparison and the associated uncertainties with possible correlations are taken into account.

3.4.2 Identification of Transfer Standards with Instabilities

All transfer standards are measured at minimum four times: A participant measures his standards twice -before and after the shipment to the pilot laboratory- with values denoted as "initial" $x_{i\,k}^{(\mathrm{i})} = C_i \cdot y_{i\,k}^{(\mathrm{i})}$ and "return" $x_{i\,k}^{(\mathrm{r})} = C_i \cdot y_{i\,k}^{(\mathrm{r})}$ and the pilot laboratory repeats all his measurements with values marked as $x_{i\,k}^{(1)}, x_{i\,k}^{(2)}$.

An instable transfer standard is identified by the participant from the analysis of the related values. The difference $y_{i\,k}^{(\mathrm{i})} - y_{i\,k}^{(\mathrm{r})}$ between his values normalised by their mean $\left(y_{i\,k}^{(\mathrm{i})} + y_{i\,k}^{(\mathrm{r})}\right)/2$ is denoted $\Delta r_{i\,k}^{(\mathrm{ir})}$. Obviously, the relative difference is independent of the magnitude of the values and the common factor C_i cancels out.

The values $\left|y_{i\,k}^{(\mathrm{i})} - y_{i\,k}^{(\mathrm{r})}\right| << 1$ are very similar and usually so are the associated uncertainties $u_{i\,k} \cong u\left(y_{i\,k}^{(\mathrm{i})}\right) \cong u\left(y_{i\,k}^{(\mathrm{r})}\right)$. Thus, the uncertainty $u\left(\Delta r_{i\,k}^{(\mathrm{ir})}\right) \cong \left|u_{i\,k}^{(\mathrm{ir})} / y_{i\,k}^{(\mathrm{ir})}\right|\sqrt{2}$ associated to the relative difference $\Delta r_{ik}^{(\mathrm{ir})}$ is simplified to the relative uncertainty of one of the original values multiplied by $\sqrt{2}$.

$$\Delta r_{i\,k}^{(\mathrm{ir})} = 2\frac{y_{i\,k}^{(\mathrm{i})} - y_{i\,k}^{(\mathrm{r})}}{y_{i\,k}^{(\mathrm{i})} + y_{i\,k}^{(\mathrm{r})}}; \qquad u\left(\Delta r_{i\,k}^{(\mathrm{ir})}\right) = \frac{u_{i\,k}^{(\mathrm{ir})}}{y_{i\,k}^{(\mathrm{ir})}}\sqrt{2} \tag{9}$$

A list of absolute differences $\left|\Delta r_{i\,k}^{(\mathrm{ir})}\right|$ for the lamps in a batch allows the ranking for stability and the worst lamp may be identified. If the stability is significantly too bad, then -on the demand of the participant- a lamp can be excluded from the comparison and the remaining number of transfer standards in a batch is reduced by one $nk_i = nk_i' - 1$.

Significance means, that the individual relative difference $\Delta r_{i\,k}^{(ir)}$ exceeds the average $\overline{\Delta r}_{i}^{(\mathrm{ir})}$ of all the relative differences of a batch by more than the associated combined uncertainty in an expanded presentation: $E_n > 1$.

$$E_n = \frac{\left|\Delta r_{i\,k}^{(\mathrm{ir})} - \overline{\Delta r}_{i}^{(\mathrm{ir})}\right|}{t_p\left(nk_i'-1\right)\sqrt{u^2\left(\Delta r_{i\,k}^{(\mathrm{ir})}\right)+u^2\left(\overline{\Delta r}_{i}^{(\mathrm{ir})}\right)}}; \qquad \begin{cases} \overline{\Delta r}_{i}^{(\mathrm{ir})} = \dfrac{1}{nk_i'}\displaystyle\sum_{k=1}^{nk_i'}\Delta r_{i\,k}^{(\mathrm{ir})} \\ u^2\left(\overline{\Delta r}_{i}^{(\mathrm{ir})}\right) = \dfrac{1}{nk_i'\left(nk_i'-1\right)}\displaystyle\sum_{k=1}^{nk_i'}\left(\Delta r_{i\,k}^{(\mathrm{ir})} - \overline{\Delta r}_{i}^{(\mathrm{ir})}\right)^2 \end{cases} \tag{10}$$

In photometry an expanded uncertainty is stated for a coverage factor $k = 2$, which refers to the student factor $t_p(v)$. The related student factor is calculated from the t-distribution function for a fraction $p = 95.45\,\%$ of probability and for the limited degrees of freedom $v = nk_i' - 1$. Values of the student factor for two probabilities and small degrees of freedom (only a few transfer standards in a batch) are listed in Table 2.

Table 2 The student factor $t_p(v)$ for two fractions p of probability and for $v = n-1$ degrees of freedom [5].

v	1	2	3	4	5	6	7	8
$p = 68.27\,\%$	1.84	1.32	1.20	1.14	1.11	1.09	1.08	1.07
$p = 95.45\,\%$	13.97	4.53	3.31	2.87	2.65	2.52	2.43	2.37

The same analysis is done for the transfer standards of the link partners with similar equations only the symbols $Li \to i$, $nk_{Li} \to nk_i$ are replaced.

Similar evaluations are performed at the pilot laboratory. The values of the measurements are marked with an additional index "R". The relative differences of the values assigned to the lamps in the batches of the participants are evaluated from $\Delta r_{\mathrm{R}i\,k}^{(12)} = 2\left(y_{\mathrm{R}i\,k}^{(1)} - y_{\mathrm{R}i\,k}^{(2)}\right)/\left(y_{\mathrm{R}i\,k}^{(1)} + y_{\mathrm{R}i\,k}^{(2)}\right)$ with associated uncertainties $u\left(\Delta r_{\mathrm{R}i\,k}^{(12)}\right)$.

In case of significant instabilities the worst lamp in a batch will be measured two more times and the average of all the measurements will be used for the further evaluations.

Significance of instability detected in the pilot laboratory will be identified using the definition of equ. (10) and the values measured by the pilot laboratory. The average and the standard deviation may be determined from the measurements of all transfer standards not only of one batch. Therefore, the degrees of freedom are so large, that $k = 2$ replaces the student factor $t_p(v)$ in the related equation.

3.5 Reported results

3.5.1 Evaluation of the EURAMET KCRV

The mean values $x_{Li\,k} = C_{Li} \cdot \left(y_{Li\,k}^{(\mathrm{i})} + y_{Li\,k}^{(\mathrm{r})} \right) / 2$ of the selected $1 \le k \le nk_{Li}$ transfer standards in the batches of the $1 \le Li \le nLi$ link partners are reported by the link partners and the related mean values $x_{\mathrm{R}\,Li\,k} = C_{\mathrm{R}\,Li} \cdot \left(y_{\mathrm{R}\,Lik}^{(1)} + y_{\mathrm{R}\,Li\,k}^{(2)} \right) / 2$ are measured by the pilot laboratory (index "R").

The calibration factor $C_{\mathrm{R}\,Li}$ in the pilot laboratory is determined from solving the definition of the DOE $D_{Li} = x_{Li} / x_{\mathrm{R}\,Li} - 1$ by using value D_{Li} known for a link partner and stated in Table 1. The averages $\bar{r}_{Li}$ of the ratios $r_{Li\,k} = x_{Li\,k} / x_{\mathrm{R}\,Li\,k} = \left(y_{Li\,k}^{(\mathrm{i})} + y_{Li\,k}^{(\mathrm{r})} \right) / \left(y_{\mathrm{R}\,Li\,k}^{(1)} + y_{\mathrm{R}\,Li\,k}^{(2)} \right)$ for the transfer standards in a batch are used additionally.

$$C_{\mathrm{R}\,Li} = \frac{C_{Li}}{D_{Li} + 1} \cdot \bar{r}_{Li}; \qquad u^2\left(C_{\mathrm{R}\,Li}\right) = C_{\mathrm{R}\,Li}^2 \left[u_{\mathrm{rel}}^2\left(C_{Li}\right) + u_{\mathrm{rel}}^2\left(\bar{r}_{Li}\right) + \left(\frac{u\left(D_{Li}\right)}{D_{Li} + 1} \right)^2 \right] \tag{11}$$

The average $\overline{C}_{\mathrm{R}}$ of the calibration factors $C_{\mathrm{R}\,Li}$ from the link partners represents the EURAMET-KCRV. Due to the different expanded uncertainties $U\left(D_{Li}\right) = 2\,u\left(D_{Li}\right)$ associated to the DOEs D_{Li} in Table 1 the average $\overline{C}_{\mathrm{R}}$ has to be evaluated as weighted mean with the variances $u^2\left(C_{\mathrm{R}\,Li}\right)$ taken as weights, which might be written as factors $w_{\mathrm{R}\,Li} = u^{-2}\left(C_{\mathrm{R}\,Li}\right) \Big/ \sum_{Li=1}^{nLi} u^{-2}\left(C_{\mathrm{R}\,Li}\right)$.

The uncertainty associated to a weighted mean can be evaluated by two different approaches, denoted as "external" or "internal" consistency with indexes "ext" and "int". The uncertainty $u_{\mathrm{ext}}\left(\overline{C}_{\mathrm{R}}\right)$ for external consistency is determined as weighted standard deviation, while the uncertainty $u_{\mathrm{int}}\left(\overline{C}_{\mathrm{R}}\right)$ for internal consistency is determined from the uncertainties $u\left(C_{\mathrm{R}\,Li}\right)$ estimated from the equ. (11).

$$\begin{aligned} \overline{C}_{\mathrm{R}} &= \sum_{Li=1}^{nLi} \left(w_{\mathrm{R}\,Li} \cdot C_{\mathrm{R}\,Li} \right) \\ w_{\mathrm{R}\,Li} &= u^{-2}\left(C_{\mathrm{R}\,Li}\right) \Big/ \sum_{Li=1}^{nLi} u^{-2}\left(C_{\mathrm{R}\,Li}\right) \end{aligned}; \qquad \begin{cases} u_{\mathrm{int}}^2\left(\overline{C}_{\mathrm{R}}\right) = 1 \Big/ \sum_{Li=1}^{nLi} u^{-2}\left(C_{\mathrm{R}\,Li}\right) \\ u_{\mathrm{ext}}^2\left(\overline{C}_{\mathrm{R}}\right) = \sum_{Li=1}^{nLi} w_{\mathrm{R}\,Li} \cdot \left(C_{\mathrm{R}\,Li} - \overline{C}_{\mathrm{R}} \right)^2 \end{cases} \tag{12}$$

The ratio $u_{\mathrm{ext}}\left(\overline{C}_{\mathrm{R}}\right) / u_{\mathrm{int}}\left(\overline{C}_{\mathrm{R}}\right)$ of the two uncertainties is called Birge-ratio $r^{(\mathrm{Birge})}\left(\overline{C}_{\mathrm{R}}\right)$ and should have a value of about unity. If the Birge-ratio exceeds unity, then the uncertainties $u\left(C_{\mathrm{R}\,Li}\right)$ are underestimated and have to be multiplied by the Birge-ratio before any further use. It should be noted, that the value of the calibration factor $\overline{C}_{\mathrm{R}}$ is not affected by the Birge ratio.

The value $\overline{C}_{\mathrm{R}}$ and the associated uncertainty $u_{\mathrm{ext}}\left(\overline{C}_{\mathrm{R}}\right)$ are evaluated from the individual calibration factors $C_{\mathrm{R}\,Li}$ and associated uncertainties in equ. (11), which are products including the averaged ratios $\bar{r}_{Li}$ of a batch of transfer standards and associated uncertainties $u(\bar{r}_{Li})$. Their evaluation is shown in the next chapter.

3.5.2 Averaged Value of a Batch

The DOE D_i is evaluated as before from ratios of $x_{ik} = C_i \cdot \left(y_{i\,k}^{(\mathrm{i})} + y_{i\,k}^{(\mathrm{r})}\right)/2$ stated by the participant and divided by the related values $x_{\mathrm{R}\,i\,k} = \overline{C}_\mathrm{R} \cdot \left(y_{\mathrm{R}\,i\,k}^{(1)} + y_{\mathrm{R}\,i\,k}^{(2)}\right)/2$ measured in the pilot laboratory. When the calibration factor $\overline{C}_\mathrm{R}$ is used by the pilot laboratory then its values represent the EURAMET-KCRV.

The $D_i = x_i / x_{\mathrm{R}\,i} - 1$ of a participant is evaluated from the average of the ratios $x_{i\,k}/x_{\mathrm{R}\,i\,k}$ for the $1 \le k \le nk_i$ individual transfer standards in a batch. It is presented using the abbreviation $\bar{r}_i$.

$$D_i = \overline{\left(\frac{x_{i\,k}}{x_{R\,i\,k}}\right)} - 1 = C_i \frac{\bar{r}_i}{\overline{C}_\mathrm{R}} - 1 \qquad \bar{r}_i = \overline{\left(y_{i\,k}^{(i)} + y_{i\,k}^{(r)}\right) / \left(y_{\mathrm{R}\,i\,k}^{(1)} + y_{\mathrm{R}\,i\,k}^{(2)}\right)}$$
$$u(D_i) \cong \sqrt{u_\mathrm{rel}^2(C_i) + u_\mathrm{rel}^2\left(\overline{C}_\mathrm{R}\right) + u_\mathrm{rel}^2(\bar{r}_i)} \tag{13}$$

Usually, the values $y_{i\,k} \cong y_{i\,k}^{(\mathrm{i})} \cong y_{i\,k}^{(\mathrm{r})}$ and associated uncertainties $u(y_{i\,k}) \cong u\left(y_{i\,k}^{(\mathrm{i})}\right) \cong u\left(y_{i\,k}^{(\mathrm{r})}\right)$ of repeated measurements of a participant are very similar and so are the values $y_{\mathrm{R}\,i\,k} \cong y_{\mathrm{R}\,i\,k}^{(1)} \cong y_{\mathrm{R}\,i\,k}^{(2)}$ and uncertainties $u(y_{\mathrm{R}\,i\,k}) \cong u\left(y_{\mathrm{R}\,i\,k}^{(1)}\right) \cong u\left(y_{\mathrm{R}\,i\,k}^{(2)}\right)$ of the pilot laboratory. Then the relative variance associated to the ratio $\bar{r}_{i\,k}$ can be approximated.

$$r_{i\,k} = \frac{y_{i\,k}^{(i)} + y_{i\,k}^{(r)}}{y_{\mathrm{R}\,i\,k}^{(1)} + y_{\mathrm{R}\,i\,k}^{(2)}} \qquad u_\mathrm{rel}^2(r_{i\,k}) = \frac{u^2\left(y_{i\,k}^{(i)}\right) + u^2\left(y_{i\,k}^{(r)}\right)}{\left(y_{i\,k}^{(i)} + y_{i\,k}^{(r)}\right)^2} + \frac{u^2\left(y_{\mathrm{R}\,i\,k}^{(1)}\right) + u^2\left(y_{\mathrm{R}\,i\,k}^{(2)}\right)}{\left(y_{\mathrm{R}\,i\,k}^{(1)} + y_{\mathrm{R}\,i\,k}^{(2)}\right)^2}$$
$$\bar{r}_i = \frac{1}{nk_i} \sum_{k=1}^{nk_i} r_{i\,k}; \qquad u_\mathrm{rel}(\bar{r}_i) \cong \sqrt{\frac{1}{2nk_i} \sum_{k=1}^{nk_i} \left(u_\mathrm{rel}^2(y_{i\,k}) + u_\mathrm{rel}^2(y_{\mathrm{R}\,i\,k})\right)} \tag{14}$$

Sometimes the uncertainties $u(r_{i\,k})$ associated to the ratios $r_{i\,k}$ within a batch are varying and a weighted mean (which turns to arithmetic for constant uncertainties) is the appropriate averaging method with the inverse variances $u^2(r_{i\,k})$ taken as weights, which are introduced as factors $v_{i\,k}$. The external consistency is the appropriate statement for the uncertainty of a batch average $\bar{r}_i$ as shown in equ. (12), and the estimated uncertainties $u(r_{i\,k})$ have to be multiplied by the Birge ratio $r^{(\mathrm{Birge})}(\bar{r}_i) = u_\mathrm{ext}(\bar{r}_i)/u_\mathrm{int}(\bar{r}_i)$, if the latter exceeds unity.

$$\bar{r}_i = \sum_{k=1}^{nk_i} \left(v_{i\,k} \cdot r_{i\,k}\right) \qquad v_{i\,k} = u^{-2}(r_{i\,k}) \Big/ \sum_{k=1}^{nk_i} u^{-2}(r_{i\,k}); \qquad \begin{cases} u_\mathrm{int}^2(\bar{r}_i) = 1 \Big/ \sum_{k=1}^{nk_i} u^{-2}(r_{i\,k}) \\ u_\mathrm{ext}^2(\bar{r}_i) = \dfrac{1}{nk_i - 1} \sum_{k=1}^{nk_i} v_{i\,k} \cdot (r_{i\,k} - \bar{r}_i)^2 \end{cases} \tag{15}$$

3.5.3 Relative Results Reported in Draft A

Depart from much general information in draft A, it is helpful to state also the relative uncertainty $u_{\mathrm{rel}}\left(\overline{C}_{\mathrm{R}}\right)$ associated to the calibration factor $\overline{C}_{\mathrm{R}}$ of the pilot laboratory evaluated from equ. (12). The value of the calibration factor is of no sense for the participants, but the associated uncertainty is just the minimum uncertainty for a DOE in this comparison, assuming negligible contributions from the measurements of the participants.

Provided the Birge-ratio $r^{(\mathrm{Birge})}\left(\overline{C}_{\mathrm{R}}\right)$ is about unity, then the uncertainty is determined from the weights using the internal consistency and one finds from the equations above:

$$\overline{C}_{\mathrm{R}} = \sum_{Li=1}^{nLi}\left(w_{\mathrm{R}\,Li}\cdot\frac{C_{Li}}{D_{Li}+1}\cdot\overline{r}_{Li}\right); \qquad w_{\mathrm{R}\,Li} = u^{-2}\left(C_{\mathrm{R}\,Li}\right)\Big/\sum_{Li=1}^{nLi}u^{-2}\left(C_{\mathrm{R}\,Li}\right)$$
$$u_{\mathrm{rel}}\left(\overline{C}_{\mathrm{R}}\right) = \left[\sum_{Li=1}^{nLi}\left[\left(\frac{C_{Li}}{\overline{C}_{\mathrm{R}}}\frac{\overline{r}_{Li}}{D_{Li}+1}\right)^2\left(u_{\mathrm{rel}}^2\left(C_{Li}\right)+\left(\frac{u\left(D_{Li}\right)}{D_{Li}+1}\right)^2+u_{\mathrm{rel}}^2\left(\overline{r}_{Li}\right)\right)\right]^{-1}\right]^{-\frac{1}{2}} \tag{16}$$

In the draft A the <u>relative variation</u> of the ratios $q_{i\,k} = r_{i\,k}/\overline{r}_{i\,k}$ is most important for the participants, because it shows the homogeneity of the transferred values. This ratio is evaluated for all lamps of a batch from the individual ratios $r_{i\,k} = \left(y_{i\,k}^{(\mathrm{i})} + y_{i\,k}^{(\mathrm{r})}\right)\Big/\left(y_{\mathrm{R}i\,k}^{(1)} + y_{\mathrm{R}i\,k}^{(2)}\right)$ in equ. (14), which are normalised by their average $\overline{r}_i$.

The normalisation eliminates the relation to the DOE and allows a participant to identify a single lamp within his batch as instable or with outlying values, provided the deviation is significantly larger than the minimum uncertainty stated before. Outlying values shall warn the participant and the pilot laboratory to start an additional check for typing errors or for any other reasons.

It should be noted, that the uncertainties $u\left(r_{i\,k}\right)$ of the individual ratios contribute to the uncertainty $u\left(\overline{r}_{i\,k}\right)$ of the average and so the uncertainty $u\left(q_{i\,k}\right)$ associated to the ratio $q_{i\,k}$ might be a little too large, if the correlation is not taken into account.

3.5.4 Results Presented in the Final Report

Assuming, neither cutoffs nor other correcting factors have to be regarded in the final report, then a small relative uncertainty $u_{\mathrm{rel}}\left(\overline{C}_{\mathrm{R}}\right)$ associated to the calibration factor $\overline{C}_{\mathrm{R}}$ of the pilot laboratory will indicate a high quality of the comparison.

This calibration factor $\overline{C}_{\mathrm{R}}$ is a constant factor for all participants with a constant relative contribution to the uncertainty associated to the individual DOEs. The latter are determined from the constant factor $C_i = 1$ and the ratios $\overline{r}_i$ in equ. (15) of the measurements carried out at the participant and the pilot laboratory and averaged for the batch of transfer standards. Usually, the value of this ratio is the dominant factor for the evaluation of the DOE of a participant.

Instead of a combination of all the earlier results to get the final equation, the principle equations are repeated now for the following discussion.

$$\begin{gathered} D_i = \frac{C_i}{\overline{C}_{\mathrm{R}}}\,\overline{r}_i - 1 \\ u(D_i) \cong \sqrt{u_{\mathrm{rel}}^2\left(\overline{C}_{\mathrm{R}}\right) + u_{\mathrm{rel}}^2(C_i) + u_{\mathrm{rel}}^2(\overline{r}_i)} \end{gathered} \qquad \left\{ \begin{aligned} \overline{r}_i &= \sum_{k=1}^{nk_i} \left(v_{i\,k} \cdot r_{i\,k}\right) \\ u^2(\overline{r}_i) &= \frac{1}{nk_i - 1} \sum_{k=1}^{nk_i} v_{i\,k} \cdot \left(r_{i\,k} - \overline{r}_i\right)^2 \end{aligned} \right. \tag{17}$$

A relative presentation of the uncertainty $u(D_i)$ of the DOE is not appropriate, because the value may be zero or close to zero. There are three origins contributing to the uncertainty of the DOE and the related uncertainties are stated in a relative presentation. The relative uncertainty $u_{\mathrm{rel}}\left(\overline{C}_{\mathrm{R}}\right)$ of the calibration factor in the pilot laboratory was explained above.

The relative uncertainty $u_{\mathrm{rel}}^2(C_i)$ is constant for all transfer standards of a participant. It was found from the constant contributions in the uncertainty budgets of the participant. More generally spoken, it is the relative uncertainty of the realised and maintained photometric unit at the participant's laboratory. Usually this uncertainty will be the most dominant contribution to the uncertainty of the DOE.

The relative uncertainty $u_{\mathrm{rel}}^2(\overline{r}_i)$ is originated in the averaging of the batch ratios, which deals with the distribution of the photometric quantity. The equ. (14) shows, that the repeatability of the measurements performed by the participant and at the pilot laboratory are contributing to this uncertainty, but mainly the stability of the transfer standards will increase for both measurements the level (this proves the factor $\sqrt{2}$ in equ. (14)). Provided, the transfer standard is stable in the electrical operation, the mechanical alignment and without severe aging, then the uncertainty $u_{\mathrm{rel}}^2(\overline{r}_i)$ would be the most unimportant contribution to the uncertainty of the DOE.

3.6 Literature

[1] CCPR Key Comparison K3a of Luminous Intensity and K4 of Luminous Flux with Lamps as Transfer Standards; PTB-Opt-62; ISBN 3-89701-471-8

[2] BIPM database: http://kcdb.bipm.org/appendixB/default.asp

[3] Mutual recognition arrangement: BIPM - CIPM MRA

[4] ISO 17025

[5] Guide to the Expression of Uncertainty in Measurement (ISO)

Annex A: Degrees of equivalence with CCPR-KCRV

The degrees of equivalence for all participants in the CCPR key comparisons for luminous intensity and luminous flux copied from the BIPM-database [2].

CCPR-K3.a and CCPR-K3.a.1

MEASURAND : Luminous intensity

The individual measurements, x_i, of the participating laboratories take the form of ratios which depend on the reference used at the pilot laboratory. This is cancelled out by normalization using the key comparison reference value, x_R. This procedure leads to normalized individual measurements: x_i / x_R.

- **Key comparison CCPR-K3.a**

The key comparison reference value, x_R, is calculated as the weighted average of the invidual results x_i, weighted by the inverse square of the individual standard uncertainties, u_i, with the application of a minimum cutoff of 0.25%.
The INTI and the BIPM are excluded from the calculation of x_R.
The standard uncertainty of x_R is u_R = 0.09%. It is negligible compared to the u_i values.

The degree of equivalence of each laboratory with respect to the reference value is given by a pair of terms: $D_i = (x_i - x_R) / x_R$ and U_i, its expanded uncertainty (k = 2), both expressed in relative units. $U_i = 2u_i$.

The degree of equivalence between two laboratories is given by a pair of terms: $D_{ij} = (x_i - x_j) / x_R$ and U_{ij}, its expanded uncertainty (k = 2), both expressed in relative units. $U_{ij} = 2(u_i^2 + u_j^2)^{1/2}$.

- **Linking CCPR-K3.a.1 to CCPR-K3.a**

The link to the key comparison CCPR-K3.a is given by the pilot laboratory CSIRO-NML that participated in both comparisons. No news results are produced by the key comparison CCPR-K3.a.1 for the CSIRO-NML.

The degree of equivalence of SPRING Singapore relative to the CCPR-K3.a key comparison reference value is given by:
$D_{\text{SPRING Singapore}}$ = 0.0003 and $U_{\text{SPRING Singapore}}$ = 0.0033, see Section 9 of the CCPR-K3.a.1 Final Report.

The degrees of equivalence between SPRING Singapore participant in CCPR-K3.a.1 and one laboratory participant in CCPR-K3.a are not calculated.

CCPR-K3.a and CCPR-K3.a.1

MEASURAND : Luminous intensity

The following table displays the **degrees of equivalence**. It includes three parts:
- "white part", list of participants (**Lab** i);
- "blue part", degrees of equivalence relative to the key comparison reference value;
- "yellow part", degrees of equivalence between pairs of participants.

The pair-wise degrees of equivalence are given as "Lab i - Lab j"

Select **Lab j** BIPM

Lab i	D_i	U_i	D_{ij}	U_{ij}
	/ 10^{-2}	/ 10^{-2}	/ 10^{-2}	/ 10^{-2}
BNM-INM	0.89	0.60	0.59	1.17
CSIR-NML	0.51	0.88	0.21	1.33
CSIRO-NML	-0.07	0.60	-0.37	1.17
ETL	-0.09	0.56	-0.39	1.15
IEN	-0.43	0.90	-0.73	1.35
IFA	-0.48	0.72	-0.78	1.23
NIM*	-0.16	0.48	-0.46	1.11
NIST*	0.12	0.48	-0.18	1.11
NPL*	0.04	0.30	-0.26	1.04
NRC	0.19	1.06	-0.11	1.46
OFMET	-0.50	1.12	-0.80	1.50
OMH	0.05	0.94	-0.25	1.37
PTB*	-0.31	0.40	-0.61	1.08
SMU	-0.36	1.40	-0.66	1.72
VNIIOFI	0.33	0.92	0.03	1.36
INTI**	-0.25	0.74	-0.55	1.24
BIPM**	0.30	1.00		
SPRING Singapore	0.03	0.33	-	-

*Cutoff uncertainty (0.25%) applied to those laboratory measurements in the calculation of the key comparison reference value, x_R.
**Laboratory excluded from the calculation of x_R.

In **black**: SPRING Singapore participant in CCPR-K3.a.1 (also indicated with " in the Lab j list).

The full matrix of equivalence is available from Summary Results in a A4 printable format.

Unless otherwise stated, in the final numbers presented here, rounding has been applied according to ISO-31-0 Annex B Rule B.

MEASURAND : Luminous flux

The individual measurements, x_i, of the participating laboratories take the form of ratios which depend on the reference used at the pilot laboratory, the PTB. This is cancelled out by normalization using the key comparison reference value, x_R. This procedure leads to normalized individual measurements: x_i / x_R.

The key comparison reference value, x_R, is calculated as the weighted average of the invidual results x_i, weighted by the inverse square of the individual standard uncertainties, u_i, with the application of a minimum cutoff of 0.30%.
NIM and BIPM are excluded from the calculation of x_R.
The standard uncertainty of x_R is u_R = 0.1%. It is negligible compared to the u_i values.

The degree of equivalence of each laboratory with respect to the reference value is given by a pair of terms: $D_i = (x_i - x_R) / x_R$ and U_i, its expanded uncertainty (k = 2), both expressed in relative units. $U_i = 2u_i$.

The degree of equivalence between two laboratories is given by a pair of terms: $D_{ij} = (x_i - x_j) / x_R$ and U_{ij}, its expanded uncertainty (k = 2), both expressed in relative units. $U_{ij} = 2(u_i^2 + u_j^2)^{1/2}$.

MEASURAND: Luminous flux

The following table displays the **degrees of equivalence**. It includes three parts:
- "white part", list of participants (**Lab** i);
- "blue part", degrees of equivalence relative to the key comparison reference value;
- "yellow part", degrees of equivalence between pairs of participants.

The pair-wise degrees of equivalence are given as "Lab i - Lab j"

Select **Lab j** BIPM

Lab i	D_i	U_i	D_{ij}	U_{ij}
	/ 10^{-2}	/ 10^{-2}	/ 10^{-2}	/ 10^{-2}
BNM-INM*	0.69	0.58	0.37	1.17
CSIR-NML	-0.01	1.06	-0.33	1.47
CSIRO-NML*	0.13	0.58	-0.19	1.17
ETL	0.18	0.68	-0.14	1.23
IEN	-0.06	0.96	-0.38	1.40
IFA	-0.43	1.70	-0.75	1.98
NIM*	-0.22	0.52	-0.54	1.14
NIST	-0.21	0.62	-0.53	1.19
NPL*	0.37	0.40	0.05	1.10
NRC	0.99	2.00	0.67	2.25
OFMET	-0.57	1.38	-0.89	1.72
OMH	0.43	1.30	0.11	1.65
PTB*	-0.42	0.56	-0.74	1.16
SMU	-0.88	2.20	-1.20	2.42
VNIIOFI	-0.51	0.66	-0.83	1.21
INTI**	-0.43	1.00	-0.75	1.43
BIPM**	0.32	1.02		

*cutoff uncertainty (0.30%) applied to those laboratory measurements in the calculation of the reference value, x_R.
**laboratory excluded from the calculation of x_R.

Unless otherwise stated, in the final numbers presented here, rounding has been applied according to ISO-31-0 Annex B Rule B.

Annex B: Mismatch correction for Planckian radiators

The illuminance $E(S(\lambda))$ is evaluated from photocurrent y divided by the luminous responsivity s_v and multiplied with the mismatch correction factor $F(S(\lambda))$. The spectral responsivity $s(\lambda)=s_0 \cdot s_\mathrm{rel}(\lambda)$ of a photometer head can be written as product of a normalisation factor s_0 and a relative spectral responsivity function $s_\mathrm{rel}(\lambda)$ matched to $V(\lambda)$. The luminous responsivity s_v and mismatch correction $F(S(\lambda))$ are evaluated from the relative function $s_\mathrm{rel}(\lambda)$ and are normalised using the Planckian distribution $P(\lambda,T_\mathrm{A})$ for CIE illuminant A with the distribution temperature $T_\mathrm{A} = 2856\ \mathrm{K}$.

$$E(S(\lambda)) = \frac{y}{s_\mathrm{v}} F(S(\lambda)); \qquad \begin{cases} \dfrac{1}{s_\mathrm{v}} = \dfrac{K_\mathrm{m}}{s_0} \dfrac{\int P(\lambda,T_\mathrm{A})\cdot V(\lambda)\mathrm{d}\lambda}{\int P(\lambda,T_\mathrm{A})\cdot s_\mathrm{rel}(\lambda)\mathrm{d}\lambda} \\ F(S(\lambda)) = \dfrac{\int S(\lambda)\cdot V(\lambda)\mathrm{d}\lambda}{\int S(\lambda)\cdot s_\mathrm{rel}(\lambda)\mathrm{d}\lambda} \Bigg/ \dfrac{\int P(\lambda,T_\mathrm{A})\cdot V(\lambda)\mathrm{d}\lambda}{\int P(\lambda,T_\mathrm{A})\cdot s_\mathrm{rel}(\lambda)\mathrm{d}\lambda} \end{cases} \tag{A1}$$

The transfer standards in this report are incandescent lamps with relative spectral distributions $S(\lambda)$ similar to Planckian radiators $P(\lambda,T)$ characterised by a distribution temperature T. The mismatch correction from equ. (A1) with $S(\lambda)$ replaced by $P(\lambda,T)$ can be evaluated for several $1 \le i \le n \ge 4$ distribution temperatures T_i and approximated by ratios T_i/T_A with a mismatch index m as exponent. The value of the mismatch index is determined by a least-mean-square fit.

$$\underset{m'\to m}{\mathrm{Min}} \sum_{i=1}^{n} \left[F(P(\lambda,T_i)) - \left(\frac{T_i}{T_\mathrm{A}}\right)^{m'} \right]^2; \qquad F(P(\lambda,T)) \approx \left(\frac{T}{T_\mathrm{A}}\right)^{m} \tag{A2}$$

Luminous responsivity s_v and mismatch index m are characteristic quantities of a photometer. They can be determined from photometric measurements, too: A luminous intensity reference lamp (index "R") is operated (at least) at two luminous intensities $I_\mathrm{R1}(T_\mathrm{R1}), I_\mathrm{R2}(T_\mathrm{R2})$ with distribution temperatures $T_\mathrm{R1}, T_\mathrm{R2}$ and generates output signals $y_\mathrm{R1}, y_\mathrm{R2}$. From these values the characteristics can be evaluated.

$$m = \log\left(\frac{I_\mathrm{R1}(T_\mathrm{R1})}{I_\mathrm{R2}(T_\mathrm{R2})}\frac{y_\mathrm{R2}}{y_\mathrm{R1}}\right) \Bigg/ \log\left(\frac{T_\mathrm{R1}}{T_\mathrm{R2}}\right); \quad s_\mathrm{v} = \frac{y_\mathrm{R}\cdot d_\mathrm{R}^2}{I_\mathrm{R}(T_\mathrm{R})}\left(\frac{T_\mathrm{R}}{T_\mathrm{A}}\right)^m \tag{A3}$$

The same method can be performed with a luminous flux reference lamp operated (at least) at two luminous fluxes $\Phi_\mathrm{R1}(T_\mathrm{R1}), \Phi_\mathrm{R2}(T_\mathrm{R2})$ generating sphere output signals $y_\mathrm{SR1}, y_\mathrm{SR2}$ and the mismatch index m_S can be determined. In principle the throughput τ_S needs more measurements (see Annex C).

$$m_\mathrm{S} = \log\left(\frac{\Phi_\mathrm{R1}(T_\mathrm{R1})}{\Phi_\mathrm{R2}(T_\mathrm{R2})}\frac{y_\mathrm{SR2}}{y_\mathrm{SR1}}\right) \Bigg/ \log\left(\frac{T_\mathrm{R1}}{T_\mathrm{R2}}\right); \qquad \frac{1}{\tau_\mathrm{S}} = \frac{y_\mathrm{SR}}{\Phi_\mathrm{R}(T_\mathrm{R})}\left(\frac{T_\mathrm{R}}{T_\mathrm{A}}\right)^m \tag{A4}$$

Annex C: Modified Substitution Methods

For the calibration of a measurement setup the transfer standards are substituted by reference standards of similar types. So, the luminous responsivity of the photometer in a goniophotometer is calibrated by a luminous flux reference standard as stated earlier in equ. (4) for the measurements at the pilot laboratory. The luminous responsivity s_v can also be calibrated by a lamp with luminous intensity $I_\mathrm{R}(T_\mathrm{R})$ at distribution temperature T_R placed in a distance d_R from the photometer and generates a photocurrent y_R, which is measured with the gain resistor R_R.

$$\Phi(T) = r_\mathrm{G}^2 \cdot \frac{y_\mathrm{G}}{w \cdot R_\mathrm{G} \cdot s_\mathrm{v}} \left(\frac{T}{T_\mathrm{A}}\right)^m \cdot G \quad \Rightarrow \quad \Phi(T) = I_\mathrm{R}(T_\mathrm{R}) \frac{r_\mathrm{G}^2}{d_\mathrm{R}^2} \frac{y_\mathrm{G}}{y_\mathrm{R}} \frac{R_\mathrm{R}}{R_\mathrm{G}} \left(\frac{T}{T_\mathrm{R}}\right)^m \cdot G \cdot corr_\mathrm{G} \qquad \text{(A5)}$$

Obviously, the distances and the integration are not compensated and will create uncertainty contributions. The effects of operational and ambient conditions are regarded additionally by the correction factor $corr_\mathrm{G}$.

The integrating sphere has to be calibrated with a luminous flux as reference -either emitted internally as shown in equ. (4) -or inserted from an external source. Let the external lamp with distribution temperature T_R illuminate $E_\mathrm{R}(T_\mathrm{R})$ a baffle with area $\pi\, r_\mathrm{R}^2$ and the luminous flux $\Phi_\mathrm{R}(T_\mathrm{R}) = \pi\, r_\mathrm{R}^2\, E_\mathrm{R}(T_\mathrm{R})$ passing through the baffle enters the sphere. Then the photocurrent y_R is measured with the gain resistor R_R and the throughput τ_R is taken into account.

The illuminance $E_\mathrm{R}(T_\mathrm{R})$ is evaluated either from the luminous intensity $I_\mathrm{R}(T_\mathrm{R})$ of a reference lamp in the distance d_R or measured by an external photometer with luminous responsivity s_ext, mismatch index m_ext and gain resistor R_ext as a photocurrent y_ext.

$$\Phi(T) = \tau_\mathrm{S} \cdot \frac{y_\mathrm{S}}{R_\mathrm{S} \cdot s_\mathrm{v}} \left(\frac{T}{T_\mathrm{A}}\right)^{m_\mathrm{S}} \quad \Rightarrow \quad \begin{cases} \Phi(T) = \pi \dfrac{r_\mathrm{R}^2}{d_\mathrm{R}^2} I_\mathrm{R}(T_\mathrm{R}) \cdot \dfrac{y_\mathrm{S}}{R_\mathrm{S}} \dfrac{R_\mathrm{R}}{y_\mathrm{R}} \left(\dfrac{T}{T_\mathrm{R}}\right)^{m_\mathrm{S}} corr_\mathrm{S} \\ \Phi(T) = \dfrac{\pi\, r_\mathrm{R}^2\, y_\mathrm{ext}}{R_\mathrm{ext} \cdot s_\mathrm{ext}} \left(\dfrac{T_\mathrm{R}}{T_\mathrm{A}}\right)^{m_\mathrm{ext}} \dfrac{y_\mathrm{S}}{R_\mathrm{S}} \dfrac{R_\mathrm{R}}{y_\mathrm{R}} \left(\dfrac{T}{T_\mathrm{R}}\right)^{m_\mathrm{S}} corr_\mathrm{S} \end{cases} \qquad \text{(A6)}$$

The illuminances $E_{\mathrm{a,R}}/E_\mathrm{a}$ (index "a") for the correction of self absorption are originated by an auxiliary lamp and measured with either the reference or the test lamp in place. The sphere response correction $f_{r,\mathrm{R}}/f_r$ (index "r") is evaluated from two relative functions, the angular throughput function $\tau_\mathrm{s}(\vartheta,\varphi)$ of the sphere and the angular distributions of the reference $g_\mathrm{R}(\vartheta,\varphi)$ and of the test lamp $g(\vartheta,\varphi)$. The effects of operational and ambient conditions are regarded additionally, by the correction factor $corr_\mathrm{S}$.

Annex D: Origin of uncertainty contributions

Output signal: The output signal y of a photometer is determined from repeated readings $\bar{y}$ corrected for a "dark signal" $\bar{y}_0$, a calibration factor c_y of the DVM and a relative stray light correction factor γ_y (No external light, only light of the standard is back reflected to the photometer). The photocurrent is written with a correction factor (in parentheses) close to unity and higher order terms $O(2)$ omitted. The equation on the left side yields for detector based measurements of equ. (3). The ratios on the right hand side are typical for the substitution method of equ. (4) and valid for the pilot laboratory.

$$y = c_y \cdot \bar{y} \cdot \left(1 - \frac{\bar{y}_0}{\bar{y}} - \gamma_y + O(2)\right) \qquad \Rightarrow \qquad \frac{y}{y_\mathrm{R}} = \frac{\bar{y}}{\bar{y}_\mathrm{R}} \cdot \left(1 - \frac{\bar{y}_0}{\bar{y}} + \frac{\bar{y}_{0\mathrm{R}}}{\bar{y}_\mathrm{R}} - \gamma_y + \gamma_{y\mathrm{R}}\right) \qquad \text{(A7)}$$

The relative stray light factor γ_y is negligible, because the contributions are small $\gamma_y << 1; \gamma_{y\mathrm{R}} << 1$ and constant $\gamma_y \cong \gamma_{y\mathrm{R}}$ (assuming an unchanged field of view of the photometer). The dark signals are small if compared with the related output signal and (nearly) compensated due to the negative sign from the substitution method.

Gain resistor: The transfer and reference standard are often measured without a change of the gain resistor and the ratio $R_{\mathrm{g,\,R}} / R_\mathrm{g}$ cancels out. Otherwise the ratio of the resistor values and the associated uncertainties has to be regarded as a factor with an associated relative uncertainty. At the pilot laboratory a constant gain is used.

$$R_{\mathrm{g,\,R}} / R_\mathrm{g} = 1 \qquad \text{(A8)}$$

Luminous responsivity: The effective luminous responsivity s_v of a photometer head depends on the ambient temperature, the alignment and a possible aging, which can modify the value $s_{\mathrm{v}0}$ found from former calibrations. In general a deviation ΔT_s from the rated ambient temperature changes the responsivity with a relative temperature coefficient α_s. The optical axis of the photometer bench should hit the centre of the entrance aperture rectangularly; any deviation by an angle ε_s changes the luminous responsivity with the cosine of that angle. The luminous responsivity of photometers is altered by aging with a relative aging coefficient β_s depending on the storage duration Δt_s and higher order terms $O(2)$ are omitted.

The substitution method used at the pilot laboratory cancels out all of these effects: the aging between the measurements of transfer standards and the standards of the link partners is negligible. At the PTB the ambient temperature is highly stabilised and the photometers are temperature controlled. The factor of the responsivity ratio is unity during a measurement campaign.

$$s_\mathrm{v} = s_{\mathrm{v}0} \cdot \left(1 + \alpha_s \cdot \Delta T_s + \beta_s \cdot \Delta t_s + O(2)\right) \cdot \cos \varepsilon_s \Rightarrow \quad \left\{ \frac{s_{\mathrm{vR}}}{s_\mathrm{v}} = 1 \right. \qquad \text{(A9)}$$

It should be noted, that the size of the transfer standards is small and their location is near the centre of the goniophotometer. Thus, the view-angle is small and cosine-correction of the photometer head is of no importance.

Distance and radius: On a photometer bench two reference planes are valid for the alignment of either lamps or photometers in a distance large enough to verify "point source behaviour" of the lamps. The effective distance d is the sum of the distance d_{B} between the reference planes and possible offsets d_{P} and d_{L} due to misalignments of photometer head and lamp, respectively.

A certificate for the length-meter states the calibration factor c_{B} for the operation at a specified ambient temperature. A deviation ΔT_d changes the distance reading with the relative temperature coefficient α_d. The alignment of lamps and photometer with respect to the reference planes is performed with zero offsets $d_{\mathrm{L}} = d_{\mathrm{LR}} = d_{\mathrm{P}} = 0$, but with non-zero associated uncertainties $u(d_{\mathrm{L}}), u(d_{\mathrm{LR}}), u(d_{\mathrm{P}})$. In the inverse squared law the distance is squared and so the result is given with higher order terms $O(2)$ omitted.

$$d^2 = c_d^2 \cdot d_{\mathrm{B}}^2 \left(1 + 2 \cdot \left(\frac{d_{\mathrm{P}} + d_{\mathrm{L}}}{d_{\mathrm{B}}} + \alpha_d \cdot \Delta T_d \right) + O(2) \right) \qquad \text{(A10)}$$

The substitution method at the pilot laboratory cancels out the calibration factor c_d. The distance d_{B} is constant during a measurement campaign, so a possible offset d_{P} of the location of the photometer head cancels out and changes of the ambient temperature are negligible. The uncertainty of the lamps alignments has to be taken into account.

$$\left(\frac{d}{d_{\mathrm{R}}} \right)^2 = \left(1 + 2 \frac{d_{\mathrm{L}} - d_{\mathrm{LR}}}{d_{\mathrm{B}}} \right) \qquad \text{(A11)}$$

Electrical conditions: An incandescent lamp operated at a rated lamp current J_{A} (nominal value without uncertainty) produces related values of luminous intensity I_{A}, distribution temperature T_{A} and lamp voltage U_{A} (in this document the character "I" refers exclusively to the luminous intensity). A deviation of the lamp current to a near-by value J changes the related quantities by coefficients $m_I = 7{,}0;\ m_T = 0{,}7;\ m_U = 1{,}9$ with the stated typical values.

$$I' = I \left(\frac{J}{J_{\mathrm{A}}} \right)^{m_I}; \qquad \Phi' = \Phi \left(\frac{J}{J_{\mathrm{A}}} \right)^{m_I}; \qquad T' = T \left(\frac{J}{J_{\mathrm{A}}} \right)^{m_T}; \qquad U' = U \left(\frac{J}{J_{\mathrm{A}}} \right)^{m_U} \qquad \text{(A12)}$$

The luminous intensity may be corrected with the factor $corr1$ for a precise setting of the lamp current using the exponent m_I. The mismatch correction deals with the distribution temperature, which is affected by the current setting, too. Finally, the correction including the mismatch reads:

$$\left(\frac{T}{T_{\mathrm{R}}}\right)^{m} corr1 = \left(\frac{J_{\mathrm{AR}}}{J_{\mathrm{A}}}\frac{J}{J_{\mathrm{R}}}\right)^{-m_I} \left(\left(\frac{J_{\mathrm{AR}}}{J_{\mathrm{A}}}\frac{J}{J_{\mathrm{R}}}\right)^{m_T}\right)^{m} = \left(\frac{J_{\mathrm{AR}}}{J_{\mathrm{A}}}\frac{J}{J_{\mathrm{R}}}\right)^{m \cdot m_T - m_I} \tag{A13}$$

The lamp current J is determined as mean value from repeated readings of the voltage drop $\overline{U}_J$ across a shunt resistor. The voltage depends on the calibration factor c_J of the DVM and a possible offset voltage $\overline{U}_{J0}$ in the electrical circuit. The shunt resistor R_0 changes with the relative temperature coefficient α_R for any deviation ΔT_R from the rated ambient temperature or due to a temperature rise by the electrical power $w\, R_0\, J_{\mathrm{A}}^2$ (thermal resistance w after thermal equilibrium).

$$\frac{J}{J_{\mathrm{A}}} = \frac{c_J}{J_{\mathrm{A}}}\frac{\overline{U}_J}{R_0}\frac{1 - \overline{U}_{J0}/\overline{U}_J}{1 + \alpha_R\left(\Delta T_R + w \cdot R_0 \cdot J_{\mathrm{A}}^2\right)} \tag{A14}$$

At the pilot laboratory the currents of all transfer standards are measured with one shunt resistor and the effects of temperature variations and offset voltages $\overline{U}_{J0}, \overline{U}_{J0\mathrm{R}}$ are found to be negligible and the ratio of the measured lamp currents depends only on the voltage drop. The photometers are matched very close to $V(\lambda)$ with a small mismatch index of about $m = 0.01$. In combination with equ. (A13) the correction of the electrical setting can be simplified using the relation $|m| \cdot m_T << m_I$.

$$\left(\frac{T}{T_{\mathrm{R}}}\right)^{m} corr1 = \left(\frac{J_{\mathrm{AR}}}{J_{\mathrm{A}}}\frac{\overline{U}_J}{\overline{U}_{J,\mathrm{R}}}\right)^{-m_I} \tag{A15}$$

Operational conditions: The luminous intensity I of a transfer standard depends on the alignment. A relative alignment factor γ_{L} is determined for the individual type of lamps. It is averaged from repeated alignments of lamps of the same type as the transfer standards and it includes the effects of alignment with respect to the burning position as well as for the direction of emittance. A luminous intensity value may change by ageing from the duration Δt_{L} of all operations with a relative aging factor β_{L}, and the correction is applied individually for each lamp.

$$\frac{I(T)}{I_{\mathrm{R}}(T_{\mathrm{R}})} corr2 = \frac{I(T)}{I_{\mathrm{R}}(T_{\mathrm{R}})}\frac{\left(1 + \beta_{\mathrm{L}} \cdot \Delta t_{\mathrm{L}} + \gamma_{\mathrm{L}} + O(2)\right)}{\left(1 + \beta_{\mathrm{LR}} \cdot \Delta t_{\mathrm{LR}} + \gamma_{\mathrm{LR}} + O(2)\right)} \tag{A16}$$

At the pilot laboratory the luminous intensities are not affected by ageing of the transfer standards $\beta_{\mathrm{L}} \cdot \Delta t_{\mathrm{L}} << 1$ and only very little by alignment-errors due to camera-supported and documented alignments. Nevertheless, for luminous intensity transfer standards the alignment is a major contribution to the combined uncertainty, because the intensity varies strongly with small alignment errors.

The luminous flux measurement is not affected by alignment of transfer standards, provided the burning position is not changed. The short time stability of the lamps during the sequential goniophotometric measurement (duration of 20 minutes) is monitored by a separate monitor channel and usually the relative variation of the monitor signal is negligible. Therefore, neither a correction of the value nor any contributions to the uncertainty will be taken into account.

$$corr2 = (1 + \gamma_{\mathrm{L}}) \tag{A17}$$

Models for the evaluation: The two models for the evaluation of the luminous intensity and the luminous flux are found from the principles in the main report combined with the contributions mentioned in the equ. (A7 to A17). It should be noted, that the goniophotometer is primarily calibrated with a luminous intensity reference standard to prove the stability, but this calibration cancels out because of the calibration factor determined from the luminous flux standards of the link partners.

$$I(T) = I_{\mathrm{R}}(T_{\mathrm{R}})\frac{\overline{y}}{\overline{y}_{\mathrm{R}}} \cdot \left(\frac{J_{\mathrm{AR}}}{J_{\mathrm{A}}}\frac{\overline{U}_J}{\overline{U}_{J,\mathrm{R}}}\right)^{-m_I}\left(1 - \frac{\overline{y}_{0g}}{\overline{y}} + \frac{\overline{y}_{0g\mathrm{R}}}{\overline{y}_{\mathrm{R}}} + 2\frac{d_{\mathrm{L}} - d_{\mathrm{LR}}}{d_{\mathrm{B}}} - \beta_{\mathrm{LR}} \cdot \Delta t_{\mathrm{LR}} + \gamma_{\mathrm{L}}\right) \tag{A18}$$

$$\Phi(T) = I_{\mathrm{G}}(T) \cdot G(\vartheta, \varphi)$$

$$I_{\mathrm{G}}(T) = I_{\mathrm{R}}(T_{\mathrm{R}}) \cdot \frac{\overline{y}_{00}}{\overline{y}_{\mathrm{R}}} \cdot \left(\frac{J_{\mathrm{AR}}}{J_{\mathrm{A}}}\frac{\overline{U}_J}{\overline{U}_{J,\mathrm{R}}}\right)^{-m_I}\left(1 - \gamma_{y00} + 2\frac{d_{\mathrm{C}} - d_{\mathrm{LR}}}{r_{\mathrm{G}}} - \beta_{\mathrm{LR}} \cdot \Delta t_{\mathrm{LR}}\right) \tag{A19}$$

$$G(\vartheta, \varphi) = \frac{2\pi}{y_{00}}\sum_{i=1}^{n_2} \overline{y}(\vartheta_i)\left[\cos\left(\vartheta_i - \tfrac{\Delta\vartheta}{2}\right) - \cos\left(\vartheta_i + \tfrac{\Delta\vartheta}{2}\right)\right]$$

A determination of the luminous intensity is independent of the type of transfer standard, thus, the structure is equal for luminous intensity and flux calibrations. The additional factor for the luminous intensity from the integration with respect to the relative angular distribution function is a weighted solid angle and thus, the product is a luminous flux.

Annex E: Uncertainty budgets

Factorisation: A value X_i assigned to a transfer standard may be calculated from equ. (A18 or A19) and the associated relative combined standard uncertainty $u_{\mathrm{rel}}(X_i)$ can be divided in two groups of contributions $X_i = Y_i \cdot Z$ one group Y_i combines contributions individual for each transfer standard and a second group Z includes all the other contributions, which are originated in the calibration process and common to all transfer standards in one batch. The associated relative uncertainties are $u_{\mathrm{rel}}(Y_i)$ and $u_{\mathrm{rel}}(Z)$, respectively. For each transfer standard yields:

$$X_i = Y_i \cdot Z; \qquad u_{\mathrm{rel}}(X_i) = \sqrt{u_{\mathrm{rel}}^2(Y_i) + u_{\mathrm{rel}}^2(Z)} \tag{A20}$$

Transfer standards in a batch of n devices transfer the unit with their individual values and the associated uncertainties $u_{\mathrm{rel}}(X_{\mathrm{i}})$. The averaged quantity $\overline{X}$ of that batch of transfer standards can be determined as the weighted mean of the individual entries and the associated uncertainty of the individual parts are averaged, too. No reduction is possible for the uncertainty associated to the common factor.

As example, let the values Y_i and the associated uncertainties $u_\mathrm{rel}(Y_\mathrm{i}) \approx u_\mathrm{rel}(Y)$ be similar, then no weighting is needed and the writing is simplified.

$$\overline{X} = Z \cdot \overline{Y} = Z \cdot \frac{1}{n} \cdot \sum_{i=1}^{n} Y_\mathrm{i}; \qquad u_\mathrm{rel}(X) = \sqrt{u_\mathrm{rel}^2(Z) + \tfrac{1}{n} u_\mathrm{rel}^2(Y)} \tag{A21}$$

This relation was already used in the main part of this report and a separation of the models of evaluation the luminous intensity and the luminous flux into the two factors is used at the pilot laboratory.

Model for Luminous Intensity: The values of individual transfer standards of a batch are depending on the same (set) of reference standards, which creates correlated uncertainties. This correlation is avoided, if the contributions in the evaluation are separated into two factors Y and Z, for individual or common effects, respectively.

$$\begin{aligned} I(T) = Y \cdot Z; \qquad & Y = \overline{y} \left(\frac{\overline{U}_J}{J_\mathrm{A}} \right)^{-m_I} \left(1 - \frac{\overline{y}_{0g}}{\overline{y}} + 2 \frac{d_\mathrm{L}}{d_\mathrm{B}} + \gamma_\mathrm{L} \right) \\ & Z = \frac{I_\mathrm{R}(T_\mathrm{R})}{\overline{y}_\mathrm{R}} \left(\frac{J_\mathrm{AR}}{\overline{U}_{J,\mathrm{R}}} \right)^{-m_I} \left(1 + \frac{\overline{y}_{0g\mathrm{R}}}{\overline{y}_\mathrm{R}} - 2 \frac{d_\mathrm{LR}}{d_\mathrm{B}} - \beta_\mathrm{LR} \cdot \Delta t_\mathrm{LR} \right) \end{aligned} \tag{A22}$$

Values and explanations for the quantities and associated uncertainties are given below to explain the origin of the information and show the symmetry for the values of the transfer and the reference standard, numbers are stated as typical for the PTB.

$I(T)$ Luminous intensity value in a specified direction of a transfer standard, corrected for operational and ambient conditions and for the nominal DC-lamp current J_A and a related distribution temperature T. The value of this quantity is the result of the calibration procedure and the associated relative expanded uncertainties has to be determined.

Y The factor contains all individual contributions, and the associated uncertainty can be reduced by averaging of the results from the members of a batch.

Z The factor is constant to all members of a batch of transfer standards and contains all common contributions. The associated uncertainty cannot be reduced by averaging processes.

J_A; J_AR The values of the lamp currents are fixed to achieve specified values of luminous intensity and of distribution temperature. The values are stated as nominal values with neither an uncertainty nor a tolerance interval.

$I_{\mathrm{R}}(T_{\mathrm{R}})$ Luminous intensity in a specified direction of a reference standard as certified for operational conditions (current J_{AR}, and distribution temperature T_{AR}) together with the associated standard uncertainties.

$\overline{U}_J, \overline{U}_{JR}$ Mean values of 30 readings of the voltage drop across the shunt resistor. The empirical standard deviation of the means are taken as associated uncertainties (the resolution of the DVM was never limiting the standard deviation).

$\bar{y}, \bar{y}_0$ Mean values of 15 readings of the current-to-voltage converter output, when measuring the light of the transfer standard. The values depend on the range setting and are given for light and dark measurements together with the empirical standard deviations of the mean taken as standard uncertainty (the resolution of the DVM was sufficiently high and never limiting the standard deviation).

$\bar{y}_{\mathrm{R}}, \bar{y}_{0\mathrm{R}}$ Mean values of 15 readings of the current-to-voltage converter output, when measuring the light of the reference standard. The values depend on the range setting and are given for light and dark measurements together with the empirical standard deviations of the mean taken as standard uncertainty (the resolution of the DVM was sufficiently high and never limiting the standard deviation).

d_{B} Distance is measured by an electronic translation meter with a resolution of $\delta d_{\mathrm{B}} = 5 \cdot 10^{-5}\ \mathrm{m}$, which is negligible, if compared to the certified interval of readings stated as expanded uncertainty $U(d_{\mathrm{B}}) = (0.0002 \pm 0.0001 \cdot d_{\mathrm{B}})\,\mathrm{m}$. The related standard uncertainty $u(d_{\mathrm{B}}) = (0.0001 + 0.00005 \cdot d_{\mathrm{B}})\mathrm{m}$ is found after division by $(k = 2)$, which gives for $d_{\mathrm{B}} = 5.0\ \mathrm{m}$ a standard uncertainty $u(d_{\mathrm{B}}) = 0.00035\,\mathrm{m}$.

$d_{\mathrm{L}}, d_{\mathrm{LR}}$ Zero-values are adjusted for the distances, but the alignments are estimated as rectangular probability distributions and converted in a standard uncertainty. The estimations depend mainly on the images made by the camera-supported alignment system.

$\gamma_{\mathrm{L}}, \gamma_{\mathrm{LR}}$ The angular alignment of unknown transfer standards is one of the most important contributions to the combined uncertainty associated to the luminous intensity. The coefficient is found from separate measurements of the intensity, when varying the alignment around a horizontal and a vertical axis through the centre of the filament, and the possible angels are estimated from the images.

$\beta_{\mathrm{L}} \cdot \Delta t_{\mathrm{L}}$ The effect of aging is not important for transfer standards, because the duration of the operation is too short, it would be detected, when series of repeated measurements were carried out.

$\beta_{\mathrm{LR}} \cdot \Delta t_{\mathrm{LR}}$ The effect of aging can become important for reference standards operated for a longer period of time. At PTB the reference standards are recalibrated annually within the network creating only small contribution to the uncertainty. In the comparison the transfer standards of the link partners eliminate any aging.

m, mI The mismatch index of a photometer can have a positive or a negative value and the associated uncertainty may be even larger than the value. At the PTB these values are well known and small $|m| \approx 0.01$. The other exponent, describing the variation of luminous intensity with a change of lamp current, is common to all incandescent lamps. The variation between different types of lamps may be included in the associated uncertainty interval.

Uncertainty budget for luminous intensity: At the pilot laboratory, the uncertainty budget is calculated using the software "Mathematica" and the three equations are given first. The symbols, values with the associated standard uncertainties and degrees of freedom (DOF) for the contributions are explained earlier and now listed below.

output quantity $I = Y \star Z$

uncorrelated $Y = y \left(\frac{U_J}{J_A}\right)^{-m_I} \left(1 + \frac{2\,d_L}{d_B} - \frac{y_0}{y} + \gamma_L\right)$

correlated $Z = \frac{I_R \left(\frac{U_{JR}}{J_{AR}}\right)^{m_I} \left(1 - \frac{2\,d_{LR}}{d_B} + \frac{y_0}{y_R} - \beta_{LR}\Delta t_{LR}\right)}{y_R}$

The calculation starts with the individual contribution, herein denoted as "uncorrelated". The budget is valid for all individual lamps, even if in this example representative values are taken instead of those for an individual lamp.

In the uncertainty budget below, the entries are sorted with the more important contributions at the top, which clearly shows, that a mechanical alignment of the lamp (without additional jigs or tools) gives the highest contribution. These values are measured with independent measurements and the extremes are used to define an interval with rectangular distribution. The value of the effective DOF ef for the combined uncertainty was calculated from the Welch-Satterthwaite-equation, but it is limited to a maximum of 1000, because larger values are more or less meaningless.

$$Y = y \left(\frac{U_J}{J_A}\right)^{-m_I} \left(1 + \frac{2\,d_L}{d_B} - \frac{y_0}{y} + \gamma_L\right)$$

symbol	value	uncertainty	DOF	sensitivity	contribution	second order
γ_L	0.	0.0046	∞	6.07762	0.0279571	0.
U_J	0.191778	9.5×10^{-6}	30.	-237.66	-0.00225777	0.
d_L	0.	0.001	∞	2.21004	0.00221004	0.
y	6.0771	0.00127619	15.	1.00009	0.0012763	0.
m_I	7.5	0.5	∞	0.0000697132	0.0000348566	0.
y_0	0.00057	6.84×10^{-6}	15.	-1.00009	-6.84059×10^{-6}	0.
d_B	5.5	0.001	∞	0.	2.84134×10^{-7}	2.84134×10^{-7}
				u(bud) =	0.028164	2.84134×10^{-7}
				u(cor) =	0.	
Y =	6.07705	ef =	1000.	u(Y) =	0.028164	
urel(Y) =	0.00463448					

The second factor for the determination of the luminous intensity as output quantity is the common contribution, also denoted as "correlated" factor. In the list all entries are sorted again with decreasing contribution. In this budget, the realisation of the units and the transfer by the reference lamp(s) gives the dominant contribution, a result just as expected for a high level transfer.

$$Z = \frac{I_R \left(\frac{U_{JR}}{J_{AR}}\right)^{m_I} \left(1 - \frac{2\, d_{LR}}{d_B} + \frac{y_0}{y_R} - \beta_{LR}\, \Delta t_{LR}\right)}{y_R}$$

symbol	value	uncertainty	DOF	sensitivity	contribution	second order
I_R	225.35	0.36056	∞	0.225786	0.0814094	0.
y_R	4.42891	0.00234732	15.	-11.4898	-0.0269703	0.
U_{JR}	0.57001	0.0000171	30.	669.474	0.011448	0.
β_{LR}	0.0003	0.00015	∞	-46.6477	-0.00699715	0.
d_{LR}	0.	0.0002	∞	-18.5049	-0.00370097	0.
Δt_{LR}	0.916667	0.0833333	∞	-0.0152665	-0.00127221	0.
m_I	7.5	0.5	∞	0.00089264	0.00044632	0.
y_0	0.00057	6.84×10^{-6}	15.	11.4901	0.000078592	0.
d_B	5.5	0.001	∞	0.	4.75815×10^{-7}	4.75815×10^{-7}
				u(bud) =	0.0868932	4.75815×10^{-7}
				u(cor) =	0.	
Z =	50.8809	ef =	1000.	u(Z) =	0.0868932	
urel(Z) =	0.00170778					

The uncertainties of the two factors depend mainly on the first few contributions, and the others may be attributed as being proofed and found to be negligible. For a discussion in this document a more detailed budget is presented. It should be noted, that the budget is sufficiently detailed but not complete, because in the discussion of the ratios several possible contributions are already cancelled out because of their unimportance at the pilot laboratory.

Finally the combination of the two factors gives the value and associated uncertainty of the luminous intensity.

uncorrelated	$Y = y \left(\frac{U_J}{J_A}\right)^{-m_I} \left(1 + \frac{2\, d_L}{d_B} - \frac{y_0}{y} + \gamma_L\right)$	= 6.07705	urel(Y) = 0.00463448
correlated	$Z = \frac{I_R \left(\frac{U_{JR}}{J_{AR}}\right)^{m_I} \left(1 - \frac{2\, d_{LR}}{d_B} + \frac{y_0}{y_R} - \beta_{LR}\, \Delta t_{LR}\right)}{y_R}$	= 50.8809	urel(Z) = 0.00170778
output quantity	I = Y*Z	= 309.206	urel(I) = 0.00493912

The values stated above show the relative uncertainties $u_{\mathrm{rel}}(Y)$, $u_{\mathrm{rel}}(Z)$ and the combined relative uncertainty $u_{\mathrm{rel}}(I)$. The factorization in the main part of this report used the two factors $y_{i\,k} = I \cdot \left(1 \pm k \cdot u_{\mathrm{rel}}^2(Y)\right) = 309.2 \cdot \left(1 \pm k \cdot 0.0046\right)$ and $C_i = 1$ with a relative uncertainty $C_i = 1 \cdot \left(1 \pm k \cdot u_{\mathrm{rel}}^2(Z)\right) = \left(1 \pm k \cdot 0.0017\right)$. Finally, the statement about the calibration of the transfer standard can be given:

The luminous intensity of the lamp transfer standard was determined with a value and associated relative expanded uncertainty for a coverage factor of $k = 2$ for an interval containing a 95,45% fraction of probability and reads:

$$I_{\mathrm{A}} = 309.21\,(1 \pm 0.0099)\ \mathrm{cd} \tag{A23}$$

Model for the evaluation of luminous flux: The integration of the angular distribution of a transfer standard can give a significant contribution to the uncertainty of a luminous flux value. At the pilot laboratory the conversion to a frequency avoids any delays and allows the (nearly) perfect averaging of the photocurrent <u>within one zone</u> (no uncertainty contribution due to φ). Similarly the angles in ϑ for start and end of a zone can produce uncertainty contributions, but the mathematic uses the angle from the end of a zone definitely as the angle to start the next zone, which averages any possible jitter. The effect of the jitter can be determined from repeated measurements, which are carried out for different traces within the zones. The measurement of angles is done with high resolution angle encoders, and the arms of the goniophotometer are stiff enough to ensure that differences between location and the indicated angle are negligible. The value g as the result of the analogue (pulse counting) and digital determination is printed from the goniophotometer and over large series of calibrations a high repeatability $u_{\mathrm{rel}}(g)$=19 10^{-4} of the integrations was found, which is minor depending on the angular distribution.

$$g = \sum_{i=1}^{n_2} \bar{y}(\vartheta_i)\left[\cos\left(\vartheta_i - \tfrac{\Delta\vartheta}{2}\right) - \cos\left(\vartheta_i + \tfrac{\Delta\vartheta}{2}\right)\right] \tag{A24}$$

The model for evaluation of luminous flux was stated in equ. (A19) and can be modified with the substitution of equ. (A21). A correlation is avoided, if the contributions are separated in two factors Y and Z for individual or common effects, respectively.

$$\Phi(T) = Y \cdot Z, \qquad Y = g \cdot \left(\frac{\overline{U}_J}{J_{\mathrm{A}}}\right)^{-m_I}$$
$$Z = 2\pi \frac{I_{\mathrm{R}}(T_{\mathrm{R}})}{\bar{y}_{\mathrm{R}}} \cdot \left(\frac{J_{\mathrm{AR}}}{\overline{U}_{J,\mathrm{R}}}\right)^{-m_I} \left(1 - \gamma_{y00} + 2\frac{d_{\mathrm{C}} - d_{\mathrm{LR}}}{r_{\mathrm{G}}} - \beta_{\mathrm{LR}} \cdot \Delta t_{\mathrm{LR}}\right) \tag{A25}$$

The values and explanations for the quantities and associated uncertainties are given below to explain the origin of the information and show the symmetry for the values of the transfer and the reference standard.

$\Phi(T)$ Luminous flux value of a transfer standard, corrected for operational and ambient conditions, for the nominal DC-lamp current J_{A} and a related distribution temperature T. The value is the result of the calibration procedure and the associated relative expanded uncertainty has to be determined.

Y The factor contains all individual contributions and the associated uncertainty can be reduced by averaging of the results from the members of a batch.

Z The factor is constant for all members of a batch and contains all common contributions. The associated uncertainty is not reduced by averaging processes.

J_{A}; J_{AR} The values of the lamp currents are fixed to achieve specified values of luminous intensity and flux and of the distribution temperature. The values are stated as nominal values with neither an uncertainty nor a tolerance interval.

$I_{\mathrm{R}}(T_{\mathrm{R}})$ Luminous intensity in a specified direction of a reference standard as certified for operational conditions (current J_{AR}, and distribution temperature T_{AR}) with associated standard uncertainties.

$\overline{U}_J$, $\overline{U}_{JR}$ Mean values of 30 readings of the voltage drop across the shunt resistor. The empirical standard deviation of the means are taken as associated uncertainties (the resolution of the DVM was never limiting the standard deviation).

g Values $\bar{y}$ of readings of current-to-frequency converter output, when measuring the angular distribution of the transfer standard. The values depend on range setting and are averaged for zones (the resolution of the frequency counter was sufficiently high and never limiting the standard deviation). The integration with the angular weighting is known to give a relative uncertainty of $u_{\mathrm{rel}}(g)$=19 10^{-4}

$\bar{y}_{\mathrm{R}}$ Mean values of 15 readings of the current-to-frequency converter output, when measuring the light of the reference standard. The values depend on the range setting and are given together with the empirical standard deviations of the mean taken as standard uncertainty (the resolution of the frequency counter was sufficiently high and never limiting the standard deviation).

r_{G} Distance $r_{\mathrm{G}} = 2.5\ \mathrm{m}$ is measured by an electronic translation meter with a sufficiently high resolution. The standard uncertainty $u(r_{\mathrm{G}}) = 0.00035\,\mathrm{m}$ is found from the certificate, but the dynamic forces of the moving frames gives a variation depending on speed and directions up to $u(r_{\mathrm{G}}) = 0.002\ \mathrm{m}$.

$d_{\mathrm{C}}, d_{\mathrm{LR}}$ Zero-values are adjusted for the distances, but the alignments are estimated as rectangular probability distributions and converted in a standard uncertainty.

γ_{y00} The straylight in the goniophotometer was determined by separate investigations and a relative error of 15 10^{-4} was found with a standard deviation 5 10^{-4} depending only minor on the angular distribution of the lamp to be tested.

$\beta_{\mathrm{LR}} \cdot \Delta t_{\mathrm{LR}}$ The effect of aging can become important for reference standards operated for a longer period of time. At PTB the reference standards are recalibrated annually within the network creating only small contribution to the uncertainty. In the comparison the transfer standards of the link partners eliminate any aging.

m, mI The same explanation as stated earlier for luminous intensity $|m| \approx 0.01$.

Uncertainty budget for luminous flux: At the PTB, the uncertainty budget is calculated using the software "Mathematica" and the three equations are given first.

output quantity $\Phi = Y \star Z$

uncorrelated $Y = g \left(\frac{U_J}{J_A} \right)^{-m_I}$

correlated $Z = \dfrac{2\pi I_R \left(\frac{U_{JR}}{J_{AR}} \right)^{m_I} \left(1 + \frac{2(d_C - d_{LR})}{r_G} - \gamma_{y00} - \beta_{LR} \Delta t_{LR} \right)}{y_R}$

The explanations given earlier for the luminous intensity calculation are valid, too.

$$\Phi = g \left(\frac{U_J}{J_A} \right)^{-m_I}$$

symbol	value	uncertainty	DOF	sensitivity	contribution	second order
g	8.125	0.0154375	1500.	0.999232	0.0154256	0.
U_J	0.19529	9.765×10^{-6}	30.	-311.796	-0.00304469	0.
m_I	7.5	0.5	∞	-0.0008315	-0.00041575	0.
				u(bud) =	0.0157287	0.
				u(cor) =	0.	
Y =	8.11876	ef =	1000.	u(Y) =	0.0157287	
urel(Y) =	0.00193733					

The second factor for the determination of the luminous flux as output quantity is the common contribution, also denoted as "correlated" factor. In the list all entries are sorted again with decreasing contribution. In this budget, the constancy of the radius due to the dynamic forces and the realisation of the units transferred by the reference lamp(s) gives the dominant contribution.

$$Z = \frac{2\pi I_R \left(\frac{U_{JR}}{J_{AR}} \right)^{m_I} \left(1 + \frac{2(d_C - d_{LR})}{r_G} - \gamma_{y00} - \beta_{LR} \Delta t_{LR} \right)}{y_R}$$

symbol	value	uncertainty	DOF	sensitivity	contribution	second order
d_c	0.	0.002	∞	255.827	0.511653	0.
I_R	225.35	0.36056	∞	1.41617	0.510613	0.
y_R	4.4289	0.00265734	15.	-72.057	-0.19148	0.
γ_{y00}	0.0015	0.0005	∞	-319.783	-0.159892	0.
U_{JR}	0.57002	0.0000285	30.	4198.97	0.119671	0.
β_{LR}	0.0004	0.00015	∞	-426.378	-0.0639567	0.
d_{LR}	0.	0.0002	∞	-255.827	-0.0511653	0.
Δt_{LR}	1.33333	0.1	∞	-0.127913	-0.0127913	0.
m_I	7.5	0.5	∞	0.0111975	0.00559873	0.
r_G	2.5	0.002	∞	0.	0.000290878	0.000290878
				u(bud) =	0.77844	0.000290878
				u(cor) =	0.	
Z =	319.133	ef =	1000.	u(Z) =	0.77844	
urel(Z) =	0.00243923					

The uncertainties of the two factors depend mainly on the first few contributions, and the others may be attributed as being proofed and found to be negligible. For a discussion in this document a more detailed budget is presented. It should be noted, that the budget is sufficiently detailed but not complete, because in the discussion of the ratios several possible contributions are already cancelled out because of their unimportance at the pilot laboratory.

Finally the combination of the two factors gives the value and associated uncertainty of the luminous flux.

uncorrelated	$Y = g\left(\frac{U_J}{J_A}\right)^{-m_I}$	= 8.11876	urel(Y) = 0.00193733
correlated	$Z = \frac{2\pi I_R \left(\frac{U_{JR}}{J_{AR}}\right)^{m_I}\left(1+\frac{2(d_C-d_{IR})}{r_G} - \gamma_{Y00} - \beta_{LR}\Delta t_{LR}\right)}{y_R}$	= 319.133	urel(Z) = 0.00243923
output quantity	$\Phi = Y*Z$ ßt	= 2590.97	urel(I) = 0.00311498

The values stated above show the relative uncertainties $u_{\mathrm{rel}}(Y)$, $u_{\mathrm{rel}}(Z)$ and the combined relative uncertainty $u_{\mathrm{rel}}(I)$. The factorization in the main part of this report used the two factors $y_{i\,k} = \Phi\cdot\left(1\pm k\cdot u_{\mathrm{rel}}^2(Y)\right) = 2591.0\cdot\left(1\pm k\cdot 0.0019\right)$ and $C_i = 1$ with a relative uncertainty $C_i = 1\cdot\left(1\pm k\cdot u_{\mathrm{rel}}^2(Z)\right) = \left(1\pm k\cdot 0.0024\right)$. Finally, the statement about the calibration of the transfer standard can be given:

The luminous flux of the lamp transfer standard was determined with a value and associated relative expanded uncertainty for a coverage factor of $k = 2$ for an interval containing a 95,45% fraction of probability and reads:

$$\Phi = 2591.0\,\left(1\pm 0.0062\right)\,\mathrm{lm} \tag{A26}$$

Annex F: Inspection of the transfer standards

Has the lamp transportation package been opened during transit ? e.g.Customs…☐Y /☐ N

If Yes please give details:

Is there any damage to the transportation package?……☐Y / ☐N.

If Yes please give details:

Are any fingerprints or contaminations visible indicating improper handling? .☐Y / ☐N

If Yes give details:

Are there any visible signs of damage to the lamp or accessories?…..☐Y / ☐N

If Yes please give details (e.g. scratches, dust, broken filament, alignment mask moved etc):

Do you believe the standard is functioning correctly ?…☐Y/ ☐N

If not please indicate your concerns

Operator:

Laboratory:

Date: Signature: ………………………………..

Annex G: Description of the measurement facility

This form should be used as a guide. It is anticipated that many of the questions will require more information than the space allocated. Please use separate sheets for a comprehensive description of the geometrical condition during the measurements (lamp, detector, bench, number, distance and size of baffles and shielding)

.

Make and type of the photometer (or equivalent)

Laboratory transfer standards used:

Description of measuring technique (please include a diagram):

Establishment or traceability route of primary scale including date of last realisation and breakdown of uncertainty:

Description of calibration laboratory conditions: e.g. temperature, humidity etc.

Operating conditions of the lamps: e.g. geometrical alignment, polarity, stray-light reduction etc.

Operator:

Laboratory:

Date: Signature: ………………………………..

Annex H: Record of lamp operating time

Lamp:

Date	**Switch-on Local Time**	**Activity (Test, Alignment, Measure)**	**Switch-off Local Time**	**Burn Hrs**	**Operator initials**

Operator:

Laboratory:

Date: Signature: ………………………………..

Annex I: Receipt confirmation

FAX

To: **Detlef Lindner**

Physikalisch-Technische Bundesanstalt
4.12 Photometry
Bundesallee 100
D 38116 Braunschweig
Germany

Fax +49 531 592 69 4123
Email: RMO-569@ptb.de

From: **Participating Laboratory**

We confirm having received the standards of the

EURAMET *RMO Comparison of Luminous Intensity/Luminous Flu*x

After visual inspection

☐ No damage has been noticed;

☐ The following damage must be reported:

Operator:

Laboratory:

Date: Signature: ………………………………..