Measuring Frequency & Steering Oscillators

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Sep 2025

Outline

NIST

- Definitions of Time and Frequency for Measurement
- Instability vs Frequency Offset
- EXAMPLE: Basic Measurements
- Impact of Clock Types and Environment
- Theory of Steering
- EXAMPLE: Steering a Rubidium
- Frequency Synthesis and Timescales
- GPS Frequency References
- DEMO: Measuring and Correcting a Quartz

Definitions of Time and Frequency

Time (T)

- Time (T) is the number of seconds that have passed and must always have a reference start point
- Period (P) is the time interval between recurring events

Frequency (f)

- Frequency (f or ν) is the number of oscillations or periods in one second
- Frequency is only a useful measurement for periodic signals

$$f = \frac{1}{P}$$

Definitions of Time and Frequency

The Relationship between Time and Frequency

- Frequency can be thought of as an instantaneous measurement or a rate
- It is often useful to conceptualize time as an integrated value as it accumulates over time

Informally:

$$f \propto \frac{d}{dT}$$

$$T \propto \int f \, dt$$

Definitions of Time and Frequency

Time and Frequency Offsets

- Time and frequency offsets are the difference between an expected or ideal value and what is measured
- Measured in Seconds and Hertz, offset is a value with units

A universal counter used for measuring time and frequency

Time Offset

$$\Delta t = t_{measured} - t_{nominal} \quad [\mathrm{s}]$$

Frequency Offset

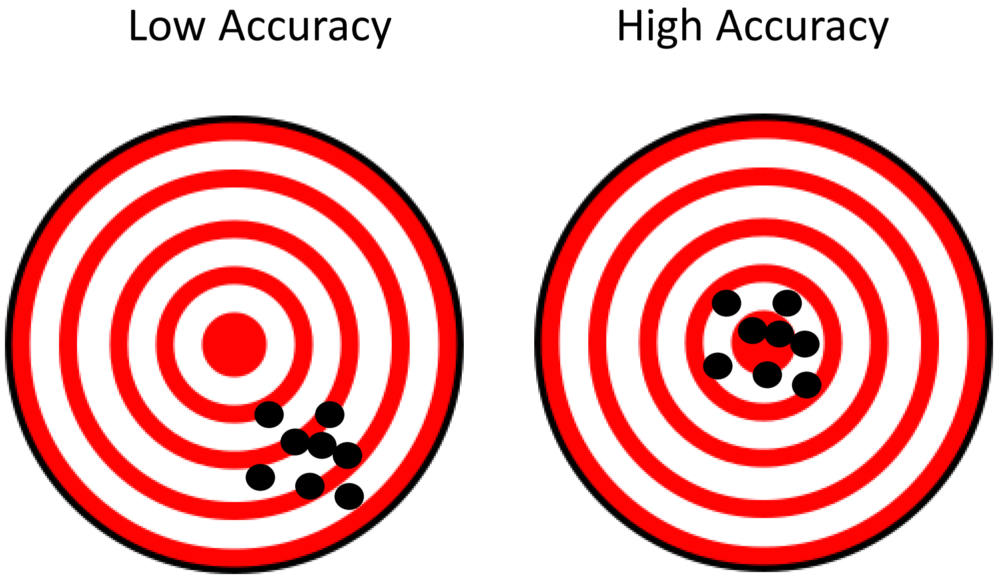
$$\Delta f = f_{measured} - f_{nominal} \, [\mathrm{Hz}]$$

Accuracy and Frequency Offset

Frequency Offset is Measurement of Accuracy

- On a dart board, it is how close to the center the darts are placed
- In a frequency measurement it is how close to the ideal frequency a measurement is
  - If 10 MHz is the goal:
  - 10.00912 MHz is not accurate
  - 9.99995 MHz is more accurate
  - 10.00000 MHz is even more accurate

Dart Board Example



Definitions of Time and Frequency

Fractional Time and Frequency Offsets

- Fractional Offsets are normalized by the ideal time interval or frequency
- Fractional values allow better inaccuracy comparison between different frequencies or lengths of time

Time Offset

$$\Delta t = t_{measured} - t_{nominal} \text{ [s]}$$

Frequency Offset

$$\Delta f = f_{measured} - f_{nominal} \text{ [Hz]}$$

Fractional Frequency Offset

$$\Delta t_{fractional} = \frac{\Delta t}{t_{nominal}} = \Delta f_{fractional} = \frac{\Delta f}{f_{nominal}} \text{ [Unitless]}$$

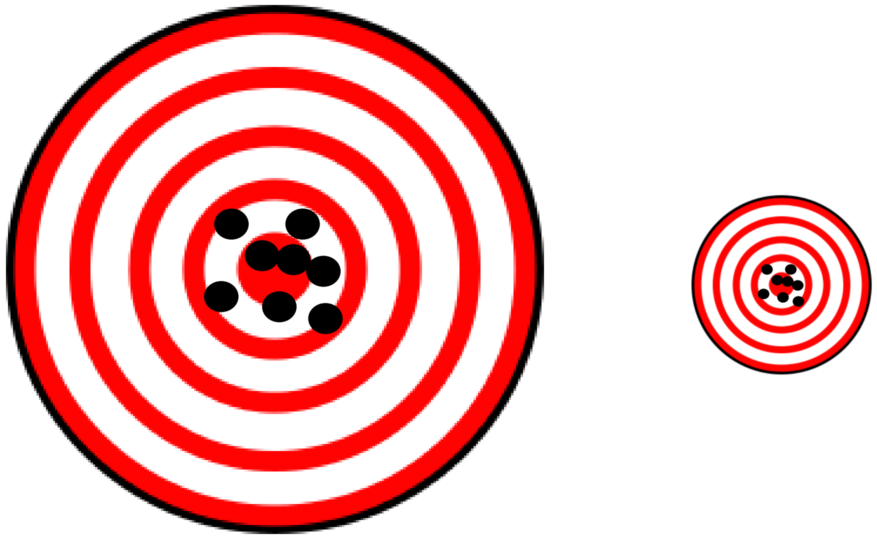
Fractional Frequency Offset

Fractional frequency makes accuracy comparison "fair" for high and low frequency oscillators

- 1.000 MHz $\pm$ 1 Hz has a fractional frequency accuracy of 1 x 10^{-6}
- 100.0 MHz $\pm$ 100 Hz also has a fractional accuracy of 1 x 10^{-6}

$$\Delta f_{fractional} = \frac{\Delta f}{f_{ideal}} = \frac{\Delta t}{T}$$

Dart Board Example



Same fractional accuracy even though the darts on the right are spread less

Fractional Time Offset

Fractional time and fractional frequency are both measuring rate offsets

- 1.000 s $\pm$ 1 μs has a fractional time accuracy of 1 x 10^{-6}
- 100.0 s $\pm$ 100 μs also has a fractional accuracy of 1 x 10^{-6}

$$\Delta t_{fractional} = \frac{\Delta t}{t_{nominal}} \quad \text{[Unitless]}$$

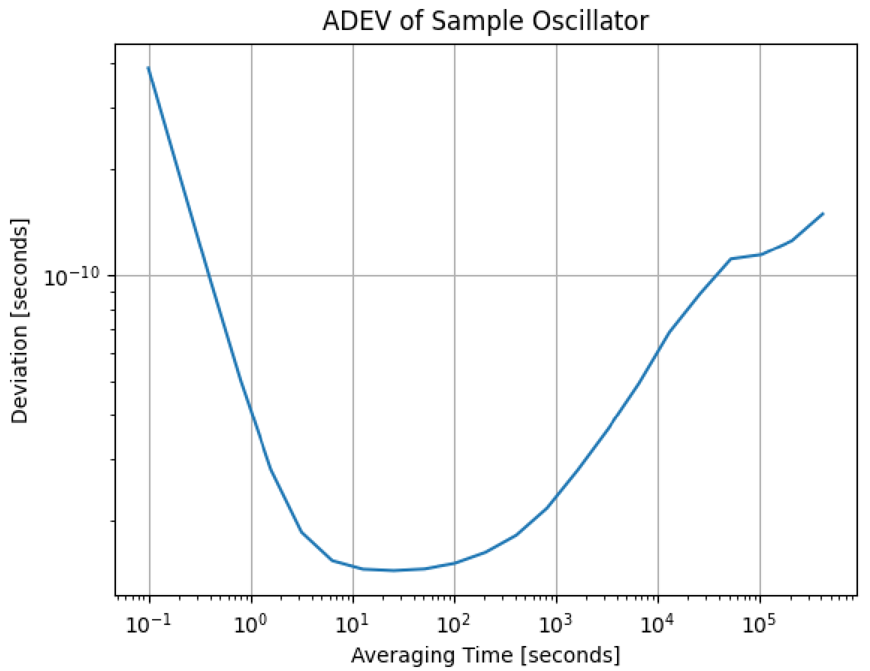
$$\Delta f_{fractional} = \frac{\Delta f}{f_{nominal}} \quad \text{[Unitless]}$$

$$\Delta f_{fractional} = \Delta t_{fractional}$$

Instability of Oscillators

Frequency Offset ≠ Instability

- Even a large frequency offset does not mean an oscillator is unstable
- Instability is only the change or uncertainty in a frequency offset over time
- Best measured using Allan Deviation or ADEV for short



Example ADEV plot showing instability

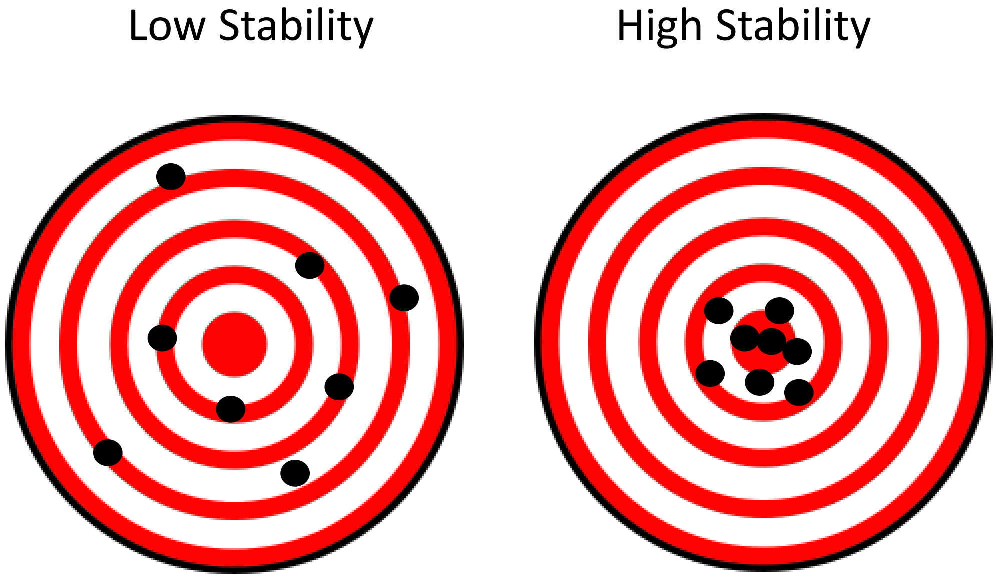
Instability of Oscillators

Stability is the spread of data points

- On a dart board, it is how wide an area the darts cover

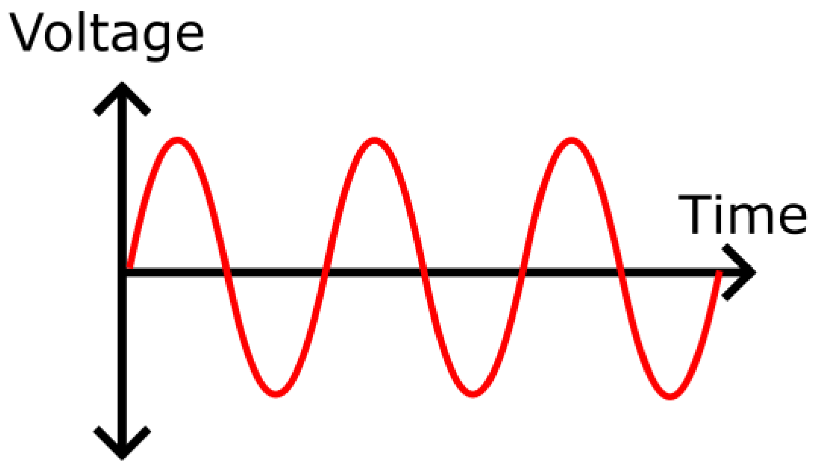
- In a frequency measurement it is how consistent the measurement is over time
  - If an oscillator measure 10.00 MHz one day and then 10.50 MHz the next, it is unstable
  - If an oscillator consistently measures 10.00 MHz every day, it is stable

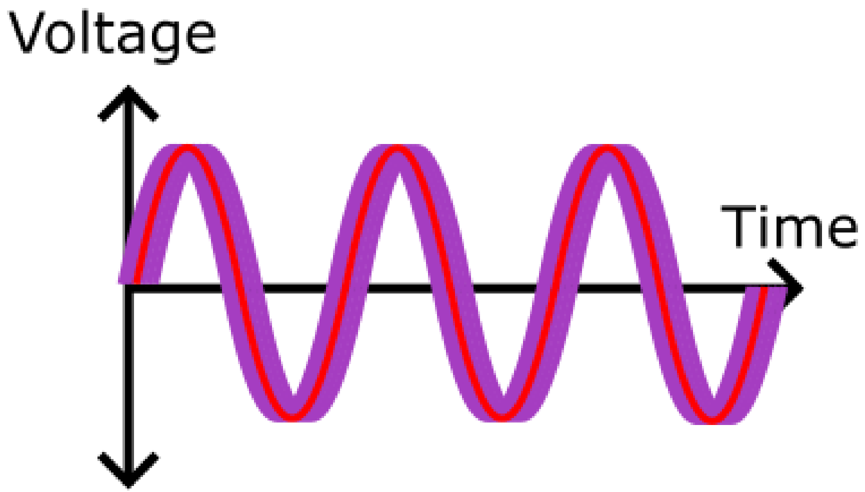
Dart Board Example



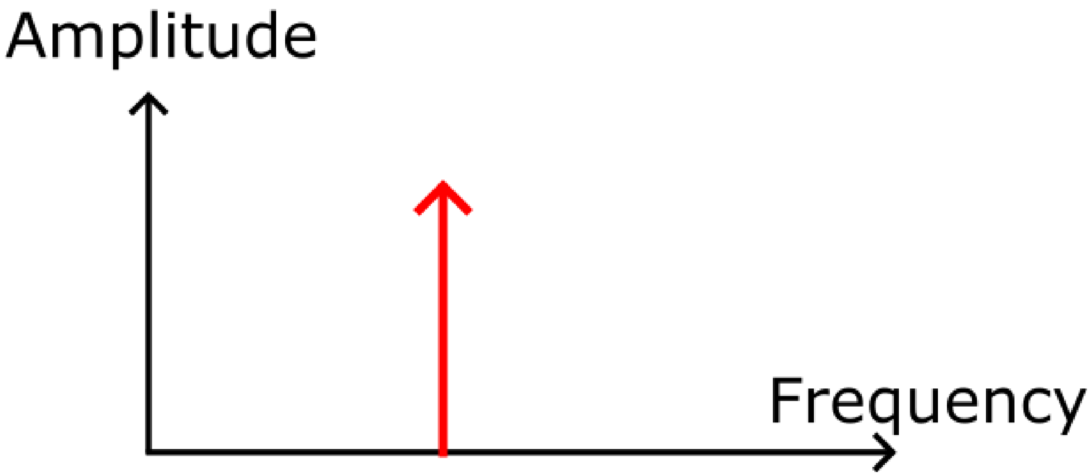
Instability of Oscillators

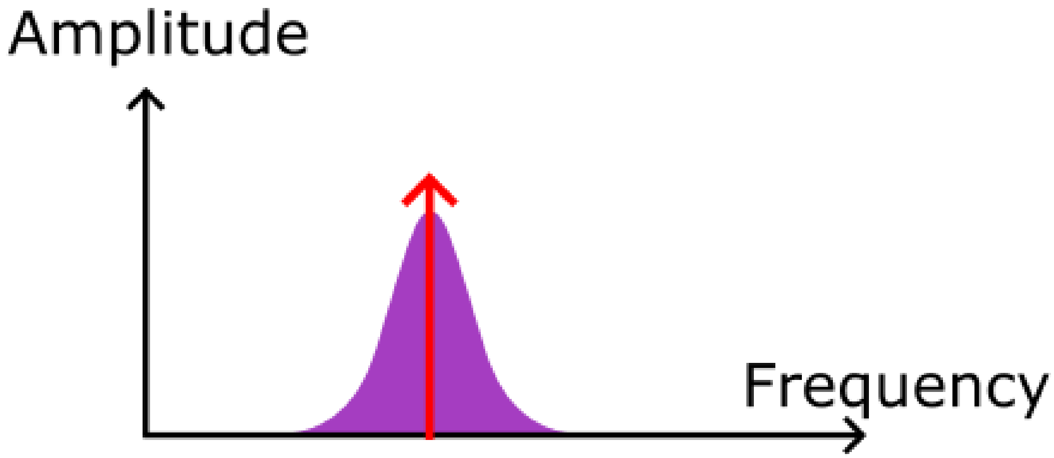
Time Domain





Frequency Domain

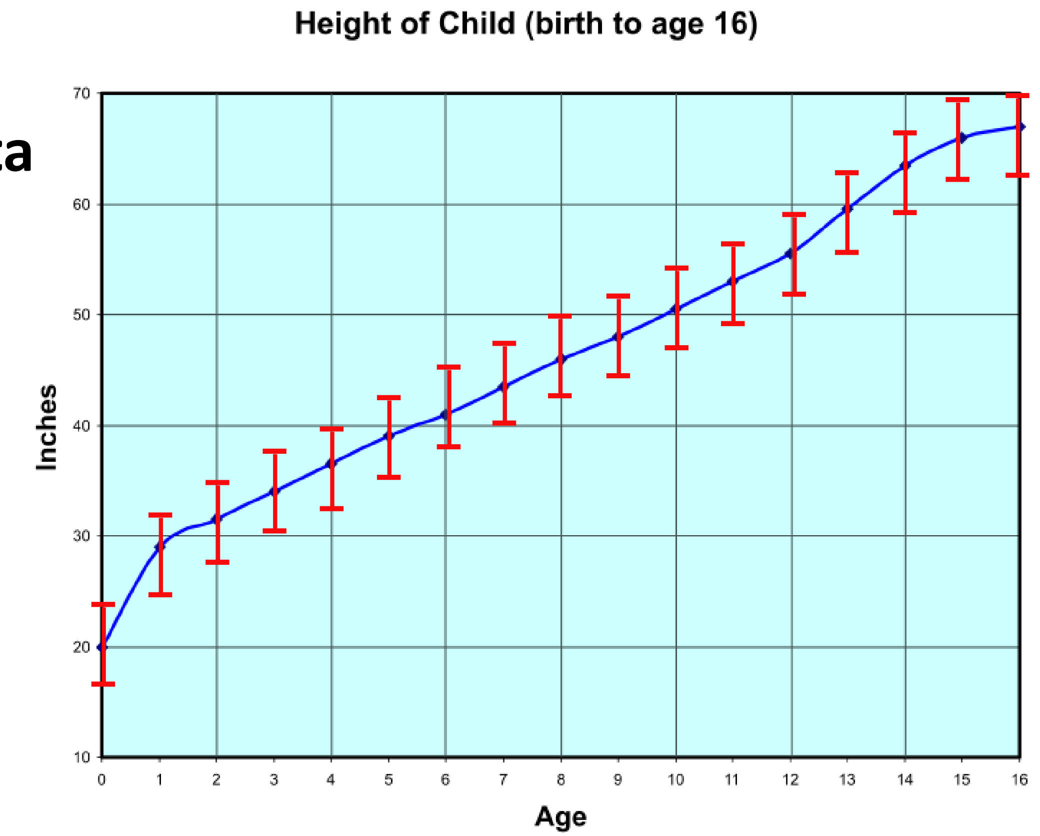


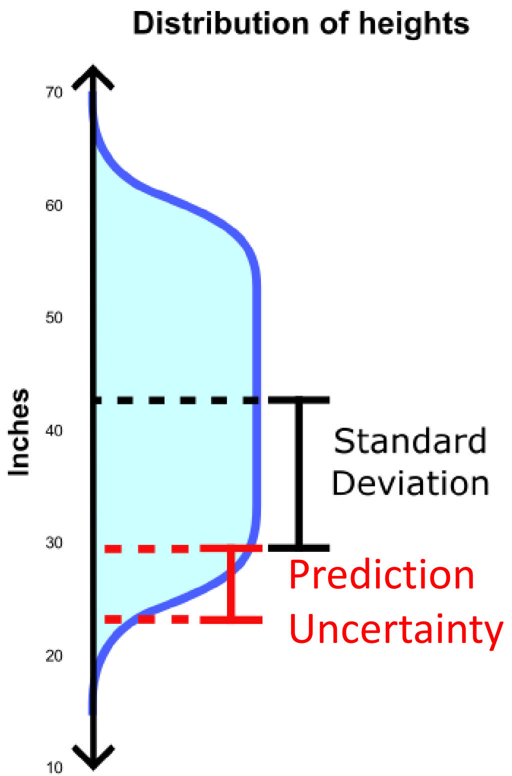


Why Not Use Standard Deviation

Standard deviation doesn't work with time-varying data

- It can still be calculated but is incomplete
- We have an additional piece of information: what time it is





Standard Deviation: 15"

Uncertainty in Height Projection: < 5"

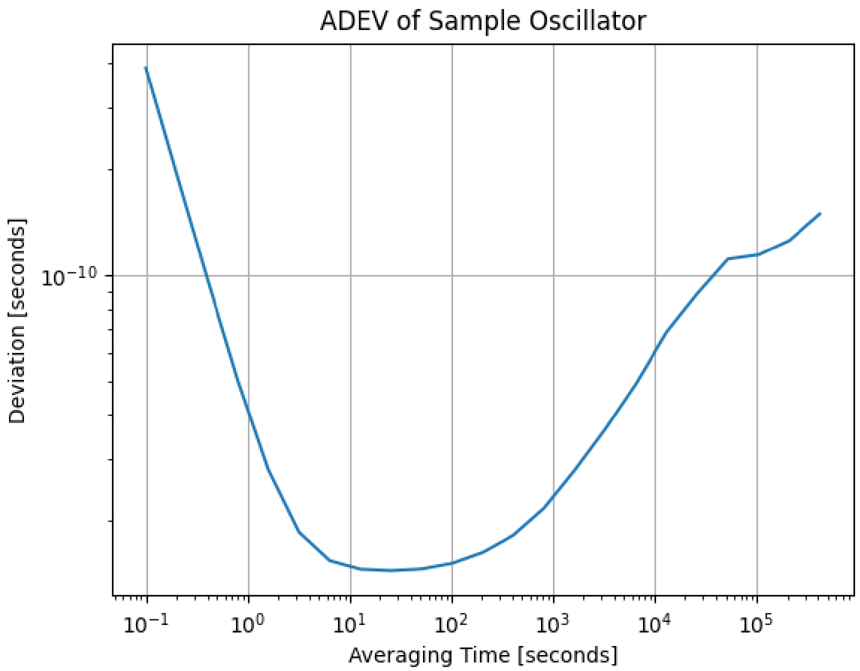
Allan Deviation (ADEV)

Standard deviation

- Estimates difference from the mean
- Useful for time-independent datasets
- Only a single value

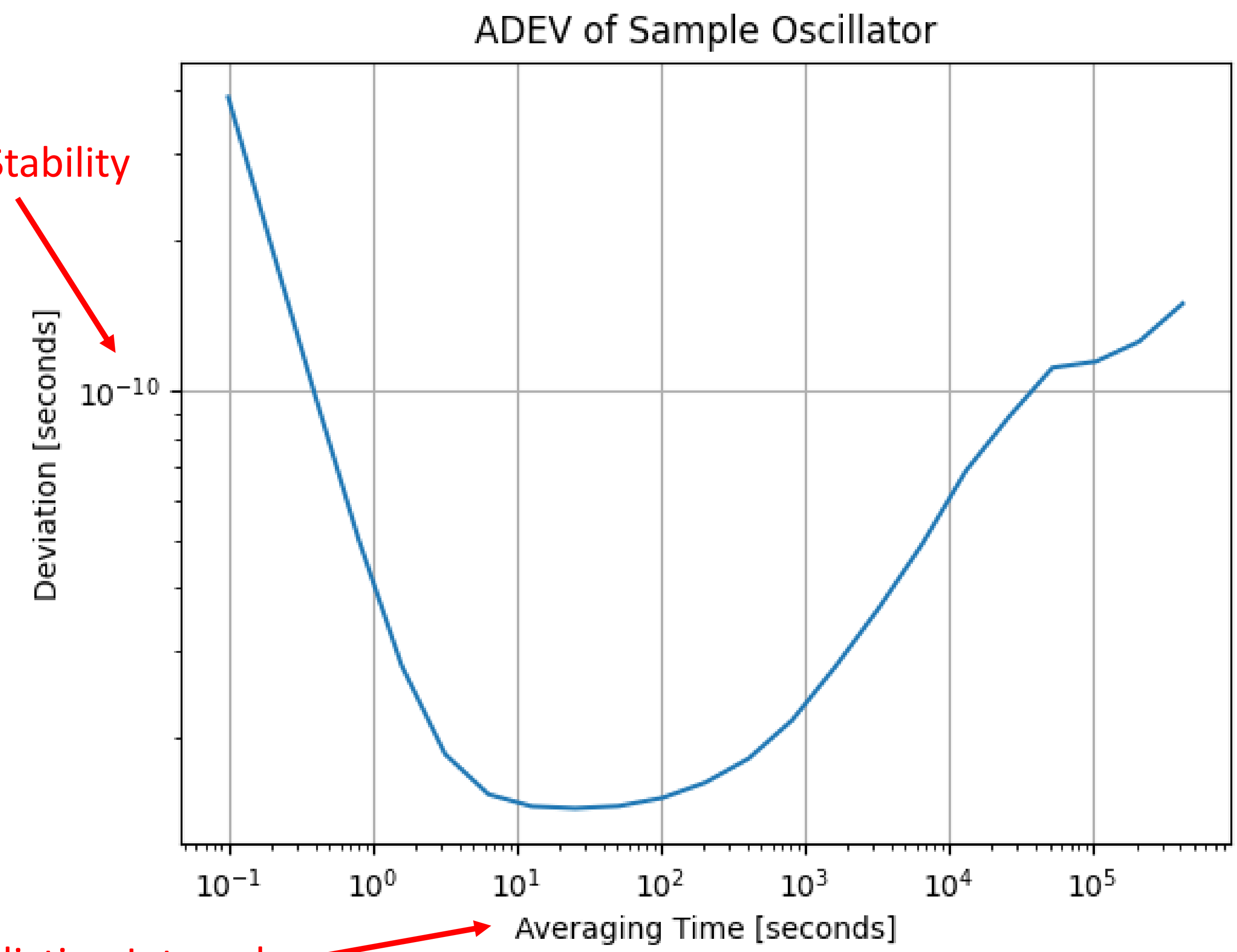
Allan deviation

- Estimates difference from point to point
- Dependent on sampling interval
- Typically plotted over multiple intervals on a log-log plot



How to Read an ADEV Plot

Fractional Frequency Stability

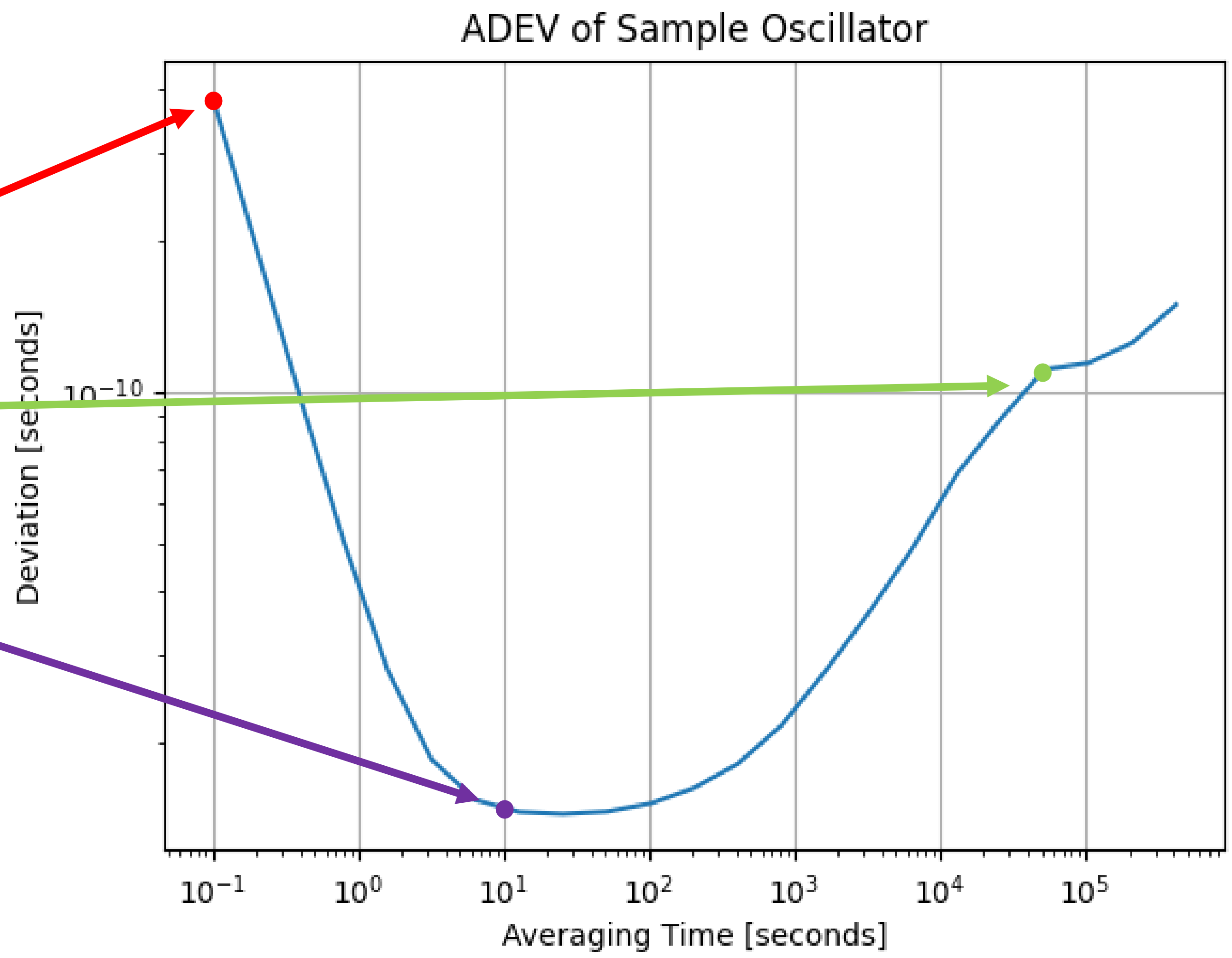


Prediction Interval

How to Read an ADEV Plot (cont.)

What does it all mean?

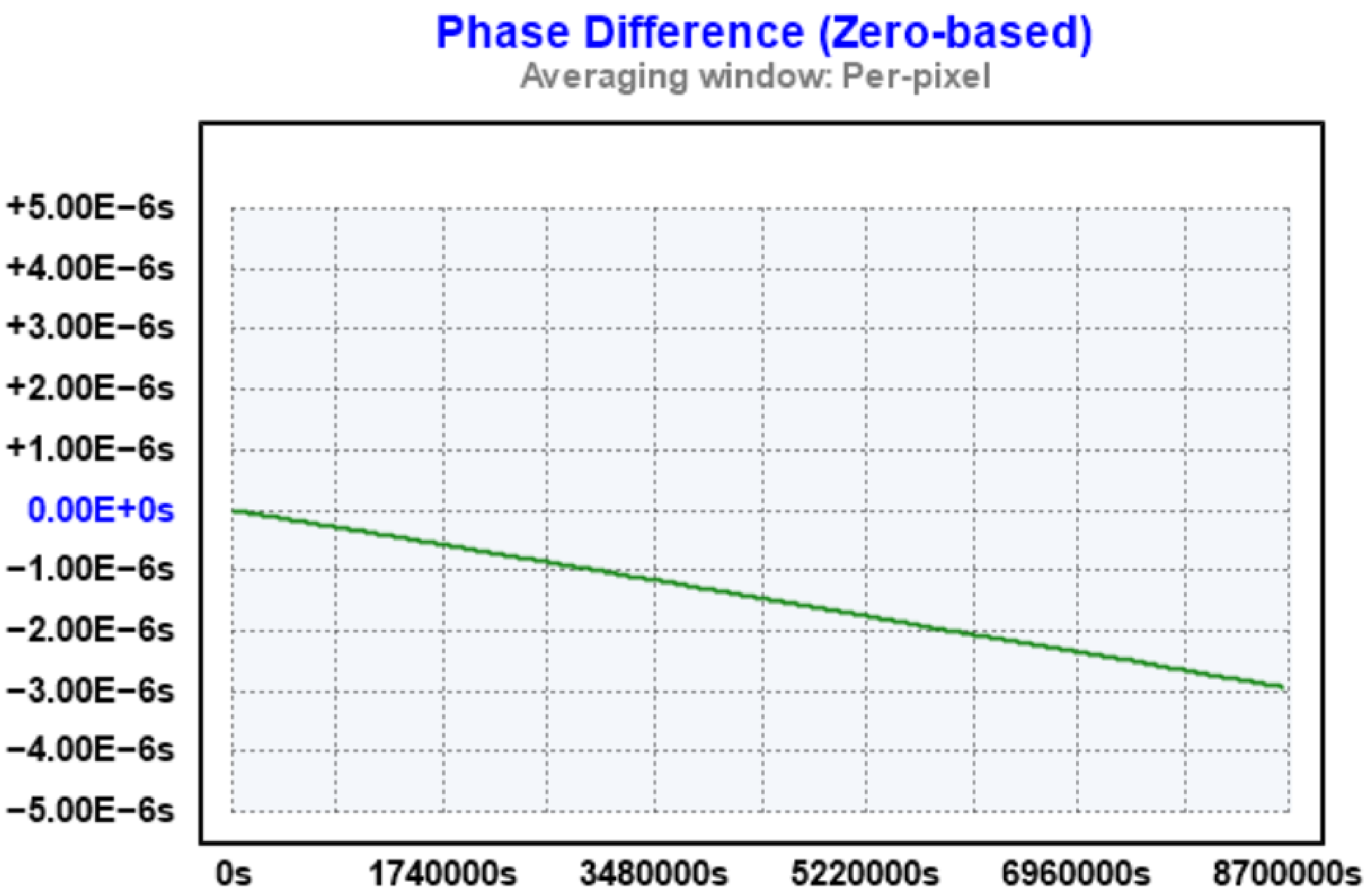
- After 100 ms, the fractional frequency will deviate by $\sim 4 \times 10^{-10}$
- After 1 day, the fractional frequency will deviate by $\sim 1 \times 10^{-10}$
- After 10 s, the fractional frequency will deviate by $\sim 1.5 \times 10^{-11}$



Measuring Frequency Offset and Instability

Oscillator Phase Data

- An oscillator's instability and frequency offset can be adequately analyzed using a time series of time offsets
- A static frequency offset show up as a linear trend



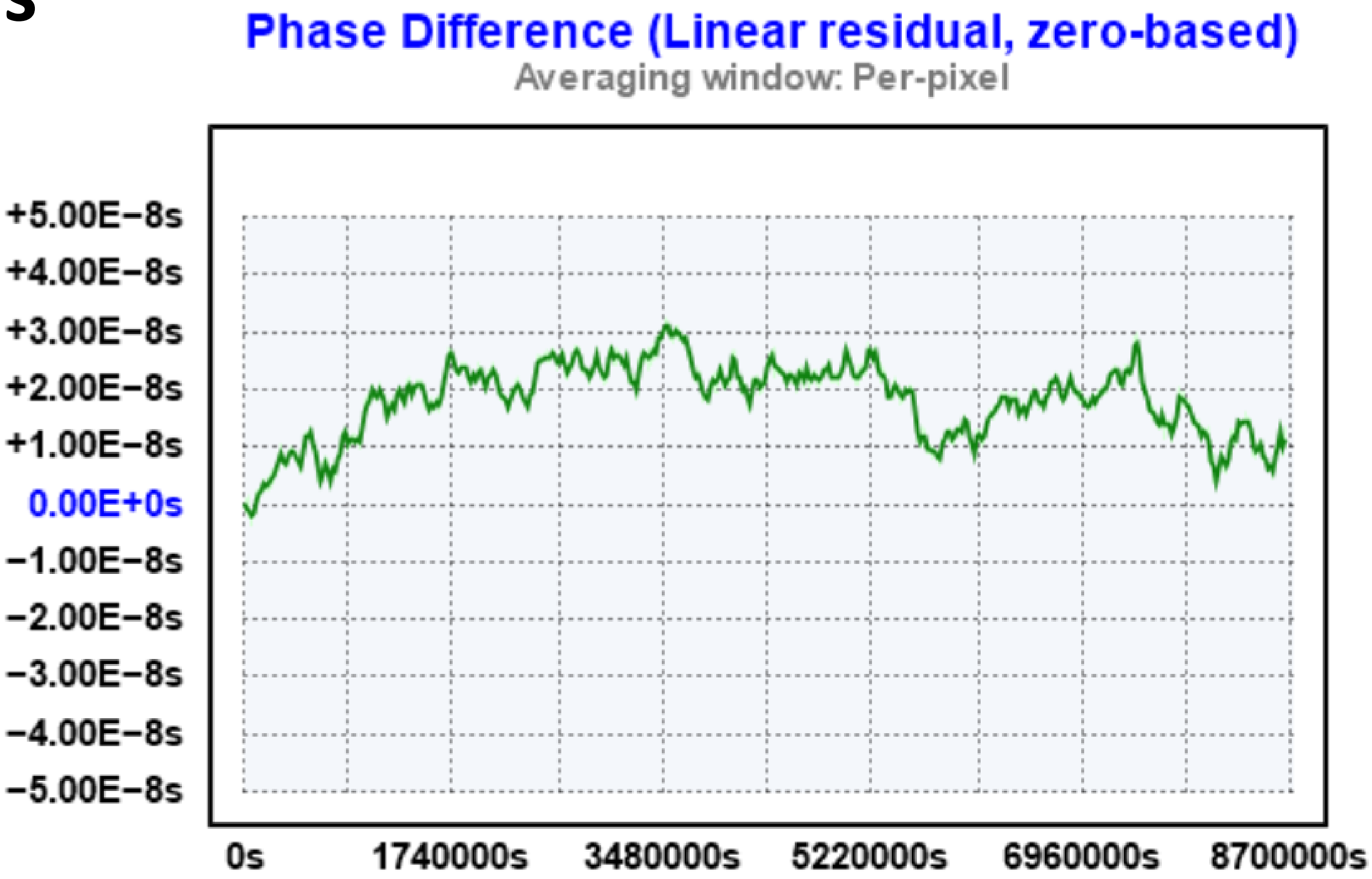
Phase record of a cesium beam tube clock output

Measuring Frequency Offset and Instability

Removing Linear Phase Trends

- Removing the best linear fit from the data shows the instability of the oscillator more clearly

- Linear phase trend represents a static frequency offset, so predictability is not impacted

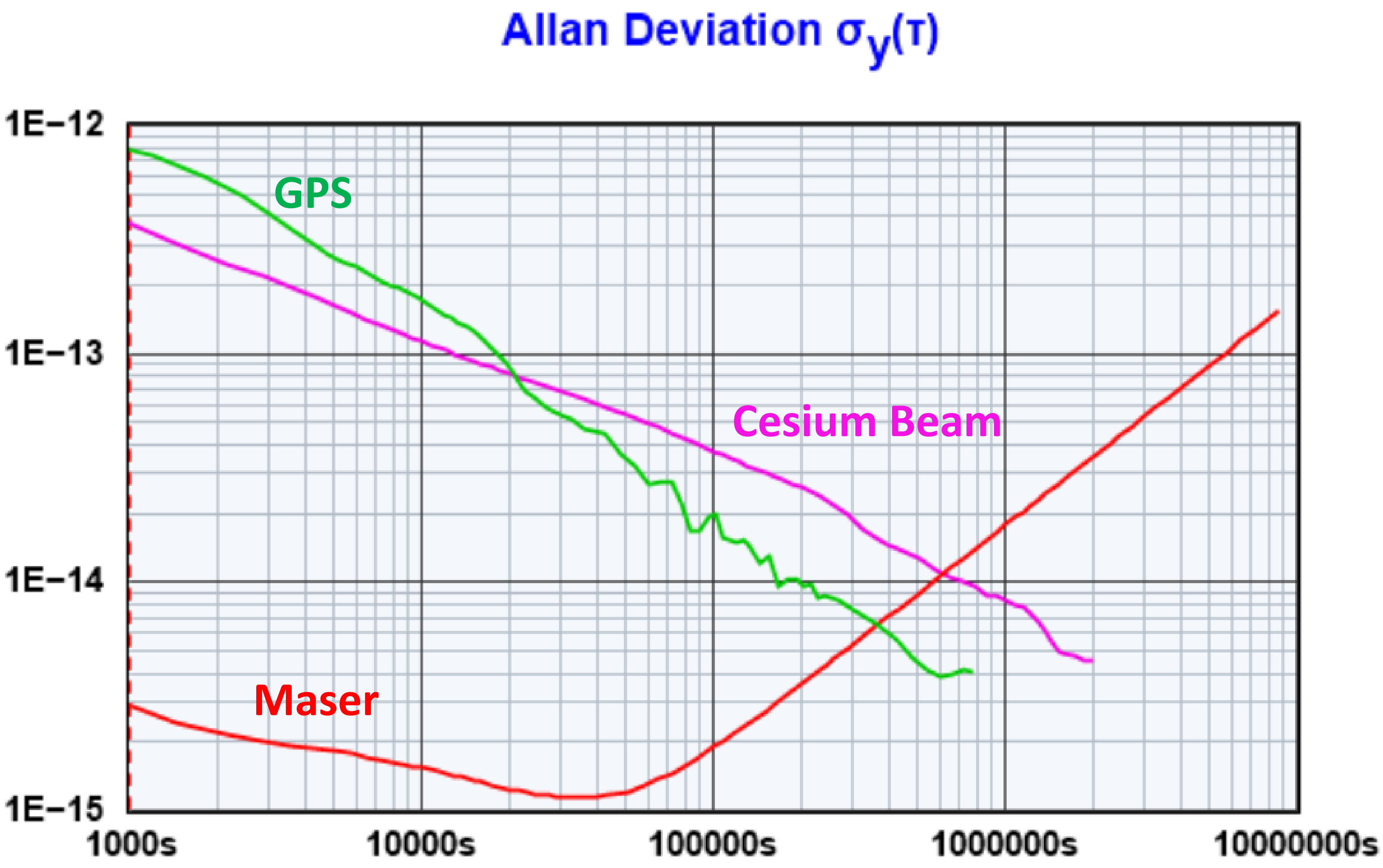


Phase record of a cesium beam tube clock output

Stability of Typical Clocks

Clock Types

- Cesium beam tubes have good short-term performance and excellent long-term performance
- Hydrogen masers have excellent short-term performance but become dominated by drift in the long-term
- GPS has poor short-term performance and approaches the stability of UTC in the long-term

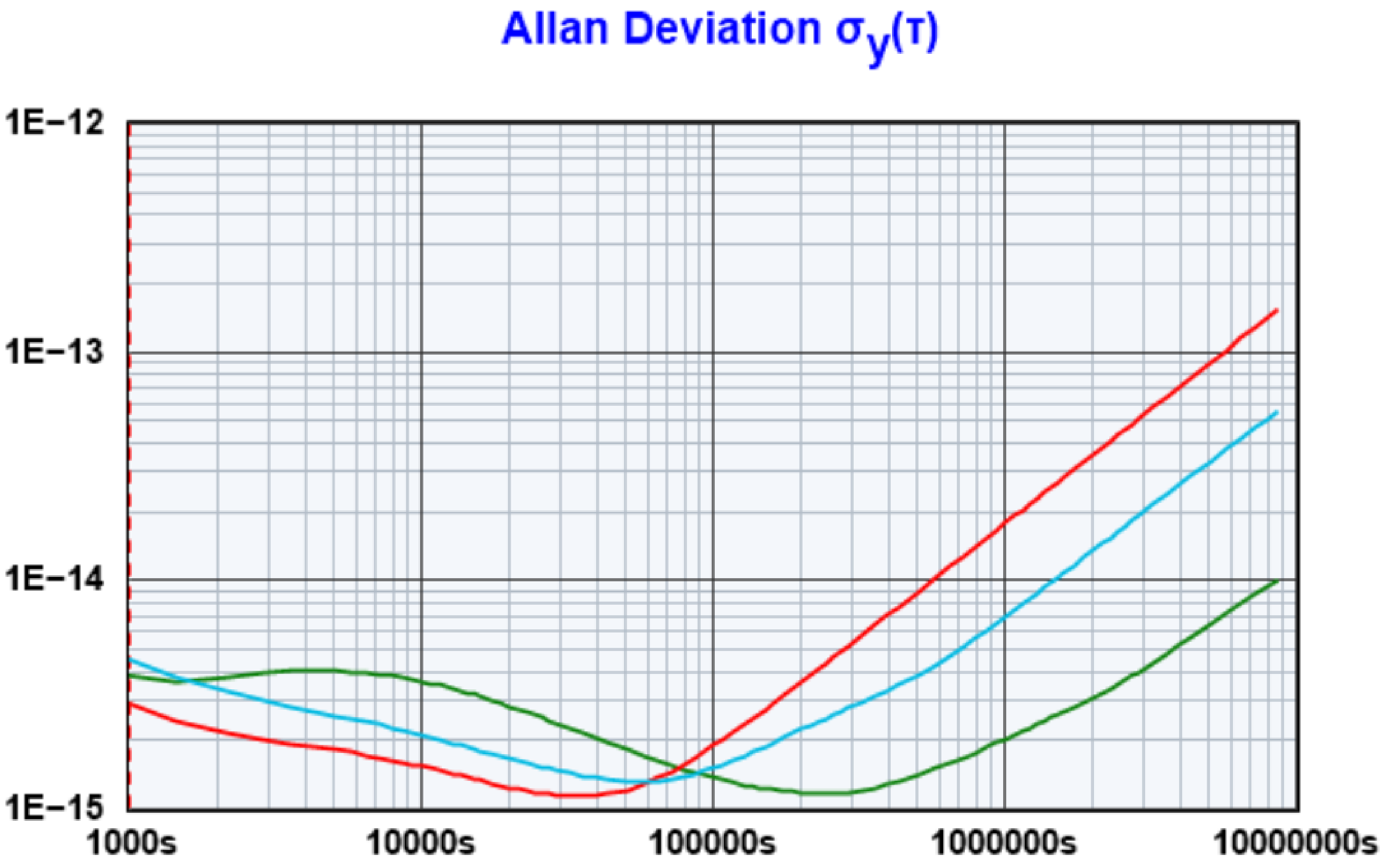


Allan Deviations of various frequency references

Stability of Typical Clocks

Stability Impacts

- Stability is a property of an oscillator in particular
- Even clocks of the same model can differ in stability significantly
- Stability is also impacted by environmental factors such as temperature, humidity, pressure, magnetic fields, and vibration, depending on the clock type

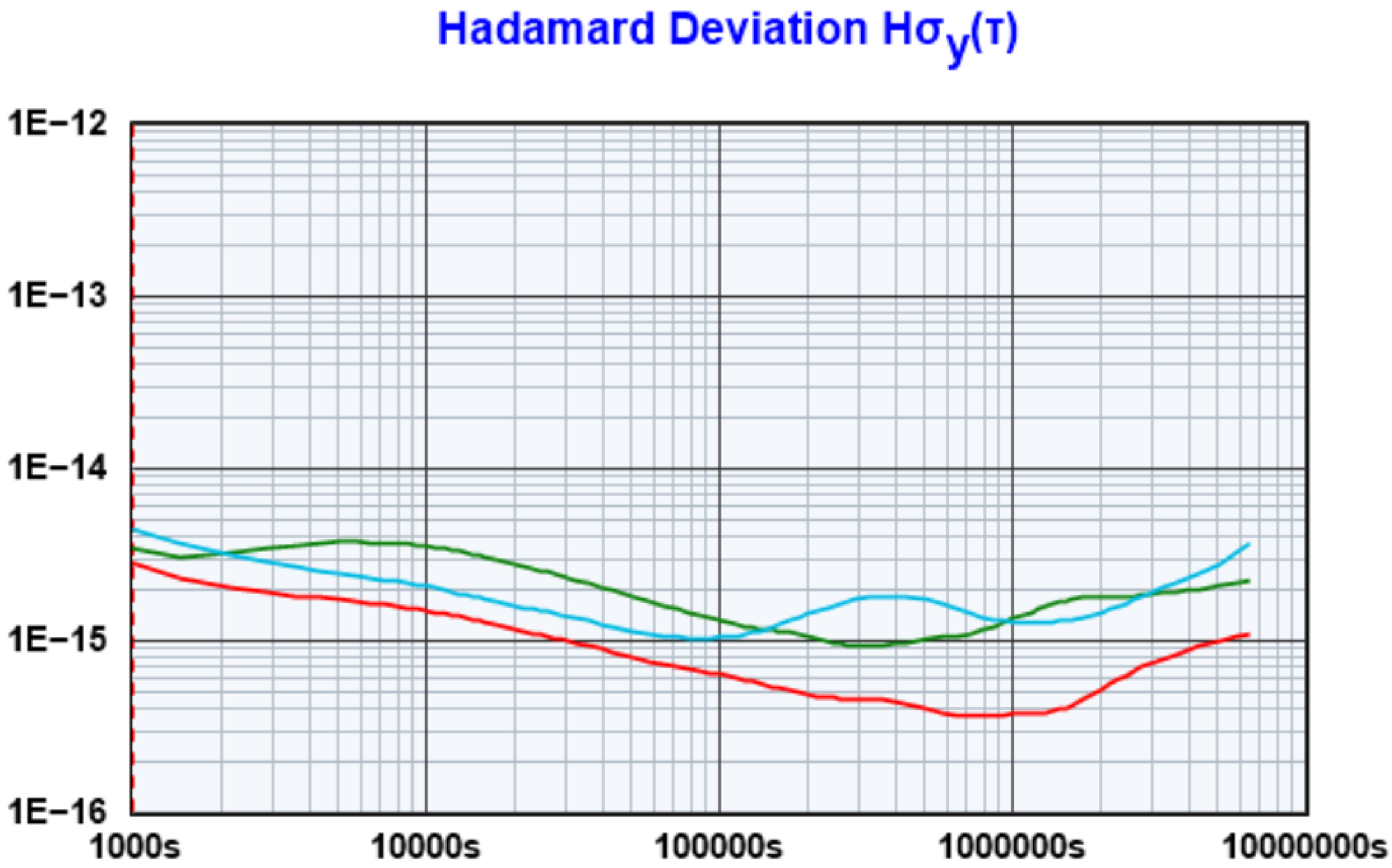


Stability of 3 hydrogen masers of the same model

Stability of Typical Clocks

Linear Drift

- Linear frequency drift is easy to measure when comparing clocks to a timescale or UTC
- Hadamard deviation approximately removes linear drifts from ADEV
- Masers have reasonably predictable drift for approximately <1 month

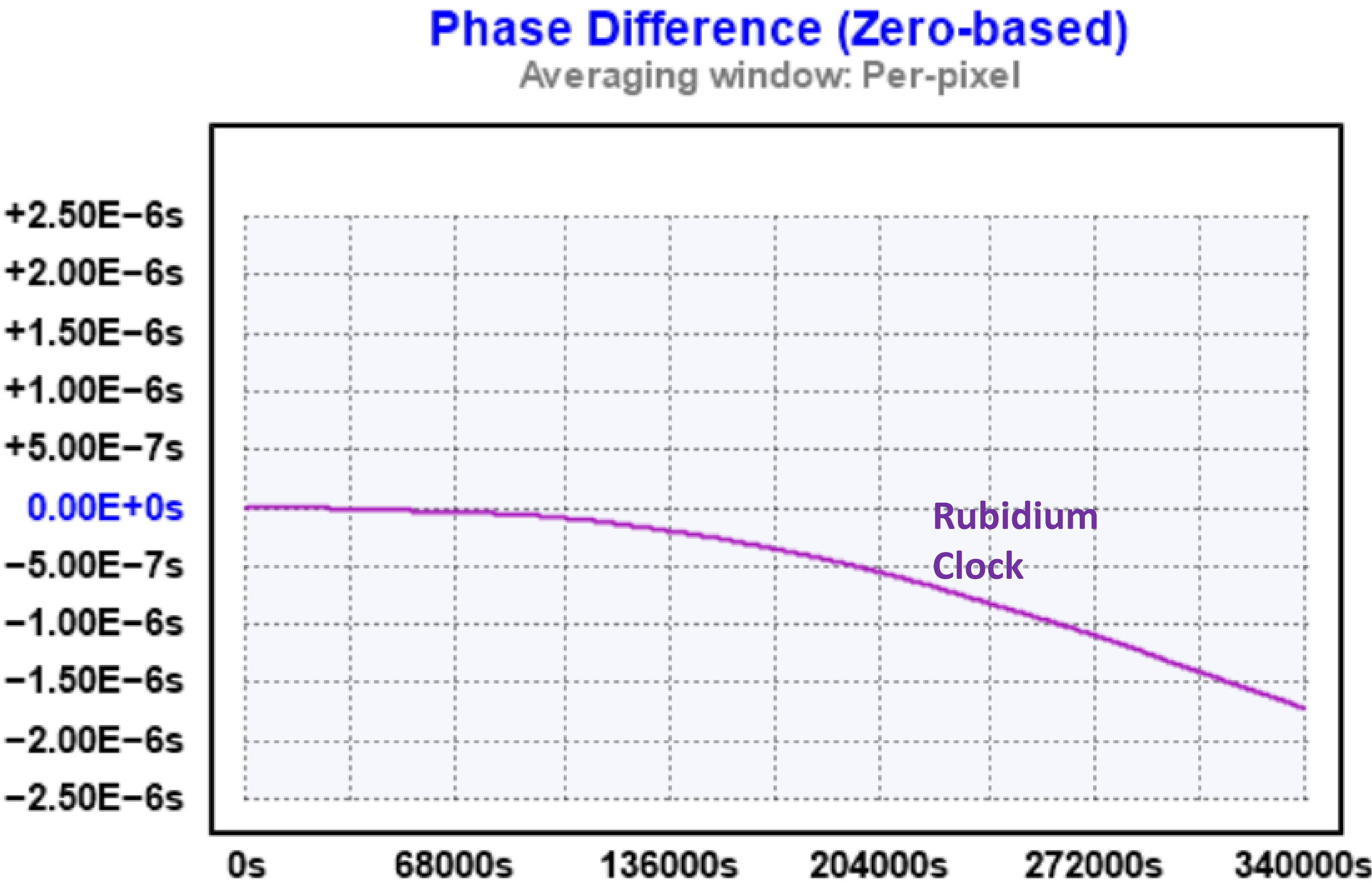


Stability of 3 hydrogen masers of the same model, drift removed

Correcting Oscillator Errors

Need for corrections

- Understanding the error in a clock only goes so far
- A physical reference signal as close to correct as possible is very important

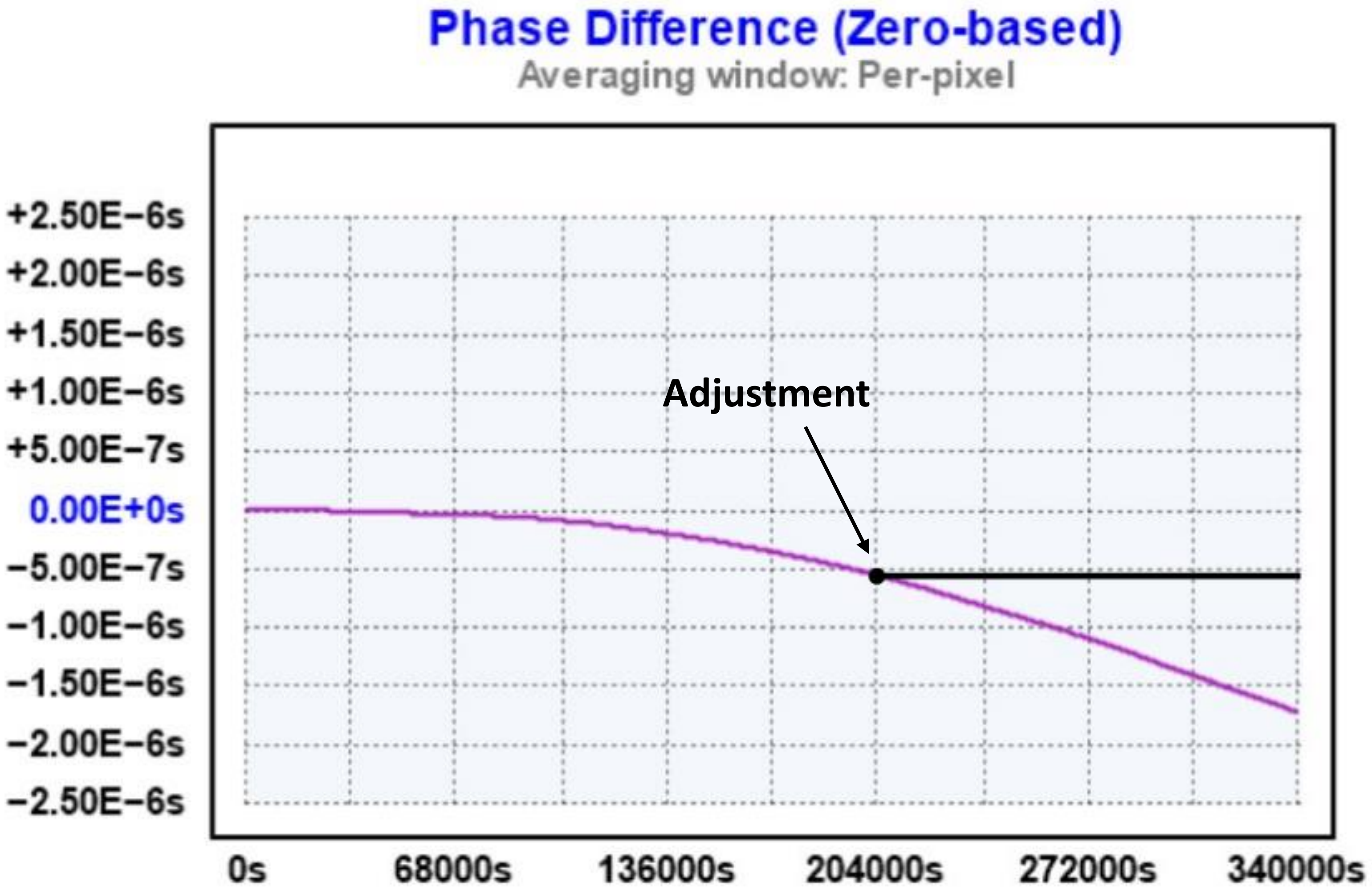


Without corrections, even a fairly stable frequency reference will accumulate large phase offsets

Correcting Oscillator Errors

Simple Steering

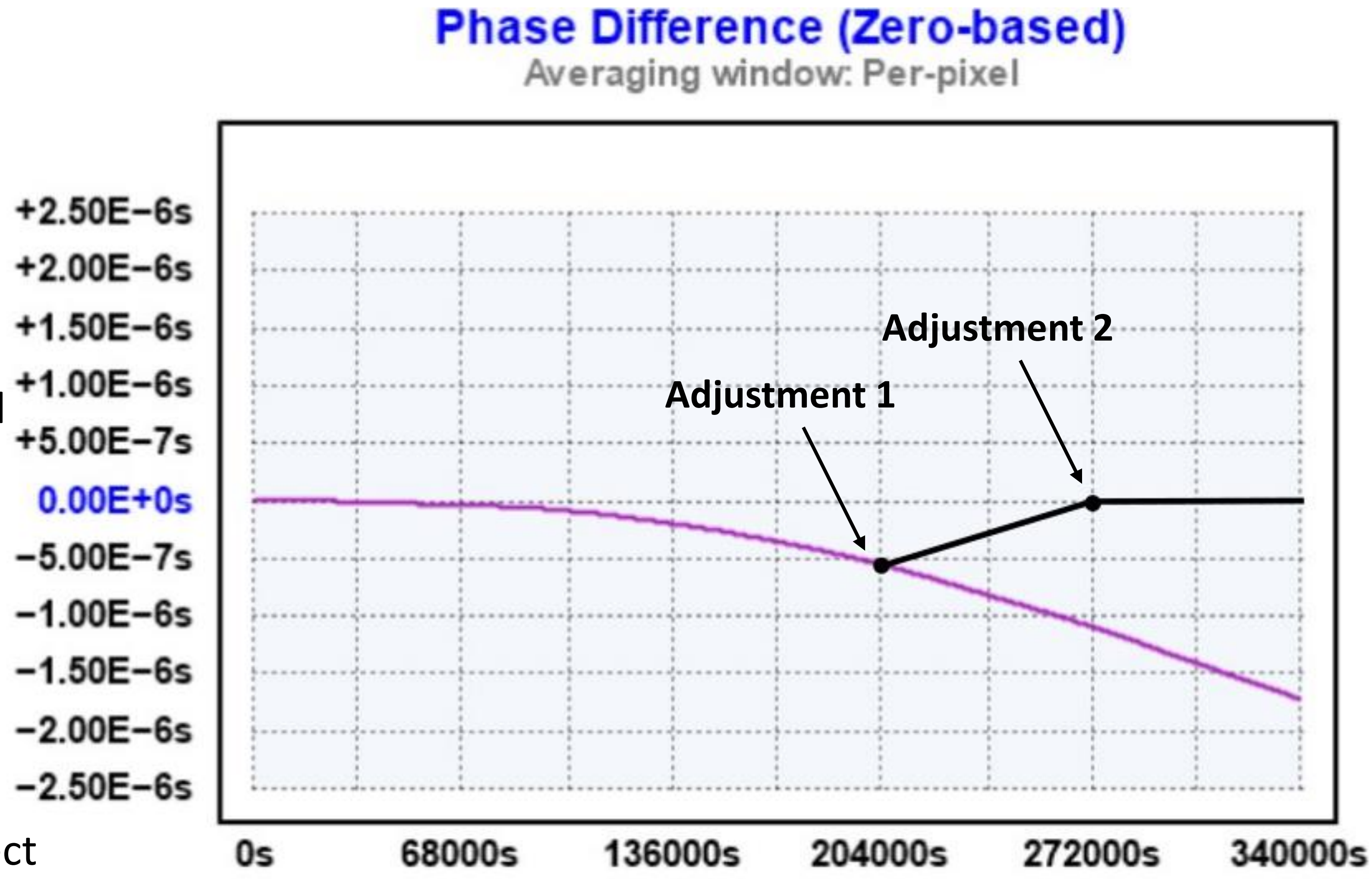
- By far the simplest method to correct a clock is to manually set its frequency in an effort to correct the error
- Just correcting the frequency will leave the phase accumulation
- This may be all that is needed depended on the system requirements



Correcting Oscillator Errors

Simple Steering (cont)

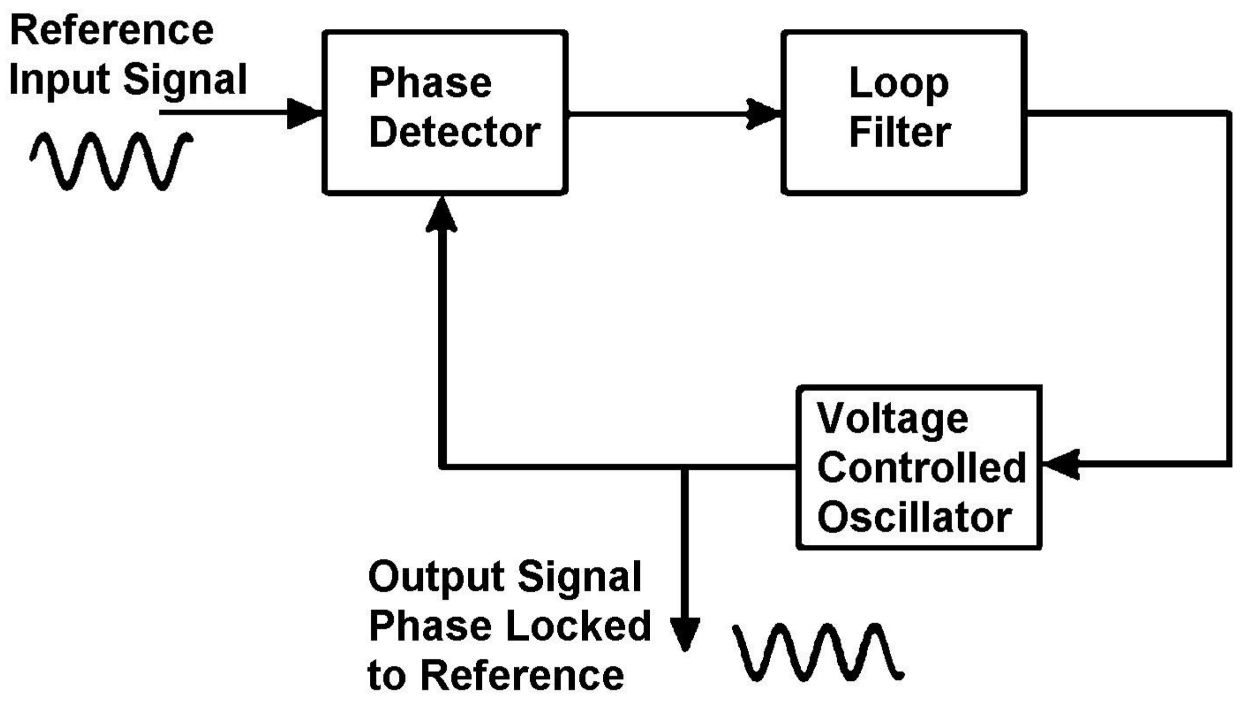
- A simple way to achieve the correct time is “Bang Bang” steering
- Effective enough that it is used by the GPS network for its in-space clocks
- Done by overcorrecting a frequency error to reduce the phase error over time
- After phase error is zero, correct frequency



Correcting Oscillator Errors

Phase Locked Loop (PLL)

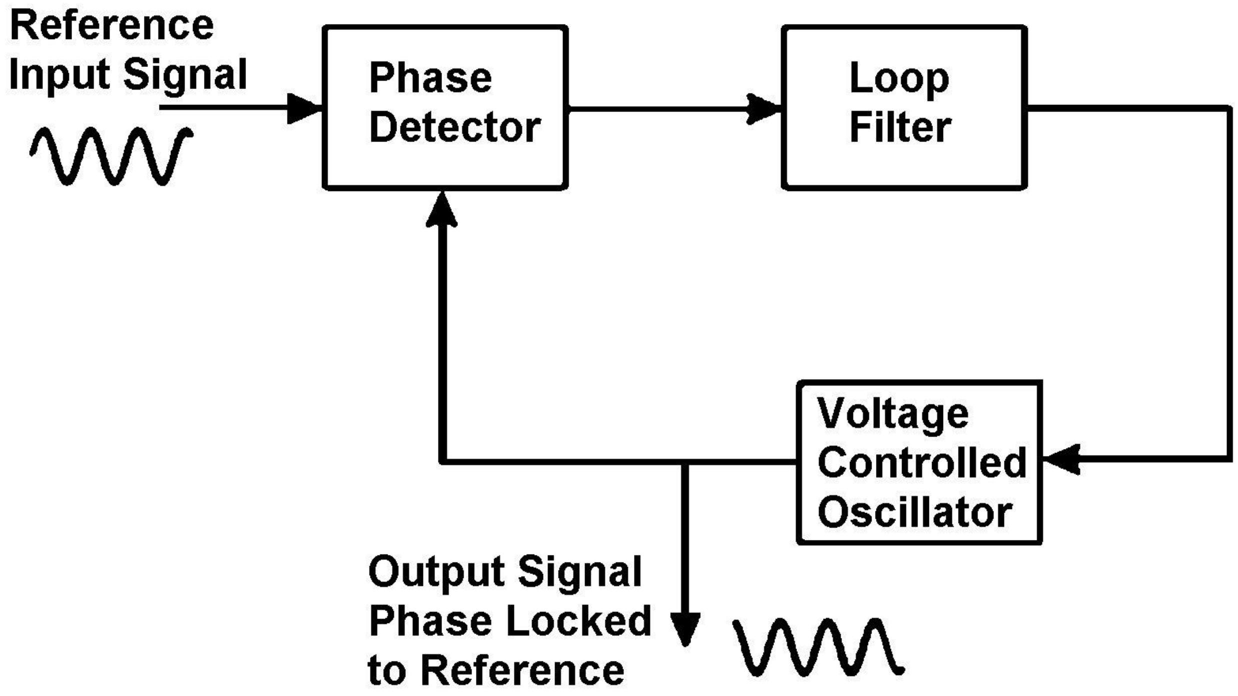
- An automated clock steering method is a PLL
- A PLL measures the difference between a reference signal and the output and drives the difference to zero
- Nearly all clock steering systems work in this way or similarly
- Allows for a combination of the qualities of multiple oscillators



Correcting Oscillator Errors

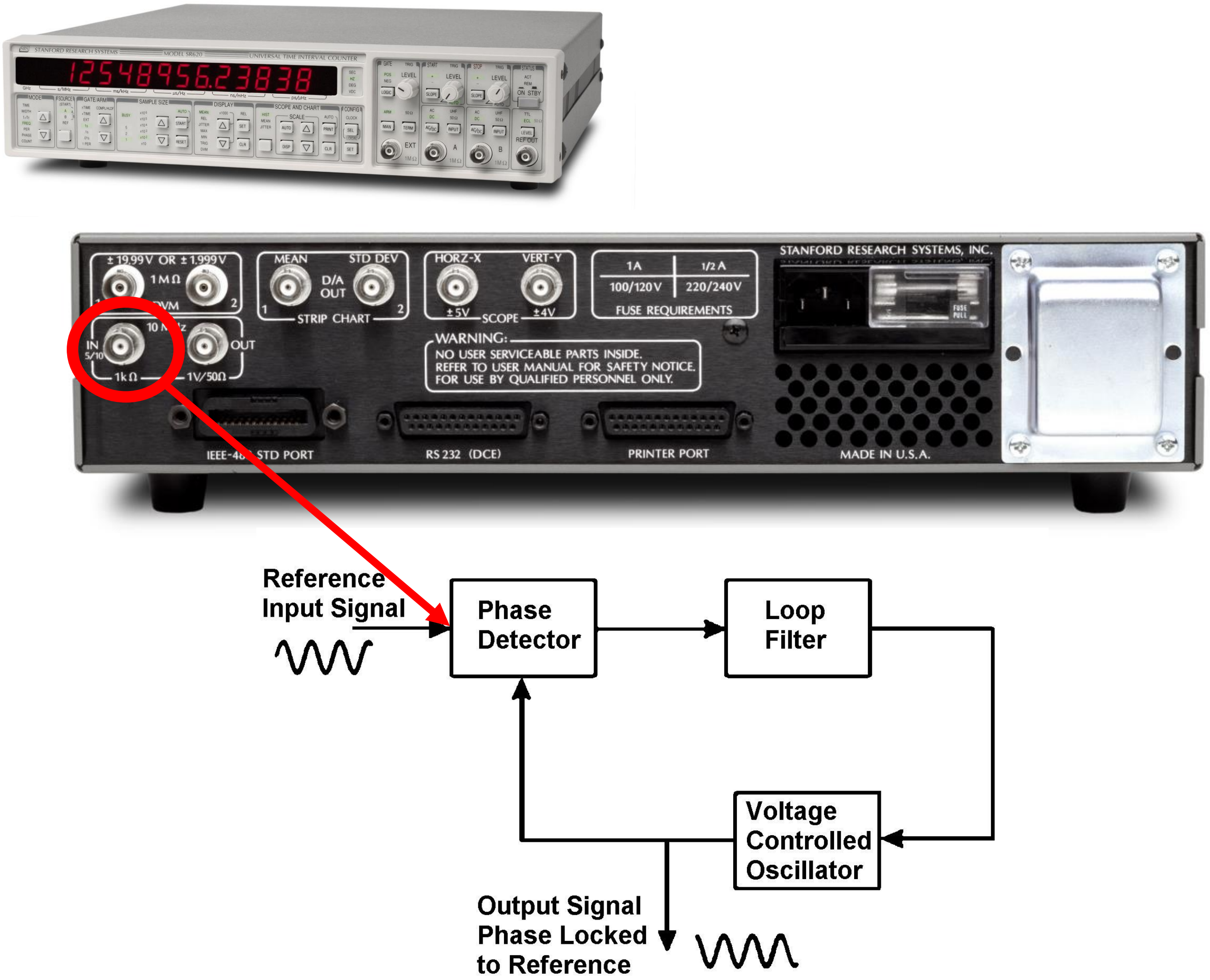
Downsides to the PLL

- Only works when the reference signal and the output are within a single cycle
- Does not have significant flexibility to correct for other errors
- Can be unstable if implemented incorrectly



PLL Use Cases

- PLLs are widely used in laboratory equipment and other devices
- PLLs can be used to “clean up” a signal provided into a device
- External signals may not have the correct amplitude, waveform, or frequency and a PLL solves these issues



Counter internal electronics

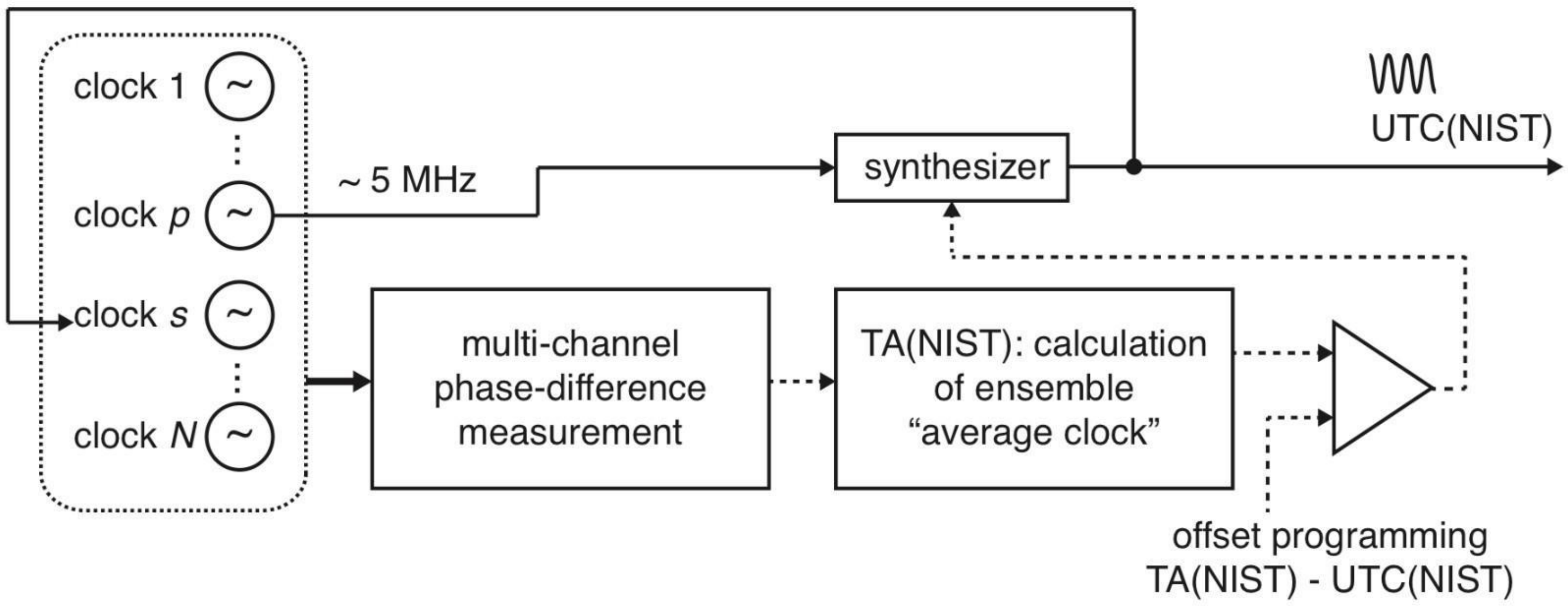
Correcting Oscillator Errors

An Auxilariary Output Generator (AOG) used in the NIST Timescale

Frequency Re-Synthesis

- Rather than controlling the oscillator itself, a generated signal can be controlled
- Generally, this process is called frequency synthesis
- Standard benchtop frequency synthesizers are insufficient
- Microsteppers, like the one shown on the left are built with timekeeping precision in mind

Correcting Oscillator Errors



A simplified diagram of the NIST timescale

Timescales

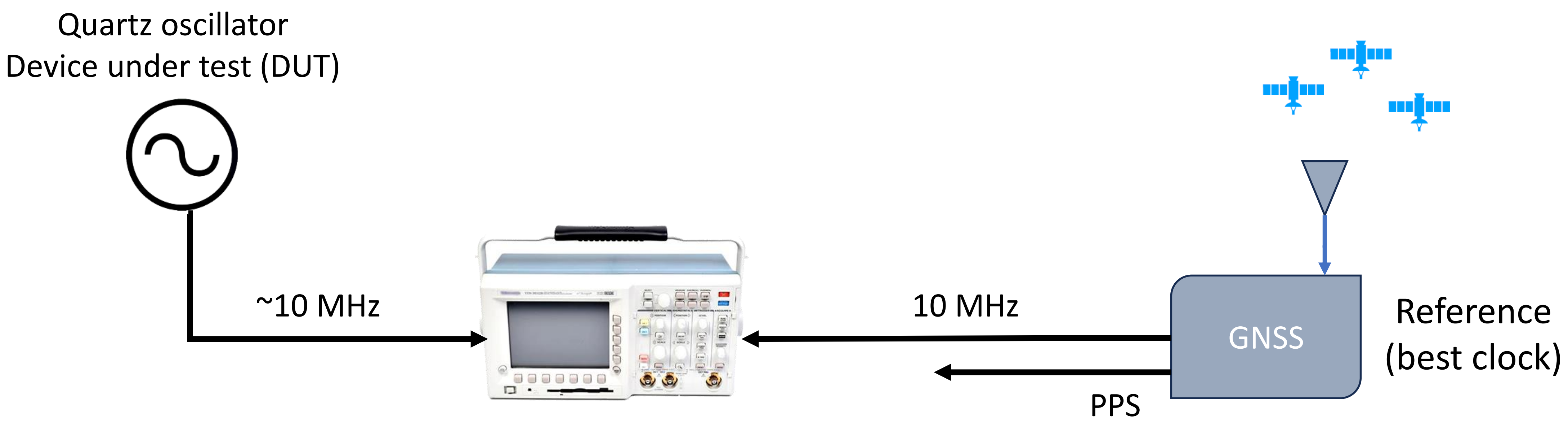
- Using a synthesizer / microstepper allows the combination of multiple clocks with more ease than steering directly
- Opens up more opportunities for alternative architectures

Practical Demo

Equipment Needed:

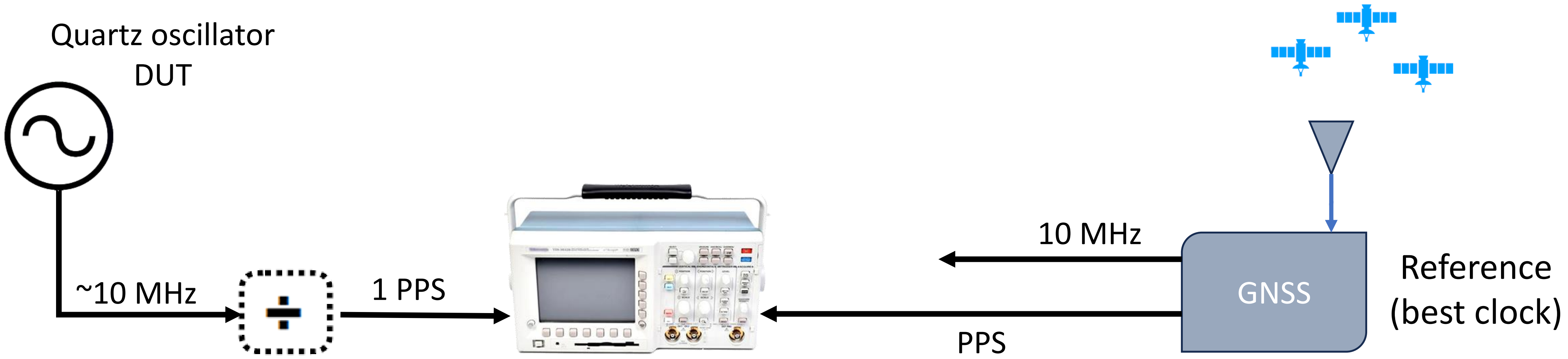
- **A frequency reference:**
  - e.g., GNSS timing receiver
- **A clock to test:**
  - e.g., variable quartz oscillator
- **A measurement device:**
  - e.g., SR620 Universal Counter
- **A frequency synthesizer:**
  - .e.g., 33120a
- **Cabling to connect equipment**
  - BNC or SMA cables

Measurement Demo - Oscilloscope



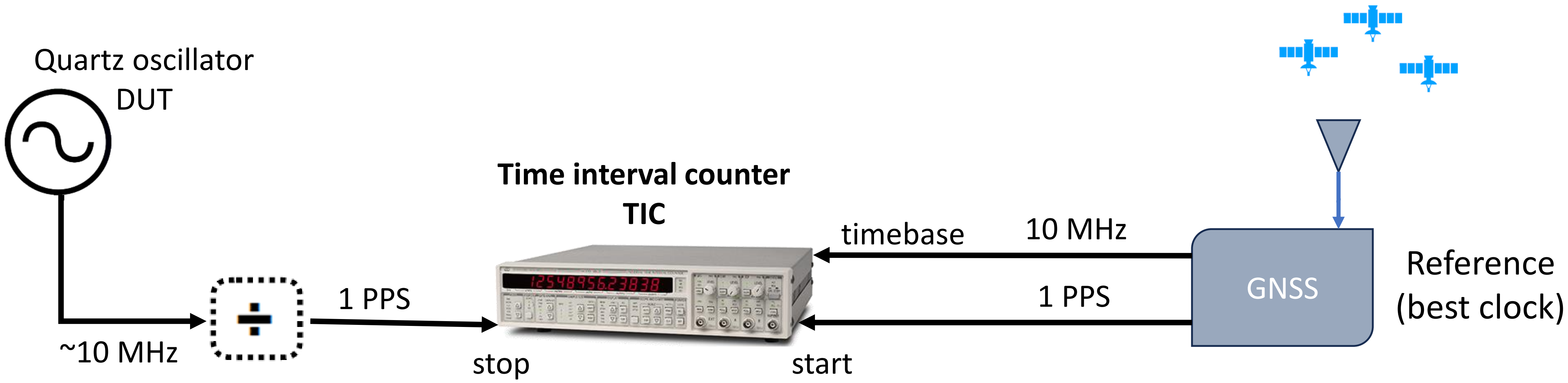
Compare DUT and reference in ***frequency*** with an oscilloscope

Measurement Demo - Oscilloscope



Compare DUT and reference in ***time*** with an oscilloscope

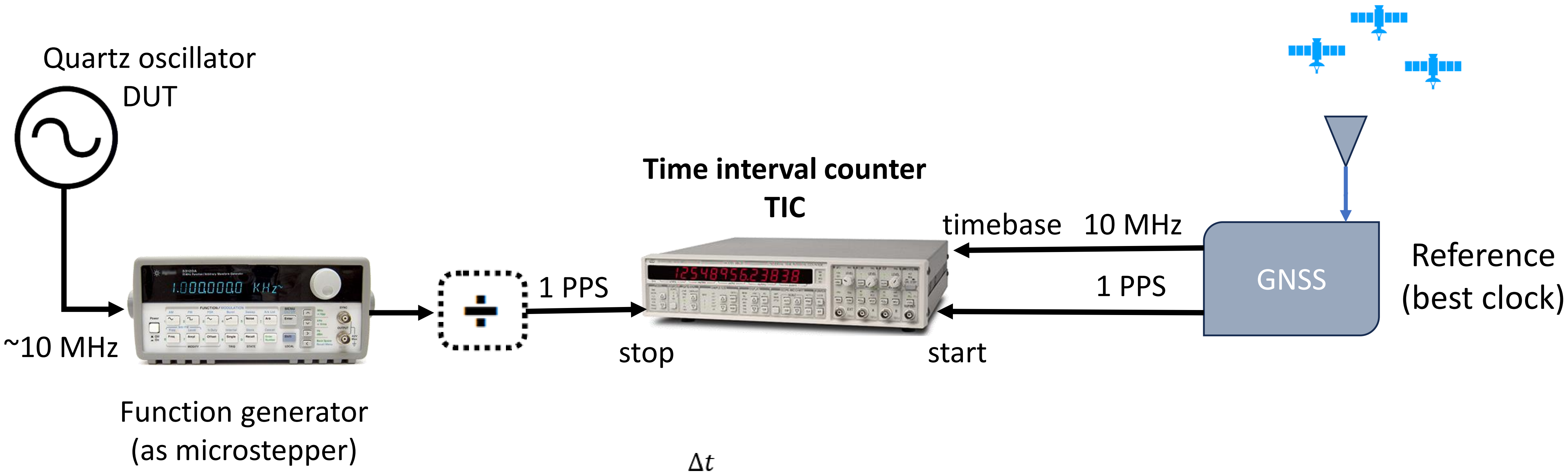
Measurement Demo – Time interval counter



Compare DUT and reference in ***time*** with a TIC

Measurement Demo – Microstepper

Compare DUT and reference in ***time***, steer in ***frequency*** using function generator as microstepper



1) Measure $\frac{\Delta t}{T}$
2) Calculate fractional time offset and fractional frequency offset
3) Calculate frequency offset and enter into function generator
4) Time permitting: steer frequency to steer the time offset towards zero

Questions?

Additional Information/ Backup Slides

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Definitions of Time and Frequency

The Fourier Transform

- Frequency is formally related to time through the Fourier Transform
- Provides knowledge of multiple frequencies contained in one signal

Formally:

$$x(f) = \int_{-\infty}^{\infty} x(t) e^{-i2\pi f t} dt$$

ADEV Formal Definition

Standard Variance

$$\sigma^2 = \frac{1}{M-1}\sum_{k=1}^{M}[x_k - \bar{x}]^2$$

Allan Variance

$$\sigma_y^2(\tau) = \frac{1}{2(M-1)}\sum_{k=1}^{M-1}[\bar{y}_{k+1} - \bar{y}_k]^2$$

Standard Deviation

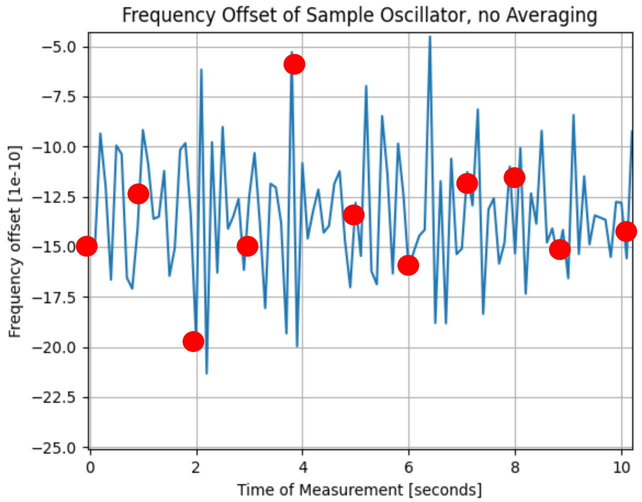
$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{1}{M-1}\sum_{k=1}^{M}[x_k - \bar{x}]^2}$$

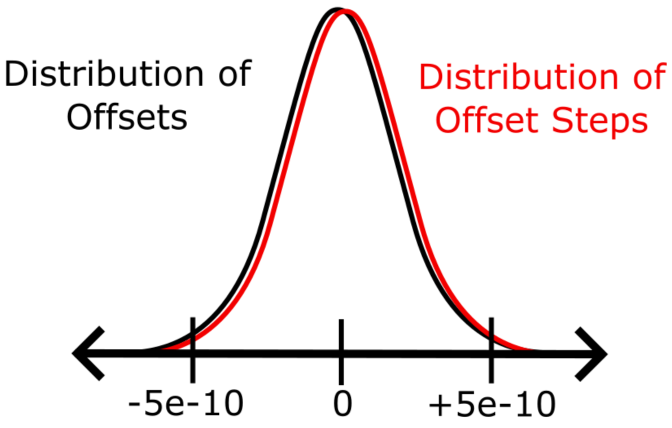
Allan Deviation

$$\sigma_y(\tau) = \sqrt{\sigma_y^2(\tau)} = \sqrt{\frac{1}{2(M-1)}\sum_{k=1}^{M-1}[\bar{y}_{k+1} - \bar{y}_k]^2}$$

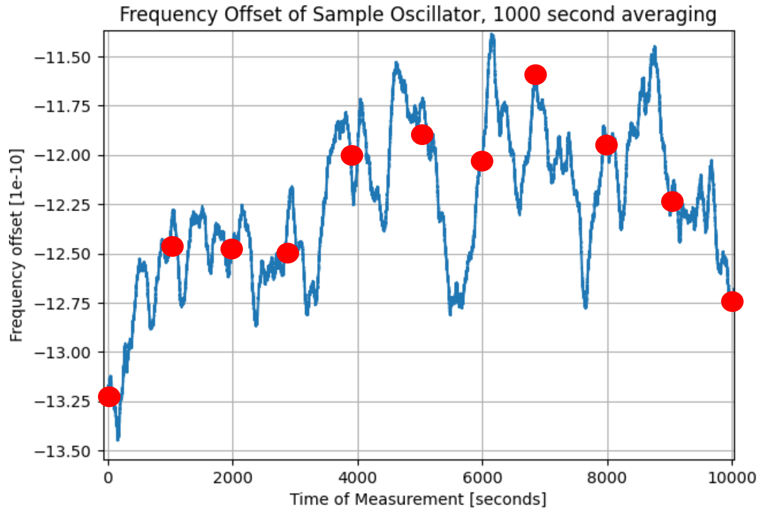
How ADEV Estimates Frequency Stability

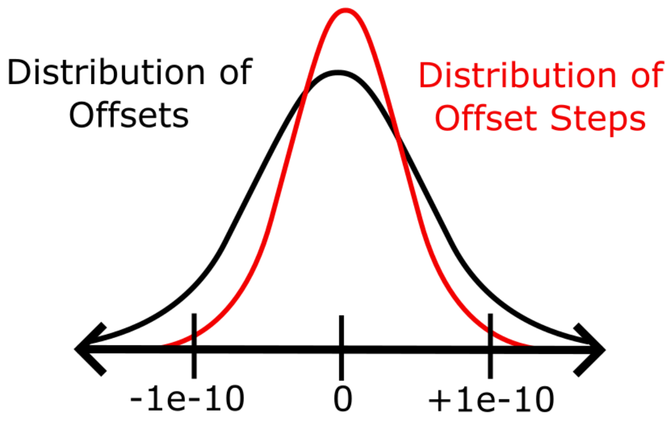
1 second interval DO AVERAGES AT 1s



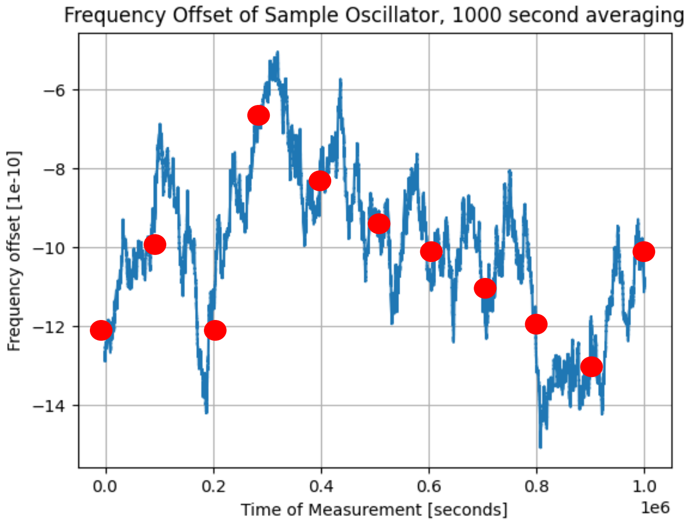


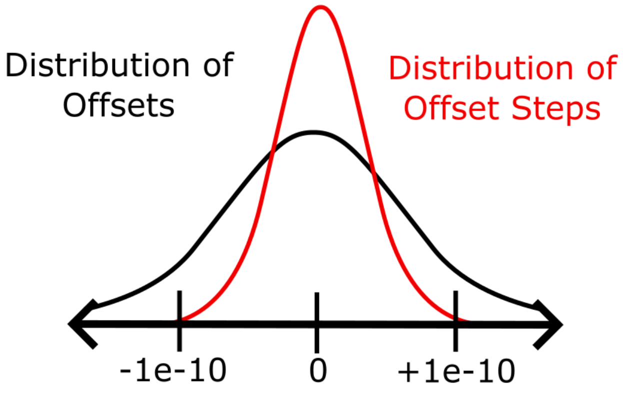
1,000 second interval





10,000 second interval





Why Does it Form that Shape?

Noise Types

- Noise comes in different types, colors, or characters
- Often described by their frequency composition
  - White noise: averages down over time

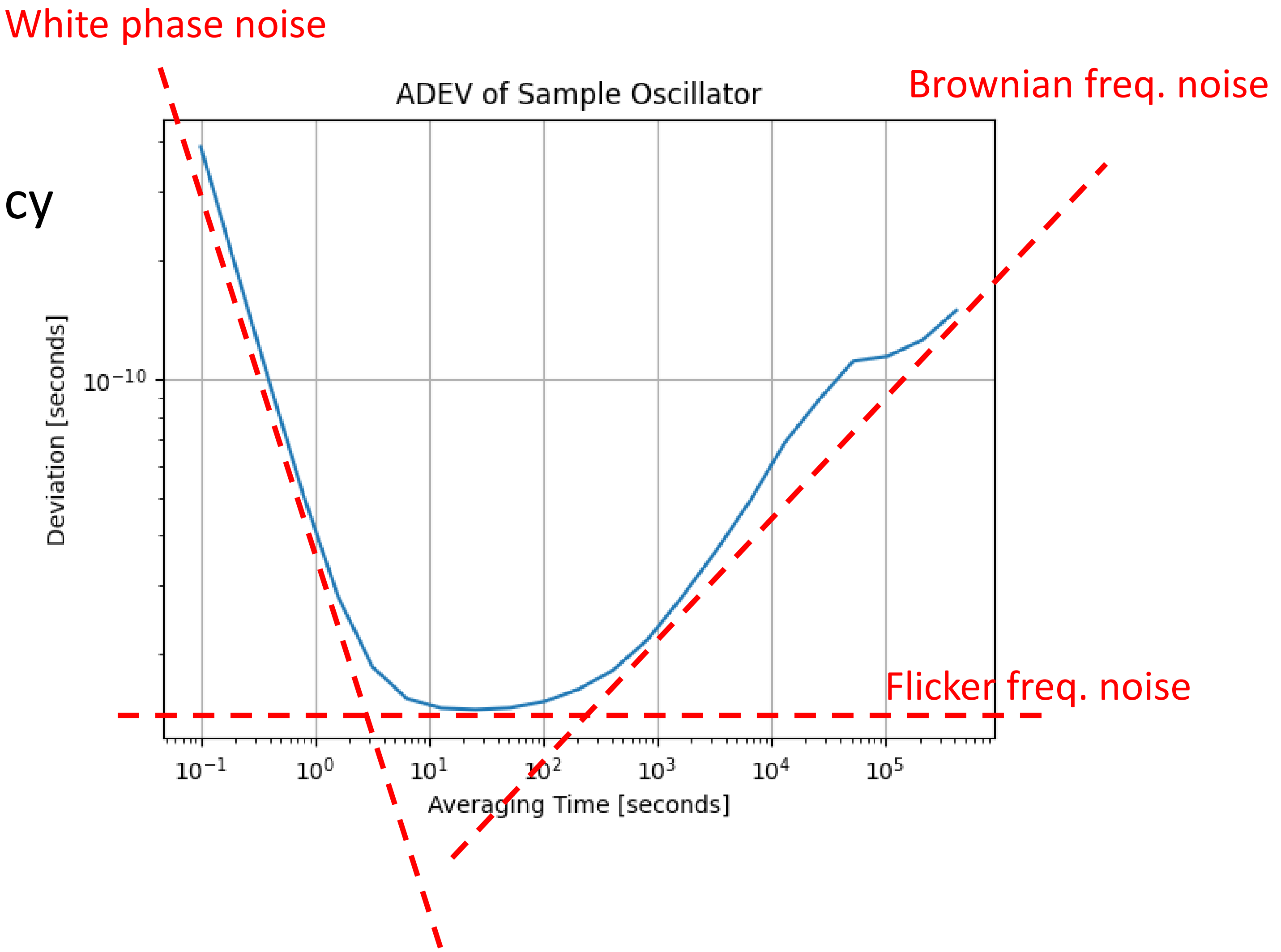
$$\sigma_y(\tau) = {}^{1}/_{\tau}$$

  - Flicker noise: lower power at higher frequencies

$$\sigma_y(\tau) = \boldsymbol{constant}$$

  - Brownian noise: even lower power at high freq.

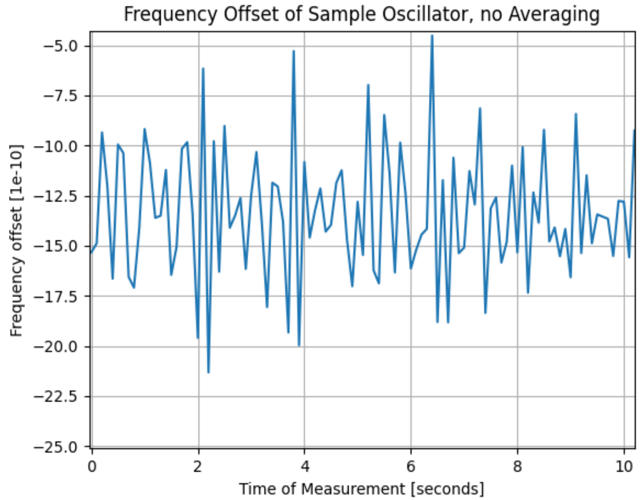
$$\sigma_y(\tau) = \boldsymbol{\tau}$$



Noise Types (cont.)

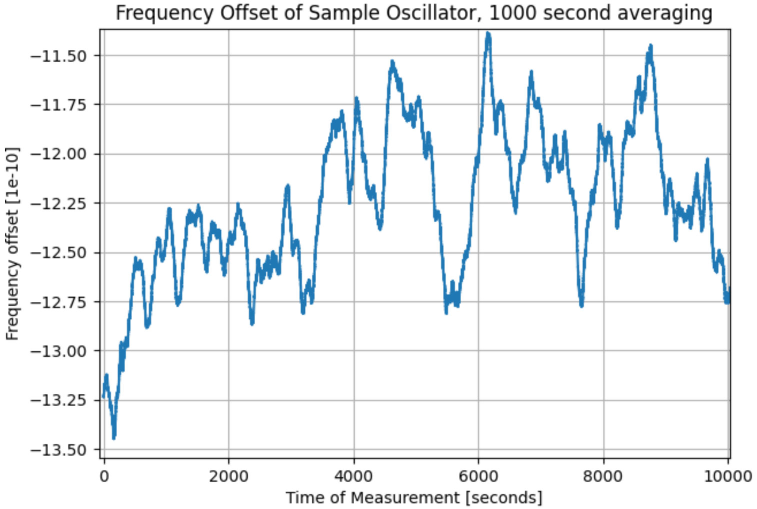
White Noise

Next value does not depend on the last



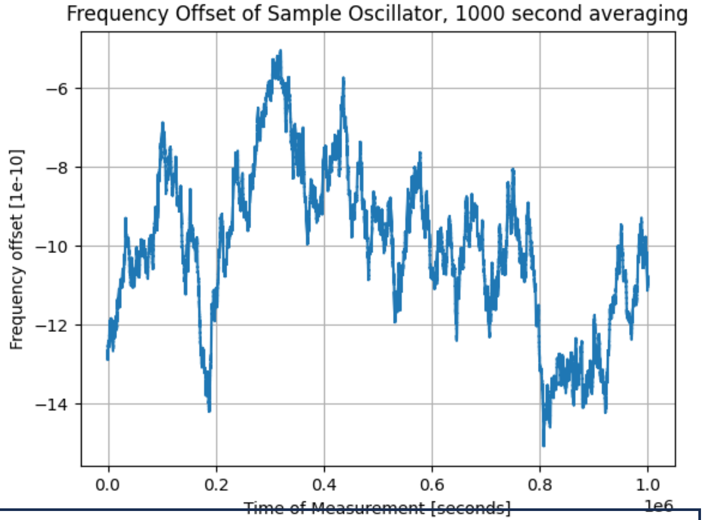
Flicker Noise

Next value depends on the last



Random Walk Noise

Next value strongly depends on the last



	White Noise	Flicker Noise	Random Walk Noise
Allan Deviation	$1/\tau$	$constant$	τ
Freq. Domain	$constant$	$1/f$	$1/f^2$

White Noise Sources

White noise is identical to a random distribution sampled at every measurement

- It is time-independent
- Often gaussian distributed but not always

Thermal Noise

Each electron has a temperature, causing it to randomly move about.

Shot Noise

Present in all devices, due to the finite number of particles in a system.

Think of it like the impact of each grain of sand in an hourglass

Flicker Noise Sources

Flicker noise has larger impact due to slower moving events

- Time-independent
- Dominant noise source in most devices

Semiconductor Noise

Semiconductors have a variety of impurity sizes, leading to a $^{1}/_{f}$ distribution of noise.

Filtered Noise

When white noise is filtered to remove fast fluctuations, only the slow-moving impact remains.

Upconverted Noise

In oscillators or radio applications, many devices will convert small white noise sources to flicker or random walk at their operating frequency

Random Walk Noise Sources

Random walk and drift are often indistinguishable

- Very long-term timescales
- Limits most devices accuracy in their lifetime

Environmental Factors

Things like temperature or humidity slowly change over time and can impact accuracy in a drift-like way

Aging

All materials decay over time, some more slowly, but this impacts the performance of a system

Phase Noise

NIST

Allan deviation

- Time domain measurement of stability
- Useful for longer intervals >1s typical
- Measured in fractional frequency stability

↔

Phase noise

- Frequency domain measurement of stability
- Useful for short intervals <1s typical
- Measured in relative power of the noise

Phase Noise (cont.)

Phase noise measures the noise power at a frequency offset

- Measurement in 1 Hz bandwidth
- Measured in dBc, values typically range from -180dBc (great) to -90dBc (not great)
- Lower values are better
- Similar analysis as with ADEV is possible