

# **Final Report of APMP.AUV.V-S1: Supplementary comparison on calibration of laser tachometers using mechanical generators**

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## 1. Introduction

This report presents the results of the first Asia Pacific Metrology Programme (APMP) comparison in the area of rotational speed, which in this case means calibration of laser tachometers.

The participants have reached consensus and considered the weighted mean as the most appropriate method for this particular comparison to compute the supplementary comparison reference values (SCRVs) and the degrees of equivalence (DoEs). Detailed analysis and application of the method for use of the weighted mean in comparisons in the field of vibration, is documented in the CCAUV.V-K1 report [1]. The calculation of the SCRVs is also in accordance with the Guidelines for CIPM key comparisons [2].

The Technical Protocol, published in 2022 [3], specifies in detail the aim, the task of the comparison, the conditions for the measurements, the transfer standard used, measurement instructions, time schedule and other items. A brief survey of the Technical Protocol is given in the following sections. Refer to Annex A for details of the TP.

It should be noticed that for this comparison, only mechanical generators are applied as rotational speed standard devices with the comparison range from 10 rpm to 99 999 rpm. The following comparison of APMP.AUV.V-S2 allows either mechanical generators or optical simulators as rotational speed standard devices with the comparison range from 30 rpm to 99 996 rpm [4]. The results of these two comparisons can support each other as direct or circumstantial evidence.

## 2. Participants

Two metrology institutes (NMIs) from APMP participated in the comparison. They are listed in chronological order of measurement in Table 2.1.

Table 2.1: List of participants and schedule of APMP.AUV.V-S1

No.	Participant Laboratory	Acronym	Country	RMO	Calibration period (week/year)
1	National Institute of Metrology	NIM	China	APMP	41/2022 to 42/2022
2	National Metrology Centre, Agency for Science, Technology and Research	NMC	Singapore	APMP	45/2022 to 46/2022

## 3. Task and purpose of the comparison

According to the rules set up by the CIPM MRA [5], the consultative committees of the CIPM have the responsibility to establish Degrees of Equivalence (DoEs) between the different measurement standards operated by the NMIs. This is done by conducting key comparisons (KCs) on different levels of the international metrological infrastructure.

However, in the sub-field of rotational speed, there is no formal comparison either at Consultative Committee (CC) level or Regional Metrology Organization Technical Committee (RMO TC) level. Therefore, during the meeting of APMP TCAUV in 2021, the decision was taken to make preparations for a supplementary comparison targeted at rotational speed (constant acceleration).

This regional supplementary comparison is organized in order to compare primary measurements of rotational speed in the range from 10 rpm to 99 999 rpm. It is the task of the comparison to calibrate two laser digital tachometers at different rotational speed values as specified in section 3 of [3]. The results of this APMP supplementary comparison may serve as supporting evidence for the registration of ‘calibration and measurement capabilities’ (CMCs) in the framework of the CIPM MRA. [5].

The measured value of rotational speed is calculated as the average of ten successive measurement results at its reflective mark. The measured value shall be given in revolution per minute (r/min or rpm) for different measurement conditions specified in section 3 of [3].

For the calibration of the laser tachometer, rotational speed standard device of mechanical generator type has to be applied.

The reported measured values and associated uncertainties will be used for the calculation of mean values of the supplementary comparison results and their associated uncertainties, as well as the deviations to the mean values with associated uncertainties.

#### **4. Transfer standard as artefacts**

For the purpose of the comparison the pilot laboratory selected two laser tachometers of which monitoring data for six months were available and not included in any published international cooperation work.

- Two laser tachometers, EMT260C, SN: NLYZ-02 and NLYZ-03.

The investigation of the long-term stability was continued after the circulation period. The results of the NIM stability measurements and other individual data of the transfer standards are given in Section 6. Note that NLYZ-03 was deliberately customized with a deviation of -0.1% from the nominal value.

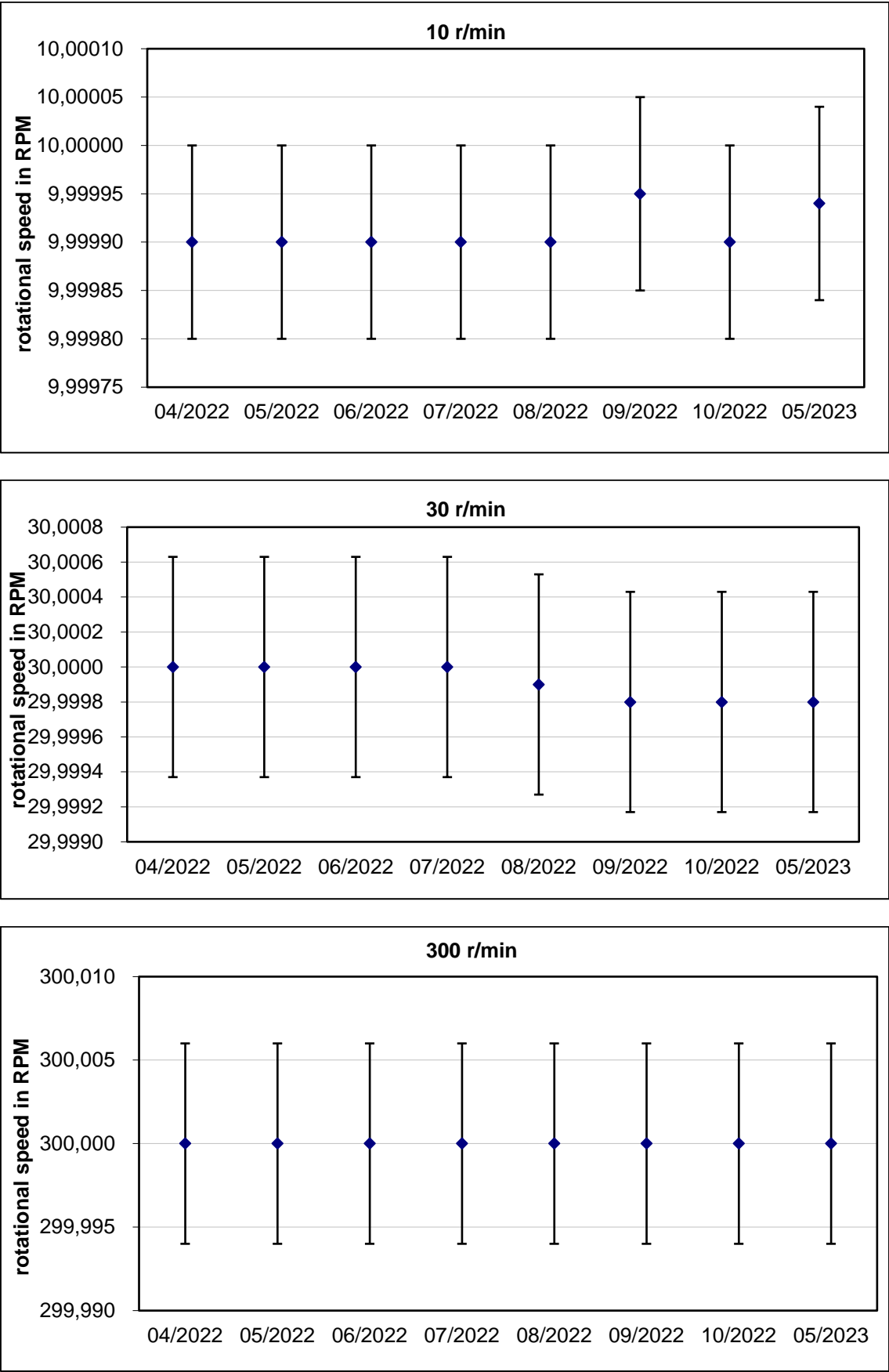
#### **5. Circulation of the artefacts**

The artefacts were circulated in a star type with a measurement period of two weeks provided for each participating laboratory. At the beginning and the end of the circulation, the artefacts were measured by the pilot laboratory in order to monitor the stability of the transfer standard.

#### **6. Results of the monitoring measurements**

Starting with calibration data in April 2022, the artefacts were monitored during the preparation period and at the end of the comparison when they were back at the pilot laboratory. The measurements at six rotational speed values are presented in Figure 6.1 and Figure 6.2. These figures depict the stability of the artefacts over time for the duration of the comparison.

Figure 6.1 Monitoring of the rotational speed of NLYZ-02 over the comparison period



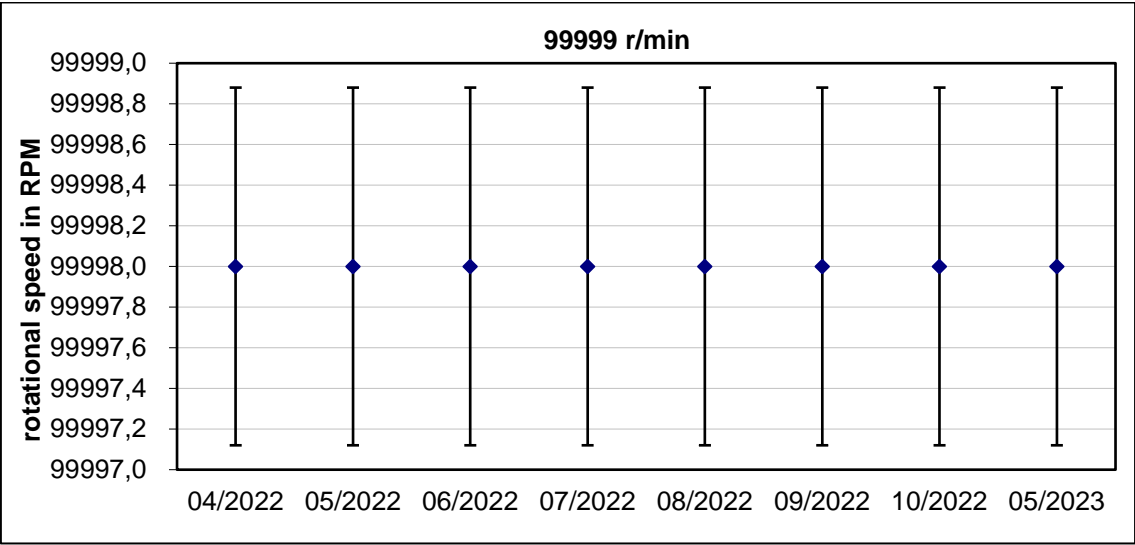
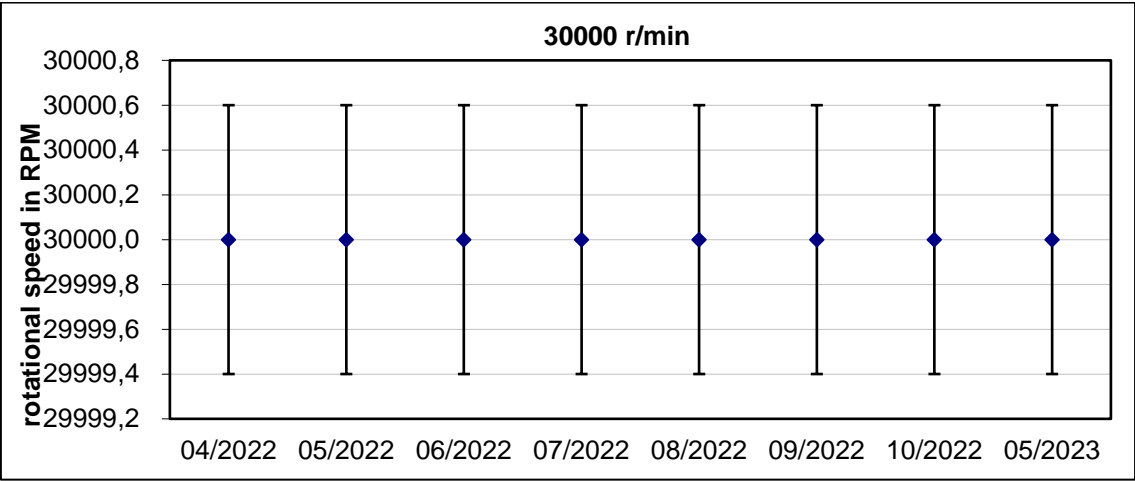
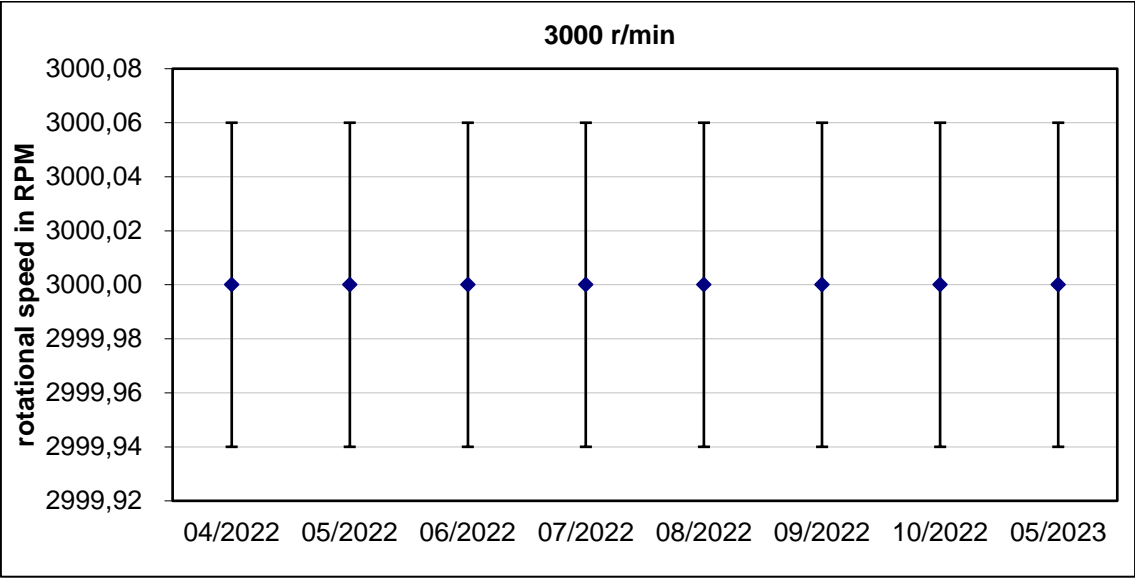
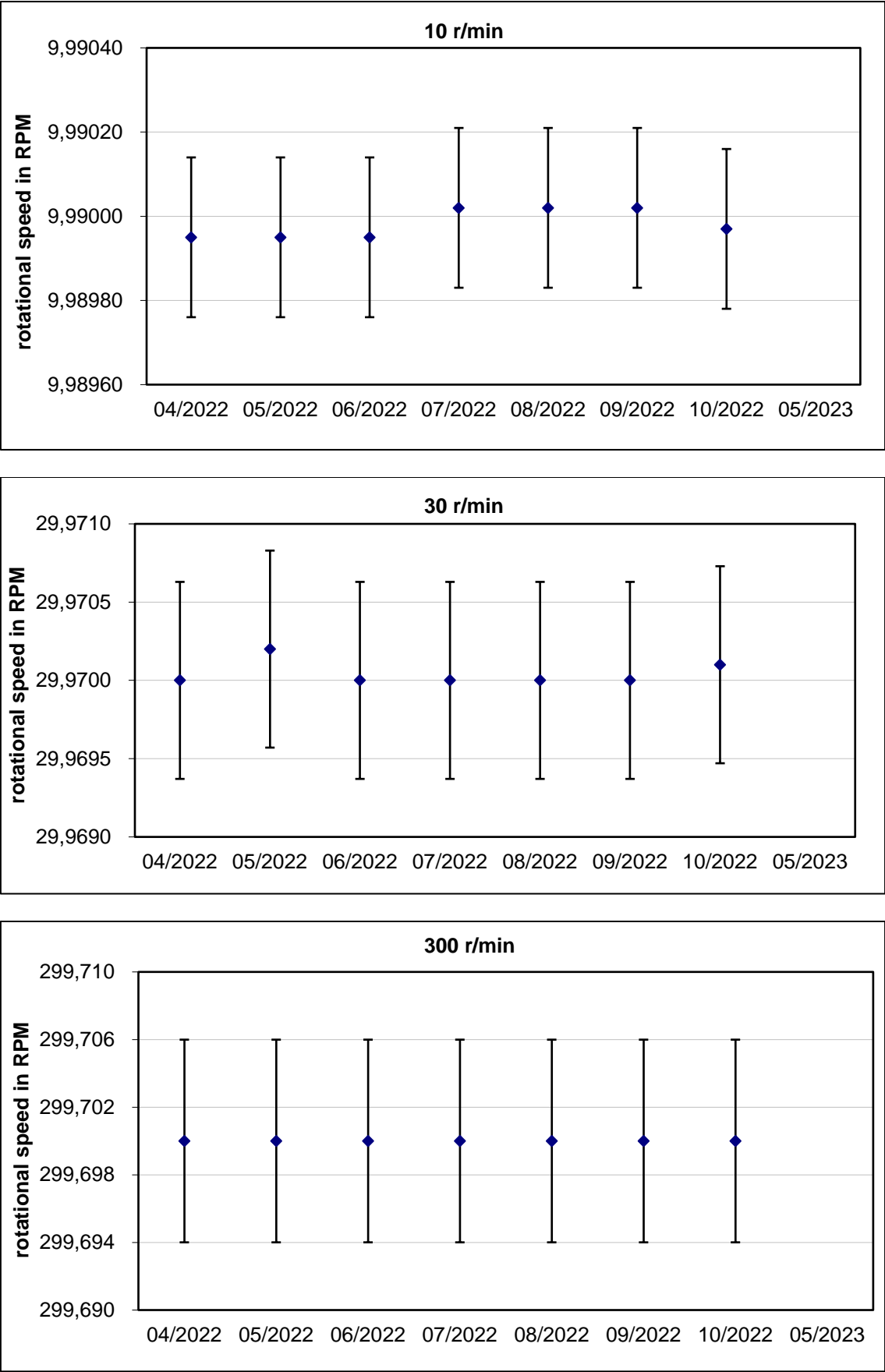
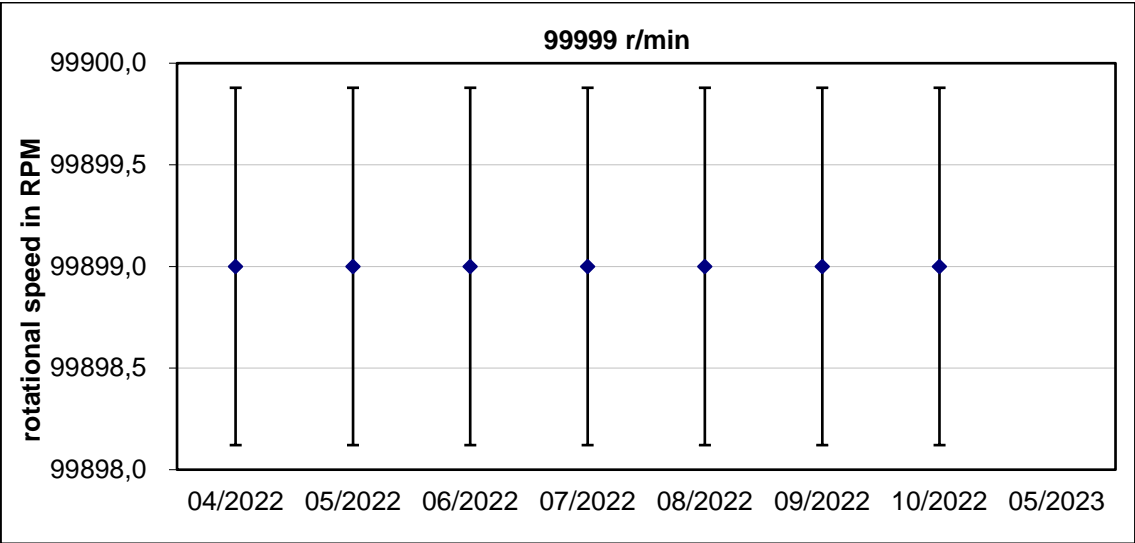
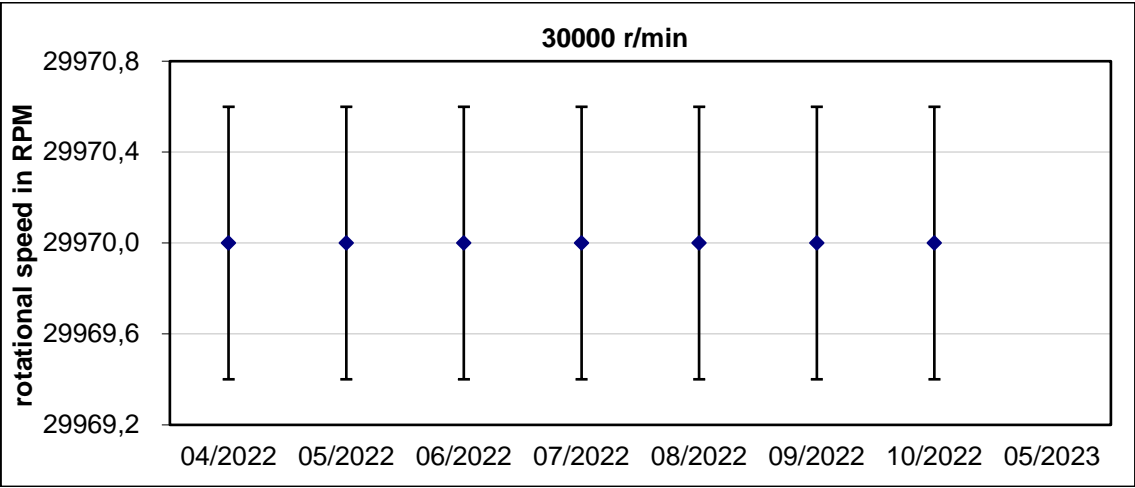
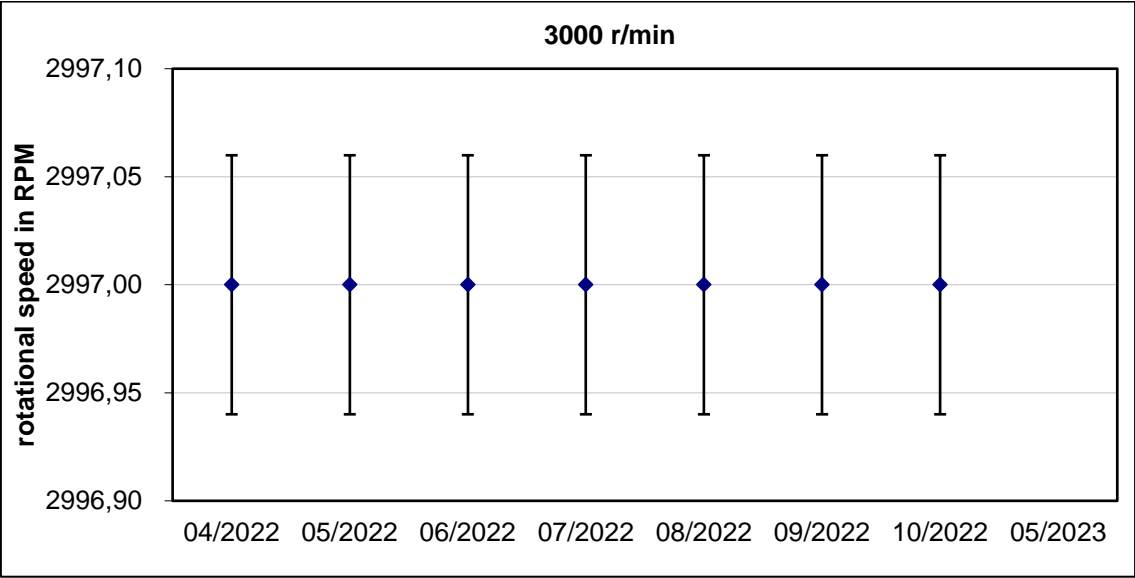


Figure 6.2 Monitoring of the rotational speed of NLYZ-03 over the comparison period





A visual inspection of the above results indicates that the artefacts were sufficiently stable during the whole period of the comparison. Note that the artefacts were also used in the following APMP supplementary comparison coded as APMP.AUV.V-S2 going



simultaneously with this comparison. Unfortunately, NLYZ-03 was opened and its laser source fixed by NIS (Egypt), the last laboratory in measurement sequence of APMP.AUV.V-S2. Therefore, the monitoring data of NLYZ-03 after that was invalid even after it was delivered back to the pilot laboratory and checked in May, 2023.

## 7. Results of the participants

The following tables report the results submitted by the participant for the comparison to the pilot laboratory using the mandatory report spreadsheet. The pilot laboratory submitted its set of results as official data to the executive secretary of CCAUV in advance to the delivery of the artefact to the participating laboratory. The results presented are in r/min for the measurement results.

Table 7.1 Participants' results for NLYZ-02 with relative expanded uncertainties ( $k = 2$ )

	NIM		NMC	
rotational speed	measurement result	rel. exp. Unc.	measurement result	rel. exp. Unc.
in r/min	in r/min	%	in r/min	%
10	9.99990	0.0010	9.98221	0.15
30	29.9998	0.0021	29.9996	0.13
50	49.9996	0.0014	49.9993	0.12
100	99.9995	0.00075	99.9970	0.14
300	300.000	0.0020	300.004	0.12
500	499.999	0.0013	499.992	0.13
1000	999.990	0.00067	999.979	0.15
3000	3000.00	0.0020	3000.06	0.12
5000	4999.96	0.0014	5000.06	0.13
10000	9999.90	0.00067	9999.94	0.12
30000	30000.0	0.0020	30000.2	0.12
50000	49999.6	0.0015	50000.4	0.13
99999	99998.0	0.00088	99998.0	0.13

Table 7.2 Participants' results for NLYZ-03 with relative expanded uncertainties ( $k = 2$ )

	NIM		NMC	
rotational speed	measurement result	rel. exp. Unc.	measurement result	rel. exp. Unc.
in r/min	in r/min	%	in r/min	%
10	9.98997	0.0019	9.95101	0.13
30	29.9701	0.0021	29.9862	0.13
50	49.9501	0.0018	49.9629	0.13
100	99.9000	0.00019	99.9783	0.14
300	299.700	0.0020	299.921	0.12
500	499.500	0.0012	499.701	0.13
1000	999.003	0.00073	999.261	0.15
3000	2997.00	0.0020	2998.01	0.12
5000	4995.00	0.0012	4995.18	0.13
10000	9990.00	0.00067	9990.35	0.12
30000	29970.0	0.0020	29971.0	0.12
50000	49950.0	0.0013	49951.3	0.13
99999	99899.0	0.00088	99901.9	0.13

## 8. Degrees of equivalence with respect to the weighted mean

The evaluation of the results was performed using a weighted mean computed with the following equations:

$$x_{WM}(n) = \sum \frac{x_i(n)}{u_i^2(n)} \cdot \left( \sum \frac{1}{u_i^2(n)} \right)^{-1} \quad (1)$$

$$u_{WM}(n) = \left( \sum \frac{1}{u_i^2(n)} \right)^{-1/2} \quad (2)$$

where the WM was calculated using the results of the participants according to [1]. In the equations above the following symbols were used:

$x_i(n)$	result of participant $i$ at rotational speed $n$
$u_i(n)$	absolute standard uncertainty of participant $i$ at rotational speed $n$
$x_{WM}(n)$	best estimate of the weighted mean (WM) at rotational speed $n$
$u_{WM}(n)$	estimated absolute standard uncertainty for the weighted mean (WM) at rotational speed $n$

Consistency checks were performed for the measurement results of NLYZ-02 and NLYZ-03. The test defined by Cox in [6, 7] was applied in order to determine the participants that are members of the largest consistent subset (LCS).

The supplementary comparison reference values (SCRVs) were finally determined by the WM using the participants that are members of the largest consistent subset (MoCS):

$x_{SCRV}(n)$	best estimate of the SCRv at rotational speed $n$
$u_{SCRV}(n)$	estimated absolute standard uncertainty of the SCRv at rotational speed $n$

Table 8.1 and 8.2 present the results of the consistency test for the NLYZ-02 and NLYZ-03 results. Cell is highlighted in yellow when  $X^2_{obs} > X^2(nu)$ .

Table 8.1: Results of the consistency test applied to all the results reported by the participants for NLYZ-02

rotational speed in r/min	number of participants	number of degrees of freedom	$X^2_{obs}$	$X^2(nu)$ with $P < 0.05$
10	2	1	5.67	3.84
30	2	1	0.00	3.84
50	2	1	0.00	3.84
100	2	1	0.00	3.84
300	2	1	0.00	3.84
500	2	1	0.00	3.84
1000	2	1	0.00	3.84
3000	2	1	0.00	3.84
5000	2	1	0.00	3.84
10000	2	1	0.00	3.84
30000	2	1	0.00	3.84
50000	2	1	0.00	3.84
99999	2	1	0.00	3.84

Table 8.2: Results of the consistency test applied to all the results reported by the participants for NLYZ-03

rotational speed in r/min	number of participants	number of degrees of freedom	$\chi^2_{obs}$	$\chi^2_{(nu)}$ with $P < 0.05$
10	2	1	36.37	3.84
30	2	1	0.64	3.84
50	2	1	0.16	3.84
100	2	1	1.30	3.84
300	2	1	1.53	3.84
500	2	1	0.36	3.84
1000	2	1	0.12	3.84
3000	2	1	0.33	3.84
5000	2	1	0.00	3.84
10000	2	1	0.00	3.84
30000	2	1	0.00	3.84
50000	2	1	0.00	3.84
99999	2	1	0.00	3.84

The results at 10 r/min presented in tables 7.1 and 7.2 were considered as not within the LCS. Considering the fact that only two participants were in this comparison, neither was excluded from the calculation of the SCR<sub>V</sub>, which should stimulate the participants to investigate the disagreement further and take appropriate corrective actions subsequently.

For the further evaluation of the comparison, the unilateral degrees of equivalence with respect to the KCRVs were calculated according to:

$$d_{i,SCRV}(n) = x_i(n) - x_{SCRV}(n) \quad (3)$$

$$u^2_{i,SCRV}(n) = \begin{cases} u^2_i(n) - u^2_{SCRV}(n) & \text{for results within the LCS} \\ u^2_i(n) + u^2_{SCRV}(n) & \text{for results not within the LCS} \end{cases} \quad (4)$$

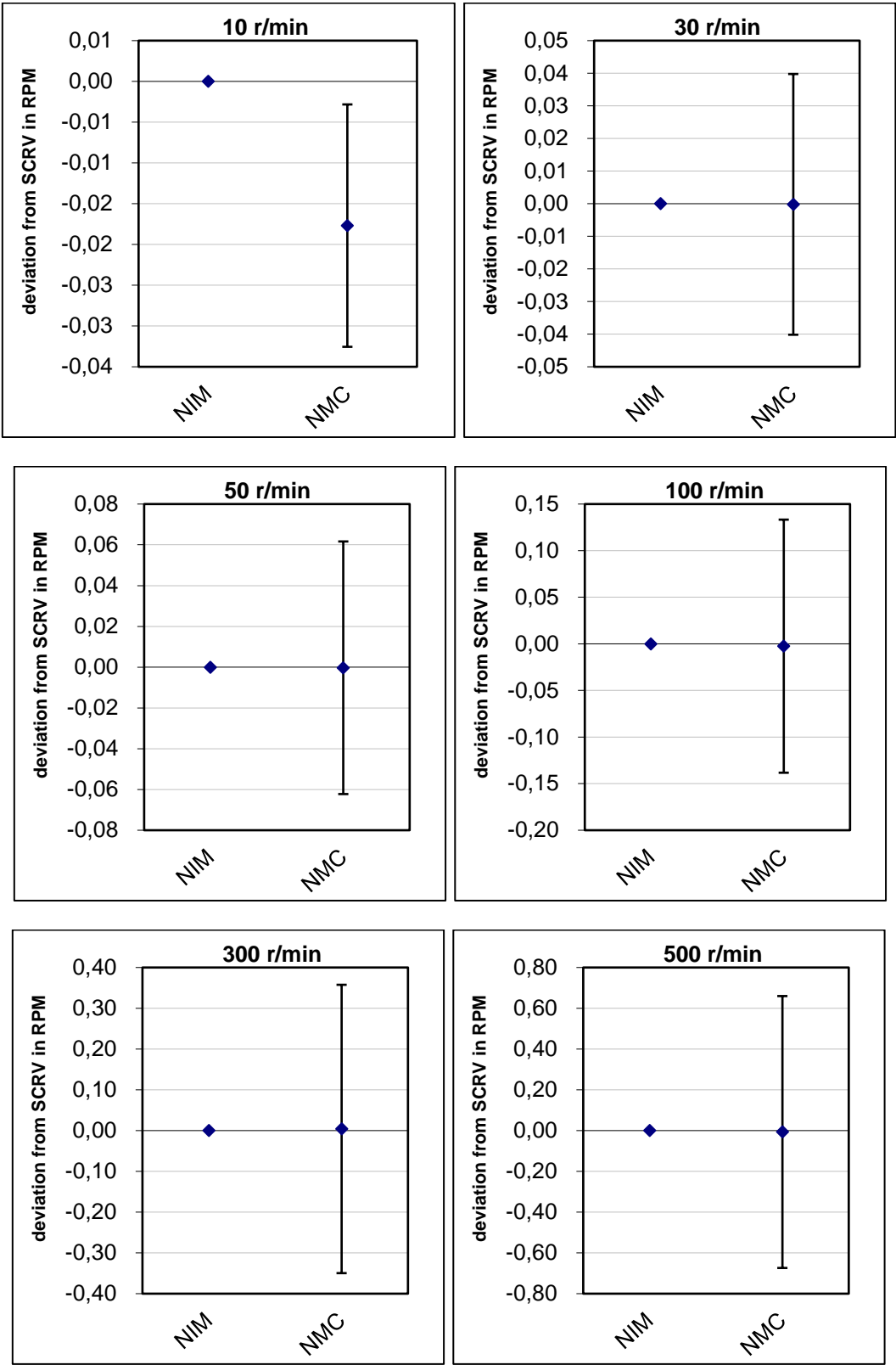
These formulas were applied for both NLYZ-02 and NLYZ-03 results. In the subsequent tables 8.3 and 8.4,  $U_i = 2u_i$  and the results are marked using a light brown background where  $|d_{i,SCRV}(n)| > 2 \cdot u_{i,SCRV}(n)$ .

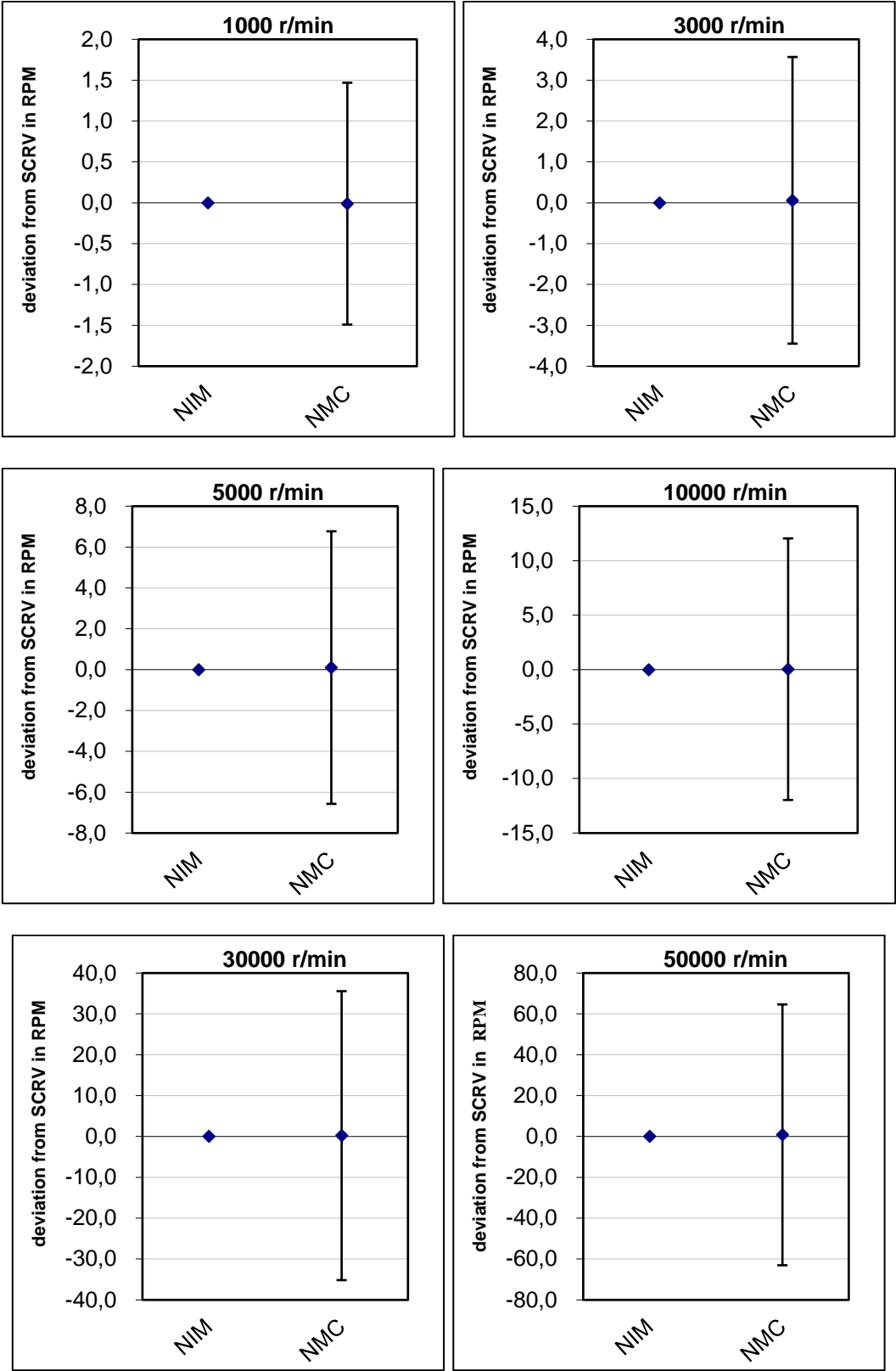
## 8.1 Results for NLYZ-02

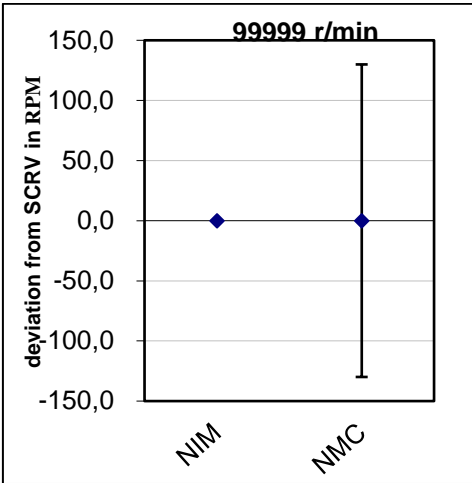
Table 8.3: Unilateral degrees of equivalence for NLYZ-02 with absolute expanded uncertainties ( $k = 2$ )

rotational speed in r/min	SCR <sub>V</sub>		NIM		NMC	
	$X_{SCRV}$	$U_{SCRV}$	$d_{L,SCRV}$	$U_{L,SCRV}$	$d_{L,SCRV}$	$U_{L,SCRV}$
	r/min		r/min		r/min	
10	9.99990	0.00010	0.00000	0.00000	-0.01769	0.01486
30	29.9998	0.00063	0.00000	0.00001	-0.00020	0.03997
50	49.9996	0.00070	0.00000	0.00001	-0.00030	0.06195
100	99.9995	0.00075	0.00000	0.00000	-0.00250	0.13579
300	300.000	0.00600	0.00000	0.00010	0.00400	0.35362
500	499.999	0.00650	0.00000	0.00006	-0.00700	0.66690
1000	999.990	0.00670	0.00000	0.00003	-0.01100	1.47983
3000	3000.000	0.05999	-0.00002	0.00103	0.05998	3.50820
5000	4999.960	0.07000	-0.00001	0.00073	0.09999	6.67095
10000	9999.900	0.06700	0.00000	0.00037	0.04000	12.0038
30000	30000.00	0.59991	-0.00006	0.01018	0.19994	35.3598
50000	49999.60	0.74994	-0.00011	0.00881	0.79989	63.8448
99999	99998.00	0.87996	0.00000	0.00596	0.00000	129.943

Figure 8.1 : Deviation of NLYZ-02 results from the SCR<sub>V</sub> for all rotational speeds of the comparison with expanded uncertainties  $U_{i,SCR_V}(k=2)$





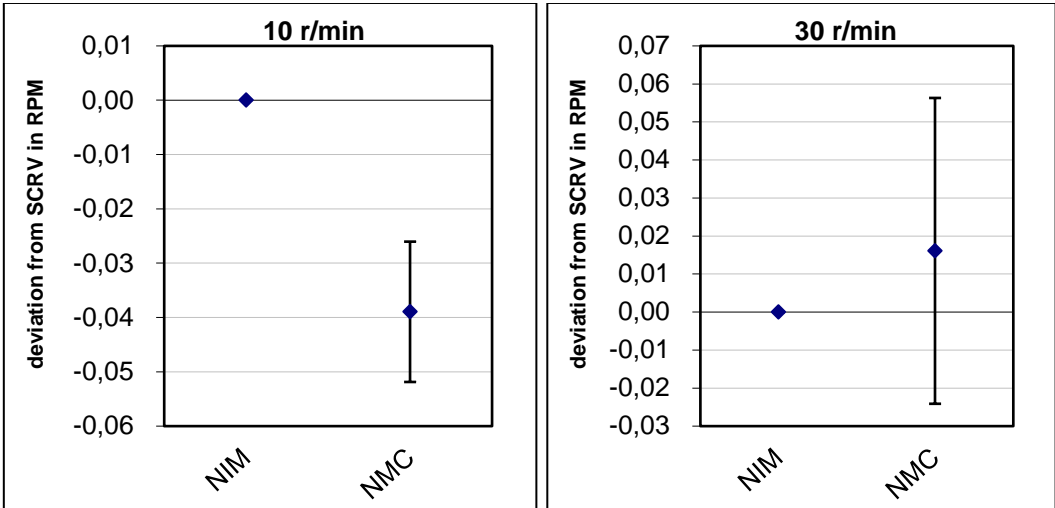


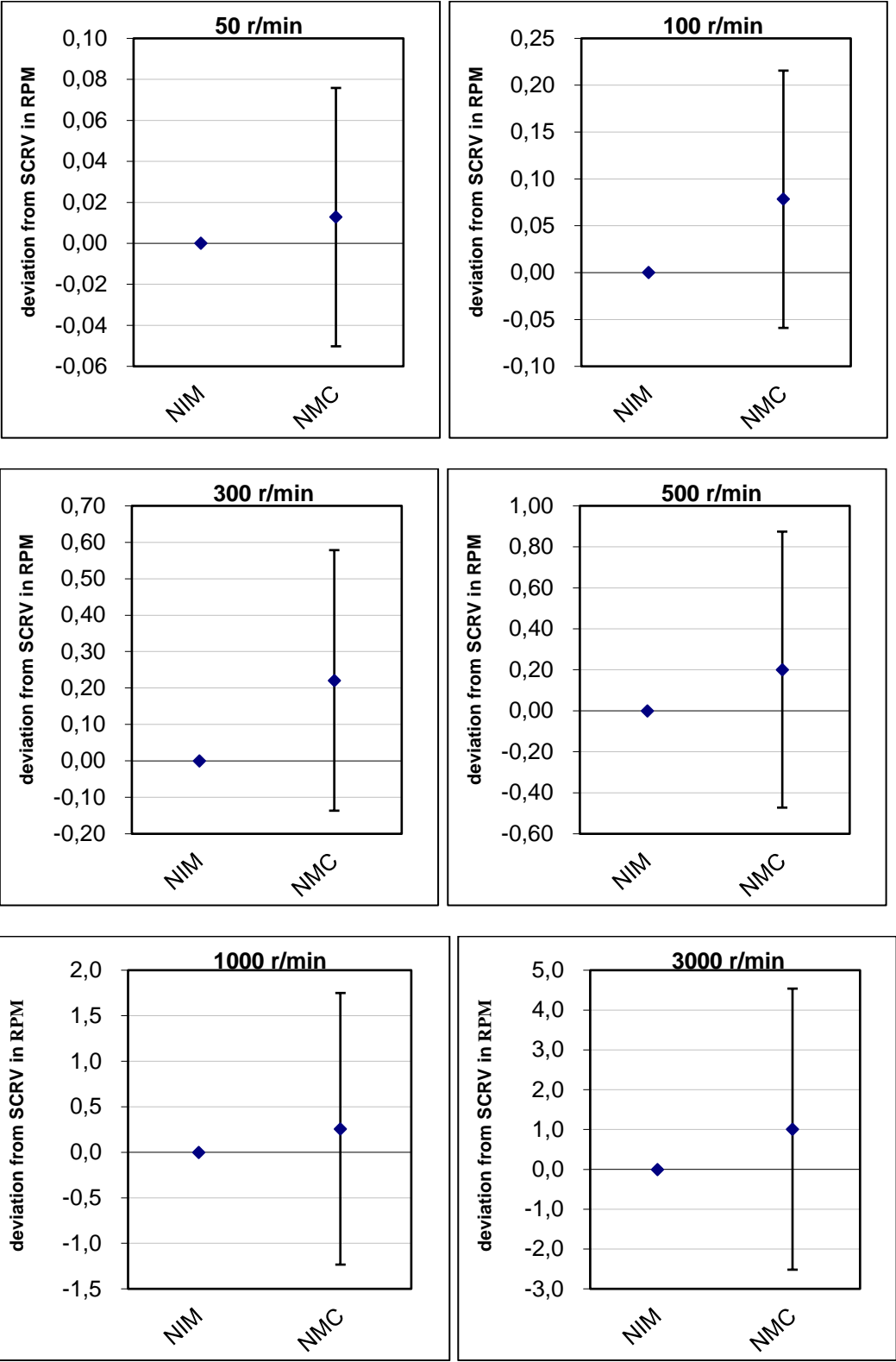
8.2 Results for NLYZ-03

Table 8.4: Unilateral degrees of equivalence for NLYZ-03 with absolute expanded uncertainties ( $k = 2$ )

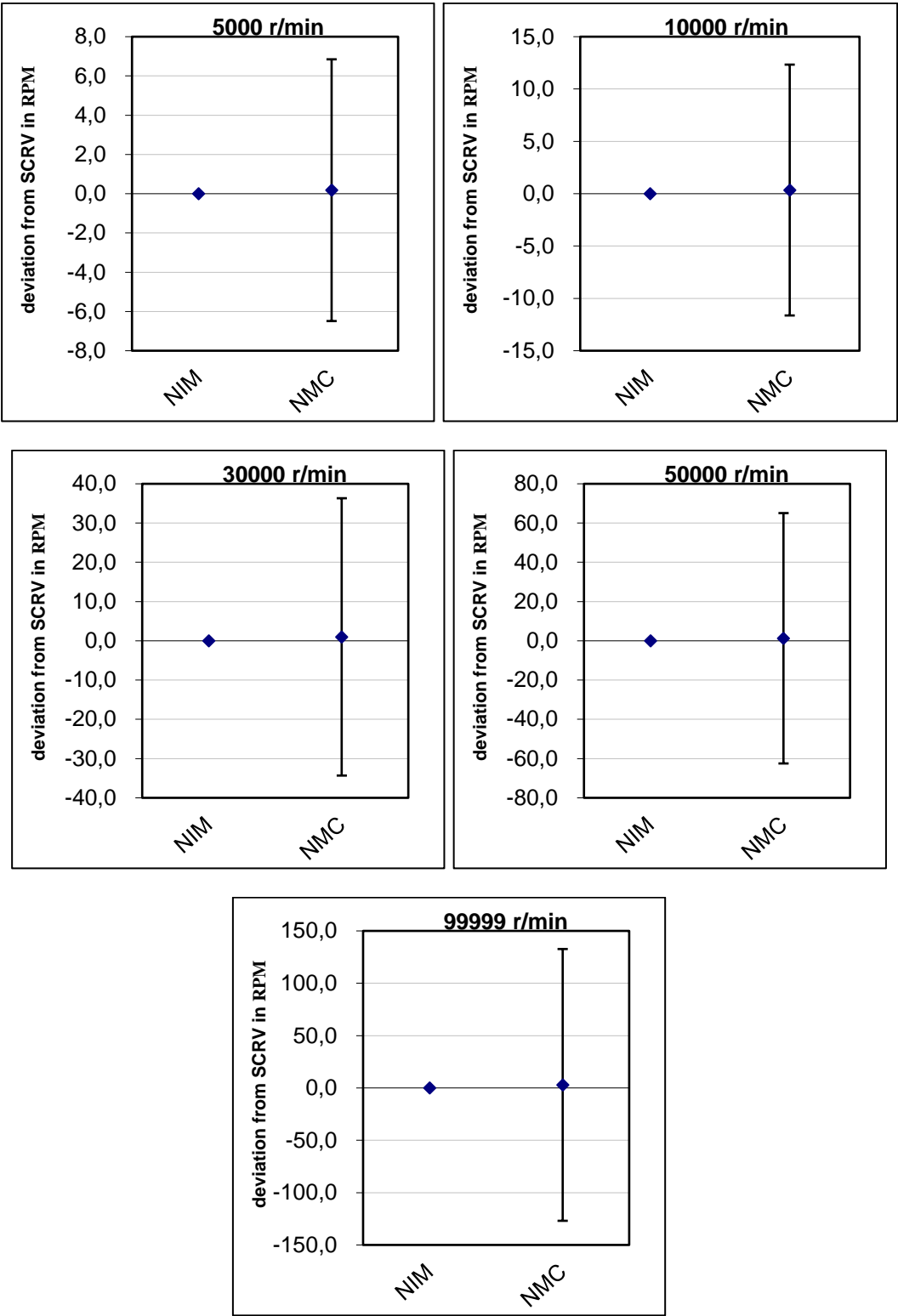
rotational speed in r/min	SCRv		NIM		NMC	
	$X_{\text{SCRv}}$	$U_{\text{SCRv}}$	$d_{\text{L,SCRv}}$	$U_{\text{L,SCRv}}$	$d_{\text{L,SCRv}}$	$U_{\text{L,SCRv}}$
	r/min		r/min		r/min	
10	9.98998	0.00019	0.00001	0.00000	-0.03895	0.01292
30	29.9701	0.00063	0.00000	0.00001	0.01610	0.04022
50	49.9501	0.00090	0.00000	0.00001	0.01280	0.06302
100	99.9000	0.00019	0.00000	0.00000	0.07830	0.13729
300	299.700	0.00599	-0.00008	0.00010	0.22094	0.35747
500	499.500	0.00599	-0.00002	0.00005	0.20098	0.67325
1000	999.003	0.00729	-0.00001	0.00004	0.25799	1.49166
3000	2997.000	0.05993	-0.00029	0.00102	1.00971	3.52832
5000	4995.000	0.05994	-0.00001	0.00054	0.17999	6.66771
10000	9990.000	0.06693	-0.00001	0.00037	0.34999	11.9816
30000	29970.00	0.59931	-0.00029	0.01017	0.9997	35.3231
50000	49950.00	0.64932	-0.00013	0.00661	1.2999	63.7797
99999	99899.00	0.87909	-0.00013	0.00595	2.8999	129.816

Figure 8.2: Deviation of NLYZ-03 results from the SCRv for all rotational speeds of the comparison with expanded uncertainties  $U_{i,\text{SCRv}}$  ( $k = 2$ )









## 9. Conclusion

The first rotational speed APMP supplementary comparison APMP.AUV.V-S1 revealed the current calibration capabilities of the 2 participants of APMP.

Both the participating laboratories provided their calibration results, which were all consistent within their declared expanded uncertainties for the measurement results except for 10 r/min. Both participants contributed to the SCRVs calculated for thirteen rotational speed comparison values.

NMC was not consistent within their declared expanded uncertainties at 10 r/min. Further improvements of their calibration devices and uncertainty evaluations will provide more accurate and reliable measurement results in the future.

## 10. Acknowledgment

This work was supported by General Administration of Quality Supervision, Inspection and Quarantine of the People's Republic of China under Contract No. ANL1613. The authors gratefully acknowledge NMC for its cooperation and supports.

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- [3] Technical Protocol of the APMP Supplementary comparison APMP.AUV.V-S1. NIM, Qiao Sun, 2022.
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## **Annex A - Technical protocol**

### **Technical Protocol of the APMP Supplementary Comparison**

#### **APMP.AUV.V-S1**

#### ***1 Task and Purpose of the Comparison***

According to the rules set up by the CIPM MRA the consultative committees of the CIPM have the responsibility to establish ‘degrees of equivalence’ (DoE) between the different measurement standards operated by the national NMIs. This is done by conducting key comparisons (KC) at different levels of the international metrological infrastructure.

However, in the sub-field of rotational speed, there is no formal comparison either at Consultative Committee (CC) level or Regional Metrology Organization Technical Committee (RMO TC) level. Therefore, during the meeting of APMP TCAUV in 2021, the decision was taken to make preparations for a supplementary comparison targeted at rotational speed (constant acceleration).

This regional supplementary comparison is organized in order to compare primary measurements of rotational speed in the range from 10 rpm to 99 999 rpm. It is the task of the comparison to calibrate two laser digital tachometers at different rotational speed values as specified in section 3. The results of this APMP supplementary comparison may serve as supporting evidence for the registration of ‘calibration and measurement capabilities’ (CMCs) in the framework of the CIPM MRA.

The measured value of rotational speed is calculated as the average of ten successive measurement results at its reflective mark. The measured value shall be given in resolution per minute (r/min or rpm) for different measurement conditions specified in section 3.

For the calibration of the laser tachometer, rotational speed standard device of mechanical generator type has to be applied.

The reported measured values and associated uncertainties will be used for the calculation of mean values of the supplementary comparison results and their associated uncertainties, as well as the deviations to the mean values with associated uncertainties.

#### ***2 Pilot Laboratory***

Pilot laboratory for this regional supplementary comparison is

Vibration and Kinematics Parameters Laboratory  
Mechanics and Acoustics Metrology Division  
National Institute of Metrology, P.R. China

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Participating laboratory for this regional supplementary comparison is

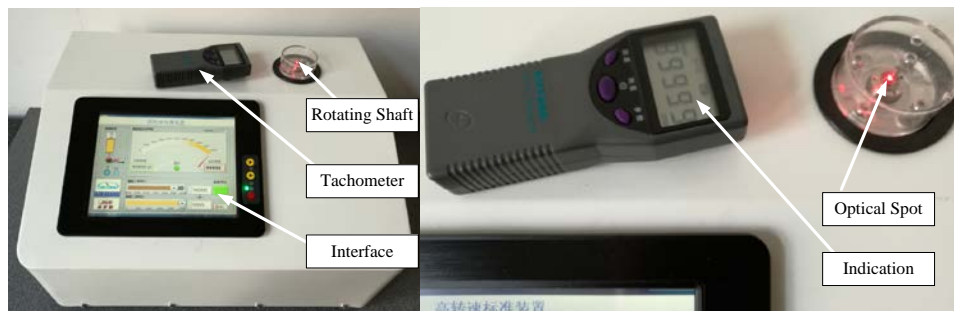
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### ***3 Device under Test and Measurement Conditions***

For the calibration task of this comparison two laser tachometers will be circulated between the participating laboratories. The laser tachometers are two EMT260C (SN: NLYZ-02 and NLYZ-03).



The measurement value of the laser tachometers is to be calibrated according to those procedures and conditions implemented by the laboratory. The measurement values reported shall be the average of ten successive measurement results at its reflective mark of a rotational speed standard device, including all reflective effects from a mechanical generator.

The rotational speed range of the measurements was agreed to be from 10 rpm to 99 999 rpm. Specifically, the laboratories are supposed to measure at the following rotational speeds (all values in rpm).

10, 30, 50, 100, 300, 500, 1 000, 3 000, 5 000, 10 000, 30 000, 50 000, 99 999.

The measurement conditions should be kept according to the laboratory's standard conditions for calibration of customers' laser tachometers for claiming their best measurement capability.

Specific conditions for the measurements of this comparison are:

- Measurement distance is within 0.1 to 1 m.
- Ambient temperature during the calibration:  $(23 \pm 3) ^\circ\text{C}$ .
- Relative humidity: max. 75 % RH.
- No flash of light affects the measurement.

#### ***4 Circulation Type, Schedule and Transportation***

The artefacts are circulated in a star type fashion with a measurement period of two weeks provided for the participating laboratory. At the beginning and the end of the circulation, the laser tachometers are measured at the pilot laboratory in order to monitor the stability.

The schedule is planned as follows:

Participant	Measurement (calendar week)	Transportation to next Participant (calendar week)
<b>NIM</b>	41-42/2022	43-44/2022
<b>NMC</b>	45-46/2022	47-48/2022
<b>NIM</b>	49-50/2022	

The cost of transportation to the next laboratory shall be covered by the participating laboratory. The laser tachometers should be delivered by an international logistic service with on-line tracking system. The transportation has to include an insurance covering a value of USD 500 in case the artefacts get damaged or lost during transportation. Hand-carry can be used as an option.

#### ***5 Measurement and Analysis Instructions***

The participating laboratories have to observe the following instructions:

- The motion of the rotational output disc of mechanical generator should be measured at the reflective mark surface in perpendicular direction.
- A tripod is recommended for installation of the laser tachometer.
- It is advised that the measurement results should be compiled from complete measurement series carried out at different days under nominally the same conditions, except that the laser tachometer is remounted. The standard deviation of the subsequent measurements should be included in the report.

#### ***6 Communication of the Results to Pilot Laboratory***

The participating laboratory will submit one printed and signed calibration report (a scanned copy sent by email to the pilot laboratory) for each laser tachometer including the following:

- a description of the calibration system used for the comparison with a photo of the system, preferably when the laser tachometer is installed,
- a description of the calibration method used and the mounting techniques for the laser tachometer,
- documented records of the ambient conditions during measurements,
- calibration results, including the relative expanded measurement uncertainty, and the

- applied coverage factor for each value,
- a detailed uncertainty budget for the system covering all components of measurement uncertainty (calculated according to GUM [1,2]). Including, among others, information on the type of uncertainty (A or B), assumed distribution function and repeatability component.

In addition, the use of the electronic spreadsheets for reporting is mandatory. The consistency between the results in electronic form and the printed and signed calibration report is the responsibility of the participating laboratories. The data submitted in the electronic spreadsheet shall be deemed as official results submitted for the comparison.

The results have to be submitted to the pilot laboratory within four weeks after the measurements have been completed.

The pilot laboratory will submit its set of results as official data to the executive secretary of CCAUV in advance to the delivery of the artefact to the participating laboratory.

## References

- [1] ISO/IEC Guide 98-3:2008 'Uncertainty of measurement -- Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)
- [2] ISO/IEC Guide 98-3:2008/Suppl 1:2008 'Propagation of distributions using a Monte Carlo method'

Results sheet for APMP.AUV.V-S1

‘Spreadsheet’ for EMT260C (NLYZ-02 and NLYZ-03)

Results sheet forAPMP.AUV.V-S1

NMI Contact Person email		<input type="checkbox"/> I confirm that the data reported here has been checked against the data reported in the NMI certificate issued for the tachometer
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rotational speed	measurement result	rel. expanded Uncertainty ( $k=2$ )
in r/min	in r/min	in %
10		
30		
50		
100		
300		
500		
1000		
3000		
5000		
10000		
30000		
50000		
99999		

Note 1: measurement results should be in 6 effective digitals.  
Note 2: relative expanded uncertainty should be in 2 effective digitals.



## **Annex B : Measurement uncertainty Budget (MUB)**

### **1 – NIM**

#### **NLYZ-02**

<i>i</i>	Uncertainty Components	Probability Distribution	10 r/min	30 r/min	50 r/min
1	Repeatability of tachometer	Normal	$3.9 \times 10^{-5}$	$1.3 \times 10^{-4}$	$1.6 \times 10^{-4}$
2	Resolution of tachometer	Rectangular	$2.9 \times 10^{-5}$	$2.9 \times 10^{-4}$	$2.9 \times 10^{-4}$
3	Uncertainty of Standard	Normal	$1.7 \times 10^{-5}$	$5.0 \times 10^{-5}$	$8.3 \times 10^{-5}$
Combined measurement uncertainty			$5.2 \times 10^{-5}$	$3.2 \times 10^{-4}$	$3.4 \times 10^{-4}$
Coverage factor			2	2	2
Expanded measurement uncertainty			$1.0 \times 10^{-4}$	$6.4 \times 10^{-4}$	$6.8 \times 10^{-4}$
Expanded relative uncertainty			$1.0 \times 10^{-5}$	$2.1 \times 10^{-5}$	$1.4 \times 10^{-5}$

<i>i</i>	Uncertainty Components	Probability Distribution	100 r/min	300 r/min	500 r/min
1	Repeatability of tachometer	Normal	$1.7 \times 10^{-5}$	$0.0 \times 10^{-3}$	$1.0 \times 10^{-3}$
2	Resolution of tachometer	Rectangular	$2.9 \times 10^{-4}$	$2.9 \times 10^{-3}$	$2.9 \times 10^{-3}$
3	Uncertainty of Standard	Normal	$1.7 \times 10^{-4}$	$5.0 \times 10^{-4}$	$8.3 \times 10^{-4}$
Combined measurement uncertainty			$3.7 \times 10^{-4}$	$2.9 \times 10^{-3}$	$3.2 \times 10^{-3}$
Coverage factor			2	2	2
Expanded measurement uncertainty			$7.5 \times 10^{-4}$	$5.9 \times 10^{-3}$	$6.3 \times 10^{-3}$
Expanded relative uncertainty			$7.5 \times 10^{-6}$	$2.0 \times 10^{-5}$	$1.3 \times 10^{-5}$

<i>i</i>	Uncertainty Components	Probability Distribution	1000 r/min	3000 r/min	5000 r/min
1	Repeatability of tachometer	Normal	$0.0 \times 10^{-3}$	$0.0 \times 10^{-3}$	$1.6 \times 10^{-2}$
2	Resolution of tachometer	Rectangular	$2.9 \times 10^{-3}$	$2.9 \times 10^{-2}$	$2.9 \times 10^{-2}$
3	Uncertainty of Standard	Normal	$1.7 \times 10^{-3}$	$5.0 \times 10^{-3}$	$8.3 \times 10^{-3}$
Combined measurement uncertainty			$3.3 \times 10^{-3}$	$2.9 \times 10^{-2}$	$3.4 \times 10^{-2}$
Coverage factor			2	2	2
Expanded measurement uncertainty			$6.7 \times 10^{-3}$	$5.9 \times 10^{-2}$	$6.8 \times 10^{-2}$
Expanded relative uncertainty			$6.7 \times 10^{-6}$	$2.0 \times 10^{-5}$	$1.4 \times 10^{-5}$

<i>i</i>	Uncertainty Components	Probability Distribution	10000 r/min	30000 r/min	50000 r/min	99999 r/min
1	Repeatability of tachometer	Normal	$0.0 \times 10^{-2}$	$0.0 \times 10^{-2}$	$1.6 \times 10^{-1}$	$0.0 \times 10^{-1}$
2	Resolution of tachometer	Rectangular	$2.9 \times 10^{-2}$	$2.9 \times 10^{-2}$	$2.9 \times 10^{-1}$	$2.9 \times 10^{-1}$
3	Uncertainty of Standard	Normal	$1.7 \times 10^{-2}$	$1.0 \times 10^{-1}$	$1.7 \times 10^{-1}$	$3.3 \times 10^{-1}$
Combined measurement uncertainty			$3.3 \times 10^{-2}$	$3.0 \times 10^{-1}$	$3.7 \times 10^{-1}$	$4.4 \times 10^{-1}$
Coverage factor			2	2	2	2
Expanded measurement uncertainty			$6.7 \times 10^{-2}$	$6.1 \times 10^{-1}$	$7.4 \times 10^{-1}$	$8.8 \times 10^{-1}$
Expanded relative uncertainty			$6.7 \times 10^{-6}$	$2.0 \times 10^{-5}$	$1.5 \times 10^{-5}$	$8.8 \times 10^{-6}$

**NLYZ-03**

<i>i</i>	Uncertainty Components	Probability Distribution	10 r/min	30 r/min	50 r/min
1	Repeatability of tachometer	Normal	$8.8 \times 10^{-5}$	$1.0 \times 10^{-4}$	$3.5 \times 10^{-4}$
2	Resolution of tachometer	Rectangular	$2.9 \times 10^{-5}$	$2.9 \times 10^{-4}$	$2.9 \times 10^{-4}$
3	Uncertainty of Standard	Normal	$1.7 \times 10^{-5}$	$5.0 \times 10^{-5}$	$8.3 \times 10^{-5}$
Combined measurement uncertainty			$0.9 \times 10^{-4}$	$3.1 \times 10^{-4}$	$4.6 \times 10^{-4}$
Coverage factor			2	2	2
Expanded measurement uncertainty			$1.9 \times 10^{-4}$	$6.2 \times 10^{-4}$	$9.2 \times 10^{-4}$
Expanded relative uncertainty			$1.9 \times 10^{-5}$	$2.1 \times 10^{-5}$	$1.8 \times 10^{-5}$

<i>i</i>	Uncertainty Components	Probability Distribution	100 r/min	300 r/min	500 r/min
1	Repeatability of tachometer	Normal	$8.7 \times 10^{-4}$	$0.0 \times 10^{-3}$	$0.0 \times 10^{-3}$
2	Resolution of tachometer	Rectangular	$2.9 \times 10^{-4}$	$2.9 \times 10^{-3}$	$2.9 \times 10^{-3}$
3	Uncertainty of Standard	Normal	$1.7 \times 10^{-4}$	$5.0 \times 10^{-4}$	$8.3 \times 10^{-4}$
Combined measurement uncertainty			$0.9 \times 10^{-3}$	$2.9 \times 10^{-3}$	$3.0 \times 10^{-3}$
Coverage factor			2	2	2
Expanded measurement uncertainty			$1.9 \times 10^{-3}$	$5.9 \times 10^{-3}$	$6.0 \times 10^{-3}$
Expanded relative uncertainty			$1.9 \times 10^{-5}$	$2.0 \times 10^{-5}$	$1.2 \times 10^{-5}$

<i>i</i>	Uncertainty Components	Probability Distribution	1000 r/min	3000 r/min	5000 r/min
1	Repeatability of tachometer	Normal	$1.5 \times 10^{-3}$	$0.0 \times 10^{-3}$	$0.0 \times 10^{-2}$
2	Resolution of tachometer	Rectangular	$2.9 \times 10^{-3}$	$2.9 \times 10^{-2}$	$2.9 \times 10^{-2}$
3	Uncertainty of Standard	Normal	$1.7 \times 10^{-3}$	$5.0 \times 10^{-3}$	$8.3 \times 10^{-3}$
Combined measurement uncertainty			$3.6 \times 10^{-3}$	$2.9 \times 10^{-2}$	$3.0 \times 10^{-2}$
Coverage factor			2	2	2
Expanded measurement uncertainty			$7.3 \times 10^{-3}$	$5.9 \times 10^{-2}$	$6.0 \times 10^{-2}$
Expanded relative uncertainty			$7.3 \times 10^{-6}$	$2.0 \times 10^{-5}$	$1.2 \times 10^{-5}$

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<i>i</i>	Uncertainty Components	Probability Distribution	10000 r/min	30000 r/min	50000 r/min	99999 r/min
1	Repeatability of tachometer	Normal	$0.0 \times 10^{-2}$	$0.0 \times 10^{-2}$	$0.0 \times 10^{-1}$	$0.0 \times 10^{-1}$
2	Resolution of tachometer	Rectangular	$2.9 \times 10^{-2}$	$2.9 \times 10^{-2}$	$2.9 \times 10^{-1}$	$2.9 \times 10^{-1}$
3	Uncertainty of Standard	Normal	$1.7 \times 10^{-2}$	$1.0 \times 10^{-1}$	$1.7 \times 10^{-1}$	$3.3 \times 10^{-1}$
Combined measurement uncertainty			$3.3 \times 10^{-2}$	$3.0 \times 10^{-1}$	$3.3 \times 10^{-1}$	$4.4 \times 10^{-1}$
Coverage factor			2	2	2	2
Expanded measurement uncertainty			$6.7 \times 10^{-2}$	$6.1 \times 10^{-1}$	$6.7 \times 10^{-1}$	$8.8 \times 10^{-1}$
Expanded relative uncertainty			$6.7 \times 10^{-6}$	$2.0 \times 10^{-5}$	$1.3 \times 10^{-5}$	$8.8 \times 10^{-6}$

2 –NMC

NLYZ-02

Calculation of Rotational Speed - Dependent MU Contribution															
i	Uncertainty Components	Probability Distribution	10 RPM (%)	30 RPM (%)	50 RPM (%)	100 RPM (%)	300 RPM (%)	500 RPM (%)	1000 RPM (%)	3000 RPM (%)	5000 RPM (%)	10000 RPM (%)	30000 RPM (%)	50000 RPM (%)	99999 RPM (%)
1	Repeatability of reference standard	normal	7.84E-03	3.39E-03	3.67E-03	1.44E-03	3.31E-04	3.38E-04	2.11E-04	9.39E-05	4.62E-05	1.41E-05	6.67E-06	1.42E-05	1.28E-05
2	Repeatability of UUT (NLYZ-02)	normal	4.17E-02	1.07E-02	6.88E-03	3.99E-03	2.19E-03	1.48E-03	7.22E-04	5.44E-04	3.27E-04	2.65E-03	1.09E-03	9.04E-04	4.47E-04
3	Resolution of reference standard	retangular	2.89E-05	9.62E-05	5.77E-05	2.89E-05	9.62E-05	5.77E-05	2.89E-05	9.62E-05	5.77E-05	2.89E-05	9.62E-05	5.77E-05	2.89E-05
4	Resolution of UUT (NLYZ-02)	retangular	2.89E-04	9.62E-04	5.77E-04	2.89E-04	9.62E-04	5.77E-04	2.89E-04	9.62E-04	5.77E-04	2.89E-04	9.62E-04	5.77E-04	2.89E-04
5	Accuracy of reference standard (Keysight 53230A)	retangular	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11
6	Stability of Tachometer Calibrator RPM (TIS)	normal	6.12E-02	6.57E-02	6.15E-02	6.78E-02	5.89E-02	6.67E-02	7.40E-02	5.85E-02	6.67E-02	6.00E-02	5.89E-02	6.38E-02	6.50E-02
Combined relative measurement uncertainty			0.074	0.067	0.062	0.068	0.059	0.067	0.074	0.058	0.067	0.060	0.059	0.064	0.065
Effective degree of freedom			91	13365	54836	742711	4831676	36542041	986275068	1197813273	15662389434	2380522	77331098	223663411	4009672416
Coverage factor			2	2	2	2	2	2	2	2	2	2	2	2	2
Expanded uncertainty (%)			0.15	0.13	0.12	0.14	0.12	0.13	0.15	0.12	0.13	0.12	0.12	0.13	0.13

NLYZ-03

Calculation of Rotational Speed -Dependent MU Contribution															
i	Uncertainty Components	Probability Distribution	10 RPM (%)	30 RPM (%)	50 RPM (%)	100 RPM (%)	300 RPM (%)	500 RPM (%)	1000 RPM (%)	3000 RPM (%)	5000 RPM (%)	10000 RPM (%)	30000 RPM (%)	50000 RPM (%)	99999 RPM (%)
1	Repeatability of reference standard	normal	8.72E-03	2.44E-03	2.59E-03	1.98E-03	3.58E-04	3.50E-04	2.29E-04	4.81E-05	5.95E-05	1.02E-05	5.85E-06	3.28E-06	1.28E-05
2	Repeatability of UUT (NLYZ-03)	normal	1.99E-02	1.34E-02	1.39E-02	1.09E-02	9.10E-03	9.62E-03	9.82E-03	6.66E-03	2.08E-03	7.93E-04	8.61E-04	6.01E-04	2.77E-04
3	Resolution of reference standard	retangular	2.89E-05	9.62E-05	5.77E-05	2.89E-05	9.62E-05	5.77E-05	2.89E-05	9.62E-05	5.77E-05	2.89E-05	9.62E-05	5.77E-05	2.89E-05
4	Resolution of UUT (NLYZ-03)	retangular	2.90E-04	9.63E-04	5.78E-04	2.89E-04	9.63E-04	5.78E-04	2.89E-04	9.63E-04	5.78E-04	2.89E-04	9.63E-04	5.78E-04	2.89E-04
5	Accuracy of reference standard (Keysight 53230A)	retangular	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11	5.77E-11
6	Stability of Tachometer Calibrator RPM (TIS)	normal	6.12E-02	6.57E-02	6.15E-02	6.78E-02	5.89E-02	6.67E-02	7.40E-02	5.85E-02	6.67E-02	6.00E-02	5.89E-02	6.38E-02	6.50E-02
Combined relative measurement uncertainty			0.065	0.067	0.063	0.069	0.060	0.067	0.075	0.059	0.067	0.060	0.059	0.064	0.065
Effective degree of freedom			987	5699	3807	14378	16539	21662	30059	54851	9455525	294172761	197147903	1149361260	27180563076
Coverage factor			2	2	2	2	2	2	2	2	2	2	2	2	2
Expanded uncertainty (%)			0.13	0.13	0.13	0.14	0.12	0.13	0.15	0.12	0.13	0.12	0.12	0.13	0.13