## Appendix B: Linking the Sensitivity Results of AFRIMETS.AUV.V-K3 to CCAUV.V-K3

## **1** Linking procedure for accelerometer voltage sensitivities

For the consistency of the procedure and therefore the comparability between the different key comparisons in the field of vibration performed so far, the linking of the accelerometer voltage sensitivity results (AVS) of the RMO comparison, AFRIMETS.AUV.V-K3, (formerly registered as supplementary regional comparison AFRIMETS.AUV.V-S2) to AVS of CCAUV.V-K3 were calculated according to the same scheme used for the linking of EUROMET.AUV.V-K1 [1] and applied for APMP.AUV.V-K3 and EURAMET.AUV.V-K3.

The linking transforms the results  $(y_i, u(y_i))$  of the participants of AFRIMETS.AUV.V-K3 to scaled values  $z_i$  and their respective uncertainties  $u(z_i)$ , which are directly comparable to the results of CCAUV.V-K3.

The scaling was done with the linking factor *R*, which was calculated from the results of the linking laboratories (NMISA and INMETRO) in the RMO comparison and the KCRV of the CIPM comparison, CCAUV.V-K3.

The measurand in the CIPM comparison is denoted by *X*. The values,  $\{(x_1, u(x_1)), ..., (x_N, u(x_N))\}$  denote the best estimates and associated standard uncertainties of the laboratories that have participated in the CIPM comparison. The measurand in the RMO comparison is denoted by *Y*. The values  $\{(y_1, u(y_1)), ..., (y_N, u(y_M))\}$  denote the best estimates and associated standard uncertainties of the laboratories that have participated in the RMO comparison.

Furthermore,  $G = \{1, ..., p\}$  ( $p \leq \min(N, M)$ ) is the index set of the linking laboratories which participated in both the CIPM and RMO comparisons. The laboratories were labeled such that any number within *G* denotes the same laboratory in both comparisons.

The value, R = X/Y denotes the transformation factor between the two measurands to establish the link between the two comparisons. The transformation factor was estimated using the KCRV of the CIPM comparison and the combined results (weighted mean) in the RMO comparison of the linking laboratories. The estimated transformation factor was then applied to the results of the RMO comparison.

The estimators  $X_1$ , ...,  $X_N$ ,  $Y_1$ , ...,  $Y_M$  were treated as being uncorrelated as no information about correlations of other participants were available.

Let x denote the KCRV of the CIPM comparison and y the weighted mean of the

linking laboratories in the RMO comparison.

$$x = \frac{\sum_{i=1}^{N} \frac{x_i}{u^2(x_i)}}{\sum_{i=1}^{N} \frac{1}{u^2(x_i)}} \quad u^2(x) = \frac{1}{\sum_{i=1}^{N} \frac{1}{u^2(x_i)}}$$
(1)

$$y = \frac{\sum_{i=1}^{G} \frac{y_i}{u^2(y_i)}}{\sum_{i=1}^{G} \frac{1}{u^2(y_i)}} \quad u^2(y) = \frac{1}{\sum_{i=1}^{G} \frac{1}{u^2(y_i)}}$$
(2)

Then *R* is estimated according to

$$r = \frac{x}{y} \qquad u^{2}(r) = \frac{u^{2}(x)}{y^{2}} + \frac{x^{2}}{y^{4}}u^{2}(y)$$
(3)

Z = RY denotes the corrected measurand in the regional comparison and

$$z_I = ry_I$$
  $u^2(z_I) = y_I^2 u^2(r) + r^2 u^2(y_I) + 2ry_I u(r, y_I),$   $I = 1..M$ 

$$u(r, y_I) = \begin{cases} -\frac{x}{y^2} u^2(y), \ I \in G\\ 0, \ \text{otherwise} \end{cases}$$
(4)

are the corresponding estimates including the associated uncertainties.

The degrees of equivalence are defined as the differences between the corrected results in the RMO comparison and the KCRV of the CIPM comparison:

$$d_i = z_i - x, \quad i = 1, \dots, M$$
 (5)

And the standard uncertainties associated with these differences are:

$$u^{2}(d_{i}) = u^{2}(z_{i}) + \left[1 - 2\frac{z_{i}}{x}\right]u^{2}(x), \quad i = 1, \dots, M$$
(6)

## 2 Degrees of Equivalence to the CCAUV Reference Value for Accelerometer Voltage Sensitivities

The linking laboratories for AVS were NMISA and INMETRO for all available measurement results at the different frequencies.

Frequency→	0.4 Hz		0.5 Hz		0.63 Hz	
	$D_i$	$U_i$	$D_i$	$U_i$	$D_i$	$U_i$
Lab $i \downarrow$	in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )	
NMISA	-0.08	0.29	-0.04	0.29	-0.07	0.29
INMETRO	0.08	0.29	0.04	0.29	0.07	0.29

Table 1: Degrees of equivalence of the participants with respect to the KCRV of

Frequency→	0.8 Hz		1.0 Hz		1.25 Hz	
	$D_i$	$U_i$	$D_i$	$U_i$	$D_i$	$U_i$
Lab $i \downarrow$	in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )	
NMISA	-0.05	0.29	-0.01	0.29	-0.03	0.29
INMETRO	0.05	0.29	0.01	0.29	0.03	0.29

Frequency→	1.6 Hz		2.0 Hz		2.5 Hz	
	$D_i$	$U_i$	$D_i$	$U_i$	$D_i$	$U_i$
Lab $i \downarrow$	in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )	
NMISA	-0.04	0.29	-0.03	0.29	-0.04	0.29
INMETRO	-0.06	0.29	-0.06	0.29	-0.02	0.29

Frequency→	3.15 Hz		4 Hz		5 Hz	
	$D_i$	$U_i$	$D_i$	$U_i$	$D_i$	$U_i$
Lab $i \downarrow$	in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )	
NMISA	-0.03	0.29	-0.02	0.29	-0.02	0.29
INMETRO	0.03	0.29	0.02	0.29	0.02	0.29

Frequency→	6.3 Hz		8 Hz		10 Hz	
	$D_i$	$D_i$	$D_i$	$U_i$	$D_i$	$U_i$
Lab $i \downarrow$	in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )	
NMISA	-0.01	0.29	0.01	0.29	0.02	0.29
INMETRO	0.01	0.29	-0.01	0.29	-0.02	0.29

Frequency→	12.5 Hz		16 Hz		20 Hz	
	$D_i$	$D_i$	$D_i$	$U_i$	$D_i$	$U_i$
Lab $i \downarrow$	in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )	
NMISA	0.02	0.29	0.03	0.29	0.05	0.29
INMETRO	-0.02	0.29	-0.03	0.29	-0.05	0.29

Frequency→	25 Hz		31.5 Hz		40 Hz	
	$D_i$	$D_i$	$D_i$	$U_i$	$D_i$	$U_i$
Lab $i \downarrow$	in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )		in mV/(m/s <sup>2</sup> )	
NMISA	0.04	0.29	0.09	0.29	0.12	0.29
INMETRO	-0.04	0.29	-0.09	0.29	-0.12	0.29

## **Reference:**

[1] Final Report of EUROMET.AUV.V-K1, Appendix A - Linking the results of the regional key comparison EUROMET.AUV.V-K1 to those of the CIPM key comparison CCAUV.V-K1,

https://www.bipm.org/utils/common/pdf/final\_reports/AUV/V-K1/EUROMET.AUV.V-K1 .pdf