Linkage for Metrosert to CCT-K9 via the EURAMET-T.K9.1 bilateral comparison

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1 Introduction

Justervesenet (JV) and Metrosert carried out a bilateral comparison, EURAMET.T-K9.1, with the purpose of linking Metrosert to the CCT-K9 reference value. Metrosert realised a subset of the fixedpoints used in the CCT-K9, namely Hg TP, Ga MP, Sn FP and Zn FP. JV participated in the EURAMET.T-K9, whose topology was a set of distinct stars with a pilot that had participated in the CCT-K9. The linkage between EURAMET laboratories was thus via the pilot degree-of-equivalence. A consequence of this organisation was that there could be no linkage for Metrosert (and bilateral links between European laboratories in different stars) until the CCT-K9 was approved and closed. The EURAMET.T-K9.1 measurements finished in 2017, and the final report was published in 2020 after a prolonged review process [1]. However, the CCT-K9 results were not ready, and therefore the EURAMET.T-K9 results could not be analysed. The consequence was that computation of the linkage had to be postponed.

The CCT-K9 final report was published in early 2023 [2]. The final EURAMET.T-K9 report followed in autumn 2024 [3]. This brief report extends [1] with details about the computation and the results of the linkage to the CCT key comparison.

2 Equipment

The star topology, together with the fact that only temperature differences are quantified in the comparison, means that the specific SPRT used as a transfer instrument is indifferent. The protocols requested that each participant chose an SPRT from their own laboratory and used as a traveling device. Hence different SPRTs were used in EURAMET.T-K9 and EURAMET.T-K9.1.

JV used two different Sn cells and two different Zn cells in EURAMET.T-K9. One of them was identical to the cells used in EURAMET.T-K9.1.

The table summarises the equipment used.

Table 1: Equipment used

Instrument	*Manufacturer model*	*Serial no*	*Info*	*ILC*	*Participant*
Fixed point cell	Fluke	Hg-07127	Hg TP	K9	JV
	Isotech	Ga442	Ga MP	K9	JV
	Pyrocontrole	Sn95k	Sn FP	K9	JV
	Fluke	Sn-05110	Sn FP	K9	JV
	Isotech	Zn281	Zn FP	K9	JV
	Fluke	Zn-06077	Zn FP	K9	JV
	Fluke	Hg-07127	Hg TP	K9.1	JV
	Isotech	Ga442	Ga MP	K9.1	JV
	Pyrocontrole	Sn95k	Sn FP	K9.1	JV
	Isotech	Zn281	Zn FP	K9.1	JV
	Fluke 5900E	000047	Hg TP	K9.1	Metrosert
	Isotech Ga232	ITL M 17401	Ga MP	K9.1	Metrosert
	Isotech 491	341564/1	Sn FP	K9.1	Metrosert
	Isotech 492	341464/2	Zn FP	K9.1	Metrosert
SPRT	Isotech 670	067		K9	JV
	Isotech 670	052		K9.1	Metrosert

3 Computing the link

Laboratories are identified via superscripts and subscripts with abbreviated forms. The three relevant laboratories here are Metrosert (M), Justervesenet (JV) and LNE-CNAM (LC).

3.1 Difference to the reference value

The calculations are identical for all fixed points and are carried out independently. For simplicity we avoid explicit notation to distinguish fixed points. Temperature deviations are written as $\Delta_{lab2}^{lab1} = T_{lab1} - T_{lab2}$.

The absolute temperatures are not observables in the comparison. The participants reported the W values provided by the traveling SPRT, and they cannot be transformed to a temperature without the use of fixed points for calibration. Since the goal in the comparison is to establish equivalence in fixed point realisations it is only possible to quantify differences. From the EURAMET.T-K9 report JV's differences with the KCRV, Δ_{KCRV}^{JV}, are stated. The differences for Metrosert compared to JV, Δ_{JV}^{M}, are available from [1]. Equation 1 states the differences for Metrosert from the KCRV:

$$\Delta_M = \Delta_{JV}^{M} + \Delta_{JV-9}^{JV-9.1} + \Delta_{KCRV}^{JV} \qquad (1)$$

The term $\Delta_{JV-9}^{JV-9.1}$ introduces a possible additional error arising at JV, since the measurements for EURAMET.T-K9 and EURAMET.T-K9.1 were carried out at different times, and for Zn and Sn two different cells were used. Staff changes also occurred at JV between the two comparisons.

3.2 Uncertainty

Standard uncertainties are represented with u, and expanded uncertainties with 95% coverage are represented with U.

The uncertainty of Δ_M can be expressed

$$u_M^2 = \boldsymbol{c}^T \boldsymbol{W} \boldsymbol{c} \tag{2}$$

The sensitivity coefficient vector $\boldsymbol{c}$ and the covariance matrix $\boldsymbol{W}$ are given by

$$\boldsymbol{c} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{3}$$

$$\boldsymbol{W} = \begin{bmatrix} u_{M,JV}^2 & 0 & w_{JV,9,9.1} \\ 0 & u_{9-9.1}^2 & 0 \\ w_{JV,9,9.1} & 0 & u_{JV,KCRV}^2 \end{bmatrix} \tag{4}$$

The covariance term accounts for systematic effects at JV. To find it requires a closer scrutiny of JV's uncertainty budget, in order to classify the contributions as systematic or random effects. The procedure is explained by Cox and Harris [3]. First, we expand the first and last terms in Eq. 1:

$$\begin{aligned} \Delta_{JV}^{M} &= \frac{W_M - W_{JV}}{\partial W/\partial T} + C_{9.1} \\ \Delta_{KCRV}^{JV} &= \frac{W_{JV} - W_{LC}}{\partial W/\partial T} + C_9 + \Delta_{KCRV}^{LC} \end{aligned} \tag{5}$$

The C terms are the corrections (with zero expected value, but a non-zero uncertainty) added based on the measurements at the participating NMI before sending the SPRT to the pilot, and after receiving it again. The important point to note is that the reported uncertainty at JV occurs with different signs in the two comparisons and hence that the covariance matrix entry is negative. The uncertainties associated with Δ_{JV}^{M} and Δ_{KCRV}^{JV} are calculated in the respective reports of Euramet.T-K9 and Euramet.T-K9.1. But in order to see how to evaluate the covariance term we have to split JV's reported uncertainty into uncorrelated (r_{JV}^2) and correlated (s_{JV}^2) contributions. One way to expand the uncertainties associated with Equations 5, without getting into too much detail, is

$$\begin{aligned} u_{M,JV}^2 &= u_M^2 + r_{JV}^2 + s_{JV}^2 + u_{9.1}^2 \\ u_{JV,KCRV}^2 &= r_{JV}^2 + s_{JV}^2 + u_9^2 + u_{LC,KCRV}^2 \end{aligned} \tag{6}$$

Note that Equations 6 uses uncertainty expressed in temperature units. The apparent sensitivity coefficient $\partial W/\partial T$ in Equations 5 cancels when the uncertainty in W is expressed in temperature units, as was required in the protocols.

Then from Cox and Harris [4]

$$w_{JV,9,9.1} = -s_{JV}^2 \tag{7}$$

When applying Equation 2 the uncertainties $u_{M,JV}$ and $u_{JV,KCRV}$ are taken from the comparison reports.

The uncertainty budget of JV is shown in Tab. 2 with a classification of the terms. The values are given in mK and taken from the EURAMET.T-K9 report.

The term $\Delta_{JV-9}^{JV-9.1}$ and its associated uncertainty $u_{9-9.1}$ takes into account any differences between the two sets of measurements at JV which are not already accounted for in the uncertainty budget. Different Zn and Sn cells were used at JV in Euramet.T-K9 and in the report one can find the differences in the values measured before and after the pilot measurements. However, for Zn and Sn the observed values changed by 0,13 mK and -0,43 mK, respectively, smaller than then uncertainty attributed to random contributions at JV. Furthermore, the differences are taken into account in the C_9 term used to compute $u_{JV,KCRV}$.

While staff was changed at JV between the K9 and K9.1 measurements, the procedures remained the same, and the new staff received careful training. There is no reason to believe a substantial change occurred in the phase transition repeatability at JV. It is possible with other changes, such as small leaks in the fixed point cells, but there are no indications that this has occurred. In conclusion, we set $\Delta_{JV-9}^{JV-9.1}$ and $u_{9-9.1}$ to 0 in the calculations.

Table 2: Random and systematic effects at Justervesenet.

Contribution	Hg TP	Ga MP	Sn FP	Zn FP
Random	**0,38**	**0,39**	**0,72**	**1,21**
Phase Transition Realization Repeatability	0,17	0,09	0,38	0,89
Propagated TPW	0,24	0,3	0,53	0,78
Reference resistor stability	0	0	0,01	0,01
Repeatability of readings (FP)	0,03	0,03	0,03	0,03
Repeatability of readings (TPW)	0,02	0,03	0,05	0,08
SPRT Pt Oxidation	0	0	0	0
SPRT self-heating	0,23	0,23	0,31	0,26
Systematic	**0,33**	**0,22**	**0,52**	**0,74**
Bridge (repeatability, non-linearity, AC quadrature)	0,06	0,06	0,06	0,07
Chemical Impurities	0,25	0,2	0,5	0,7
Gas pressure	0,18	0,07	0,11	0,15
Heat Flux	0,06	0,03	0,09	0,16
Hydrostatic-head	0,08	0,01	0,03	0,03
Insulation leakage	0	0	0	0

4 Results

The results in Euramet.T-K9.1 and JVs results in Euramet.T-K9 are summarised in the table below. JVs degrees of equivalence are taken from the Euramet.T-K9 report issued in October 2024. The calculations and results data can be downloaded from [5].

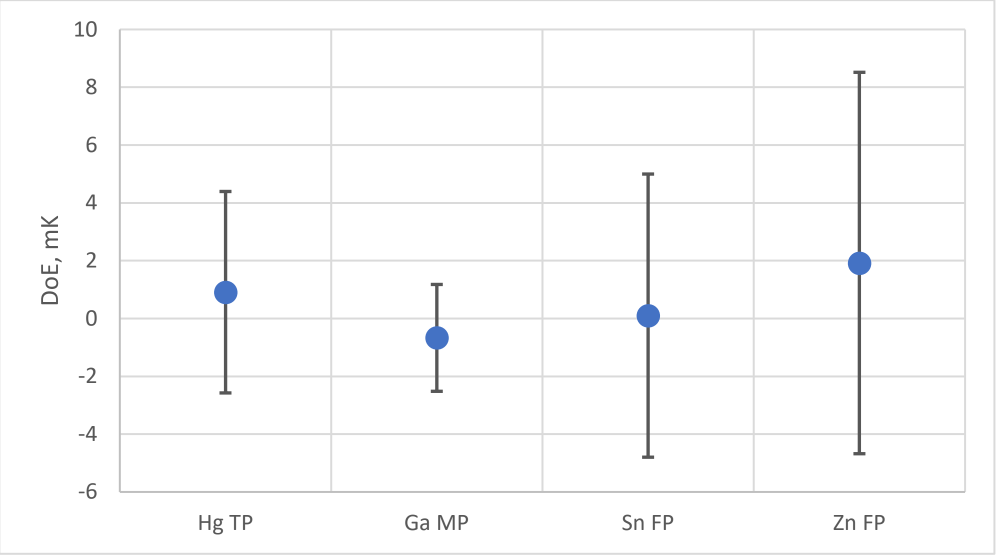
Table 3: Comparison results

Fixed point	Δ^{JV}_{KCRV}, *mK*	$u_{JV,KCRV}$, *mK*	Δ^{M}_{JV}, *mK*	$u_{M,JV}$, *mK*
Hg TP	0,60	0,37	0,31	1,57
Ga MP	-0,30	0,28	-0,37	0,76
Sn FP	0,34	0,32	-0,24	2,19
Zn FP	1,19	0,60	0,73	2,89

The linkage results for Metrosert are summarised in Tab. 3 and shown graphically below.

Table 4: Linkage results

Fixed point	Δ^{M}_{KCRV}, *mK*	*u, mK*	*Coverage*	*95% U, mK*
Hg TP	0,91	1,74	2	3,48
Ga MP	-0,67	0,92	2	1,85
Sn FP	0,1	2,45	2	4,90
Zn FP	1,92	3,30	2	6,60



5 References

[1] Å. A. F. Olsen, K. Opel og K. Tammik, «EURAMET-T.K9.1 bilateral comparison of ITS-90 SPRT calibration from the Hg TP to Zn FP,» *Metrologia,* vol. 57, p. 03001, 2020.

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