EUROMET Key Comparison
EUROMET.L-K4
(EUROMET Project 812)
Calibration of diameter standards
GROUP 2
Version B
FINAL v. 1.2



MKEH Hungarian Trade Licencing Office

> MKEH Budapest, May 2010 Edit Bánréti

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1. Introduction

The metrological equivalence of national measurement standards and of calibration certificates issued by national metrology institutes is established by a set of key comparisons chosen and organized by the Consultative Committees of the CIPM or by the regional metrology organizations (RMOs) in collaboration with the Consultative Committees.

At its meeting in October 2003, the EUROMET TC Length decided upon a key comparison on diameter standards, numbered EUROMET.L-K4. This comparison follows the previous 384 (EUROMET.L-K4) comparison.

Due to the large number of the participants, it has been decided to have 2 groups in the project. The participants for the 2 groups were separated according to the claimed uncertainties. Those whose expanded uncertainties are less that or equal to 0.3 micrometer (for 50 mm gauge) belong to the group 1, the others to the group 2. The link between the two groups was served by the laboratories who took part in both the group 1 and group 2.

Mr. Gian Bartolo Picotto, INRIM (formerly IMGC) Italy acts as the pilot laboratory for the group 1, Ms. Edit Banreti, MKEH (formerly OMH) Hungary acts as the pilot for the group 2.

The measurements started in 2005, after purchasing and the preliminary measurements of the gauges.

A goal of the EUROMET key comparisons for topics in dimensional metrology is to demonstrate the equivalence of routine calibration services offered by NMIs to clients, as listed in Appendix C of the Mutual Recognition Agreement (MRA) [BIPM, 1999]. To this end, participants in this comparison agree to use the same apparatus and methods as routinely applied to client artefacts.

2. Organisation

2.1. Participants in the group 2

		Table 1
Laboratory	Address	Contact person /tel/fax/e-mail
BEV	Bundesamt für Eich – und	Dr. Michael Matus
	Vermessungswesen	+43 1 21110 6540
	Arltgasse 35	+43 1 21110 6000
	A-1160 Wien	michael.matus@bev.gv.at
	Austria	
CEM	Centro Espanol de Metrologia	Mr. Emilio Prieto
	Alfar, 2	+34 91 807 47 16
	ES-28760 Tres Cantos (Madrid)	+34 91 807 48 07/809
	Spain	eprieto@cem.mityc.es
CMI	Czech Metrology Institute	Dr. Petr Balling
	V Botanice 4	+420 257 288 326
	CZ 150 70 Praha 5	+420 257 32 80 77
	Czech Republic	pballing@cmi.cz
DTI	Danish Technological Institute	Mr. Jens Bo Toftegaard
	Gregersensvej 1	+45 7220 3034
	DK-2630 Taastrup	+45 7220 2999
	Denmark	jens.bo.toftegaard@teknologisk.d
		k 3
EIM	Hellenic Institute of Metrology	Dr. Christos Bandis
	Industrial Area of Thessaloniki	+30 2310 56 99 99
	Block 45	+30 2310 56 99 96
	GR-57 022 Sindos	bandis@eim.gr
	Thessaloniki	
	Greece	
FSB	University of Zagreb	Mr. Vedran Mudronja
	Faculty of Mechanical Eng. and	+385 1 616 83 27
	Naval Architecture	+385 1 616 85 99
	Ivana Lucica 5	vedran.mudronja@fsb.hr
	HR 10000 Zagreb	
	Croatia	
GUM	Central Office of Measures	Mr. Zbigniew Ramotowski
	ul. Elektoralna 2	+48 22 581 9543
	PL-00950 Warszawa	+48 22 620 8378
	Poland	length@gum.gov.pl
INRIM	Instituto Nazionale di Ricerca	Mr. Gian Bartolo Picotto
(formerly IMGC) -link	Metrologica	+39 011 3977 469
	Strada delle cacce, 91	+39 011 3977 459
	IT-10135 Torino	g.picotto@inrim.it
	Italy	
INM	National Institute of Metrology	Mr. Alexandru Duta
	11, Sos. Vitan-Barzest	+40 21 334 55 20
	RO-042122	+40 21 334 55 33
	Bucharest 4	alexandru.duta@inm.ro
	Romania	
LNMC	Latvian National Metrology Centre	Mis. Edite Turka
	157, K. Valdemara Str.	+3/1 / 362 086
	Riga, LV-1013	+3/1 / 362 805
	Latvia	edite.turka@Inmc.lv

METAS (link)	METAS	Dr. Rudolf Thalmann
(Lindenweg 50	+41 31 32 33 385
	CH-3003 Bern-Wabern	+41 31 32 33 210
	Switzerland	rudolf.thalmann@metas.ch
METROSERT	AS Metrosert	Mr. Lauri Lillepea
	Aru 10	+372 6 019 508
	EE-10317 Tallinn	+372 6 020 081
	Estonia	lauri.lillepea@metrosert.ee
MIRS	University of Maribor	Dr. Bojan Acko
	Faculty of Mech. Eng.	+386 2 220 7581
	Smetanova 17	+386 2 220 7990
	SI-2000 Maribor	bojan.acko@uni-mb.si
	Slovenia	
MKEH	Hungarian Trade Licencing Office	Ms. Edit Banreti
(formerly OMH)	H-1124 Budapest	+36 1 458 59 97
	Németvölgyi út 37-39	+36 1 458 59 27
	Hungary	banretie@mkeh.hu
NSAI	NSAI National Metrology Laboratory	Mr. Paul Turner
(formerly NML)	Claremont Avenue, Glasnevin	+353 1 808 2611
	IE-9 Dublin	+353 1 808 2603
	Ireland	paul.turner@nsai.ie
NPL (link)	National Physical Laboratory	Dr. Andrew Lewis
	Queens Road	+44 20 8943 6124
	Teddington Middlesex	+44 20 8614 0533
	TW 11 OLW	andrew.lewis@npl.co.uk
	United Kingdom	-
NRC	Institute for National Measurement	Mr. Kostadin Doytchinov
	Standards (INMS)	+613 991 0265
	National Research Council Canada	+613 952 1394
	(NRC)	Kostadin.Doytchinov@nrc-
	1200 Montreal Road	cnrc.gc.ca
	Ottawa, ON, Canada	
	K1A OR6	
UME	Ulusal Metroloji Enstitüsü	Dr. Tanfer Yandayan
	Tübitak, UME	+90 262 646 63 55/235
	Anibal Cad. Besevler	+90 262 646 59 14
	TR-Gebze, Kocaeli 41470	tanfer.yandayan@ume.tubitak.gov
	Turkey	.tr

2.2. Schedule

Table 2 indicates the timetable for the group 2 with the dates for performing the measurements.

The gauges have been purchased from Switzerland. METAS had measured the gauges before the technical protocol was available in order to check if the gauges were in suitable condition for the comparison.

At the EURAMET TC-L meeting held in October 2008, METAS kindly offered to measure the gauges according to the technical protocol, to have better link between the two groups and also to the CCL comparison.

		Table 2
Laboratory	Country	Date
MKEH (OMH)	Hungary	May-June 2005
BEV	Austria	July 2005
-	-	August 2005
CMI	Czech Republic	September 2005
GUM	Poland	October 2005
NSAI (NML)	Ireland	November 2005
DTI	Denmark	December 2005
NPL (link)	United Kingdom	January 2006 and
		March-Apr 2009
MIRS	Slovenia	February 2006
EIM	Greece	March 2006
METROSERT	Estonia	April 2006
LNMC	Latvia	May 2006
MKEH (OMH)	Hungary	June 2006
INRIM (link)	Italy	July 2006
FSB	Croatia	August 2006
INM	Romania	September 2006
UME	Turkey	October 2006
NRC	Canada	November 2006
CEM	Spain	December 2006
MKEH	Hungary	January 2007
MKEH	Hungary	September 2008
METAS (link)	Switzerland	February 2009

The comparison was organized in two loops because of the limited validity of the ATA Carnet. The first loop was organized to have the circulation within the EU countries, than an ATA Carnet was used to circulate in the second loop. There was 1 month delay in the first loop, but we had more than 6 months delay as a consequence of custom problems, delay in shipment to the next participant, etc.

The gauges were measured 4 times by the pilot laboratory, before the circulation, after the first loop, after the second loop and one year later in order to check the stability of the gauges.

3. Standards

				Table 3
Туре	Manufacturer	Identification	Dimension [mm]	Material
Ring	Microtool	2619	5	Steel
Ring	Microtool	2618	40	Steel
Plug	Microtool	2621	5	Steel
Plug	Microtool	2620	50	Steel
Sphere	SWIP	D 4901	30	Ceramic (Al ₂ O ₃)

The gauges that were taken part in the comparison are listed in the table 3 below.

Inscriptions:

<u>Ring gauges</u>: the two ring gauges were marked on their upper surface with their identification and two lines indicating the measurement direction.

<u>Plug gauges</u>: the two plug gauges were marked on their handle with their identification and their upper surface two lines indicating the measurement direction.

<u>Sphere:</u> the identification number and two lines indicating the measurement direction were marked on the bottom of the ball near the shaft.

4. Measurement instructions and reporting of results

4.1. Traceability

Length measurements had to be traceable to the latest realisation of the metre as set out in the current "*Mise en Pratique*".

Temperature measurements had to be made using the International Temperature Scale of 1990 (ITS-90).

4.2. Measurand

The measurand was the diameter of each gauge at 20 °C and corrected to zero force. The diameter of the ring and plug gauges had to be measured at the marked lines in 3 different heights according to the table below. The diameter of the sphere had to be measured at the marked lines.

 $x mm \uparrow^{*}$ and $x mm \downarrow^{*}$ refer to the required measurement locations x mm above and below the mid height of the cylinder. The upper side of the rings was defined by the inscription, for the plugs the handle or the holding cylinder were assumed to be below.

The measurement locations for the cylindrical gauges are shown in table 4:

			Table 4
\varnothing 5 mm ring gauge	\varnothing 40 mm ring gauge	\varnothing 5 mm plug gauge	Ø 50 mm plug gauge
3 mm ↑	10 mm ↑	2 mm ↑	6 mm ↑
middle	middle	middle	middle
3 mm↓	10 mm ↓	2 mm↓	6 mm ↓

The roundness of the ring and plug gauges had also to be measured at the same heights as the diameter measurements and one roundness measurement for the sphere on the equator.

The straightness of the ring and plug gauges had to be measured in the marked lines (0 and 180) °.

The roundness and straightness measurements were required only if they are done normally for the customers as well.

The calibration had to be carried out as for a normal customer. It means that there was no information about the form error of the artefacts. The measurements had to be reported for zero measuring force and at the reference temperature of 20 °C, using a thermal expansion coefficient of $11,6\cdot10^{-6}$ 1/K for the cylindrical standards and $8,1\cdot10^{-6}$ 1/K for the ceramic ball, which were both assumed values because they have not been measured.

The participants were invited to report the deviation from roundness at given cut-off frequencies (in UPR) of the long-pass filter, in order to achieve a better comparability of the results. Gaussian filter had to be used, where it was available, but in any case the participants were asked to specify which type of filter they used.

For the 30 mm sphere, the participants were invited to report the deviation from roundness at 500 UPR, 150 UPR and 50 UPR.

In addition, following a decision at the 2005 EUROMET TC-L meeting, it was decided to use the results of roundness measurement on the sphere as evidence for proving CMC claims of those NMIs offering a roundness measuring service, or planning to submit a CMC for such a service, especially where such NMIs have no previous comparison evidence available. Participants were therefore invited to report the deviation from roundness of the sphere at 500, 150 and 50 UPR. These measurements had to be performed using the roundness measuring service for which the CMC claims are to be tested.

The uncertainty of measurement had to be estimated according to the ISO Guide for the Expression of Uncertainty in Measurement.

5. Measurement methods and instruments used by the participants

The methods and instruments used by the participants for the diameter measurements are listed in table 5, together with the traceability of those who used reference rings, plugs. The instruments for the roundness measurements are in table 6.

Toble 4

					Table 5
Laboratory	Equipment	Reference	Probe	Temp. [⁰C]	Traceability of rings used
МКЕН	SIP 550M length measuring machine.	Reference rings (12 and 40 mm) and plugs (5 and 50 mm)	Spherical ϕ 1.2 for ring 5, hooked for ring 40, flat for plugs and sphere	19.5-20.5	METAS
BEV	SIP 3002 length measuring machine with laser interferometer	Plug 50 mm, rings 20 and 40 mm	Spherical ϕ 1.2 , flat for plugs and sphere	19.99-20.26	METAS
СМІ	SIP 1002M length measuring machine with laserinterferometer	Gauge blocks	Spherical for rings and plugs, flat for sphere	19.7-20.3	-
GUM	SIP 3002M length measuring machine	Gauge blocks ring 5 mm	Spherical ϕ 2 for ring 5, hooked for ring 40, flat for plugs and sphere	20.02-20.5	Ring 5 mm: METAS
NML	OKM OPAL 600 Universal Measuring Machine	Gauge blocks	1500 mN	19.69-20.10	-
DTI	ULM		Spherical ϕ 2, ϕ 3, flat for plugs and sphere	20.0-20.1	DFM DKD, Zeiss
NPL	NPL modified Zeiss metroscope (plug 50 and sphere), machine with capacitive sensors (plug 5) and NPL designed internal measuring machine (rings)	Gauge blocks and fused silica box standard	5 mm / 2 mm diameter ball- ended 0.06 N, 2 mm / 0.5 mm flat, 0.15 N/2.5 N	19.915-20.3	-
MIRS	Zeiss ULM with laserinterferometer	External: absolute, internal: gauge blocks	Spherical $\phi 2$ for rings, flat for plugs and sphere	19.71-19.95	(gauge blocks BEV)
EIM	Mahr 828 with laserinterferometer	External: absolute, internal: gauge blocks	Spherical ϕ 3for rings, ϕ 5 for plugs and sphere	20.2-20.3	-
METROSERT	ULM Opal with laserinterferometer	Ring Plug	Spherical ϕ 0.62 for 5 mm, ϕ 10 for 40 mm, plane for plugs and sphere	19.9-20.5	MIKES
LNMC	Ring: ULM 828 CiM Plug: Optical vertical length comparator	Rings 10 and 50 mm	Spherical ϕ 3 for rings, flat for plugs	19.8-20.1	no information

INRIM	Moore measuring machine modified at INRIM with laserinterferometer and LVDT probe	Gauge block	Spherical, $\phi 2$ and 3 mm	19.92-20.00	-
FSB	JOINT length meas. machine	Ring 50 for ring 50 mm	Spherical ϕ 1.5 for ring; plain ϕ 2 for plugs and sphere	19.95-20.15	INRIM
INM	SIP-MUL 1000 with HP5526 laserinterferometer	Gauge block 20 mm	Spherical, ø3 mm	20.00-20.60	-
UME	Modified Mahr 828 CIM 1000	Ref. ring 50 for rings, gauge blocks for plugs	Spherical, ϕ 3 and 5 mm	19.7-20.3	METAS
NRC	Mitutoyo Legex 707 CMM with MPP 300 probe	Gauge blocks	Spherical, $\phi 2$ and 4 mm	19.94-19.97	-
CEM	Mitutoyo Ultra Quick Vision CMM	Gauge blocks, grade K	Spherical $\phi 2$ and 4 mm	19.89-20.10	-
METAS	Cylindrical gauges: Length measuring machine designed by SIP and METAS Sphere: SIP 3002 length measurement machine (Abbe comparator) with laser interferometer and flat probes.	Gauge blocks	Spherical \operatorname{0}.5 for ring 5, \operatorname{4} for ring 40 and plugs and flat for sphere	19.98-20.07	-

	Table 6.
Laboratory	Equipment
CMI	Taylor Hobson Talyrond 3-PC
GUM	Taylor Hobson Talyrond 210
DTI	Formtester FAG 2100
NPL	Taylor Hobson Talycenta (ring and plugs) and Taylor Hobson Talyrond 73 (sphere)
EIM	Taylor Hobson Talyrond 290
METROSERT	Taylor Hobson TR 265
INRIM	RTH TR30 modified at INRIM
FSB	Mahr MMQ 3 (ref: ϕ 50 hemisphere)
INM	Mahr MMQ 44 (round), TH Form Talysurf (str.) with roughness standard
UME	Mahr MFU 800/ MMQ 40 (results on MFU 800 was used)
CEM	Mitutoyo RA-H5000 0.01 μm resolution for rings and plugs, 0.001 μm with multi-step error sep. method
METAS	Taylor Hobson TR300 (cylindrical gauges), TR73 (sphere)

6. Stability of the gauges

The gauges were measured four times by the pilot laboratory, at the dates indicated in the graphs. The following diagrams show the measured diameters with the stated expanded uncertainties (k=2).



6.1. Stability of the ring 5 mm





6.2. Stability of the ring 40 mm







6.3. Stability of the plug 5 mm







6.4. Stability of the plug 50 mm







6.5. Stability of the sphere 30 mm



7. Analysis of the results

The weighted mean is used as the reference value in the comparison.

For each laboratory (*i*) the normalised weight, w_i was calculated by the following formula:

$$w_i = C \cdot \frac{1}{u^2(x_i)} \tag{1}$$

where $u(x_i)$ is the standard uncertainty given by the laboratory "*i*" and *C* is the normalizing factor and is calculated by the following formula:

$$C = \frac{1}{\sum_{i=1}^{n} \frac{1}{u^2(x_i)}}$$
 (2)

where *n* is the number of the laboratories

The weighted mean (reference value) is:

$$\overline{x}_{w} = \sum_{i=1}^{n} w_{i} \cdot x_{i}$$
(3)

The uncertainty of the deviation from the weighted mean is:

$$u(x_i - \overline{x}_w) = \sqrt{u^2(x_i) - u_{\text{int}}^2(\overline{x}_w)}$$
(4)

The analysis of the results of each participant can be done by calculating the deviation of the given result from the weighted mean $(x_i - \overline{x}_w)$ and the uncertainty of this deviation.

The statistical consistency of the results with the uncertainties given by the participants can be checked by the E_n value for each laboratory.

$$E_{n} = \frac{x_{i} - \bar{x}_{w}}{2\sqrt{u^{2}(x_{i}) - u_{int}^{2}(\bar{x}_{w})}} \qquad (k=2)$$
(5)

where $x_i - \overline{x}_w$ is the deviation from the weighted mean for a result of a laboratory,

 u_{int} is the so called internal standard deviation that is based on the estimated standard uncertainties as reported by the participants:

$$u_{\rm int}(\bar{x}_w) = \sqrt{C} \tag{6}$$

En values are expected to be less than 1 for a coverage factor of k=2.

The statistical consistency of the comparison can be analysed by the so called Birge ratio test. The Birge ratio compares the observed spread of the results with the spread expected from the individual reported uncertainties.

The Birge ratio is:

$$R_B = \frac{u_{ext}(\bar{x}_w)}{u_{int}(\bar{x}_w)} \tag{7}$$

where u_{ext} is the so called external standard deviation and can be calculated by the following formula:

$$u_{ext}(\bar{x}_{w}) = \sqrt{\frac{1}{I-1} \cdot \frac{\sum_{i=1}^{n} \frac{(x_{i} - \bar{x}_{w})^{2}}{u^{2}(x_{i})}}{\sum_{i=1}^{n} \frac{1}{u^{2}(x_{i})}}}$$
(8)

The Birge ratio has an expectation value of $R_B=1$, when considering standard uncertainties.

For a coverage factor of k = 2, the expectation value is increased and the data in a comparison are consistent provided that

$$R_B < \sqrt{1 + \sqrt{8/(I-1)}} \tag{9}$$

Where *I* is the number of the results that are taken in the calculation. For the cases of I = 18, 17 and 16, $R_B < 1,298$, 1,307 and 1,315 respectively indicates consistency.

If the calculation of a gauge shows inconsistent dataset, the largest consistent subset is determined by elimination, starting with excluding the result having the largest E_n value that makes the largest contribution to the overall chi-squared value. The iteration runs until $R_B < R_{B\,crit}$.

When a result x_i is excluded from the reference value, it is not correlated to it and its E_n value is calculated by:

$$E_{n} = \frac{x_{i} - \bar{x}_{w}}{2\sqrt{u^{2}(x_{i}) + u_{int}^{2}(\bar{x}_{w})}}$$
(10)

The iteration method to determine the largest subset has been only done for the final results (chapter 9).

Correlations:

Some laboratories have traceability for the reference rings used in the ring measurements from other laboratory (see table 6). This can effect the results. It was asked to check whether there is correlation between different participants. This calculation was kindly offered by Michael Matus, BEV. The outcome can be found in Appendix 2.

8. Results as it was submitted by the participants

The following laboratories has changed some of their results because of different reasons:

- 1. NPL, GB remeasured some of the gauges before the results were available (new results: 2009.07.29). The original results showed some problems for some items. NPL was requested to make re-measurements because they are one of the linking laboratories between the two groups. The new measurements were made with different operator and in the new laboratory after they had moved to the new NPL building.
- 2. INM, RO revised the uncertainty budgets that are in agreement with their CMC claims (2009.03.23). After checking their results, it turned out that the instrument resolution and the repeatability had not been taken into consideration in the uncertainty calculation. The laboratory will use a new device for diameter calibrations, that is to be validated with a new comparison.
- 3. FSB, CR introduced new value for the diameter of the ring 40 mm (2009.07.20). The reference ring of 50 mm that was used in the comparison has a new calibration certificate issued by another NMI. Both calibration certificates has been sent as a proof.
- 4. NML, IR sent revised uncertainties (2009. 08.31). In late 2008 NML piloted a trilateral intercomparison (No. 1057) with METAS and NPL in the area of pin gauge calibration. One of the main outcomes of this comparison was that the laboratory seriously underestimated the performance of their ULM OPAL single axis measuring machine and also the influence of compression on the measurement of ring gauges and plugs.

For the four laboratories the original results are kept and evaluated in the chapter 8. The evaluation with the corrected values can be found in the chapter 9. The revised results are to be considered as final.

The graphs belonging to the measurands show the measured results with the expanded uncertainties (k=2).

As it was agreed at the EURAMET TC-L meeting in Lisbon, the evaluation of the roundness and straightness data of the cylindrical gauges is not shown because these parameters have been measured mainly for the diameter uncertainty estimation of the ring and plug gauges, as contributions in the uncertainty calculation. The roundness measurements of the sphere are considered to prove the CMC claims for roundness measurements.

8.1. Ring 5 mm - diameter

r								oipunte				
		ORIGINA	AL VALUES	5				(CORRECTI	ED VALU	ES	
	+	unc	middle	unc	-	unc	+	unc	middle	unc	-	unc
LABORATORY	mm	μm	mm	μm	mm	μm	mm	μm	mm	μm	mm	μm
METAS	5,00022	0,04	5,00034	0,04	5,00028	0,04						
BEV	4,99966	0,25	4,99966	0,25	4,99969	0,25						
CMI	5,0000	0,3	5,0000	0,3	4,9996	0,3						
GUM	5,00047	0,25	5,00065	0,25	5,00041	0,25						
NML	4,99980	0,115	4,99988	0,115	4,99990	0,115		0,475		0,475		0,475
DTI	4,9999	0,45	5,0003	0,45	5,0001	0,45						
NPL	5,00013	0,045	5,00028	0,042	5,00018	0,039						
MIRS					1	NOT MEAS	URED					
EIM	5,00010	0,25	5,00020	0,25	5,00022	0,25						
METROSERT	4,99949	0,341	4,99968	0,341	4,99973	0,341						
LNMC	5,00010	0,5	5,00023	0,5	5,00013	0,5						
MKEH	4,99992	0,13	4,99997	0,13	4,99999	0,13						
INRIM	5,00018	0,050	5,00031	0,050	5,00025	0,050						
FSB					١	NOT MEAS	URED					
INM	4,9997	0,20	4,9997	0,20	4,9998	0,20		0,39		0,39		0,39
UME	5,00028	0,115	5,00028	0,115	5,00027	0,115						
NRC	5,00013	0,1	5,00025	0,1	5,00020	0,1						
CEM	5,00018	0,14	5,00038	0,14	5,00036	0,14						

Table 7: Ring 5 mm results given by the participants:

Table 8: Results evaluated with the original values (E_n : k=2)

			<u> </u>	``````````````````````````````````````	, ,	1
weighted mean:	5,00015	mm	5,00027	mm	5,00020	mm
^u int	0,02	μm	0,02	μm	0,02	μm
u ext	0,03	μm	0,04	μm	0,03	μm
No of labs	16		16		16	
R _B (uext/uint)	1,466		1,664		1,441	
R _B crit	1,315		1,315		1,315	
	+		middle		-	
	xi-xw	En	xi-xw	En	xi-xw	En
METAS	0,07	1,12	0,07	1,08	0,08	1,12
BEV	-0,49	-0,98	-0,61	-1,22	-0,51	-1,03
CMI	-0,15	-0,24	-0,27	-0,45	-0,60	-1,01
GUM	0,32	0,65	0,38	0,77	0,21	0,41
NML	-0,35	-1,53	-0,39	-1,72	-0,30	-1,35
DTI	-0,25	-0,27	0,03	0,04	-0,10	-0,12
NPL	-0,02	-0,20	0,01	0,17	-0,02	-0,37
MIRS			NOT MEASU	JRED		
EIM	-0,05	-0,09	-0,07	-0,14	0,02	0,03
METROSERT	-0,66	-0,96	-0,59	-0,86	-0,47	-0,70
LNMC	-0,05	-0,05	-0,04	-0,04	-0,07	-0,07
MKEH	-0,23	-0,88	-0,30	-1,16	-0,21	-0,84
INRIM	0,03	0,38	0,04	0,47	0,05	0,51
FSB			NOT MEASU	JRED		
INM	-0,45	-1,12	-0,57	-1,43	-0,40	-1,02
UME	0,13	0,60	0,01	0,05	0,07	0,29
NRC	-0,02	-0,08	-0,02	-0,09	0,00	-0,02
CEM	0,03	0,12	0,11	0,41	0,16	0,56



The following graphs show the original results of the ring 5 mm with the expanded uncertainties (k=2).





8.2. Ring 40 mm - diameter

		ORIGIN	AL VALUES						CORRECT	ED VALUE	S	
	+	unc	middle	unc	-	unc	+	unc	middle	unc	-	unc
LABORATORY	mm	μm	mm	μm	mm	μm	mm	μm	mm	μm	mm	μm
METAS	39,99974	0,04	39,99971	0,04	39,99974	0,04						
BEV	39,99976	0,25	39,99977	0,25	39,99978	0,25						
CMI	39,9996	0,4	39,9997	0,4	39,9996	0,4						
GUM	39,99980	0,13	39,99977	0,13	39,99986	0,13						
NML	40,00008	0,100	40,00010	0,100	40,00009	0,100		0,332		0,332		0,332
DTI	40,0002	0,55	40,0003	0,55	39,9999	0,55						
NPL	39,99975	0,043	39,99972	0,041	39,99979	0,041						
MIRS	39,9996	0,31	39,9996	0,31	39,9995	0,31						
EIM	39,99969	0,25	39,99970	0,25	39,99969	0,25						
METROSERT	39,99986	0,344	39,99991	0,344	39,99990	0,344						
LNMC	39,99936	0,5	39,99932	0,5	39,99922	0,5						
MKEH	39,99977	0,15	39,99977	0,15	39,99976	0,15						
INRIM	39,99970	0,055	39,99968	0,055	39,99971	0,055						
FSB	39,99935	0,26	39,99935	0,26	39,99935	0,26	39,99953		39,99953		39,99953	
INM	39,9994	0,22	39,9994	0,22	39,9991	0,22		0,40		0,40		0,40
UME	39,99998	0,143	39,9999	0,143	40,00006	0,143						
NRC	39,99968	0,1	39,99961	0,1	39,99971	0,1						
CEM	39,99971	0,14	39,99963	0,14	39,99962	0,14						

Table 9: Ring 40 mm results given by the participants:

weighted mean:	39,99975	mm	39,99972	mm	39,99976	mm
u _{int}	0,022	μm	0,022	μm	0,022	μm
u _{ext}	0,025	μm	0,027	μm	0,031	μm
No of labs	18		18		18	
R _B (uext/uint)	1,133		1,210		1,384	
R _B crit	1,298		1,298		1,298	
	+		middle		-	
	xi-xw	En	xi-xw	En	xi-xw	En
METAS	-0,01	-0,16	-0,01	-0,19	-0,02	-0,34
BEV	0,01	0,02	0,05	0,09	0,02	0,03
CMI	-0,15	-0,19	-0,02	-0,03	-0,16	-0,20
GUM	0,05	0,19	0,05	0,18	0,10	0,38
NML	0,33	1,69	0,38	1,93	0,33	1,68
DTI	0,45	0,41	0,58	0,53	0,14	0,12
NPL	0,00	0,00	0,00	-0,04	0,03	0,40
MIRS	-0,15	-0,24	-0,12	-0,20	-0,26	-0,42
EIM	-0,06	-0,12	-0,02	-0,05	-0,07	-0,15
METROSERT	0,11	0,16	0,19	0,27	0,14	0,20
LNMC	-0,39	-0,39	-0,40	-0,40	-0,54	-0,54
MKEH	0,02	0,07	0,05	0,16	0,00	-0,01
INRIM	-0,05	-0,50	-0,04	-0,43	-0,05	-0,52
FSB	-0,40	-0,77	-0,37	-0,72	-0,41	-0,80
INM	-0,35	-0,80	-0,32	-0,74	-0,66	-1,51
UME	0,23	0,81	0,18	0,63	0,30	1,05
NRC	-0,07	-0,36	-0,11	-0,58	-0,05	-0,27
CEM	-0,04	-0,15	-0,09	-0,34	-0,14	-0,52



The following graphs show the original results of the ring 40 mm with the expanded uncertainties (k=2).





8.3. Plug 5 mm - diameter

	ORIGINAL VALUES							CORRECTED VALUES				
	+	unc	middle	unc	-	unc	+	unc	middle	unc	-	unc
LABORATORY	mm	μm	mm	μm	mm	μm	mm	μm	mm	μm	mm	μm
METAS	4,99992	0,04	4,99992	0,04	4,99998	0,04						
BEV	4,99992	0,15	4,99997	0,15	4,99997	0,15						
CMI	4,9996	0,3	4,9996	0,3	4,9996	0,3						
GUM	4,999844	0,090	4,999819	0,090	4,999821	0,090						
NML	4,99986	0,082	4,99983	0,082	4,99983	0,082		0,075		0,075		0,075
DTI	4,9997	0,45	4,9999	0,45	4,9997	0,45						
NPL	5,00030	0,0407	5,00030	0,0409	5,00032	0,0411	4,99986	0,046	4,99981	0,047	4,99987	0,046
MIRS	4,9997	0,15	4,9997	0,15	4,9997	0,15						
EIM	4,99991	0,17	4,99997	0,17	5,00007	0,17						
METROSERT	4,99975	0,156	4,99984	0,156	4,99980	0,156						
LNMC	4,99946	0,7	4,99946	0,7	4,9995	0,7						
MKEH	4,99988	0,12	4,99988	0,12	4,99988	0,12						
INRIM	4,99993	0,055	4,99996	0,050	5,00000	0,050						
FSB	4,99975	0,30	4,99975	0,30	4,99978	0,30						
INM	4,9991	0,20	4,9991	0,20	4,9991	0,20		0,39		0,39		0,39
UME	4,99977	0,150	4,99978	0,150	4,99974	0,150						
NRC	4,99979	0,1	4,99984	0,1	4,99993	0,1						
CEM	5,00004	0,14	5,00011	0,14	5,00018	0,14						

Table 11: Plug 5 mm results given by the participants:

Table	12: Results	evaluated	with the	original	values	$(E_n: k=2)$
1 0010	1 El 1 toodatto	oralaatoa		engina	10100	$(-m \cdot \cdot \cdot -)$

weighted mean:	4,99999	mm	4,99999	mm	5,00003	mm
u _{int}	0,021	μm	0,021	μm	0,021	μm
u _{ext}	0,051	μm	0,050	μm	0,051	μm
No of labs	18		18		18	
R _B (uext/uint)	2,475		2,443		2,463	
R _B crit	1,298		1,298		1,298	
	+		middle		-	
	xi-xw	En	xi-xw	En	xi-xw	En
METAS	-0,07	-0,97	-0,07	-1,06	-0,05	-0,66
BEV	-0,07	-0,22	-0,02	-0,08	-0,06	-0,19
CMI	-0,39	-0,65	-0,39	-0,66	-0,43	-0,71
GUM	-0,14	-0,81	-0,17	-0,99	-0,20	-1,16
NML	-0,13	-0,79	-0,16	-1,03	-0,20	-1,23
DTI	-0,29	-0,32	-0,09	-0,10	-0,33	-0,36
NPL	0,31	4,49	0,31	4,34	0,29	4,14
MIRS	-0,29	-0,96	-0,29	-0,99	-0,33	-1,09
EIM	-0,08	-0,23	-0,02	-0,07	0,04	0,13
METROSERT	-0,24	-0,76	-0,15	-0,50	-0,23	-0,73
LNMC	-0,53	-0,38	-0,53	-0,38	-0,53	-0,38
MKEH	-0,11	-0,45	-0,11	-0,48	-0,15	-0,61
INRIM	-0,06	-0,55	-0,03	-0,36	-0,03	-0,27
FSB	-0,24	-0,39	-0,24	-0,41	-0,25	-0,41
INM	-0,89	-2,23	-0,89	-2,24	-0,93	-2,32
UME	-0,22	-0,73	-0,21	-0,72	-0,29	-0,96
NRC	-0,20	-1,00	-0,15	-0,78	-0,10	-0,49
CEM	0,05	0,19	0,12	0,42	0,15	0,56



The following graphs show the original results of the plug 5 mm with the expanded uncertainties (k=2).





8.4. Plug 50 mm - diameter

	ORIGINAL VALUES						CORRECTED VALUES					
	+	unc	middle	unc	-	unc	+	unc	middle	unc	-	unc
LABORATORY	mm	μm	mm	μm	mm	μm	mm	μm	mm	μm	mm	μm
METAS	49,99940	0,07	49,99925	0,07	49,99928	0,07						
BEV	49,99947	0,15	49,99936	0,15	49,99940	0,15						
CMI	49,9992	0,4	49,9992	0,4	49,9992	0,4						
GUM	49,99933	0,12	49,99918	0,12	49,99920	0,12						
NML	49,99923	0,098	49,99922	0,098	49,99925	0,098		0,1		0,1		0,1
DTI	49,9986	0,55	49,9985	0,55	49,9986	0,55						
NPL	49,99912	0,0686	49,99910	0,0644	49,99911	0,0674	49,99921	0,058	49,99911	0,0583	49,99905	0,0638
MIRS	49,9986	0,26	49,9985	0,26	49,9986	0,26						
EIM	49,99948	0,18	49,99939	0,18	49,99938	0,18						
METROSERT	49,99932	0,239	49,99921	0,239	49,99921	0,239						
LNMC	49,99894	1	49,99896	1	49,99910	1						
MKEH	49,99931	0,14	49,99936	0,14	49,99945	0,14						
INRIM	49,99946	0,060	49,99932	0,075	49,99940	0,060						
FSB	49,99923	0,3	49,99917	0,3	49,99923	0,3						
INM	49,9986	0,23	49,9986	0,23	49,9987	0,23		0,41		0,41		0,41
UME	49,99913	0,156	49,99914	0,156	49,99924	0,156						
NRC	49,99949	0,1	49,99943	0,1	49,99942	0,1						
CEM	49,99951	0,14	49,99933	0,14	49,99940	0,14						

Table 13: Plug 50 mm results given by the participants:

weighted mean:	49,99932	mm	49,99923	mm	49,99928	mm
u _{int}	0,028	μm	0,029	μm	0,028	μm
u _{ext}	0,045	μm	0,039	μm	0,038	μm
No of labs	18		18		18	
R _B (uext/uint)	1,601		1,335		1,350	
R _B crit	1,298		1,298		1,298	
	+		middle		-	
	xi-xw	En	xi-xw	En	xi-xw	En
METAS	0,08	0,59	0,02	0,18	0,00	0,02
BEV	0,15	0,49	0,13	0,45	0,12	0,42
CMI	-0,12	-0,16	-0,03	-0,03	-0,08	-0,10
GUM	0,01	0,02	-0,05	-0,20	-0,08	-0,33
NML	-0,09	-0,50	-0,01	-0,04	-0,03	-0,15
DTI	-0,72	-0,66	-0,73	-0,66	-0,68	-0,62
NPL	-0,20	-1,63	-0,13	-1,10	-0,17	-1,37
MIRS	-0,72	-1,40	-0,73	-1,41	-0,68	-1,31
EIM	0,16	0,44	0,16	0,46	0,10	0,29
METROSERT	0,00	-0,01	-0,02	-0,04	-0,07	-0,14
LNMC	-0,38	-0,19	-0,27	-0,13	-0,18	-0,09
MKEH	-0,01	-0,05	0,13	0,49	0,17	0,63
INRIM	0,14	1,28	0,09	0,67	0,12	1,15
FSB	-0,09	-0,16	-0,06	-0,10	-0,05	-0,08
INM	-0,72	-1,59	-0,63	-1,37	-0,58	-1,27
UME	-0,19	-0,63	-0,09	-0,28	-0,04	-0,12
NRC	0,17	0,86	0,20	1,06	0,14	0,74
СЕМ	0,19	0,68	0,10	0,38	0,12	0,45



The following graphs show the original results of the plug 50 mm with the expanded uncertainties (k=2).





8.5. Sphere 30 mm - diameter

ORIGINA	AL VALUES		CORRECTED VALUES			
	diameter	unc	diameter	unc		
LABORATORY	mm	μm	mm	μm		
METAS	29,98623	0,07				
BEV	29,98611	0,15				
СМІ	29,9858	0,4				
GUM	29,986056	0,090				
NML	29,98600	0,066		0,09		
DTI	29,9855	0,55				
NPL	29,98630	0,0564	29,98636	0,0538		
MIRS	29,9851	0,15				
EIM	29,98634	0,18				
METROSERT	29,98632	0,196				
LNMC		NOT N	MEASURED			
MKEH	29,98617	0,14				
INRIM	29,98631	0,065				
FSB	29,98603	0,30				
INM	29,98530	0,21		0,40		
UME	29,98616	0,100				
NRC	29,98629	0,1				
CEM	29,98609	0,08				

Table 15: Sphere 30 mm results given by the participants:

Table 16: Results evaluated with the original values ($E_n: k=2$)

weighted mean:	29,98615	mm
u _{int}	0,02	μm
u _{ext}	0,06	
No of labs	17	
R _B (uext/uint)	2,409	
R _B crit	1,307	
Lab	xi-xw	En
METAS	0,08	0,61
BEV	-0,04	-0,13
CMI	-0,35	-0,44
GUM	-0,09	-0,54
NML	-0,15	-1,22
DTI	-0,65	-0,59
NPL	0,15	1,48
MIRS	-1,05	-3,54
EIM	0,19	0,53
METROSERT	0,17	0,44
LNMC	NOT MEASU	IRED
MKEH	0,02	0,07
INRIM	0,16	1,33
FSB	-0,12	-0,20
INM	-0,85	-2,04
UME	0,01	0,05
NRC	0,14	0,72
CEM	-0,06	-0,39



The following graphs show the original results of the sphere 30 mm with the expanded uncertainties (k=2).

8.6. Sphere 30 mm - roundness

500 UPR	round.	u(k=1)			
LABORATORY	μm	μm			
METAS	0,035	0,015			
CMI	0,09	0,05			
GUM	0,102	0,038			
DTI	0,10	0,045			
NPL	not measured				
EIM	0,091	0,07			
METROSERT	0,09	0,05			
INRIM	0,039	0,004			
FSB	0,097	0,055			
INM	0,10	0,04			
UME	0,078	0,038			
CEM	0,034	0,005			

Table 17: Sphere 30 mm results given by the participants (500 UPR)

Table 18: Results evaluated with	the original values	$(E_n: k=2)$
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500 UPR		
weighted mean:	0,039	μm
u _{int}	0,003	μm
u _{ext}	0,003	μm
No of labs	11	
R _B (uext/uint)	1,130	
R _B crit	1,376	
	X _i -X _w	En
METAS	-0,004	-0,13
СМІ	0,051	0,51
GUM	0,063	0,83
DFM	0,061	0,68
DFM NPL	0,061 not me	0,68 asured
DFM NPL EIM	0,061 not me 0,052	0,68 asured
DFM NPL EIM METROSERT	0,061 not me 0,052 0,051	0,68 asured 0,37 0,51
DFM NPL EIM METROSERT INRIM	0,061 not me 0,052 0,051 0,000	0,68 asured 0,37 0,51 0,01
DFM NPL EIM METROSERT INRIM FSB	0,061 not me 0,052 0,051 0,000 0,058	0,68 asured 0,37 0,51 0,01 0,53
DFM NPL EIM METROSERT INRIM FSB INM	0,061 not me 0,052 0,051 0,000 0,058 0,061	0,68 asured 0,37 0,51 0,01 0,53 0,77
DFM NPL EIM METROSERT INRIM FSB INM UME	0,061 not me 0,052 0,051 0,000 0,058 0,061 0,039	0,68 asured 0,37 0,51 0,01 0,53 0,77 0,52

150 UPR	round.	u(k=1)
LABORATORY	μm	μm
METAS	0,032	0,015
CMI	0,10	0,05
GUM	0,078	0,036
DTI	0,12	0,045
NPL	0,038	0,003
EIM	0,076	0,07
METROSERT	0,07	0,04
INRIM	0,039	0,004
FSB	0,097	0,055
INM	0,08	0,04
UME	0,051	0,035
CEM	0,029	0,005

Table 19: Sphere 30 mm results given by the participants (150 UPR)

Table 20: Results evaluated with the original values ($E_n: k=2$)

150 UPR		
weighted mean:	0,037	μm
u _{int}	0,002	μm
u _{ext}	0,002	μm
No of labs	12	
R _B (uext/uint)	1,078	
R _B crit	1,361	
	X _i -X _w	En
METAS	-0,005	-0,18
CMI	0,063	0,63
GUM	0,041	0,57
DTI	0,083	0,92
NPL	0,001	0,15
EIM	0,039	0,28
METROSERT	0,033	0,41
INRIM	0,002	0,24
FSB	0,060	0,54
INM	0,043	0,53
UME	0,014	0,20
CEM	-0,008	-0,92

50 UPR	round.	u(k=1)
LABORATORY	μm	μm
METAS	0,026	0,015
CMI	0,10	0,05
GUM	0,067	0,036
DTI	0,14	0,045
NPL	0,034	0,003
EIM	0,058	0,07
METROSERT	0,06	0,03
INRIM	0,035	0,004
FSB	0,097	0,055
INM	0,04	0,04
UME	0,039	0,032
CEM	0,024	0,004

Table 21: Sphere 30 mm results given by the participants (50 UPR)

Table 22: Results evaluated with the original values ($E_n: k=2$)

50 UPR		
weighted mean:	0,032	μm
u _{int}	0,002	μm
u ext	0,002	μm
No of labs	12	
R _B (uext/uint)	1,214	
R _B crit	1,361	
	X _i -X _w	En
METAS	-0,006	-0,21
CMI	0,068	0,68
GUM	0,035	0,48
DTI	0,108	1,20
NPL	0,002	0,40
EIM	0,026	0,18
METROSERT	0,028	0,46
INRIM	0,003	0,40
FSB	0,065	0,59
INM	0,008	0,10
UME	0,007	0,11
CEM	-0,008	-1,19

The following graphs show the original results of the roundness of the sphere 30 mm with the expanded uncertainties (k=2).







8.7. Roundness and straightness of the cylindrical gauges

The following tables show the roundness and straightness data submitted by the participants for the cylindrical gauges. The roundness of the ring and plug gauges had to be measured at the same heights as the diameter measurements. The straightness of the ring and plug gauges had to be measured at the marked lines (0 and 180) °.

The roundness and straightness measurements were required only if they are done normally for the customers.

The roundness measurements on the sphere are to be kept as evidence for proving CMC claims of those NMIs offering a roundness measuring service, or planning to submit a CMC for such a service. The roundness and straightness data for the cylindrical gauges are measured to determine the uncertainty contribution of these parameters, so they are introduced only for information without showing the complete evaluation of the results.

It is worth noting that most of the gauges suffered some scratches or damage during the comparison.

8.7.1 Ring 5 mm roundness and straightness

	Original values								Corrected	l values		
ROUNDNESS	+ 3 mm	unc	middle	unc	-3 mm	unc	+ 3 mm	unc	middle	unc	-3 mm	unc
LABORATORY	μm	μm	μm	μm	μm	μm	μm	μm	μm	μm	μm	μm
METAS	0,03	0,03	0,16	0,03	0,03	0,03		d	amaged at	middle!!		
СМІ	0,07	0,05	0,04	0,05	0,04	0,05						
GUM	0,034	0,032	0,053	0,032	0,057	0,033						
DTI	0,05	0,04	0,05	0,04	0,08	0,04						
NPL	0,19	0,0443	0,12	0,0409	0,09	0,0407	0,03	0,0365	0,06	0,0371	0,05	0,0361
EIM	0,058	0,07	0,048	0,07	0,058	0,07						
METROSERT	0,05	0,03	0,07	0,03	0,05	0,04						
INRIM	0,029	0,050	0,051	0,050	0,048	0,050						
INM	0,06	0,04	0,10	0,04	0,04	0,04						
UME	0,058	0,028	0,069	0,028	0,080	0,028						
CEM	0,04	0,02	0,06	0,02	0,03	0,02						

Table 23: Ring 5 mm roundness as given by the participants

METAS reported damage at middle prior to measurements

0	unc	180	unc
μm	μm	μm	μm
0,09	0,05	0,94	0,05
0,12	0,04	0,13	0,04
0,06	0,175	0,07	0,175
0,12	0,07	0,13	0,07
0,12	0,05	0,14	0,05
0,162	0,107	0,147	0,107
0,07	0,02	0,09	0,02
	0 μm 0,09 0,12 0,06 0,12 0,12 0,12 0,162 0,07	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $

Table 24: Ring 5 mm straightness as given by the participants

METAS reported damage at 180° prior to measurements

8.7.2 Ring 40 mm roundness and straightness

ROUNDNESS	+ 10 mm	u (k=1)	middle	u (k=1)	-10 mm	u (k=1)
LABORATORY	μm	μm	μm	μm	μm	μm
METAS	0,15	0,04	0,08	0,04	0,08	0,04
СМІ	0,08	0,05	0,07	0,05	0,12	0,05
GUM	0,110	0,035	0,115	0,035	0,106	0,034
DTI	0,11	0,045	0,11	0,045	0,14	0,045
NPL	0,11	0,0410	0,10	0,0420	0,08	0,0430
EIM	0,158	0,07	0,108	0,07	0,170	0,07
METROSERT	0,11	0,04	0,1	0,04	0,10	0,04
INRIM	0,086	0,050	0,074	0,050	0,084	0,050
FSB	0,083	0,055	0,065	0,055	0,082	0,055
INM	0,14	0,04	0,16	0,04	0,16	0,04
UME	0,104	0,035	0,074	0,035	0,100	0,035
CEM	0,13	0,02	0,10	0,02	0,16	0,02

Table 25: Ring 40 mm roundness as given by the participants

UME reported scratches at the middle prior to measurements

STRAIGHTNESS	0	unc	180	unc
LABORATORY	μm	μm	μm	μm
METAS	0,05	0,05	0,09	0,05
DTI	0,14	0,09	0,13	0,09
EIM	0,21	0,175	0,15	0,175
METROSERT	0,20	0,07	0,25	0,07
INRIM	0,13	0,05	0,11	0,05
INM	0,15	0,08	0,21	0,08
UME	0,094	0,107	0,074	0,107
CEM	0,05	0,02	0,07	0,02

Table 26: Ring 40 mm straightness as given by the participants

UME reported scratches at the middle prior to measurements

8.7.3 Plug 5 mm roundness and straightness

	Original values						Corrected values					
ROUNDNESS	+ 2 mm	unc	middle	unc	-2 mm	unc	+ 2 mm	unc	middle	unc	-2 mm	unc
LABORATORY	μm	μm	μm	μm	μm	μm	μm	μm	μm	μm	μm	μm
METAS	0,03	0,03	0,05	0,03	0,03	0,03						
CMI	0,17	0,05	0,11	0,05	0,13	0,05						
GUM	0,056	0,037	0,062	0,035	0,048	0,035						
DTI	0,04	0,04	0,03	0,04	0,05	0,04						
NPL	0,08	0,0609	0,08	0,0607	0,31	0,0604	0,04	0,0361	0,07	0,0364	0,31	0,0358
EIM	0,071	0,07	0,067	0,07	0,078	0,07						
METROSERT	0,21	0,04	0,17	0,03	0,38	0,04						
INRIM	0,040	0,050	0,041	0,050	0,044	0,050						
INM	0,04	0,04	0,04	0,04	0,03	0,04						
UME	0,254	0,097	0,051	0,097	0,064	0,097						
CEM	0,06	0,02	0,08	0,02	0,04	0,02						

Table 27: Plug 5 mm roundness as given by the participants

UME and METROSERT reported scratches prior to measurements that may affect the measured values

STRAIGHTNESS	0	unc	180	unc
LABORATORY	μm	μm	μm	μm
METAS	0,09	0,05	0,03	0,05
DTI	0,08	0,04	0,07	0,04
EIM	0,05	0,175	0,05	0,175
METROSERT	0,17	0,07	0,17	0,07
INRIM	0,08	0,05	0,14	0,05
UME	0,176	0,107	0,174	0,107
CEM	0,02	0,02	0,05	0,02

Table 28: Plug 5 mm straightness as given by the participants

UME reported scratches at the straightness lines prior to measurements
8.7.4 Plug 50 mm roundness and straightness

ROUNDNESS	+ 6 mm	unc	middle	unc	-6 mm	unc	
LABORATORY	μm	μm	μm	μm	μm	μm	
METAS	0,16	0,04	0,20	0,04	0,14	0,04	
CMI	0,09	0,05	0,11	0,05	0,12	0,05	
GUM	0,197	0,035	0,171	0,037	0,167	0,036	
DTI	0,22	0,045	0,22	0,045	0,21	0,045	
NPL	0,13	0,0366	0,16	0,0371	0,17	0,0382	
EIM	0,130	0,07	0,135	0,07	0,141	0,07	
METROSERT	0,13	0,04	0,14	0,04	0,14	0,04	
INRIM	0,114	0,050	0,132	0,050	0,137	0,050	
FSB	0,118	0,055	0,106	0,055	0,108	0,055	
INM	0,28	0,04	0,19	0,04	0,20	0,04	
UME	0,137	0,035	0,177	0,035	0,110	0,035	
CEM	0,18	0,03	0,12	0,03	0,17	0,03	

Table 29: Plug 50 mm roundness as given by the participants

More laboratories reported scratches prior to measurements but anybody reported possible affects on the measured data

<u>0</u>		U		
STRAIGHTNESS	0	unc	180	unc
LABORATORY	mm	μm	mm	μm
METAS	0,07	0,05	0,05	0,05
DTI	0,19	0,045	0,22	0,045
EIM	0,13	0,175	0,17	0,175
METROSERT	0,18	0,05	0,16	0,05
INRIM	0,17	0,05	0,13	0,05
INM	0,06	0,08	0,09	0,08
UME	0,066	0,053	0,087	0,053
CEM	0,10	0,03	0,08	0,03

Table 30: Plug 50 mm straightness as given by the participants

9. FINAL RESULTS

The following chapter is dealing with the evaluation with the corrected submitted values as final results. Four laboratories submitted corrected results as described in the chapter 8.

The corrected values are indicated with colours.

The tables with the data submitted by the laboratories contain the standard uncertainties (k=1) as they were asked in the protocol.

The E_n calculations are performed with the expanded uncertainties (k=2). The uncertainty bars in the graphs are also with k=2.

9.1. Ring 5 mm - diameter

	SUBMITTED VALUES							
	+	unc	middle	unc	-	unc		
LABORATORY	mm	μm	mm	μm	mm	μm		
METAS	5,00022	0,04	5,00034	0,04	5,00028	0,04		
BEV	4,99966	0,25	4,99966	0,25	4,99969	0,25		
CMI	5,0000	0,3	5,0000	0,3	4,9996	0,3		
GUM	5,00047	0,25	5,00065	0,25	5,00041	0,25		
NML	4,99980	0,475	4,99988	0,475	4,99990	0,475		
DTI	4,9999	0,45	5,0003	0,45	5,0001	0,45		
NPL	5,00013	0,045	5,00028	0,042	5,00018	0,039		
MIRS			NOT ME	ASURED				
EIM	5,00010	0,25	5,00020	0,25	5,00022	0,25		
METROSERT	4,99949	0,341	4,99968	0,341	4,99973	0,341		
LNMC	5,00010	0,5	5,00023	0,5	5,00013	0,5		
MKEH	4,99992	0,13	4,99997	0,13	4,99999	0,13		
INRIM	5,00018	0,050	5,00031	0,050	5,00025	0,050		
FSB			NOT ME	ASURED				
INM	4,9997	0,39	4,9997	0,39	4,9998	0,39		
UME	5,00028	0,115	5,00028	0,115	5,00027	0,115		
NRC	5,00013	0,1	5,00025	0,1	5,00020	0,1		
CEM	5,00018	0,14	5,00038	0,14	5,00036	0,14		

Table 31: Ring 5 mm final results as given by the participants:

Table 32: Results evaluated with the final values

weighted mean:	5,00016	mm	5,00029	mm	5,00022	mm
u _{int}	0,02	μm	0,02	μm	0,02	μm
u _{ext}	0,03	μm	0,03	μm	0,03	μm
No of labs	16		16		16	
R _B (uext/uint)	1,140		1,269		1,187	
R _B crit	1,315		1,315		1,315	
	+		middle		-	
	xi-xw	En	xi-xw	En	xi-xw	En
METAS	0,06	0,87	0,05	0,80	0,06	0,92
BEV	-0,50	-1,01	-0,63	-1,26	-0,53	-1,06
CMI	-0,16	-0,27	-0,29	-0,48	-0,62	-1,03
GUM	0,31	0,62	0,36	0,73	0,19	0,38
NML	-0,36	-0,38	-0,41	-0,43	-0,32	-0,34
DTI	-0,26	-0,29	0,01	0,01	-0,12	-0,13
NPL	-0,03	-0,43	-0,01	-0,10	-0,04	-0,60
MIRS			NOT MEASU	JRED		
EIM	-0,06	-0,13	-0,09	-0,17	0,00	0,00
METROSERT	-0,67	-0,99	-0,61	-0,89	-0,49	-0,72
LNMC	-0,06	-0,06	-0,06	-0,06	-0,09	-0,09
MKEH	-0,24	-0,95	-0,32	-1,24	-0,23	-0,89
INRIM	0,02	0,19	0,02	0,26	0,03	0,35
FSB			NOT MEASU	JRED		
INM	-0,46	-0,59	-0,59	-0,75	-0,42	-0,54
UME	0,12	0,52	-0,01	-0,03	0,05	0,23
NRC	-0,03	-0,17	-0,04	-0,19	-0,02	-0,09
CEM	0,02	0,06	0,09	0,34	0,14	0,51









9.2. Ring 40 mm - diameter

	SUBMITTED VALUES								
	+	unc	middle	unc	-	unc			
LABORATORY	mm	μm	mm	μm	mm	μm			
METAS	39,99974	0,04	39,99971	0,04	39,99974	0,04			
BEV	39,99976	0,25	39,99977	0,25	39,99978	0,25			
CMI	39,9996	0,4	39,9997	0,4	39,9996	0,4			
GUM	39,99980	0,13	39,99977	0,13	39,99986	0,13			
NML	40,00008	0,332	40,00010	0,332	40,00009	0,332			
DTI	40,0002	0,55	40,0003	0,55	39,9999	0,55			
NPL	39,99975	0,043	39,99972	0,041	39,99979	0,041			
MIRS	39,9996	0,31	39,9996	0,31	39,9995	0,31			
EIM	39,99969	0,25	39,99970	0,25	39,99969	0,25			
METROSERT	39,99986	0,344	39,99991	0,344	39,99990	0,344			
LNMC	39,99936	0,5	39,99932	0,5	39,99922	0,5			
MKEH	39,99977	0,15	39,99977	0,15	39,99976	0,15			
INRIM	39,99970	0,055	39,99968	0,055	39,99971	0,055			
FSB	39,99953	0,26	39,99953	0,26	39,99953	0,26			
INM	39,9994	0,4	39,9994	0,4	39,9991	0,4			
UME	39,99998	0,143	39,9999	0,143	40,00006	0,143			
NRC	39,99968	0,1	39,99961	0,1	39,99971	0,1			
CEM	39,99971	0,14	39,99963	0,14	39,99962	0,14			

Table 33: Ring 40 mm final results as given by the participants:

Table 34: Results evaluated with the final values

waighted man:	20.00074	mm	20 00071	mm	20.00075	mm
weighted mean.	39,99974	111111	39,99971	111111	39,99973	111111
u _{int}	0,023	μm	0,023	μm	0,023	μm
u _{ext}	0,016	μm	0,016	μm	0,021	μm
No of labs	18		18		18	
R _B (uext/uint)	0,697		0,710		0,927	
R _B crit	1,298		1,298		1,298	
	+		middle		-	
	xi-xw	En	xi-xw	En	xi-xw	En
METAS	0,00	0,02	0,00	0,02	-0,01	-0,21
BEV	0,02	0,04	0,06	0,12	0,03	0,05
CMI	-0,14	-0,17	-0,01	-0,01	-0,15	-0,19
GUM	0,06	0,24	0,06	0,24	0,11	0,42
NML	0,34	0,52	0,39	0,59	0,34	0,51
DTI	0,46	0,42	0,59	0,54	0,15	0,13
NPL	0,01	0,16	0,01	0,16	0,04	0,53
MIRS	-0,14	-0,22	-0,11	-0,18	-0,25	-0,41
EIM	-0,05	-0,10	-0,01	-0,02	-0,06	-0,13
METROSERT	0,12	0,18	0,20	0,29	0,15	0,21
LNMC	-0,38	-0,38	-0,39	-0,39	-0,53	-0,53
MKEH	0,03	0,11	0,06	0,21	0,01	0,02
INRIM	-0,04	-0,39	-0,03	-0,29	-0,04	-0,44
FSB	-0,21	-0,40	-0,18	-0,35	-0,22	-0,43
INM	-0,34	-0,42	-0,31	-0,39	-0,65	-0,82
UME	0,24	0,86	0,19	0,68	0,31	1,08
NRC	-0,06	-0,30	-0,10	-0,51	-0,04	-0,22
CEM	-0,03	-0,10	-0,08	-0,29	-0,13	-0,48



The following graphs show the final results of the ring 40 mm with the expanded uncertainties (k=2).





9.3. Plug 5 mm - diameter

SUBMITTED VALUES									
	+	unc	middle	unc	-	unc			
LABORATORY	mm	μm	mm	μm	mm	μm			
METAS	4,99992	0,04	4,99992	0,04	4,99998	0,04			
BEV	4,99992	0,15	4,99997	0,15	4,99997	0,15			
CMI	4,9996	0,3	4,9996	0,3	4,9996	0,3			
GUM	4,999844	0,090	4,999819	0,090	4,999821	0,090			
NML	4,99986	0,075	4,99983	0,075	4,99983	0,075			
DTI	4,9997	0,45	4,9999	0,45	4,9997	0,45			
NPL	4,99986	0,046	4,99981	0,047	4,99987	0,046			
MIRS	4,9997	0,15	4,9997	0,15	4,9997	0,15			
EIM	4,99991	0,17	4,99997	0,17	5,00007	0,17			
METROSERT	4,99975	0,156	4,99984	0,156	4,99980	0,156			
LNMC	4,99946	0,7	4,99946	0,7	4,9995	0,7			
MKEH	4,99988	0,12	4,99988	0,12	4,99988	0,12			
INRIM	4,99993	0,055	4,99996	0,050	5,00000	0,050			
FSB	4,99975	0,30	4,99975	0,30	4,99978	0,30			
INM	4,9991	0,39	4,9991	0,39	4,9991	0,39			
UME	4,99977	0,150	4,99978	0,150	4,99974	0,150			
NRC	4,99979	0,1	4,99984	0,1	4,99993	0,1			
CEM	5,00004	0,14	5,00011	0,14	5,00018	0,14			

Table 35: Plug 5 mm final results as given by the participants:

Table 36: Results with the final values

weighted mean:	4,99988	mm	4,99988	mm	4,99992	mm
u _{int}	0,021	μm	0,021	μm	0,021	μm
u _{ext}	0,018	μm	0,021	μm	0,025	μm
No of labs	18		18		18	
R _B (uext/uint)	0,865		1,016		1,180	
R _B crit	1,298		1,298		1,298	
	+		middle		-	
	xi-xw	En	xi-xw	En	xi-xw	En
METAS	0,04	0,62	0,04	0,59	0,06	0,87
BEV	0,04	0,14	0,09	0,30	0,05	0,17
CMI	-0,28	-0,47	-0,28	-0,47	-0,32	-0,54
GUM	-0,03	-0,20	-0,06	-0,35	-0,10	-0,57
NML	-0,02	-0,13	-0,05	-0,34	-0,09	-0,63
DTI	-0,18	-0,20	0,02	0,02	-0,22	-0,25
NPL	-0,02	-0,23	-0,07	-0,83	-0,05	-0,62
MIRS	-0,18	-0,60	-0,18	-0,60	-0,22	-0,74
EIM	0,03	0,09	0,09	0,27	0,15	0,44
METROSERT	-0,13	-0,42	-0,04	-0,13	-0,12	-0,39
LNMC	-0,42	-0,30	-0,42	-0,30	-0,42	-0,30
MKEH	0,00	0,01	0,00	0,00	-0,04	-0,17
INRIM	0,05	0,51	0,08	0,89	0,08	0,87
FSB	-0,13	-0,21	-0,13	-0,22	-0,14	-0,23
INM	-0,78	-1,00	-0,78	-1,00	-0,82	-1,05
UME	-0,11	-0,36	-0,10	-0,34	-0,18	-0,61
NRC	-0,09	-0,45	-0,04	-0,20	0,01	0,05
CEM	0,16	0,58	0,23	0,83	0,26	0,94

-



The following graphs show the final results of the plug 5 mm with the expanded uncertainties (k=2).





9.4. Plug 50 mm - diameter

SUBMITTED VALUES								
	+	unc	middle	unc	-	unc		
LABORATORY	mm	μm	mm	μm	mm	μm		
METAS	49,99940	0,07	49,99925	0,07	49,99928	0,07		
BEV	49,99947	0,15	49,99936	0,15	49,99940	0,15		
CMI	49,9992	0,4	49,9992	0,4	49,9992	0,4		
GUM	49,99933	0,12	49,99918	0,12	49,99920	0,12		
NML	49,99923	0,1	49,99922	0,1	49,99925	0,1		
DTI	49,9986	0,55	49,9985	0,55	49,9986	0,55		
NPL	49,99921	0,058	49,99911	0,0583	49,99905	0,0638		
MIRS	49,9986	0,26	49,9985	0,26	49,9986	0,26		
EIM	49,99948	0,18	49,99939	0,18	49,99938	0,18		
METROSERT	49,99932	0,239	49,99921	0,239	49,99921	0,239		
LNMC	49,99894	1	49,99896	1	49,99910	1		
MKEH	49,99931	0,14	49,99936	0,14	49,99945	0,14		
INRIM	49,99946	0,060	49,99932	0,075	49,99940	0,060		
FSB	49,99923	0,3	49,99917	0,3	49,99923	0,3		
INM	49,9986	0,41	49,9986	0,41	49,9987	0,41		
UME	49,99913	0,156	49,99914	0,156	49,99924	0,156		
NRC	49,99949	0,1	49,99943	0,1	49,99942	0,1		
CEM	49,99951	0,14	49,99933	0,14	49,99940	0,14		

Table 37: Plug 50 mm final results as given by the participants:

Table 38: Plug 50 mm results evaluated with the final values

weighted mean:	49,99934	mm	49,99923	mm	49,99927	mm
u _{int}	0,03	μm	0,03	μm	0,03	μm
u _{ext}	0,04	μm	0,03	μm	0,04	μm
No of labs	18		18		18	
R _B (uext/uint)	1,361		1,220		1,396	
R _B crit	1,298		1,298		1,298	
	+		middle		-	
	xi-xw	En	xi-xw	En	xi-xw	En
METAS	0,06	0,48	0,02	0,15	0,01	0,09
BEV	0,13	0,45	0,13	0,44	0,13	0,45
CMI	-0,14	-0,17	-0,03	-0,04	-0,07	-0,09
GUM	-0,01	-0,04	-0,05	-0,22	-0,07	-0,29
NML	-0,11	-0,56	-0,01	-0,05	-0,02	-0,10
DTI	-0,74	-0,67	-0,73	-0,66	-0,67	-0,61
NPL	-0,13	-1,26	-0,12	-1,19	-0,22	-1,91
MIRS	-0,74	-1,43	-0,73	-1,41	-0,67	-1,29
EIM	0,14	0,40	0,16	0,45	0,11	0,31
METROSERT	-0,02	-0,04	-0,02	-0,04	-0,06	-0,12
LNMC	-0,40	-0,20	-0,27	-0,14	-0,17	-0,08
МКЕН	-0,03	-0,10	0,13	0,47	0,18	0,66
INRIM	0,12	1,14	0,09	0,65	0,13	1,24
FSB	-0,11	-0,18	-0,06	-0,10	-0,04	-0,06
INM	-0,74	-0,90	-0,63	-0,77	-0,57	-0,70
UME	-0,21	-0,68	-0,09	-0,30	-0,03	-0,09
NRC	0,15	0,79	0,20	1,04	0,15	0,79
CEM	0,17	0,62	0,10	0,36	0,13	0,48

The results show interesting situation. The dataset is consistent at the mid-height, but inconsistent at the upper and lower heights.

Three method is introduced below to see the different approaches to exclude results from the reference value.

The first approach is to concentrate on the value at the mid-height as the most important parameter of the diameter gauges. In this case, the dataset is considered to be consistent at the mid-height and the results can be found in the table 38.

The second approach is to consider the largest E_n value to be excluded from the calculation of the reference value and the reference values for all three heights are calculated from the remaining laboratories. The recomputed results are shown in the table 39.

weighted mean:	49,99938	mm	49,99927	mm	49,99932	mm
u _{int}	0,03	μm	0,03	μm	0,03	μm
u _{ext}	0,04	μm	0,04	μm	0,03	μm
No of labs	17		17		17	
R _B (uext/uint)	1,254		1,109		1,077	
R _B crit	1,307		1,307		1,307	
	+		middle		-	
	xi-xw	En	xi-xw	En	xi-xw	En
METAS	0,02	0,19	-0,02	-0,15	-0,04	-0,33
BEV	0,09	0,32	0,09	0,31	0,08	0,27
CMI	-0,18	-0,22	-0,07	-0,09	-0,12	-0,15
GUM	-0,05	-0,20	-0,09	-0,38	-0,12	-0,52
NML	-0,15	-0,77	-0,05	-0,26	-0,07	-0,37
DTI	-0,78	-0,71	-0,77	-0,70	-0,72	-0,66
NPL	-0,17	-1,26	-0,17	-1,26	-0,27	-1,91
MIRS	-0,78	-1,50	-0,77	-1,49	-0,72	-1,40
EIM	0,10	0,29	0,12	0,34	0,06	0,17
METROSERT	-0,06	-0,12	-0,06	-0,12	-0,11	-0,23
LNMC	-0,44	-0,22	-0,31	-0,15	-0,22	-0,11
МКЕН	-0,07	-0,24	0,09	0,34	0,13	0,47
INRIM	0,08	0,82	0,05	0,38	0,08	0,77
FSB	-0,15	-0,24	-0,10	-0,17	-0,09	-0,15
INM	-0,78	-0,95	-0,67	-0,82	-0,62	-0,76
UME	-0,25	-0,80	-0,13	-0,42	-0,08	-0,26
NRC	0,11	0,60	0,16	0,85	0,10	0,52
CEM	0,13	0,49	0,06	0,23	0,08	0,29

Table 39: Plug 50 mm evaluation with excluding NPL results at all 3 heights to get consistent dataset

The third approach is to consider the three dataset independent at all three heights and the calculation is done separately. In this case, MIRS result is excluded at +6 mm, NPL result is excluded at -6 mm and left unchanged at middle. This calculation is shown in the table 40.

weighted mean:	49,99935	mm	49,99923	mm	49,99932	mm
u _{int}	0,03	μm	0,03	μm	0,03	μm
u _{ext}	0,03	μm	0,04	μm	0,03	μm
No of labs	17		18		17	
R _B (uext/uint)	1,208		1,238		1,077	
R _B crit	1,307		1,298		1,307	
	+		middle		-	
	xi-xw	En	xi-xw	En	xi-xw	En
METAS	0,05	0,41	0,02	0,17	-0,04	-0,33
BEV	0,12	0,42	0,13	0,45	0,08	0,27
CMI	-0,15	-0,18	-0,03	-0,04	-0,12	-0,15
GUM	-0,02	-0,07	-0,05	-0,21	-0,12	-0,52
NML	-0,12	-0,61	-0,01	-0,04	-0,07	-0,37
DTI	-0,75	-0,68	-0,73	-0,66	-0,72	-0,66
NPL	-0,14	-1,34	-0,13	-1,26	-0,27	-1,91
MIRS	-0,75	-1,43	-0,73	-1,41	-0,72	-1,40
EIM	0,13	0,37	0,16	0,46	0,06	0,17
METROSERT	-0,03	-0,06	-0,02	-0,04	-0,11	-0,23
LNMC	-0,41	-0,20	-0,27	-0,13	-0,22	-0,11
MKEH	-0,04	-0,13	0,13	0,48	0,13	0,47
INRIM	0,11	1,06	0,09	0,66	0,08	0,77
FSB	-0,12	-0,20	-0,06	-0,10	-0,09	-0,15
INM	-0,75	-0,91	-0,63	-0,77	-0,62	-0,76
UME	-0,22	-0,71	-0,09	-0,29	-0,08	-0,26
NRC	0,14	0,74	0,20	1,05	0,10	0,52
CEM	0,16	0,59	0,10	0,37	0,08	0,29

Table 40: Plug 50 mm evaluation with excluding MIRS at +6 mm and NPL at -6 mm

The following graphs show the final results of the plug 50 mm with the expanded uncertainties (k=2).







9.5. Sphere 30 mm - diameter

SUBMITTED VALUES				
	diameter unc			
LABORATORY	mm	μm		
METAS	29,98623	0,07		
BEV	29,98611	0,15		
CMI	29,9858	0,4		
GUM	29,986056	0,090		
NML	29,98600 0,09			
DTI	29,9855 0,55			
NPL	29,98636	0,0538		
MIRS	29,9851	0,15		
EIM	29,98634	0,18		
METROSERT	29,98632 0,19			
LNMC	NOT MEASU	JRED		
MKEH	29,98617	0,14		
INRIM	29,98631	0,065		
FSB	29,98603	0,30		
INM	29,98530	0,40		
UME	29,98616	0,100		
NRC	29,98629	0,1		
CEM	29,98609	0,08		

Table 41: Sphere 30 mm final results given by the participants:

Table 42: Results evaluated with the final values

weighted mean:	29,98618	mm
u _{int}	0,02	μm
u _{ext}	0,06	μm
No of labs	17	
R _B (uext/uint)	2,305	
R _B crit	1,307	
Lab	xi-xw	En
METAS	0,05	0,35
BEV	-0,07	-0,25
CMI	-0,38	-0,48
GUM	-0,13	-0,74
NML	-0,18	-1,06
DTI	-0,68	-0,62
NPL	0,18	1,85
MIRS	-1,08	-3,66
EIM	0,16	0,44
METROSERT	0,14	0,35
LNMC	NOT MEA	ASURED
MKEH	-0,01	-0,05
INRIM	0,13	1,05
FSB	-0,15	-0,26
INM	-0,88	-1,11
UME	-0,02	-0,12
NRC	0,11	0,55
CEM	-0,09	-0,62

The dataset is not consistent. MIRS results are excluded first from the reference value determination. The results are shown in the table 43.

weighted mean:	29,98621	mm
u _{int}	0,03	μm
u _{ext}	0,04	μm
No of labs	16	
R _B (uext/uint)	1,445	
R _B crit	1,315	
Lab	xi-xw	En
METAS	0,02	0,12
BEV	-0,10	-0,35
CMI	-0,41	-0,52
GUM	-0,16	-0,92
NML	-0,21	-1,24
DTI	-0,71	-0,65
NPL	0,15	1,53
MIRS	-1,11	-3,66
EIM	0,13	0,35
METROSERT	0,11	0,27
LNMC	NOT MEA	ASURED
MKEH	-0,04	-0,16
INRIM	0,10	0,80
FSB	-0,18	-0,31
INM	-0,91	-1,15
UME	-0,05	-0,28
NRC	0,08	0,39
CEM	-0,12	-0,82

Table 43: Results evaluated with the final values – MIRS excluded

The dataset is still not consistent, the results with excluding NPL is shown in the table 44.

weighted mean:	29,98617	mm
u _{int}	0,03	μm
u _{ext}	0,04	μm
No of labs	15	
R _B (uext/uint)	1,251	
R _B crit	1,325	
Lab	xi-xw	En
METAS	0,06	0,45
BEV	-0,06	-0,21
CMI	-0,37	-0,47
GUM	-0,12	-0,69
NML	-0,17	-1,01
DTI	-0,67	-0,61
NPL	0,19	1,53
MIRS	-1,07	-3,51
EIM	0,17	0,47
METROSERT	0,15	0,38
LNMC	NOT MEA	SURED
MKEH	0,00	-0,01
INRIM	0,14	1,17
FSB	-0,14	-0,24
INM	-0,87	-1,09
UME	-0,01	-0,07
NRC	0,12	0,61
CEM	-0,08	-0,56

Table 44: Results evaluated with the final values – MIRS and NPL excluded to get consistent dataset

The following graphs show the final results of the sphere 30 mm with the expanded uncertainties (k=2).



9.6. Sphere 30 mm - roundness

500 UPR	round.	u(k=1)	
LABORATORY	μm	μm	
METAS	0,035	0,015	
CMI	0,09	0,05	
GUM	0,102	0,038	
DTI	0,10 0,04		
NPL	not measured		
EIM	0,091 0,07		
METROSERT	0,09	0,05	
INRIM	0,039	0,004	
FSB	0,097	0,055	
INM	0,10	0,04	
UME	0,078	0,038	
CEM	0,034	0,005	

Table 45: Sphere 30 mm final results given by the participants (500 UPR)

Table 46: Results evaluated with the final	values
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500 UPR		
weighted mean:	0,039	μm
u _{int}	0,003	μm
u _{ext}	0,003	μm
No of labs	11	
R _B (uext/uint)	1,130	
R _B crit	1,376	
	X _i -X _w	En
METAS	-0,004	-0,13
CMI	0,051	0,51
GUM	0,063	0,83
DFM	0,061	0,68
NPL	not me	asured
EIM	0,052	0,37
METROSERT	0,051	0,51
INRIM	0,000	0,01
FSB	0,058	0,53
INM	0,061	0,77
UME	0,039	0,52
CEM	-0,005	-0,62

150 UPR	round.	u(k=1)
LABORATORY	μm	μm
METAS	0,032	0,015
СМІ	0,10	0,05
GUM	0,078	0,036
DTI	0,12	0,045
NPL	0,038	0,003
EIM	0,076	0,07
METROSERT	0,07	0,04
INRIM	0,039	0,004
FSB	0,097	0,055
INM	0,08	0,04
UME	0,051	0,035
CEM	0,029	0,005

Table 47: Sphere 30 mm final results given by the participants (150 UPR)

Table 48: Results evaluated with the final values

150 UPR		
weighted mean:	0,037	μm
u _{int}	0,002	μm
u _{ext}	0,002	μm
No of labs	12	
R _B (uext/uint)	1,078	
R _B crit	1,361	
	X _i -X _w	En
METAS	-0,005	-0,18
CMI	0,063	0,63
GUM	0,041	0,57
DTI	0,083	0,92
NPL	0,001	0,15
EIM	0,039	0,28
METROSERT	0,033	0,41
INRIM	0,002	0,24
FSB	0,060	0,54
INM	0,043	0,53
UME	0,014	0,20
CEM	-0,008	-0,92

50 UPR	round.	u(k=1)
LABORATORY	μm	μm
METAS	0,026	0,015
CMI	0,10	0,05
GUM	0,067	0,036
DTI	0,14	0,045
NPL	0,034	0,003
EIM	0,058	0,07
METROSERT	0,06	0,03
INRIM	0,035	0,004
FSB	0,097	0,055
INM	0,04	0,04
UME	0,039	0,032
CEM	0,024	0,004

Table 49: Sphere 30 mm final results given by the participants (50 UPR)

Table 50: Results evaluated with the final values

50 UPR		
weighted mean:	0,032	μm
u _{int}	0,002	μm
u _{ext}	0,002	μm
No of labs	12	
R _B (uext/uint)	1,214	
R _B crit	1,361	
	X _i -X _w	En
METAS	-0,006	-0,21
CMI	0,068	0,68
GUM	0,035	0,48
DTI	0,108	1,20
NPL	0,002	0,40
EIM	0,026	0,18
METROSERT	0,028	0,46
INRIM	0,003	0,40
FSB	0,065	0,59
INM	0,008	0,10
UME	0,007	0,11
CEM	-0,008	-1,19









10. DISCUSSION OF RESULTS

The results of diameter measurements are consistent for the rings of 5 mm and 40 mm and also for the plug of 5 mm. An interesting situation can be seen for the plug 50 mm, where the dataset is consistent at mid-height but inconsistent at +6 mm and -6 mm caused by different laboratories. Three approaches has been introduced to handle the situation to be able to compare the different methods.

The dataset for the diameter of the sphere is not consistent, after two steps of iteration reached consistency.

The roundness data of the sphere shows good consistency at different filters (500, 150 and 50 UPR).

It is worth noting that although the evaluation of the roundness and straightness data of the cylindrical gauges is not included in the protocol, they show good consistency.

The main problem of the comparison can be that one of the linking laboratory's results for the diameter of the plug 50 mm and the sphere seems to be outliers and also the second linking laboratory's E_n value for the sphere is larger than 1, so the linking procedure between the two groups might be more difficult.

10. 1. Histograms of E_n values

10.1.1. The following diagram contains the histogram of all E_n values calculated from the total dataset of the diameter measurements, altogether from 227 results.



Histogram of E_n values (k=2) from the total 227 elements of the original dataset

A number of 21 E_n values exceed the range of ± 1 (*k*=2), which gives 7,6 %, larger than the expected 5% consistency check.

10.1.2. The following diagram contains the histogram of the E_n values after excluding 5 results from calculation of KCRD (3 for plug 50 mm, 2 for sphere).



Histogram of E_n values (k=2) from 227 elements, after excluding 5 data from the calculation of KCRD

In this case a number of 10 E_n values exceed the range of ±1 (k=2), which gives 4,4 %, less than the expected 5% consistency check.

10.1.3 The histogram of E_n values from the submitted results of the sphere roundness measurements are shown in the following diagram.





A number of 2 from the total 34 data exceeds the expected range of ± 1 that gives 5,9 %, that exceeds the expected 5% consistency check.

11. CONCLUSIONS

The diameter comparison was carried out in two groups. This protocol deals with the results of group 2.

The comparison was organized to have two loops, one within EU and another for non EU countries. The measurements were taken according to the scheduled dates in the first loop but than more delay came from the custom problems, *etc*.

Altogether 18 laboratories took part in the comparison, 17 from EURAMET and NRC, Canada.

Newly purchased gauges were used in the comparison. The pilot laboratory measured the gauges four times to monitor the stability of the gauges. The results show good stability for every gauge so no additional artefact uncertainty was applied in calculation of the KCRV. The cylindrical gauges suffered visible scratches that might cause some problems in the roundness measurements. No scrathes on the sphere.

Overall, the comparison has been successful. The comparison showed that these kind of measurements that could be kept "simple" measurements can cause trouble for the laboratories and makes the participants to think over the estimated uncertainty and the procedure.

12. ACKNOWLEDGEMENT

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14. Appendix 1

DEVIATIONS FROM THE REFERENCE VALUES WITH THEIR UNCERTAINTIES

The reference values of the measured parameters were determined as weighted means.

$$\overline{x}_{w} = \sum_{i=1}^{n} w_{i} \cdot x_{i}$$

The deviation of the individual results from the weighted mean was calculated $(x_i - x_w)$ and the uncertainty of this deviation:

$$U(x_{i} - \bar{x}_{w}) = 2\sqrt{u^{2}(x_{i}) - u_{int}^{2}(\bar{x}_{w})} \quad (k=2)$$

The repeated measurements of the gauges shows good stability, there is no need to introduce an additional artefact uncertainty.

The deviations from the reference values together with their uncertainties can be found in the following tables and graphs.

14.1 Ring 5 mm – diameter

	+3 mm		middle		-3 mm	
	X _i -X _w	U (xi-xw)	X _i -X _w	U (xi-xw)	$X_i - X_w$	U (xi-xw)
METAS	0,06	0,07	0,05	0,07	0,06	0,07
BEV	-0,50	0,50	-0,63	0,50	-0,53	0,50
CMI	-0,16	0,60	-0,29	0,60	-0,62	0,60
GUM	0,31	0,50	0,36	0,50	0,19	0,50
NML	-0,36	0,95	-0,41	0,95	-0,32	0,95
DTI	-0,26	0,90	0,01	0,90	-0,12	0,90
NPL	-0,03	0,08	-0,01	0,07	-0,04	0,06
EIM	-0,06	0,50	-0,09	0,50	0,00	0,50
METROSERT	-0,67	0,68	-0,61	0,68	-0,49	0,68
LNMC	-0,06	1,00	-0,06	1,00	-0,09	1,00
MKEH	-0,24	0,26	-0,32	0,26	-0,23	0,26
INRIM	0,02	0,09	0,02	0,09	0,03	0,09
INM	-0,46	0,78	-0,59	0,78	-0,42	0,78
UME	0,12	0,23	-0,01	0,23	0,05	0,23
NRC	-0,03	0,19	-0,04	0,19	-0,02	0,20
CEM	0,02	0,28	0,09	0,28	0,14	0,28

Table 51. Deviation from the reference value with its uncertainty







14.2 Ring 40 mm – diameter

	+10) mm	middle		-10 mm	
	x _i -x _w	U (xi-xw)	X _i -X _w	U (xi-xw)	X _i -X _w	U (xi-xw)
METAS	0,00	0,07	0,00	0,07	-0,01	0,07
BEV	0,02	0,50	0,06	0,50	0,03	0,50
CMI	-0,14	0,80	-0,01	0,80	-0,15	0,80
GUM	0,06	0,26	0,06	0,26	0,11	0,26
NML	0,34	0,66	0,39	0,66	0,34	0,66
DTI	0,46	1,10	0,59	1,10	0,15	1,10
NPL	0,01	0,07	0,01	0,07	0,04	0,07
MIRS	-0,14	0,62	-0,11	0,62	-0,25	0,62
EIM	-0,05	0,50	-0,01	0,50	-0,06	0,50
METROSERT	0,12	0,69	0,20	0,69	0,15	0,69
LNMC	-0,38	1,00	-0,39	1,00	-0,53	1,00
MKEH	0,03	0,30	0,06	0,30	0,01	0,30
INRIM	-0,04	0,10	-0,03	0,10	-0,04	0,10
FSB	-0,21	0,52	-0,18	0,52	-0,22	0,52
INM	-0,34	0,80	-0,31	0,80	-0,65	0,80
UME	0,24	0,28	0,19	0,28	0,31	0,28
NRC	-0,06	0,19	-0,10	0,19	-0,04	0,19
CEM	-0,03	0,28	-0,08	0,28	-0,13	0,28

Table 52. Deviation from the reference value with its uncertainty







14.3 Plug 5 mm – diameter

	+2	mm	middle		-2 mm	
	$X_i - X_w$	U (xi-xw)	X _i -X _w	U (xi-xw)	x_i - x_w	U (xi-xw)
METAS	0,04	0,07	0,04	0,07	0,06	0,07
BEV	0,04	0,30	0,09	0,30	0,05	0,30
CMI	-0,28	0,60	-0,28	0,60	-0,32	0,60
GUM	-0,03	0,17	-0,06	0,17	-0,10	0,18
NML	-0,02	0,14	-0,05	0,14	-0,09	0,14
DTI	-0,18	0,90	0,02	0,90	-0,22	0,90
NPL	-0,02	0,08	-0,07	0,08	-0,05	0,08
MIRS	-0,18	0,30	-0,18	0,30	-0,22	0,30
EIM	0,03	0,34	0,09	0,34	0,15	0,34
METROSERT	-0,13	0,31	-0,04	0,31	-0,12	0,31
LNMC	-0,42	1,40	-0,42	1,40	-0,42	1,40
MKEH	0,00	0,24	0,00	0,24	-0,04	0,24
INRIM	0,05	0,10	0,08	0,09	0,08	0,09
FSB	-0,13	0,60	-0,13	0,60	-0,14	0,60
INM	-0,78	0,78	-0,78	0,78	-0,82	0,78
UME	-0,11	0,30	-0,10	0,30	-0,18	0,30
NRC	-0,09	0,20	-0,04	0,20	0,01	0,20
CEM	0,16	0,28	0,23	0,28	0,26	0,28

Table 53. Deviation from the reference value with its uncertainty







14.4 Plug 50 mm – diameter

	+6	mm	middle		-6 mm	
	X_i - X_w	U (xi-xw)	X_i - X_w	U (xi-xw)	X_i - X_w	U (xi-xw)
METAS	0,06	0,13	0,02	0,13	0,01	0,13
BEV	0,13	0,29	0,13	0,29	0,13	0,29
CMI	-0,14	0,80	-0,03	0,80	-0,07	0,80
GUM	-0,01	0,23	-0,05	0,23	-0,07	0,23
NML	-0,11	0,19	-0,01	0,19	-0,02	0,19
DTI	-0,74	1,10	-0,73	1,10	-0,67	1,10
NPL	-0,13	0,10	-0,12	0,10	-0,22	0,11
MIRS	-0,74	0,52	-0,73	0,52	-0,67	0,52
EIM	0,14	0,36	0,16	0,36	0,11	0,36
METROSERT	-0,02	0,47	-0,02	0,47	-0,06	0,47
LNMC	-0,40	2,00	-0,27	2,00	-0,17	2,00
MKEH	-0,03	0,27	0,13	0,27	0,18	0,27
INRIM	0,12	0,11	0,09	0,14	0,13	0,11
FSB	-0,11	0,60	-0,06	0,60	-0,04	0,60
INM	-0,74	0,82	-0,63	0,82	-0,57	0,82
UME	-0,21	0,31	-0,09	0,31	-0,03	0,31
NRC	0,15	0,19	0,20	0,19	0,15	0,19
CEM	0,17	0,27	0,10	0,27	0,13	0,27

Table 54. Deviation from the reference value with its uncertainty from the complete dataset (approach 1)

Table 55	Deviation from	the reference v	alue with its	uncertainty	excluding NPL
		(approa	ach 2)	-	-

	+6	mm	mic	middle		-6 mm	
	X _i -X _w	U (xi-xw)	x _i -x _w	U (xi-xw)	X _i -X _w	U (xi-xw)	
METAS	0,02	0,13	-0,02	0,12	-0,04	0,13	
BEV	0,09	0,29	0,09	0,29	0,08	0,29	
CMI	-0,18	0,80	-0,07	0,80	-0,12	0,80	
GUM	-0,05	0,23	-0,09	0,23	-0,12	0,23	
NML	-0,15	0,19	-0,05	0,19	-0,07	0,19	
DTI	-0,78	1,10	-0,77	1,10	-0,72	1,10	
NPL	-0,17	0,10	-0,17	0,10	-0,27	0,11	
MIRS	-0,78	0,52	-0,77	0,52	-0,72	0,52	
EIM	0,10	0,35	0,12	0,35	0,06	0,35	
METROSERT	-0,06	0,47	-0,06	0,47	-0,11	0,47	
LNMC	-0,44	2,00	-0,31	2,00	-0,22	2,00	
MKEH	-0,07	0,27	0,09	0,27	0,13	0,27	
INRIM	0,08	0,10	0,05	0,13	0,08	0,10	
FSB	-0,15	0,60	-0,10	0,60	-0,09	0,60	
INM	-0,78	0,82	-0,67	0,82	-0,62	0,82	
UME	-0,25	0,31	-0,13	0,31	-0,08	0,31	
NRC	0,11	0,19	0,16	0,19	0,10	0,19	
CEM	0,13	0,27	0,06	0,27	0,08	0,27	

	+6	mm	middle		-6 mm	
	x_i - x_w	U (xi-xw)	x_i - x_w	U (xi-xw)	x_i - x_w	U (xi-xw)
METAS	0,05	0,13	0,02	0,13	-0,04	0,13
BEV	0,12	0,29	0,13	0,29	0,08	0,29
CMI	-0,15	0,80	-0,03	0,80	-0,12	0,80
GUM	-0,02	0,23	-0,05	0,23	-0,12	0,23
NML	-0,12	0,19	-0,01	0,19	-0,07	0,19
DTI	-0,75	1,10	-0,73	1,10	-0,72	1,10
NPL	-0,14	0,10	-0,13	0,10	-0,27	0,00
MIRS	-0,75	0,00	-0,73	0,52	-0,72	0,52
EIM	0,13	0,36	0,16	0,36	0,06	0,35
METROS	-0,03	0,47	-0,02	0,47	-0,11	0,47
LNMC	-0,41	2,00	-0,27	2,00	-0,22	2,00
MKEH	-0,04	0,27	0,13	0,27	0,13	0,27
INRIM	0,11	0,11	0,09	0,14	0,08	0,10
FSB	-0,12	0,60	-0,06	0,60	-0,09	0,60
INM	-0,75	0,82	-0,63	0,82	-0,62	0,82
UME	-0,22	0,31	-0,09	0,31	-0,08	0,31
NRC	0,14	0,19	0,20	0,19	0,10	0,19
CEM	0,16	0,27	0,10	0,27	0,08	0,27

 Table 56. Deviation from the reference value with its uncertainty excluding NPL at

 -6 mm, MIRS at +6 mm (approach 3)

The following graphs show the deviations from the reference values with the total dataset (approach 1)







14.5 Sphere 30 mm – diameter

	x _i -x _w	U (xi-xw)			
METAS	0,046	0,131			
BEV	-0,074	0,296			
CMI	-0,384	0,798			
GUM	-0,128	0,173			
NML	-0,184	0,173			
DTI	-0,684	1,099			
NPL	0,176	0,095			
MIRS	-1,084	0,296			
EIM	0,156	0,357			
METROSERT	0,136	0,389			
MKEH	-0,014	0,276			
INRIM	0,126	0,120			
FSB	-0,154	0,598			
INM	-0,884	0,798			
UME	-0,024	0,194			
NRC	0,106	0,194			
CEM	-0,094	0,152			

Table 57.	Deviation from the reference value with	its uncertainty
	with total dataset	-

Table 58. Deviation from the reference value with its une	certainty
MIRS and NPL excluded	-

	X _i -X _w	U (xi-xw)
METAS	0,057	0,128
BEV	-0,063	0,294
CMI	-0,373	0,798
GUM	-0,117	0,171
NML	-0,173	0,171
DTI	-0,673	1,099
NPL	0,187	0,091
MIRS	-1,073	0,294
EIM	0,167	0,355
METROSERT	0,147	0,388
MKEH	-0,003	0,274
INRIM	0,137	0,117
FSB	-0,143	0,597
INM	-0,873	0,798
UME	-0,013	0,192
NRC	0,117	0,192
CEM	-0,083	0,149





14.6 Sphere 30 mm – roundness

500 UPR	xi-xw	U (xi-xw)	
METAS	-0,004	0,029	
CMI	0,051	0,100	
GUM	0,063	0,076	
DFM	0,061	0,090	
NPL	not measured		
EIM	0,052	0,140	
METROSERT	0,051	0,100	
INRIM	0,000	0,005	
FSB	0,058	0,110	
INM	0,061	0,080	
UME	0,039	0,076	
CEM	-0,005	0,008	

Table 59. Deviation from the reference value with its uncertainty at 500 UPR

Table 60. Deviation from the reference value with its uncertainty at 150 UPR

150 UPR	x _i -x _w	U (xi-xw)
METAS	-0,005	0,030
CMI	0,063	0,100
GUM	0,041	0,072
DTI	0,083	0,090
NPL	0,001	0,004
EIM	0,039	0,140
METROSERT	0,033	0,080
INRIM	0,002	0,007
FSB	0,060	0,110
INM	0,043	0,080
UME	0,014	0,070
CEM	-0,008	0,009

Table 61. Deviation from the reference value with its uncertainty at 50 UPR

50 UPR	X _i -X _w	U (xi-xw)
METAS	-0,006	0,030
CMI	0,068	0,100
GUM	0,035	0,072
DFM	0,108	0,090
NPL	0,002	0,004
EIM	0,026	0,140
METROSERT	0,028	0,060
INRIM	0,003	0,007
FSB	0,065	0,110
INM	0,008	0,080
UME	0,007	0,064
CEM	-0,008	0,007






15. Appendix 2

CORRELATIONS (by Michael Matus)

The evaluation of comparison data in the presence of correlation between different participants can be handled using a straightforward generalization of the standard technique [1]. The theoretical background and formulas are outlined in [2, 3]. For the application the knowledge of the complete covariance matrix is necessary, that is not only the individual uncertainties must be known, but also the covariances between the results of different laboratories. These numerical values are not provided by the participants therefore the pilot must somehow deduce them using existing information.

The physical cause for correlation in this comparison originates from the traceability chain. Some participants use standard gauges calibrated by another participant. Therefore pair wise correlation is expected between {METAS, BEV, MKEH, UME} and {INRIM, FSB}, respectively.

15.1 Generalized Birge test

References [2,3] present a generalization of the classical Birge test, where correlations can be covered in a natural way. The necessary information are the complete covariance matrix **D** and the vector **x** of the results. Then the following holds:

• The KCRV:

$$m = \mathbf{B}^{\mathrm{T}}\mathbf{x}$$
 with $\mathbf{B}^{\mathrm{T}} = (\mathbf{1}^{\mathrm{T}}\mathbf{D}^{-1}\mathbf{1})^{-1}\mathbf{1}^{\mathrm{T}}\mathbf{D}^{-1}$

• The internal uncertainty

$$u_{\rm int} = \sqrt{\left(\mathbf{1}^{\rm T} \, \mathbf{D}^{-1} \mathbf{1}\right)^{-1}}$$

• The generalized Birge ratio

$$R_{\rm B} = \sqrt{\frac{1}{n-1} \left(\mathbf{x} - \mathbf{1}m\right)^{\rm T} \mathbf{D}^{-1} \left(\mathbf{x} - \mathbf{1}m\right)}$$

The external uncertainty u_{ext} can be get by multiplying R_B with u_{int} . However its value is not really needed. It is not completely clear for me if a simple formula for the E_n values exists also, taking into account the different correlations (both between the x_i and the x_i and *m* respectively). For the time being the classical evaluation was used.

15.2 Estimation of covariance values

Only the diagonal elements (variances) of **D** are given by the participants, the nondiagonal values (covariances) must be deduced by other ways. The following general example explains how numerical values for them were found here. Assume:

- Lab 0 performs a measurement x_0 with uncertainty $u(x_0)$
- Lab A performs a measurement x_A with uncertainty $u(x_A)$
- Lab B performs a measurement x_{B} with uncertainty $u(x_{B})$
- Lab A gets traceability by Lab 0 via an artefact
- Lab B gets traceability by Lab 0 via an (different) artefact

Then it is clear that:

- Results of Lab A correlate with results of Lab 0, covariance $u(x_A, x_0) \neq 0$
- Results of Lab B correlate with results of Lab 0, covariance $u(x_{\rm B}, x_0) \neq 0$
- Results of Lab A correlate with results of Lab B, covariance $u(x_A, x_B) \neq 0$, even if Lab 0 does not take part in this comparison

The correlation originates by the standards (calibrated by the same Lab 0 and used by Lab A and Lab B, respectively) by an unknown parameter δ_0 with uncertainty $u(\delta_0)$.

This parameter can be seen as a bias produced by Lab 0 during its calibration. It is (assumed) constant but unknown (otherwise Lab 0 would have corrected for it) so the best estimate is 0. The uncertainty $u(\delta_0)$ can only be estimated by Lab 0 who knows which part of his uncertainty budget includes components common in the respective measurements. Without this special knowledge all that can be concluded is $u(\delta_0) \le u(x_0)$. Now the results of the two labs can be modeled by this common bias $x_A = x'_A + \delta_0$ and $x_B = x'_B + \delta_0$. With this model it is possible to calculate the covariance $u(x_A, x_0)$ according to GUM, clause F.1.2.3. Using its terminology one finds:

$$\begin{aligned} x_{\rm A} &= F\left(x'_{\rm A}, x'_{\rm B}, \delta_0\right) \equiv x'_{\rm A} + \delta_0 \\ x_{\rm B} &= G\left(x'_{\rm A}, x'_{\rm B}, \delta_0\right) \equiv x'_{\rm B} + \delta_0 \\ u\left(x_{\rm A}, x_{\rm B}\right) &= \frac{\partial F}{\partial x'_{\rm A}} \frac{\partial G}{\partial x'_{\rm A}} u^2\left(x'_{\rm A}\right) + \frac{\partial F}{\partial x'_{\rm B}} \frac{\partial G}{\partial x'_{\rm B}} u^2\left(x'_{\rm B}\right) + \frac{\partial F}{\partial \delta_0} \frac{\partial G}{\partial \delta_0} u^2\left(\delta_0\right) = u^2\left(\delta_0\right) \end{aligned}$$

The correlation coefficient which is more readily interpreted than the covariance is:

$$r(x_{\rm A}, x_{\rm B}) = \frac{u^2(\delta_0)}{u(x_{\rm A})u(x_{\rm B})}$$

It is perfectly valid for Lab 0 to take over the role of one of the labs, in this case the correlation coefficient simplifies to:

$$r(x_{\rm A}, x_{\rm 0}) = \frac{u(\delta_{\rm 0})}{u(x_{\rm A})}$$

This way the full correlation matrices have been calculated. They are shown in the Appendix.

15.3 Results

The results for the calculation without and with (maximum) correlation are summarized in the following tables. Presented values are rounded to the next nm, R_B and E_n values are calculated with non-rounded values. Since the uncertainties are quite small, even a 0.1 nm change can have a significant impact on these values. For the 5 mm ring it proved necessary to remove results from BEV (with the highest E_n value) to fulfil the R_B criterion. (This is no problem since the deviation is too large, I know the reason for it, and have taken corrective actions already)

Ring 5 mm middle section	no correlation	full correlation all labs	full correlation BEV removed
X _{ref}	287 nm	303 nm	303 nm
U int	23 nm	24 nm	24 nm
Uext	29 nm	32 nm	29 nm
$R_{\rm B}$	1.269	1.372	1.215
R _{B crit}	1.315	1.315	1.325
n	16	16	15

Ring 40 mm middle section	no correlation	full correlation
X _{ref}	–291 nm	–297 nm
U int	23 nm	23 nm
Uext	16 nm	17 nm
$R_{\rm B}$	0.710	0.708
R _{B crit}	1.298	1.298
n	18	18

15.4 Conclusion

It is clear that the outcome of this comparison does not depend on whether or not correlation between participants is considered. Even with the extreme correlation coefficients, still compatible with physical principles, the numerical values are effected only marginally. In reality the correlations will be smaller and consequently the results will change even less. The "=" in $u(\delta_0) \le u(x_0)$ is very unrealistic, even when there is only little time between the respective measurements. For example the standard gauges of BEV were calibrated by METAS in September 2004 but METAS took part in the comparison in February 2009. It is very likely that METAS recalibrated in between temperature and environmental sensors, gauge blocks for probe constants, etc. or even changed the calibration procedure.

"For the calculations presented in the following, correlations between different participants have been neglected. To check if this assumption is justifiable, data for the 5 mm and 40 mm rings was evaluated both ways, classical as well as taking into account an extreme (but physical reasonable) correlation derived from the stated traceability chains. The techniques outlined in reference [2,3] were used for this evaluation. The results changed only marginally and did not affect conclusions drawn from the data. Since in reality the correlations are even smaller it is justifiable to evaluate the comparison data without taking correlation into account."

	METAS	BEV	CMI	GUM	NML	DTI	NPL	EIM	METROSERT	LNMC	MKEH	INRIM	WN	UME	NRC	CEM
METAS	1	0.160	0	0	0	0	0	0	0	0	0.308	0	0	0.348	0	0
BEV	0.160	1	0	0	0	0	0	0	0	0	0.049	0	0	0.056	0	0
CMI	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
GUM	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
NML	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
ITD	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
NPL	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
EIM	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
METROSERT	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
LNMC	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
MKEH	0.308	0.049	0	0	0	0	0	0	0	0	1	0	0	0.107	0	0
INRIM	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
INM	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
UME	0.348	0.056	0	0	0	0	0	0	0	0	0.107	0	0	1	0	0
NRC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
CEM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

15.5 Correlation matrix for the results of the 5 mm ring (middle section)

	METAS	BEV	cMI	GUM	NML	E	NPL	MIRS	EIM	METROSERT	LNMC	MKEH	INRIM	FSB	WN	UME	NRC	CEM
METAS	1	0.160	0	0	0	0	0	0	0	0	0	0.267	0	0	0	0.280	0	0
BEV	0.160	1	0	0	0	0	0	0	0	0	0	0.043	0	0	0	0.045	0	0
CMI	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
GUM	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NML	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
DTI	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
NPL	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
MIRS	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
EIM	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
METROSERT	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
LNMC	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
MKEH	0.267	0.043	0	0	0	0	0	0	0	0	0	1	0	0	0	0.075	0	0
INRIM	0	0	0	0	0	0	0	0	0	0	0	0	1	0.212	0	0	0	0
FSB	0	0	0	0	0	0	0	0	0	0	0	0	0.212	1	0	0	0	0
INM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
UME	0.280	0.045	0	0	0	0	0	0	0	0	0	0.075	0	0	0	1	0	0
NRC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
CEM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

15.6 Correlation matrix for the results of the 40 mm ring (middle section)

15.7 References

- [1] Cox M G 2002 The evaluation of key comparison data Metrologia 39 589-595
- [2] Kacker R N, Forbes A B, Kessel R and Sommer K-D 2008 Bayesian posterior predictive p-value of statistical consistency in interlaboratory evaluations *Metrologia* 45 512–523
- [3] Kacker R N, Forbes A B, Kessel R and Sommer K-D 2008 Classical and Bayesian interpretation of the Birge test of consistency and its generalized version for correlated results from interlaboratory evaluations *Metrologia* 45 257–264