EURAMET.M.D-K4.2020.1
Bilateral Key Comparison in calibration of hydrometers from 1 000 kg/m^3 to 2 000 kg/m^3
(EURAMET Key Comparison 1787)

Final report

May 2026

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Abstract

This report presents the results of the key comparison EURAMET.M.D-K4.2020.1 (EURAMET 1787) that covered the calibration of high-resolution hydrometers for liquid in the density range 1 000 kg/m^3 to 2 000 kg/m^3 at the reference temperature of 20 °C.

Two laboratories participated in this key comparison, INRIM and IPQ. The results were linked to the CIPM key comparison CCM.D-K4 comparison using the measurements from INRIM, acting as pilot laboratory, which participated in both comparisons. The measurement range of this comparison has been reduced to the range from 1 000 kg/m^3 to 2 000 kg/m^3, when comparing to the CCM.D-K4 comparison.

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1. Introduction

This comparison is a bilateral the EURAMET Regional Key Comparison between INRIM and IPQ, with the aim of improving the current IPQ CMCs on the calibration of hydrometers. It is based on a decision at the EURAMET TC-M in 2025. The TC-M agreed that INRIM should organize this key comparison as the pilot laboratory and link the results to the CIPM key comparison CCM.D-K4.

The CIPM key comparison CCM.D-K4 was coordinated by INRIM, the transfer standards involved in the CIPM comparison were hydrometers in the range between 600 kg/m^3 to 2 000 kg/m^3. The measurements were performed from January 2011 to April 2012. The Final Report was published in 2016 [1].

As requested by IPQ, the scope of this key comparison has been limited to the range from 1 000 kg/m^3 to 2 000 kg/m^3, while the protocol remained essentially equivalent to the one of the EURAMET.M.D-K4. 2020 [2].

2. Participants and schedule

The measurements were performed from January 2026 to April 2026. The stability check was performed by INRIM, which performed the measurements at the beginning and end of the comparison.

Table 1 lists the circulation scheme.

Table 1. List of the participating NMIs and the circulation scheme.

5th January	29th January	INRIM
2nd February	10th March	IPQ
16th March	30st April	INRIM

3. Transfer Standards

For the comparison, INRIM and IPQ provided three hydrometers to cover the measurement range from 1.000 g/cm^3 to 2.000 g/cm^3, with similar characteristics to those used in the CCM.D-K4 comparison. Details are shown in Table 2. Even though a hydrometer with a resolution of 0.000 2 g/cm^3 was used for the range from 1.480 g/cm^3 - 1.500 g/cm^3 (instead of 0.000 1 g/cm^3 as used for the CCM comparison), the resolution was considered adequate for the uncertainties reported by IPQ.

Table 2. Transfer standards.

Range	Resolution	Serial Number
[0.990; 1.000] g/cm^3	0.000 1 g/cm^3	19726673
[1.480; 1.500] g/cm^3	0.000 2 g/cm^3	17658983
[1.980; 2.000] g/cm^3	0.000 2 g/cm^3	17674615

INRIM did not detect any significant changes in calibration on any transfer standards after returning from circulation. The stability of the transfer standard was checked and confirmed for values less than 7×10^{-6}, see Chapter 6.

4. Comparison Protocol

In a hydrometer calibration, the measurand is the correction *C* related to the reference liquid and reference temperature, evaluated at each calibration point:

$$\mathrm{C}= \rho_\mathrm{x} - \rho_\mathrm{r}, \tag{1}$$

where ρ_x is the density of the buoyant reference liquid in which the hydrometer would freely float at the scale mark ρ_r.

In this comparison, the results were given at the reference temperature of 20 ºC and the cubic expansion coefficient for all hydrometers was assumed to be $25\cdot10^{-6}$ °C^{-1}, with an uncertainty of $1.2\cdot10^{-6}$ °C^{-1}.

In Table 3, the calibration points and the reference surface tension of the liquid, in which each hydrometer was intended to be used, are shown.

Table 3. Calibration points and reference surface tensions.

Range	Reading scale/ g/cm^3	Ref. surface tension/ mN/m
[0.990; 1.000] g/cm^3	0.991	35.0
	0.995	35.0
	0.999	35.0
[1.480; 1.500] g/cm^3	1.481	75.0
	1.490	75.0
	1.499	75.0
[1.980; 2.000] g/cm^3	1.981	75.0
	1.990	75.0
	1.999	75.0

5. Calibration method and equipment used

According to the protocol, Cuckow's method was used by all participating NMIs. For each hydrometer, at least five weighing sequences were performed in air and in the reference fluid at each of the three scale marks. Scale readings were matched to the liquid level when the centre of the line was aligned with the horizontal liquid plane and detected with the specified visual alignment. A summary of the equipment used at both NMIs is given in Table 4. The equipment was found to have very similar characteristics.

Table 4. Summary of the equipment for all participating NMIs.

Laboratory	Balance Maximum load, resolution	Thermometer resolution	Working liquid	Visualisation
INRIM	520 g, 0.01 mg	0.001 °C	n-Nonane	CCD camera
IPQ	400 g, 0.1 mg	0.001 °C	n-Dodecane	CCD camera

6. Results of check measurements

The stability of the transfer standard was checked by the pilot laboratory comparing the correction of hydrometer at each scale mark, before starting (C_s) and at the end (C_e) of the comparison. The results are shown in Table 5.

The maximum relative variation $\Delta_C/\rho = (C_s – C_e)/\rho$, observed between the first and the second measurement, was less than 7×10^{-6}. This difference can be considered negligible and is coherent with the typical uncertainty of reproducibility.

Consequently, the INRIM values for the comparison have been calculated as mean values of the two measurements and used in the following evaluation.

Table 5. Measurements performed by the INRIM, as pilot laboratory.

Hydrometer S/N	Value ρ / g/cm^3	C_s $\times 10^{-6}$ g/cm^3	$U(C_s)$ $\times 10^{-6}$ g/cm^3	C_e $\times 10^{-6}$ g/cm^3	$U(C_e)$ $\times 10^{-6}$ g/cm^3	$\Delta_C = C_s - C_e$ $\times 10^{-6}$ g/cm^3	Δ_C/ρ $\times 10^{-6}$
19726673	0.991	-4	24	-5	24	-1	-1
	0.995	-5	24	-1	24	-4	-4
	0.999	-9	24	-5	24	-4	-4
17658983	1.481	86	37	88	37	-2	-1
	1.490	78	37	83	37	-5	-3
	1.499	65	37	69	37	-4	-3
17674615	1.981	53	48	50	48	4	2
	1.990	42	48	55	48	-13	-7
	1.999	42	48	51	48	-9	-4

7. Results of participants

For each hydrometer, five weighing sequences were carried out in air and in the reference liquid at each of the three scale readings. Each participating laboratory calculated the average value of the correction C at 20 °C and the standard deviation u_c at the indicated scale reading. The measurement results presented by each participant are shown in Table 6 and Figure 1 (error bars given in term of expanded uncertainty U_{95}).

The uncertainty of measurement for the corrections was calculated in the provided Excel sheet, as well as the effective degrees of freedom ν_{eff} of the combined standard uncertainty u_c, the t-factor $t_{95}(\nu_{eff})$ taken from the t-distribution for a 95% confidence level and the expanded uncertainty for the corrections as $U_{95} = t_{95}(\nu_{eff}) \cdot u_c$. The summary of the uncertainty budget for each participant is shown in Appendix Tables 15, 16 and 17.

Table 6. Measurements results reported by participants for the hydrometer 19726673 with standard uncertainty u_c and expanded uncertainty U_{95} at each calibrated mark.

Hydrometer 19726673 0.990 – 1.000 g/cm^3		INRIM	IPQ
0.991	x 10^{-6} / g/cm^3	-4	-3
0.995		-3	5
0.999		-7	15
u_c		12	41
U_{95}		24	81

Figure 1. Correction C reported by participants for the hydrometer 19726673.

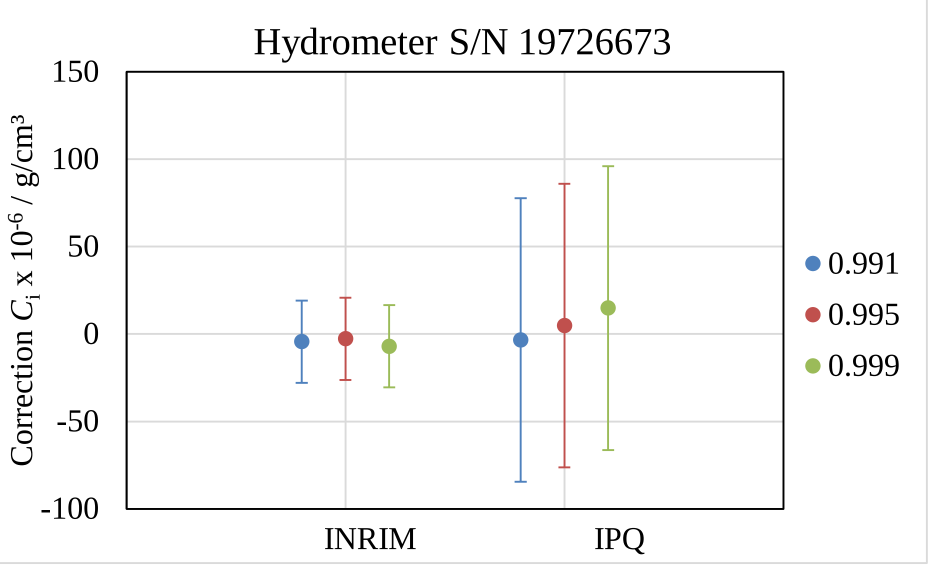


Table 7. Measurements results reported by participants for the hydrometer 17658983 with standard uncertainty u_c and expanded uncertainty U_{95} at each calibrated mark.

Hydrometer 17658983 1.480 – 1.500 g/cm^3		INRIM	IPQ
1.481	x 10^{-6} / g/cm^3	87	80
1.490		81	82
1.499		67	47
u_c		18	72
U_{95}		36	143

Figure 2. Correction C reported by participants for hydrometer 17658983.

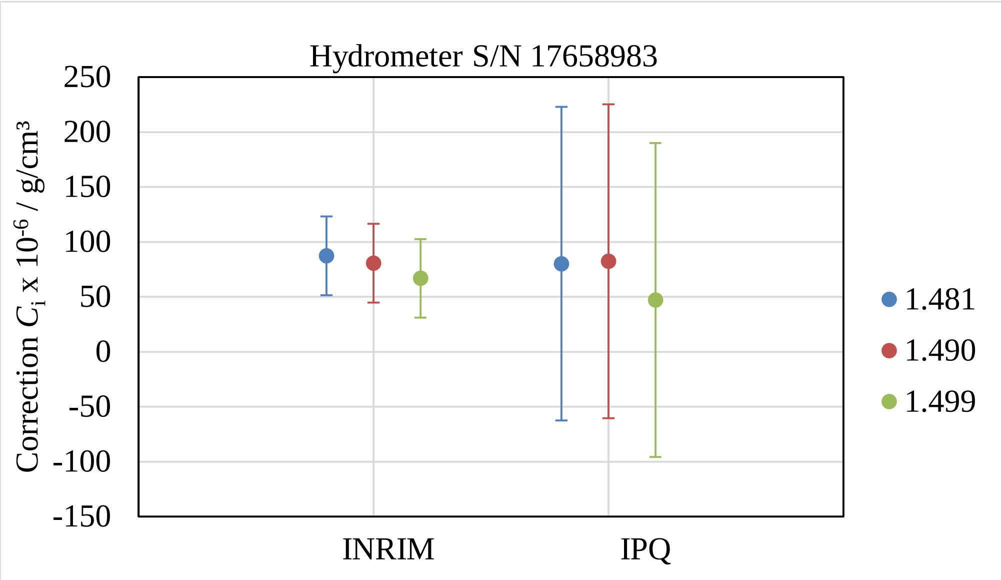
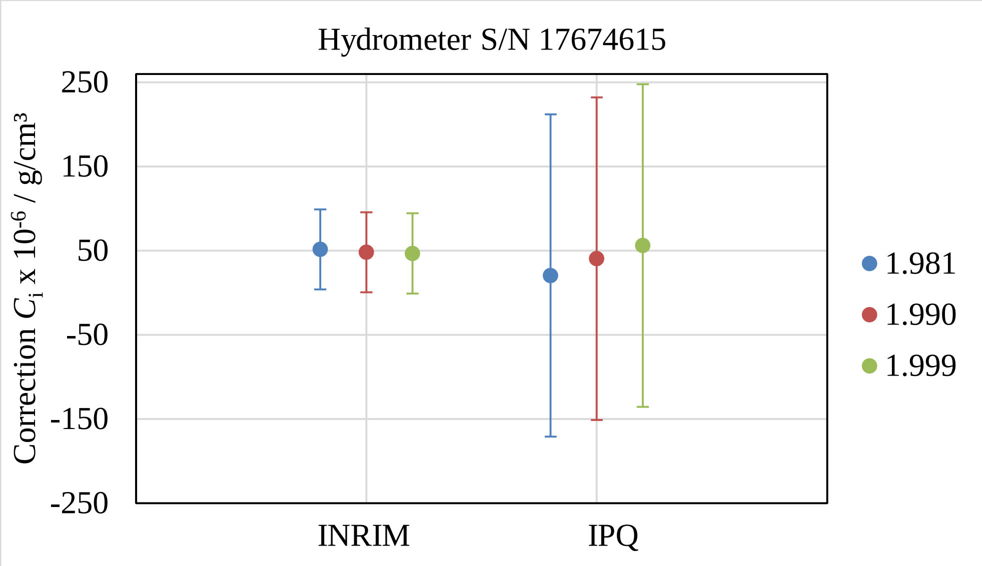


Table 8. Measurements results reported by participants for the hydrometer 17674615 with standard uncertainty u_c and expanded uncertainty U_{95} at each calibrated mark.

Hydrometer 17674615 1.980 – 2.000 g/cm^3		INRIM	IPQ
1.981	x 10^{-6} / g/cm^3	52	21
1.990		48	41
1.999		47	56
u_c		24	81
U_{95}		48	192

Figure 3. Correction C as reported by participants for hydrometer 17674615.



8. Link to CCM.D-K4 and Degree of Equivalence

According to the EURAMET Guide No. 4 [3], in this EURAMET key comparison, no reference value was determined, the degrees of equivalence were calculated by linking to the *KCRV* of the CIPM key comparison CCM.D-K4 [1] in which similar hydrometers were used as travelling standards.

The link has been provided by the differences between the values measured by the INRIM linking laboratory that took part in both interlaboratory comparisons.

The link between measured artefacts in the CIPM and EURAMET comparisons was made by the differences of the measured corrections of the linking laboratory in the two comparisons. For each measurement point i, the linking laboratory yielded a value for the difference $\Delta\rho_i$ between the measurand of the CIPM hydrometer $x_i(KCRV)$ and the measurand of the EURAMET hydrometer $x_i(EUR)$:

$$\Delta\rho_i = x_i(KCRV) - x_i(EUR). \quad (2)$$

Where

- $x_i(KCRV)$ is the reference value (*KCRV*) for the CIPM comparison.
- $x_i(EUR)$ is the correction obtained from the linking laboratory in the EURAMET comparison.

The degree of equivalence D_i [4] is calculated as the difference between the measurement performed by the laboratory in this comparison $\rho_{i\ \mathrm{EUR}}$ (corrected by $\Delta\rho$), and the *KCRV* of the CCM.D-K4 comparison:

$$D_i = \rho_{i\ \mathrm{CIPM}} - KCRV = \rho_{i\ \mathrm{EUR}} + \Delta\rho_i - KCRV \tag{3}$$

and

$$U(D_i) = 2\,u(D_i), \tag{4}$$

where

$$u(D_i) = \sqrt{u^2(\rho_{i\ \mathrm{EUR}}) + u^2(\Delta\rho_i) + u^2(KCRV)}. \tag{5}$$

For the linking laboratories, due to the correlation:

$$u(D_i) = \sqrt{u^2(\rho_{i\ \mathrm{EUR}}) + u^2(\Delta\rho) - u^2(KCRV)}. \tag{6}$$

The uncertainty of the difference $u(\Delta\rho_i)$ is evaluated considering that $x_i(KCRV)$ and $x_i(EUR)$ are correlated.

As the measuring range of each hydrometer was very small (between 0.010 g/cm^3 and 0.020 g/cm^3), a nominal density range ρ_j (1.0 g/cm^3 – 1.5 g/cm^3 – 2.0 g/cm^3) has been defined in both comparisons.

The differences $\Delta\rho_i$ has been evaluated considering the same measuring range of the comparison CCM.D-K4.

As in the comparison EURAMET.M.D-K4.2020 [5] the INRIM laboratory, together with the PTB, acted as linking laboratories, and the reproducibility of the differences $\Delta\rho_i$ between the two EURAMET and CCM comparisons was evaluated, it is possible to use the same uncertainty value $u(\Delta\rho_i)$ for this comparison.

In the comparison EURAMET.M.D-K4.2020 the relative uncertainty $u(\Delta)/\rho$ was similar enough for the whole range, and it was decided to use the maximum value of $u(\Delta)/\rho = 5.1\times10^{-6}$ for all points. This value was also used for this comparison.

The link was calculated with respect to petal A of the CIPM comparison CCM.D-K4. The *KCRV* reference values, the associated uncertainties, and the correction $\Delta\rho$ and uncertainty $u(\Delta\rho)$ for each calibration point and for the three hydrometers are shown in Table 9.

Table 9. Linking values of the CIPM comparison CCM.D-K4 petal A by the reference value *KCRV*, the associated uncertainties *u*(*KCRV*) to this EURAMET comparison with the correction $\boldsymbol{\Delta\rho}$ and uncertainty $\boldsymbol{u}(\boldsymbol{\Delta\rho})$ for each calibration measurement point.

Value g/cm³	*KCRV* × 10^{-6} g/cm³	*u*(*KCRV*) (*k* = 1) × 10^{-6} g/cm³	$\Delta\rho$ × 10^{-6} g/cm³	$u(\Delta\rho)$ (*k* = 1) × 10^{-6} g/cm³
0.98592	-29	6.5	-25	5.0
0.99106	-41	6.5	-38	5.1
0.9967	-22	6.5	-15	5.1
1.491	75	7.5	-12	7.6
1.495	85	7.5	4	7.6
1.499	73	7.5	6	7.6
1.981	108	10.5	56	10.1
1.990	91	10.5	43	10.1
1.999	148	10.5	101	10.2

The degree of equivalence D_i, the associated uncertainty $U(D_i)$ and normalized error $E_n = D_i / U(D_i)$, obtained by each participant, are shown in Table 10 and from Figure 4 to Figure 6, where the error bars are presented in terms of expanded uncertainty.

Table 10. Degree of equivalence and normalized error for each measurement point.

Range	INRIM			IPQ		
	D_i	$U(D_i)$	E_n	D_i	$U(D_i)$	E_n
g/cm³	× 10^{-6}	g/cm³		× 10^{-6}	g/cm³	
0.985	0	23	0	1	83	0.01
0.991	0	23	0	8	83	0.09
0.996	0	23	0	22	83	0.26
1.491	0	36	0	-7	144	0.05
1.495	0	36	0	2	144	0.01
1.499	0	36	0	-20	144	0.14
1.981	0	48	0	-31	194	0.16
1.990	0	48	0	-8	194	0.04
1.999	0	48	0	9	194	0.05

Figure 4. D_i of each participating NMI in the density range of 0.985 g/cm³ to 0.996 g/cm³.

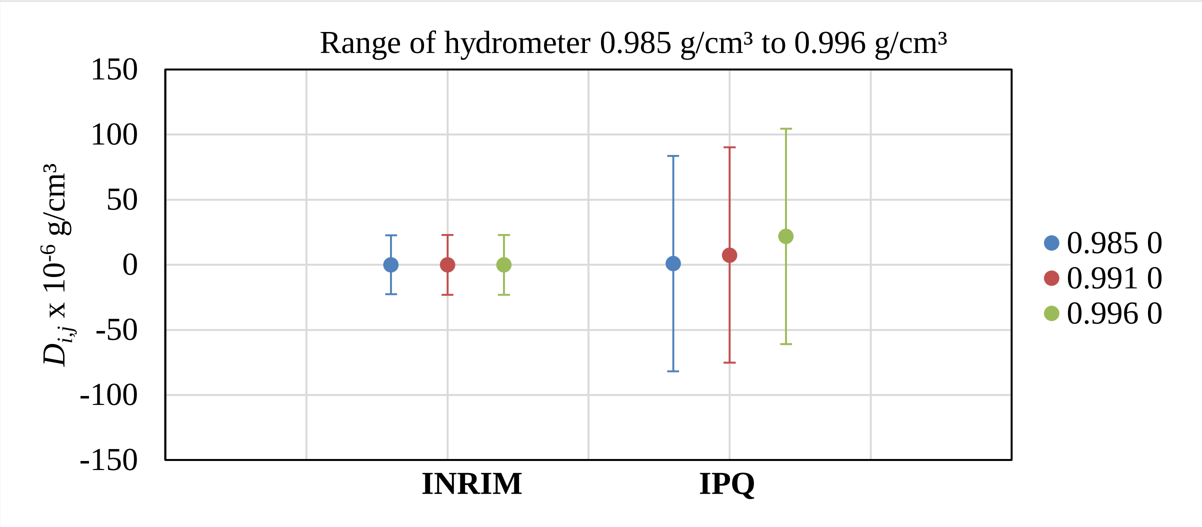


Figure 5. D_i of each participating NMI in the density range of 1.490 g/cm³ to 1.500 g/cm³.

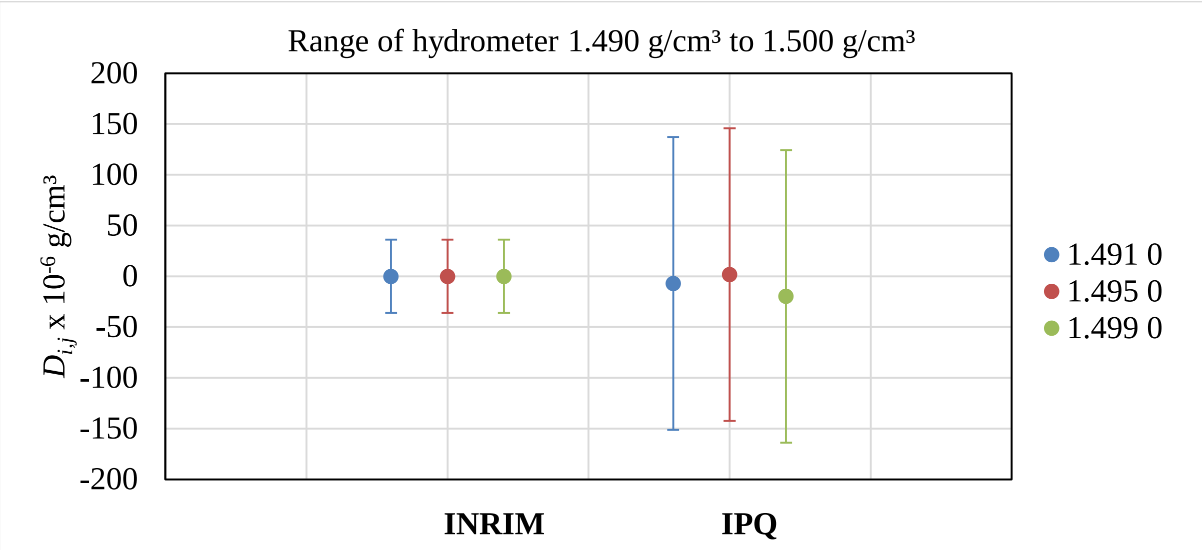
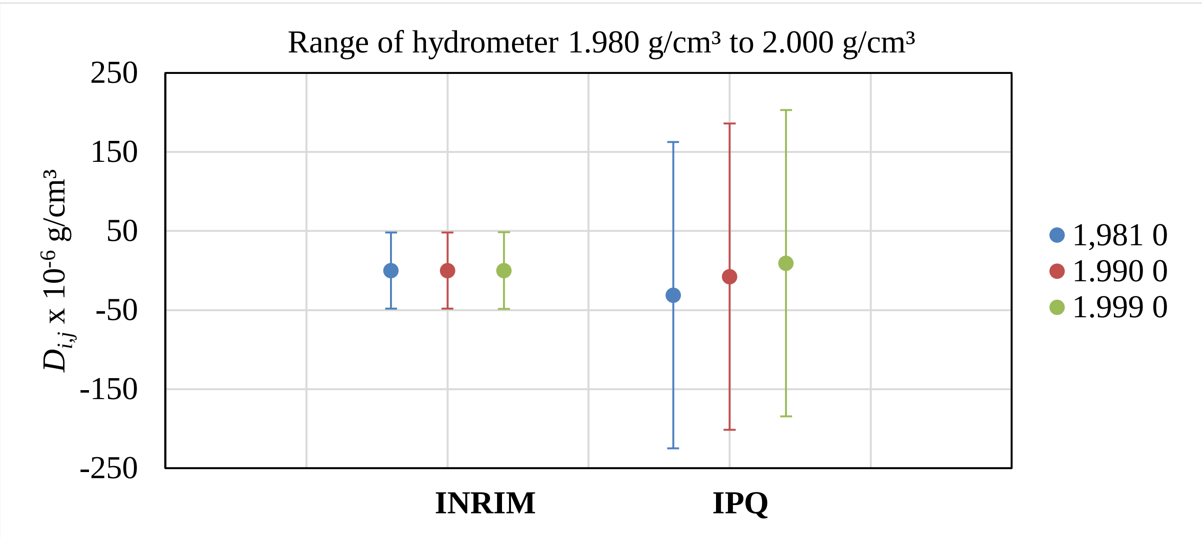


Figure 6. D_i of each participating NMI in the density range of 1.490 g/cm³ to 1.500 g/cm³.



9. Degrees of Equivalence between laboratories

The degrees of equivalence between laboratories have been evaluated, considering the values $\rho_{i\ \mathrm{CIPM}}$ (of equation 8). For two laboratories i and j

$$D_{i,j} = \rho_{i\ \mathrm{CIPM}} - \rho_{j\ \mathrm{CIPM}}. \tag{7}$$

The uncertainty depends on whether the laboratories belong to the same petal or to different petals. For IPQ and INRIM:

$$U(D_{i,j}) = 2\sqrt{u^2(x_i) + u^2(x_j)}. \tag{8}$$

The results are shown in Appendix Tables 12, 13 and 14.

10. Conclusions

The comparison EURAMET Key Comparison 1787 (EURAMET.M.D-K4.2020.1) was a bilateral EURAMET Regional Key Comparison of the CIPM comparison CCM.D-K4 between INRIM and IPQ laboratories. The density ranged from 1.000 g/cm^3 to 2.000 g/cm^3, using one hydrometer with 0.000 1 g/cm^3 and two hydrometers with 0.000 2 g/cm^3 resolution, similar to those in the CCM.D-K4.

The measurements of this comparison were conducted during January 2026 and April 2026.

The degrees of equivalence of all participating laboratories have been evaluated by linking to the INRIM results of *KCRV* values of the CCM.D-K4. In order to assess the support of CMCs entries provided by this comparison, the obtained degrees of equivalence and the declared uncertainties have been compared against the uncertainty values of the CMCs - tables *U*(CMC), the results are in Appendix Table 11.

11. References

[1] S. Lorefice et al. "CCM key comparison CCM.D-K4 'Hydrometer'", Metrologia, 2016, 53, 07003.

[2] Malengo. A. Technical Protocol for Bilateral Key Comparison on hydrometers calibration from 1 000 kg/m^3 to 2 000 kg/m^3, 2025. https://www.bipm.org/kcdb/comparison?id=2100

[3] EURAMET Guide No. 4 Version 2.0 (04/2021).

[4] M.G. Cox, The evaluation of key comparison data, Metrologia, 2002, **39**, 589-595.

[5] A. Maelngo et al. EURAMET.M.D-K4.2020, Metrologia 2022 59 Tech. Suppl. 07005

12. Appendix

Table 11. Comparison of the results with the CMCs.

NMI	Range g/cm³	U(CMC) × 10^{-6} g/cm³	U × 10^{-6} g/cm³	$U \leq$ CMC	Passed with U	Passed with CMC
INRIM	1.0	24	24	yes	yes	yes
	1.5	36	36	yes	yes	yes
	2.0	48	48	yes	yes	yes
IPQ	1.0	100	81	yes	yes	yes
	1.5	500	143	yes	yes	yes
	2.0	500	192	yes	yes	yes

Table 12. Degrees of equivalence between laboratories, range of 1.000 g/cm^3.

x_j	INRIM		IPQ	
0.991 g/cm^3	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	-1	84
IPQ	1	84	-	-

x_j	INRIM		IPQ	
0.995 g/cm^3	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	-8	84
IPQ	8	84	-	-

x_j	INRIM		IPQ	
0.999 g/cm^3	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	-22	84
IPQ	22	84	-	-

Table 13. Degrees of equivalence between laboratories, range of 1.500 g/cm^3.

x_j	INRIM		IPQ	
1.491 g/cm^3	D_{ij}	U	D_{ij}	U
x_i	g/cm^3	×10^{-6}	g/cm^3	×10^{-6}
INRIM	-	-	7	147
IPQ	-7	147	-	-

x_j	INRIM		IPQ	
1.495 g/cm^3	D_{ij}	U	D_{ij}	U
x_i	g/cm^3	×10^{-6}	g/cm^3	×10^{-6}
INRIM	-	-	-2	147
IPQ	2	147	-	-

x_j	INRIM		IPQ	
1.499 g/cm^3	D_{ij}	U	D_{ij}	U
x_i	g/cm^3	×10^{-6}	g/cm^3	×10^{-6}
INRIM	-	-	20	147
IPQ	-20	147	-	-

Table 14. Degrees of equivalence between laboratories, range of 2.000 g/cm^3.

x_j	INRIM		IPQ	
1.981 g/cm^3	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	31	197
IPQ	-31	197	-	-

x_j	INRIM		IPQ	
1.990 g/cm^3	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	8	197
IPQ	-8	197	-	-

x_j	INRIM		IPQ	
1.999 g/cm^3	D_{ij}	U	D_{ij}	U
x_i	g/cm^3 ×10^{-6}		g/cm^3 ×10^{-6}	
INRIM	-	-	-9	197
IPQ	9	197	-	-

Table 15. Uncertainty budget for the range of 1.000 g/cm^3 (in density g/cm^3×10^{-6}, k =1).

Influence quantity	INRIM	IPQ
	g/cm^3×10^{-6}	
Weighing in air	0.13	3.4
Weighing in liquid	0.94	16.1
Additional weights	0.00	0.00
Cubic thermal expansion	0.03	0.02
Diameter of stem	0.14	0.1
Air density	0.78	2.8
Liquid density	7.0	33.0
Temperature of liquid	9.1	2.7
Thermal expansion coefficient of liquid	0.03	6.8
Compressibility of liquid	0.0	0.0
Surface tension of liquid	1.5	0.1
Gravitation acceleration	0.0	0.0
Gradient of gravitational	0.0	0.0
Height difference of weights and hydrometer	0.0	0.0
Readings error	3.0	10.5
Standard deviation of the mean of corrections	1.3	11.5

Table 16. Uncertainty budget for the range of 1.500 g/cm^3 (in density g/cm^3×10^{-6}, k =1).

Influence quantity	INRIM	IPQ
	g/cm³×10^{-6}	
Weighing in air	0.88	12.3
Weighing in liquid	2.0	31.5
Additional weights	0.0	0.0
Cubic thermal expansion	0.02	0.2
Diameter of stem	1.2	1.4
Air density	3.2	2.3
Liquid density	10.4	55.8
Temperature of liquid	13.7	2.0
Thermal expansion coefficient of liquid	0.02	8.0
Compressibility of liquid	0.0	0.0
Surface tension of liquid	2.5	0.1
Gravitation acceleration	0.0	0.0
Gradient of gravitational	0.0	0.0
Height difference of weights and hydrometer	0.0	0.0
Readings error	4.0	24.1
Standard deviation of the mean of corrections	1.4	15.6

Table 17. Uncertainty budget for the range of 2.000 g/cm^3 (in density g/cm^3×10^{-6}, k =1).

Influence quantity	INRIM	IPQ
	g/cm³×10^{-6}	
Weighing in air	1.2	17.1
Weighing in liquid	1.9	32.4
Additional weights	0.0	0.0
Cubic thermal expansion	0.05	0.3
Diameter of stem	0.5	0.4
Air density	5.3	3.1
Liquid density	13.9	82.8
Temperature of liquid	18.3	2.4
Thermal expansion coefficient of liquid	0.05	11.9
Compressibility of liquid	0.0	0.0
Surface tension of liquid	2.7	0.1
Gravitation acceleration	0.0	0.0
Gradient of gravitational	0.0	0.0
Height difference of weights and hydrometer	0.0	0.0
Readings error	4.0	26.0
Standard deviation of the mean of corrections	1.8	15.3