Bilateral Comparison of 1.018 V and 10 V Standards
between the EMI (United Arab Emirates) and the BIPM,
January to March 2025
(part of the ongoing BIPM key comparison BIPM.EM-K11.a and b)

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Introduction

As part of the ongoing BIPM key comparison BIPM.EM-K11.a and b, a comparison of the 1.018 V and 10 V voltage reference standards of the BIPM and the *Emirates Metrology Institute* (EMI), Abu Dhabi, United Arab Emirates, was carried out from January to March 2025. Two BIPM Zener diode-based travelling standards (Fluke 732B), BIPM_A (ZA) and BIPM_D (ZD), were transported by freight to EMI and back to BIPM. In order to keep the Zeners powered during the transportation, a voltage stabilizer developed by the BIPM was connected in parallel to the internal battery. The voltage stabilizer consists of a set of two batteries, electrically protected against overcurrent surges, easy to recharge and is designed to power two transfer standards for a least 12 consecutive days.

At EMI, the reference standard for DC voltage is a 732B Zener standard traceable to a primary voltage standard by means of a calibration service requested from a National Metrology Institute (NPL, United Kingdom). The output electromotive force (EMF) of each travelling standard was measured against the Zener voltage standard by means of an accurate multimeter, which is used as a null detector.

At the BIPM, the output EMF of each travelling standard was calibrated before and after the measurements at EMI against a Programmable Josephson Voltage (PJVS) developed at the BIPM around a PTB programmable SNS (Superconductor/Normal Metal/Superconductor) array.

Results of all measurements were corrected by the BIPM for the dependence of the output voltages of the Zener standards on internal temperature and ambient atmospheric pressure.

Outline of the measuring method

EMI 1.018 V and 10 V measurements

The travelling standards were received in the laboratory on 24 January 2025 and a visual check was made to ensure that the standards were in working condition. The “IN-CAL” light was on and the “LOW BAT” light was off. The travelling standards were connected to AC power and left over the weekend to stabilize. A check measurement was made on 27 January 2025 to ensure that there was no unexpected deviation from the nominal value.

The travelling standards were disconnected from AC line power each morning and measurements began 2 hours after disconnecting the standards from AC line power. The binding posts “GUARD” and “CHASSIS” were connected together and to the laboratory earth. The travelling standards were connected to the measurement instruments using screened twisted pair cable which was connected to the chassis of the measuring instrument and to the laboratory earth.

The resistance of the travelling standard internal thermistor was measured using Fluke 8508A* Reference Multimeter, Serial Number 218865636, using the “LO I” option. The Reference Multimeter had been calibrated by EMI. The temperature, relative humidity and pressure were measured using Extech SD700 Datalogger, Serial Number A079587. The Datalogger had been calibrated by EMI.

One measurement on each output was made on each day. The AC line power was reconnected at the end of the measurements.

Measurements were made using a Measurements International 8000A Automatic Potentiometer, with “8000A version 4.3.8” software in accordance with EMI Procedure CP-E-03 between 30 January 2025 and 5 February 2025.

The 8000A Automatic Potentiometer is a computer controlled binary resistive voltage divider, based on the design by R.D Cutkosky [1].

* Certain commercial equipment, instruments, or materials are identified in this report to facilitate understanding. Such identification does not imply recommendation or endorsement by BIPM or EMI, nor does it imply that the materials or equipment that are identified are necessarily the best available for the purpose.

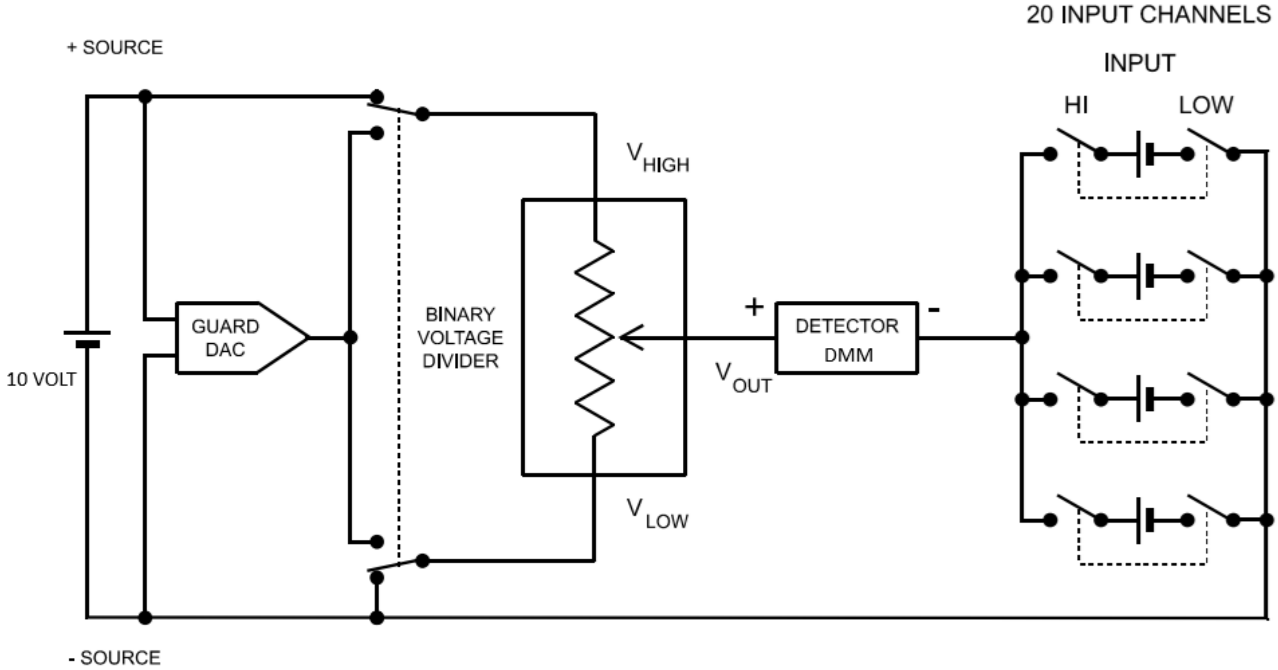

Figure 1: Schematic of the measurement set up operated at EMI where the BIPM Voltage Transfer Standards are connected across HI to LOW input channels.

During each measurement of V_{IN} the ratio of the divider R_{NOM} is adjusted such that the divided source voltage V_{OUT} is set to be within 1.2 mV of the measured voltage V_{IN}. The difference V_{DIF} between the input voltage and divided source voltage is measured using the Digital Multimeter (DMM). The divider is then switched off and the offset voltage V_{OFFSET} of the DMM is measured.

$$V_{IN} = V_{OUT} - (V_{DIF} - V_{OFFSET})$$

$$V_{OUT} = V_{SOURCE} \times R_{NOM}$$

Therefore

$$V_{IN} = (V_{SOURCE} \times R_{NOM}) - (V_{DIF} - V_{OFFSET})$$

The measured value of V_{IN} and the measured ratio $\frac{V_{IN}}{V_{SOURCE}}$ are available in the data file produced by the 8000A. The values of R_{NOM}, V_{DIF} and V_{OFFSET} are internal to the 8000A software and are not available to the user.

The first step in the measurement process is to perform a self-calibration of the 8000A. This process measures and corrects for the errors in the 13 stages of the binary voltage divider.

The second step in the measurement process is source calibration. The calibrated Standard Reference Fluke 732B is connected to Input “1” and connected with reversed polarity to input “2” to allow measurements of negative voltage. The source is used to measure the value of the calibrated reference V_{REF}, and the voltage of the source V_{SOURCE} is calculated based on this measurement.

These two steps need to be performed every 24 hours to minimize the uncertainty of the subsequent measurements due to changes in the 8000A and the source.

The next step in the measurement process is measurement of the Units under test (UUTs). The UUTs are connected to the 8000A inputs. The source is used to measure the value of the UUT V_{UUT}. The reported value is the mean of 30 individual measurements.

Measurements using the 8000A indicate that there is a zero offset. The value of this offset is removed during 8000A calibration and is stable between calibrations. Measurements performed indicate that residual zero offset after calibration, V_{ZERO}, is less than 0.2 μV.

The mathematical model representing the measurement of the voltage of the travelling standard can be expressed as:

$$V_{UUT} = [(R_{UUT} + R_{RS} + R_{ER}) \times (V_{REF} + \delta V_{DR} + \delta V_{TR} + \delta V_{TC} + \delta V_{NO})] + V_{TH} + V_{ZERO}$$

Where:

V_{UUT} is the measured voltage of the UUT

R_{UUT} is the mean ratio of the UUT voltage to the reference standard voltage

R_{RS} is the resolution of the 8000A

R_{ER} is the ratio error of the 8000A

V_{ZERO} is the residual zero offset of the 8000A

V_{REF} is the calibrated value of the reference standard

δV_{DR} is the relative drift of the reference standard since the last calibration

δV_{TR} is the relative change of the reference standard due to transportation

δV_{TC} is the relative change of the reference standard due to temperature

δV_{NO} is the relative noise of the reference standard

V_{TH} is the thermoelectric voltage

BIPM Measurements for 1.018 V and 10 V

The output voltage of the Zener standard to be measured was connected in series opposition to the BIPM PJVS - PTB 10 V SNS array (S/N: 2013-02/4a) [2], through a low thermal EMF multiplexer [3, 4]. The binding post terminals “GUARD” and “CHASSIS” of the Zener standard were connected together and connected to a single point which is the grounding reference point of the measurement setup.

The measurements started at least two hours after the mains plug at the rear of the Zeners had been disconnected in order for the Zener internal temperature to stabilize.

In this comparison, the BIPM detector was a digital nanovoltmeter *Keithley 2182A* operated on its 10 mV range. A computer was used to monitor, record the measurements, acquire the data, correct for temperature and pressure dependence, and calculate results.

The BIPM array biasing frequency was adjusted in such a way that the voltage difference between the primary and the secondary voltage standards was always below 1 µV for both nominal voltages.

One individual measurement point was acquired according to the following:

1- The Zener and the BIPM array are set in their positive polarity, connected in series opposition and the detector data reading sequence starts;
2- The polarity of the detector is reversed and a reading sequence is carried out. The number of measurements is twice the number acquired in step 1;
3- The polarity of the detector is reversed again to match the conditions of step 1 and the reading sequence restarts;
4- The Zener and the BIPM array are set in their negative polarity, connected in series opposition and the detector data reading sequence starts;
5- The polarity of the detector is reversed and a reading sequence is carried out. The number of measurements is twice the number acquired in step 4;
6- The polarity of the detector is reversed again to match the conditions of step 4 and the reading sequence restarts.

The reversal of the array polarity (by reversing the bias current) is always accompanied by a reversal of the Zener voltage standard using the multiplexer. The reversal of the detector polarity is done to cancel out any internal detector thermal EMF with a constant drift rate.
Each data acquisition step consists of 50 preliminary measurements followed by 100 measurements. Each of these should not differ from the mean of the preliminary measurements by more than four times their standard deviation. If so, the software warns the operator with a beep. If many beeps occur, it means that the Zener output value has changed. The decision on restarting the “Data Acquisition” step in progress is based on considering the graphical representation of the measurements on the computer screen. The procedure to acquire one individual measurement point is repeated five times in a row and the mean value corresponds to one result on the graph (cf. Fig. 2, 3, 4, and 5).

Results at 10 V

Figure 2 shows the measured values obtained for the two standards by the two laboratories at 10 V. Figure 3 presents the voltage evolution of the arithmetic mean of the two standards which is used to compute the final result at 10 V. A linear least squares fit is applied to all of the individual BIPM results, and to the mean value of both transfer standards. The comparison result is the voltage difference between the BIPM fitted value at the mean date of the EMI measurements (02/02/2025) and the mean value of the EMI measurements, and the related uncertainties.

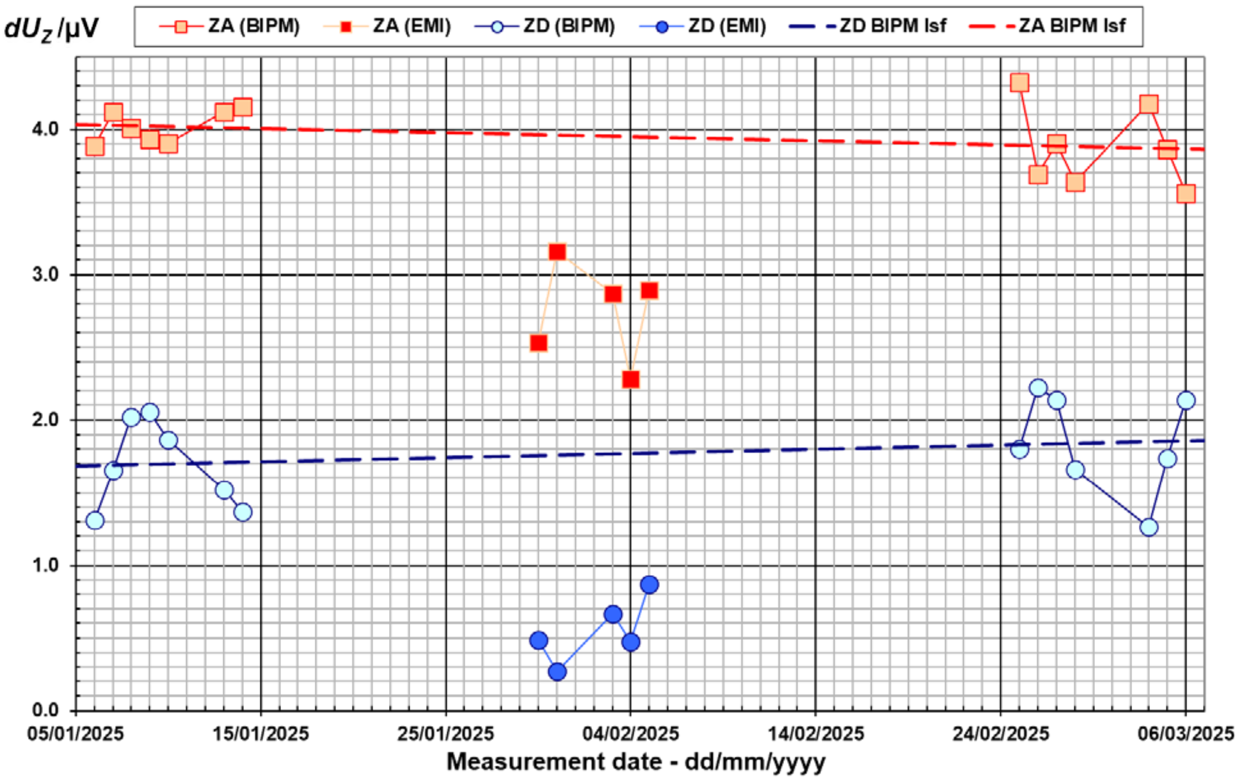

Figure 2: Voltage of ZA (squares) and ZD (disks) at 10 V measured at both institutes (light markers for BIPM and dark markers for EMI), referred to an arbitrary offset, as a function of the measurement date with a linear least squares fit (lsf) to the BIPM measurements.

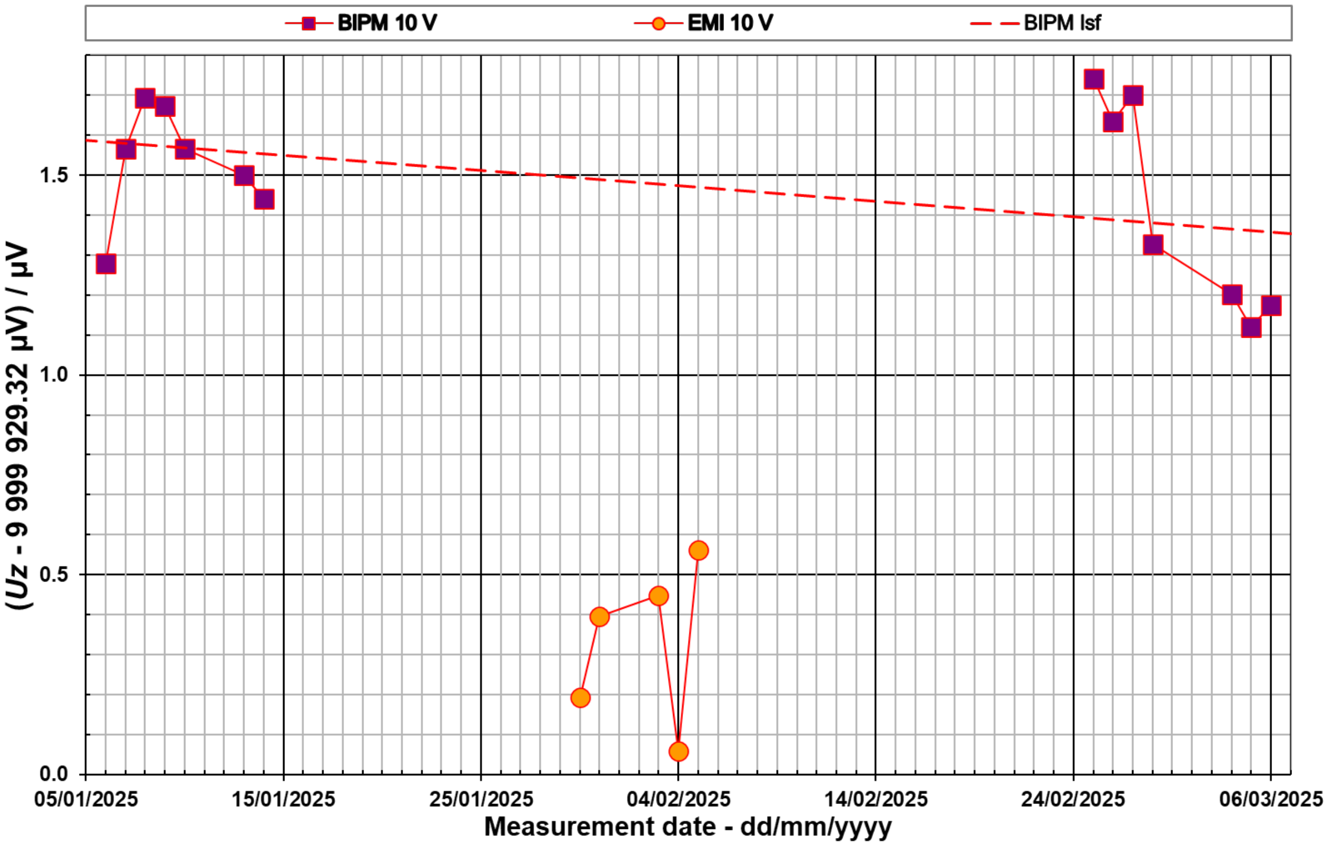

Figure 3: Voltage evolution of the arithmetic mean of the two standards at 10 V. EMI measurements are represented by disks and BIPM measurements by squares. A least squares fit is applied to the BIPM measurements.

Uncertainty Budgets at 10 V

BIPM uncertainty budget at 10 V

Table 1 summarizes the uncertainties related to the calibration of a Zener standard against the Programmable Josephson Voltage Standard at the BIPM at the level of 10 V.

Experience has shown that flicker or 1/*f* noise ultimately limits the stability characteristics of Zener standards and it is not appropriate to use the standard deviation divided by the square root of the number of observations to characterize the dispersion of measured values. For the present standards, the relative value of the voltage noise floor due to flicker noise is about 1 part in 10^8 [5]. The Type A standard uncertainty in the Table 1 therefore has a lower limit of 100 nV. However, if the standard deviation of the measurements at the mean date of the participant is larger than the flicker noise floor, it is this standard deviation which is considered to be the Type A standard uncertainty.

PJVS & detector uncertainty components	**Standard uncertainty (nV)**
Noise of the measurement loop that includes the residual thermal EMF including the residual EMF of the reversing switch (Type A)	2
Detector gain (Type B)	negligible
Leakage resistance (Type B)	4
Frequency (Type B)	0.1
Zener noise (Type A)	Not lower than the 1/*f* noise estimated as 100 nV, included in the comparison uncertainty budget (Table 3)
Zener pressure and temperature correction	Included in the comparison uncertainty budget (Table 3)

Table 1: Estimated standard uncertainties arising from the PJVS and the measurement setup for Zener calibrations with the BIPM equipment at the level of 10 V.

EMI uncertainty budget at 10 V

Tables 2a and 2b list the uncertainties related to the calibration of the Zeners at EMI for ZA and ZD, respectively.

Quantity	Type	Distribution	Standard uncertainty µV	Sensitivity	Uncertainty contribution µV	Degrees of freedom
Repeatability	A	Normal	0.212	1	0.212	29
732B Standard Calibration	B	Rectangular	0.150	1	0.150	∞
732B Transportation	B	Rectangular	0.635	1	0.635	∞
732B Drift	B	Rectangular	1.097	1	1.097	∞
732B Temperature	B	Rectangular	0.693	1	0.693	∞
732B Noise	B	Rectangular	0.346	1	0.346	∞
Thermoelectric Voltage	B	Rectangular	0.173	1	0.173	∞
8000A Zero Offset	B	Rectangular	0.116	1	0.116	∞
8000A Resolution	B	Rectangular	0.058	1	0.058	∞
8000A Ratio	B	Rectangular	0.254*	1	0.254	∞
[7]	**Combined uncertainty**…… $\boldsymbol{u(U_z)}$ **= 1.544 µV**					
	Relative combined uncertainty…… $\boldsymbol{u(U_z) / U_z}$ **= 0.154 µV/V**					
	Effective degrees of freedom …… $\boldsymbol{\nu_{eff}} = \infty$					
	Coverage factor …… $k_{0.9545} = 2$					
	Expanded uncertainty (95.45%)…… $U(U_Z) = k_{0.9545} \times u(U_Z) = 3.088$ µV					
	Relative expanded uncertainty…… $U(U_Z) / U_Z = 0.309$ µV/V					

Table 2a: Estimated standard uncertainties of U_z for a Zener calibration with the EMI equipment at the level of 10 V for Zener ZA.

* The ratio error of the 8000A is measured annually using a special "Ratio Verification" mode of the 8000A and a resistive voltage divider constructed from calibrated standard resistors. We reverse the connection of the resistors to the bridge and calculate half the sum of the forward and reverse ratio errors measured by the bridge.

The measured error is of similar magnitude to the uncertainty of the measurement and no correction is made for this term. The uncertainty is estimated by adding the magnitude of the uncorrected error of the 8000A and the uncertainty of the measurement.

Quantity	Type	Distribution	Standard uncertainty µV	Sensitivity	Uncertainty contribution µV	Degrees of freedom
Repeatability	A	Normal	0.211	1	0.211	29
732B Standard Calibration	B	Rectangular	0.150	1	0.150	∞
732B Transportation	B	Rectangular	0.635	1	0.635	∞
732B Drift	B	Rectangular	1.097	1	1.097	∞
732B Temperature	B	Rectangular	0.693	1	0.693	∞
732B Noise	B	Rectangular	0.346	1	0.346	∞
Thermoelectric Voltage	B	Rectangular	0.173	1	0.173	∞
8000A Zero Offset	B	Rectangular	0.116	1	0.116	∞
8000A Resolution	B	Rectangular	0.058	1	0.058	∞
8000A Ratio	B	Rectangular	0.254	1	0.254	∞
[7]	**Combined uncertainty**………………………………………… $\boldsymbol{u(U_z)}$ **= 1.544 µV** **Relative combined uncertainty**………………..………… $\boldsymbol{u(U_z)\ /\ U_z}$ **= 0.154 µV/V**					
	Effective degrees of freedom ………………..…… ………………….……….. $\boldsymbol{v_{eff}}$ = ∞					
	Coverage factor ………………………………......………..………. $k_{0.9545}$ = 2					
	Expanded uncertainty (95.45%)………...……$U(U_Z) = k_{0.9545} \times u(U_Z)$ = 3.088 µV					
	Relative expanded uncertainty…………………………….. $U(U_Z)\ /\ U_Z$ = 0.309 µV/V					

Table 2b: Estimated standard uncertainties of U_z for a Zener calibration with the EMI equipment at the level of 10 V for Zener ZD.

Uncertainty contributions for the comparison EMI/BIPM at 10 V

Table 3 lists the results and the uncertainty contributions for the comparison EMI/BIPM at 10 V.

		Results/µV		Uncertainty/µV	
		ZA	ZD	ZA	ZD
1	EMI (U_{EMI} – 10 V)	-43.75	-96.95		
2	Type A uncertainty			0.212	0.211
3	correlated (Type B) unc.			1.530	
4	BIPM (U_{BIPM} – 10 V)	-42.55	-95.74		
5	Type A uncertainty			0.100	0.100
6	correlated (Type B) unc.			<0.005	
7	pressure and temperature correction uncertainty			0.030	0.026
8	(U_{EMI} – U_{BIPM})	-1.20	-1.21		
9	Total uncorrelated uncertainty			0.236	0.235
10	Total correlated uncertainty			1.530	
11	< U_{EMI} – U_{BIPM} >	**-1.20**			
12	*a priori* uncertainty			0.167	
13	*a posteriori* uncertainty			0.005	
14	**comparison total standard uncertainty/µV**			**1.54**	

Table 3: Results and uncertainties of EMI (United Arab Emirates)/BIPM bilateral comparison of 10 V standards using two Zener travelling standards: reference date 2 February 2025. Standard uncertainties are used throughout.

In Table 3, the following elements are listed:

(1) the value attributed by EMI to each Zener, U_{EMI}, computed as the arithmetic mean of all data from EMI and corrected for temperature and pressure differences between both laboratories by the BIPM.

(2) EMI combined Type A uncertainty (cf. Tables 2a and 2b).

(3) the uncertainty component arising from the realization and maintenance of the volt at EMI: it is the quadratic combination of the Type B components of the participant uncertainty budget listed in Tables 2a and 2b. This uncertainty is completely correlated between the different Zeners used for the comparison.

(4-6) the corresponding quantities for the BIPM referenced to the mean date of the EMI measurements. In this case, the Type A uncertainty is limited by the flicker noise level of 100 nV.

(7) the uncertainty due to the combined effects of the pressure and temperature coefficients [8, 9] and to the differences of the mean pressures and temperatures in the participating laboratories is calculated as follows:

The uncertainty of the temperature correction $u_{T,i}$ of Zener *i* is determined for the difference ΔR_i between the mean values of the thermistor resistances measured at both institutes which is then multiplied by the uncertainty $u(c_{T,i})$ of the relative temperature coefficients of each Zener standard:

$$u_{T,i} = U \times u(c_{T,i}) \times \Delta R_i$$

where U = 10 V, $u(c_{T,ZA})$ = 0.244 × 10^{-7} / kΩ, $u(c_{T,ZD})$ = 0.288 × 10^{-7} / kΩ,

ΔR_{ZA} = 0.122 kΩ and ΔR_{ZD} = 0.089 kΩ.

The same procedure is applied for the uncertainty $u_{P,i}$ of the pressure correction for the difference ΔP_i between the mean values of the pressure measured at both institutes:

$$u_{P,i} = U \times u(c_{P,i}) \times \Delta P_i$$

where U = 10 V, $u(c_{P,ZA})$ = 0.048 × 10^{-9} / hPa, $u(c_{P,ZD})$ = 0.068 × 10^{-9} / hPa, ΔP_{ZA} = 7.3 hPa and ΔP_{ZD} = 8.2 hPa.

The uncertainties of the temperature and the pressure measurements are negligible.

(8) the difference (U_{EMI} – U_{BIPM}) for each Zener, and (9) the uncorrelated part of the uncertainty, calculated as the quadratic sum of lines 2, 5 and 7.

(10) the correlated part of the uncertainty, calculated as the quadratic sum of lines 3 and 6, for each travelling standard.

(11) the result of the comparison is the arithmetic mean of the differences of the calibration results for the different standards.

(12 and 13) the uncertainty related to the transfer, estimated by comparing the following uncertainties:

(12) the *a priori* uncertainty, determined as the standard uncertainty of the mean, obtained by propagating the uncorrelated uncertainties for both Zeners;

(13) the *a posteriori* uncertainty, which is the standard deviation of the mean of the two results.

(14) the total uncertainty of the comparison, which is the root sum square of the correlated part of the uncertainty (10) and the larger of (12) and (13).

To estimate the uncertainty related to the stability of the standards during transportation, we have calculated the "*a priori*" uncertainty of the mean of the results obtained for the two standards (also called statistical internal consistency). It consists of the quadratic combination of the uncorrelated uncertainties of each result. We compared this component to the "*a posteriori*" uncertainty (also called statistical external consistency) which consists of the experimental standard deviation of the mean of the results from the two travelling standards[*].

If the "*a posteriori*" uncertainty is significantly larger than the "*a priori*" uncertainty, we assume that a standard has changed in an unusual way, probably during their transportation. We use the larger of these two estimates in calculating the final uncertainty.

In the present comparison, the "*a posteriori*" uncertainty is largely smaller than the "*a priori*" uncertainty at 10 V, indicating that the results for both Zeners are consistent within the uncertainties and that no unexpected change occurred during the transportation.

The comparison result is presented as the difference between the value assigned to a 10 V standard by EMI, at EMI, U_{EMI}, and that assigned by the BIPM, at the BIPM, U_{BIPM}, on the reference date of the 2nd of February 2025:

$$\boldsymbol{U_{\mathrm{EMI}} - U_{\mathrm{BIPM}} = -1.20\ \mu\mathrm{V}; \qquad u_{\mathrm{c}} = 1.54\ \mu\mathrm{V}}$$

where u_{c} is the combined standard uncertainty associated with the measured difference, including the uncertainty of the realization of the volt at EMI, at the BIPM, and the uncertainty related to the comparison.

[*] With only two travelling standards, the uncertainty of the standard deviation of the mean is comparable to the value of the standard deviation of the mean itself.

Results at 1.018 V

Figure 4 shows the measured values obtained for the two standards by the two laboratories at 1.018 V and Figure 5 presents the voltage evolution of the arithmetic mean of the two standards which is used to compute the final result at 1.018 V.

A linear least squares fit is applied to the results of the BIPM, before and after the measurements at EMI, to obtain the results for both standards and their uncertainties at the mean date of the EMI measurements (02/02/2025).

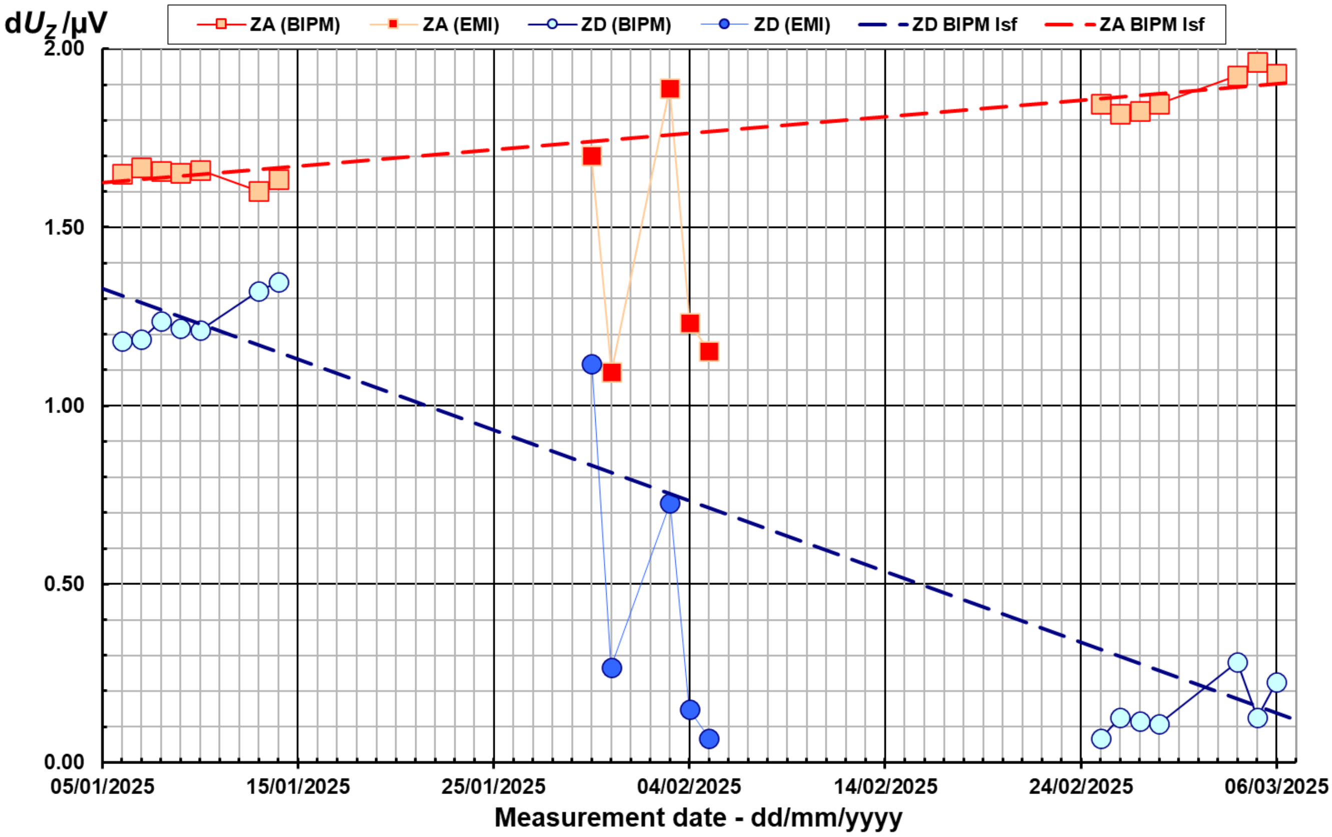

Figure 4: Voltage of ZA (squares) and ZD (disks) at 1.018 V measured at both institutes (light markers for BIPM and dark markers for EMI), referred to an arbitrary offset, as a function of the measurement date with a linear least squares fit (lsf) to the BIPM measurements**.**

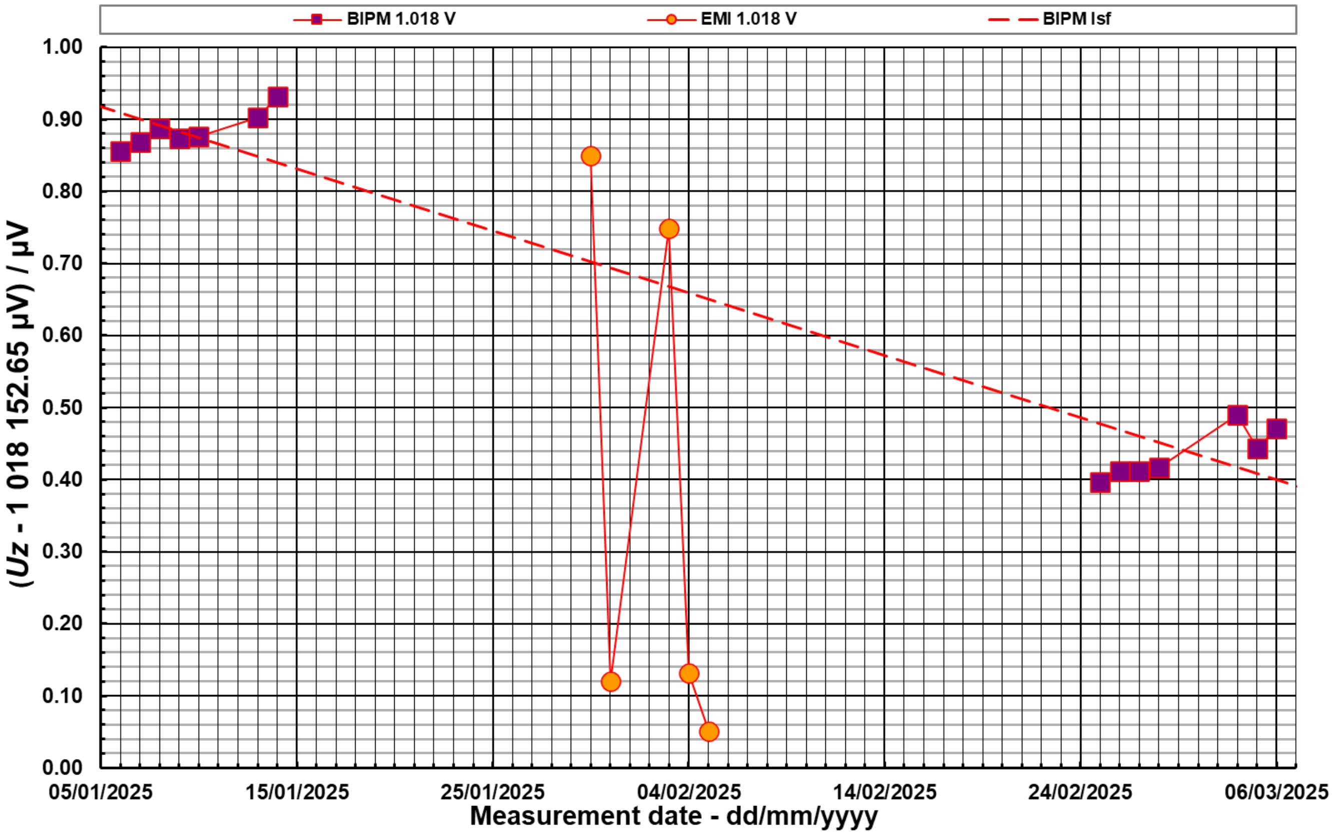

Figure 5: Voltage evolution of the arithmetic mean of the two standards at 1.018 V. EMI measurements are represented by disks and BIPM measurements by squares. A least squares fit is applied to the BIPM measurements.

Uncertainty Budgets at 1.018 V

BIPM uncertainty budget at 1.018 V

Table 4 summarizes the uncertainties related to the calibration of a Zener standard against the Programmable Josephson Voltage Standard at the BIPM at the level of 1.018 V.

PJVS & detector uncertainty components	**Standard uncertainty (nV)**
Noise of the measurement loop that includes the residual thermal EMF including the residual EMF of the reversing switch (Type A)	2
Detector gain (Type B)	negligible
Leakage resistance (Type B)	0.4
Frequency (Type B)	0.01
Zener noise (Type A)	Not lower than the 1/*f* noise estimated as 10 nV, included in the comparison uncertainty budget (Table 6)
Zener pressure and temperature correction	Included in the comparison uncertainty budget (Table 6)

Table 4: Estimated standard uncertainties arising from the PJVS and the measurement setup for Zener calibrations with the BIPM equipment at the level of 1.018 V.

EMI uncertainty budget at 1.018 V

Tables 5a and 5b list the uncertainties related to the calibration of the Zeners at EMI for ZA and ZD, respectively.

Quantity	Type	Distribution	Standard uncertainty µV	Sensitivity	Uncertainty contribution µV	Degrees of freedom
Repeatability	A	Normal	0.071	1	0.071	29
732B Standard Calibration	B	Rectangular	0.015	1	0.015	∞
732B Transportation	B	Rectangular	0.065	1	0.065	∞
732B Drift	B	Rectangular	0.112	1	0.112	∞
732B Temperature	B	Rectangular	0.071	1	0.071	∞
732B Noise	B	Rectangular	0.018	1	0.018	∞
Thermoelectric Voltage	B	Rectangular	0.176	1	0.176	∞
8000A Zero Offset	B	Rectangular	0.118	1	0.118	∞
8000A Resolution	B	Rectangular	0.059	1	0.059	∞
8000A Ratio	B	Rectangular	0.259	1	0.259	∞
[7]	**Combined uncertainty**……………………………………………. $\boldsymbol{u(U_z)}$ **= 0.378 µV** **Relative combined uncertainty**………………...………… $\boldsymbol{u(U_z)\ /\ U_z}$ **= 0.371 µV/V**					
	Effective degrees of freedom ………………..…… …………………….…………..$\boldsymbol{v_{eff}}$ = ∞					
	Coverage factor ………………………………………...………..………… $k_{0.9545}$ = 2					
	Expanded uncertainty (95.45%)………..……. $U(U_Z) = k_{0.9545} \times u(U_Z)$ = 0.756 µV					
	Relative expanded uncertainty…………………………….. $U(U_Z)\ /\ U_Z$ = 0.743 µV/V					

Table 5a: Estimated standard uncertainties for a Zener calibration with the EMI equipment at the level of 1.018 V for Zener ZA.

Quantity	Type	Distribution	Standard uncertainty µV	Sensitivity	Uncertainty contribution µV	Degrees of freedom
Repeatability	A	Normal	0.070	1	0.070	29
732B Standard Calibration	B	Rectangular	0.015	1	0.015	∞
732B Transportation	B	Rectangular	0.065	1	0.065	∞
732B Drift	B	Rectangular	0.112	1	0.112	∞
732B Temperature	B	Rectangular	0.071	1	0.071	∞
732B Noise	B	Rectangular	0.018	1	0.018	∞
Thermoelectric Voltage	B	Rectangular	0.176	1	0.176	∞
8000A Zero Offset	B	Rectangular	0.118	1	0.118	∞
8000A Resolution	B	Rectangular	0.059	1	0.059	∞
8000A Ratio	B	Rectangular	0.259	1	0.259	∞
[7]	**Combined uncertainty**…………………………………………. $\boldsymbol{u(U_z)}$ **= 0.378 µV** **Relative combined uncertainty**………………..………… $\boldsymbol{u(U_z)\ /\ U_z}$ **= 0.371 µV/V**					
	Effective degrees of freedom ………………..…… ………………………..$\boldsymbol{v_{eff}}$ = ∞					
	Coverage factor ………………………………………..………..………… $k_{0.9545} = 2$					
	Expanded uncertainty (95.45%)………...……$U(U_Z) = k_{0.9545} \times u(U_Z) = 0.756$ µV					
	Relative expanded uncertainty…………………………….. $U(U_Z)\ /\ U_Z = 0.743$ µV/V					

Table 5b: Estimated standard uncertainties for a Zener calibration with the EMI equipment at the level of 1.018 V for Zener ZD.

Uncertainty contributions for the comparison EMI/BIPM at 1.018 V

Table 6 lists the results and the uncertainty contributions for the comparison EMI/BIPM at 1.018 V.

		Results/µV		Uncertainty/µV	
		ZA	**ZD**	**ZA**	**ZD**
1	EMI (U_{EMI} – 1.018 V)	194.26	111.80		
2	Type A uncertainty			0.071	0.070
3	correlated (Type B) unc.			0.371	
4	BIPM (U_{BIPM} – 1.018 V)	194.61	112.06		
5	Type A uncertainty			0.010	0.010
6	correlated (Type B) unc.			<0.005	
7	pressure and temperature correction uncertainty			0.003	0.004
8	(U_{EMI} – U_{BIPM})	-0.35	-0.26		
9	Total uncorrelated uncertainty			0.072	0.071
10	Total correlated uncertainty			0.371	
11	< U_{EMI} – U_{BIPM} >	**-0.31**			
12	*a priori* uncertainty			0.051	
13	*a posteriori* uncertainty			0.045	
14	**comparison total standard uncertainty/µV**			**0.37**	

Table 6: Results and uncertainties of EMI (United Arab Emirates)/BIPM bilateral comparison of 1.018 V standards using two Zener travelling standards: reference date 2 February 2025. Standard uncertainties are used throughout.

In Table 6, the following elements are listed:

(1) the value attributed by EMI to each Zener U_{EMI}, computed as the arithmetic mean of all data from EMI and corrected for temperature and pressure differences between both laboratories by the BIPM.

(2) the EMI Type A uncertainty (cf. Tables 5a and 5b).

(3) the uncertainty component arising from the realization and maintenance of the volt at EMI: it is the quadratic combination of the Type B components of the participant uncertainty budget listed in Tables 5a and 5b. This uncertainty is completely correlated between the different Zeners used for the comparison.

(4-6) the corresponding quantities for the BIPM referenced to the mean date of EMI measurements. In this case, the Type A uncertainty is limited by the flicker noise level of 10 nV.

(7) the uncertainty due to the combined effects of the pressure and temperature coefficients [8, 9] and to the differences of the mean pressures and temperatures in the participating laboratories is calculated as follows:

The uncertainty of the temperature correction $u_{T,i}$ of Zener *i* is determined for the difference ΔR_i between the mean values of the thermistor resistances measured at both institutes which is then multiplied by the uncertainty $u(c_{T,i})$ of the relative temperature coefficients of each Zener standard:

$$u_{T,i} = U \times u(c_{T,i}) \times \Delta R_i$$

where U = 1.018 V, $u(c_{T,ZA})$ = 0.237 × 10^{-7} / kΩ, $u(c_{T,ZD})$ = 0.400 × 10^{-7} / kΩ,

ΔR_{ZA} = 0.122 kΩ and ΔR_{ZD} = 0.091 kΩ.

The same procedure is applied for the uncertainty $u_{P,i}$ of the pressure correction for the difference ΔP_i between the mean values of the pressure measured at both institutes:

$$u_{P,i} = U \times u(c_{P,i}) \times \Delta P_i$$

where U = 1.018 V, $u(c_{P,ZA})$ = 0.071 × 10^{-9} / hPa, $u(c_{P,ZD})$ = 0.068 × 10^{-9} / hPa,

ΔP_{ZA} = 6.8 hPa and ΔP_{ZD} = 8.6 hPa.

The uncertainties of the temperature and the pressure measurements are negligible.

(8) the difference (U_{EMI} – U_{BIPM}) for each Zener, and (9) the uncorrelated part of the uncertainty, calculated as the quadratic sum of lines 2, 5 and 7.

(10) the correlated part of the uncertainty, calculated as the quadratic sum of lines 3 and 6, for each travelling standard.

(11) the result of the comparison is the arithmetic mean of the differences of the calibration results for the different standards.

(12 and 13) the uncertainty related to the transfer, estimated by comparing the following uncertainties:

(12) the *a priori* uncertainty, determined as the standard uncertainty of the mean, obtained by propagating the uncorrelated uncertainties for both Zeners;

(13) the *a posteriori* uncertainty, which is the standard deviation of the mean of the two results.

(14) the total uncertainty of the comparison, which is the root sum square of the correlated part of the uncertainty (10) and the larger of (12) and (13).

In this case the *a priori* uncertainty is comparable to the *a posteriori* uncertainty. We conclude that at 1.018 V both Zeners behaved consistently within the uncertainty of the comparison.

The result of the comparison is presented as the difference between the value assigned to a 1.018 V standard by EMI, at EMI, U_{EMI}, and that assigned by the BIPM, at the BIPM, U_{BIPM}, on the reference date of the 2nd of February 2025:

$$\boldsymbol{U_{\mathrm{EMI}} - U_{\mathrm{BIPM}} = -0.31\ \mu\mathrm{V}; \qquad u_{\mathrm{c}} = 0.37\ \mu\mathrm{V}}$$

where u_{c} is the combined standard uncertainty associated with the measured difference, including the uncertainty of the realization of the volt at EMI, at the BIPM, and the uncertainty related to the comparison.

Conclusion

The final result of the comparison is presented as the difference between the values assigned to DC voltage standards by EMI, at the level of 1.018 V and 10 V, at EMI, U_{EMI}, and those assigned by the BIPM, at the BIPM, U_{BIPM}, at the reference date of the 2nd of February 2025.

$$\boldsymbol{U_{\mathrm{EMI}} - U_{\mathrm{BIPM}} = -0.31\ \mu\mathrm{V}; \qquad u_{\mathrm{c}} = 0.37\ \mu\mathrm{V},\ \text{at } 1.018\ \mathrm{V}}$$

$$\boldsymbol{U_{\mathrm{EMI}} - U_{\mathrm{BIPM}} = -1.20\ \mu\mathrm{V}; \qquad u_{\mathrm{c}} = 1.54\ \mu\mathrm{V},\ \text{at } 10\ \mathrm{V}}$$

where u_{c} is the combined standard uncertainty associated with the measured difference, including the uncertainty of the realization of the volt at EMI, at the BIPM, and the uncertainty related to the comparison.

The comparison results show that the voltage standards maintained by EMI and the BIPM were equivalent, within their stated standard uncertainties at 1.018 V and 10 V. EMI's results improved compared to the results obtained in GULFMET.BIPM.EM-K11 (2018) [10] and this reflects the quality of the work achieved at the EMI.

References

[1] **R. D. Cutkosky**, A New Switching Technique for Binary Resistive Dividers, *IEEE Trans. Instrum. Meas.*, vol. IM 27, No. 4, pp. 421 422, 1978.

[2] **F. Mueller et al.**, 1 V and 10 V SNS Programmable Voltage Standards for 70 GHz *IEEE Trans. On Applied Superconductivity,* Vol. 19, NO. 3, JUNE 2009 pp.981-86.

[3] **Chayramy R. and Solve S**., A very low thermal EMF computer-controlled scanner, *Meas. Sci. Technol*. 24 025008 (2013).

[4] **Chayramy R. et al.**, Metrology of Zener-based secondary voltage standards, *Meas. Sci. Technol*. 32 (2021) 105019 (18pp) DOI: 10.1088/1361-6501/ac055e.

[5] **Witt T. J.**, Maintenance and dissemination of voltage standards by Zener-diode-based instruments, *IEE Proc.-Sci. Meas. Technol*., vol. 149, pp. 305-312, 2002.

[7] Evaluation of measurement data, Guide to the expression of uncertainty in measurement, JCGM 100:2008, pp 70-78. https://www.bipm.org/documents/20126/2071204/JCGM_100_2008_E.pdf/cb0ef43f-baa5-11cf-3f85-4dcd86f77bd6.

[8] **Solve S., Chayramy R. and Power O.**, Temperature sensitivity coefficients of the BIPM secondary voltage standards, *CPEM 2016 Digest, Ottawa*, DOI: 10.1109/CPEM.2016.7540702.

[9] **Solve S., Chayramy R. and Yang S.**, Pressure Sensitivity Coefficients of the BIPM secondary voltage standards, *CPEM 2018 Digest, Paris* DOI: 10.1109/CPEM.2018.8501074.

[10] **Yang S. et al,** GULFMET Key Comparison of DC Voltage Standards at 1.018 V and 10 V**,** *Metrologia* 2020 **57** *Tech. Suppl.* 01012**,** DOI 10.1088/0026-1394/57/1A/01012.