## Notes on the analysis procedure used in the 2025 update to the CIPM list "Recommended values of standard frequencies"

The analysis underpinning the update to the list was performed by a sub-group of the CCL-CCTF Frequency Standards Working Group (WGFS), consisting of Sebastien Bize (LNE-OP), Francesca Collini (BIPM), Tetsuya Ido (NICT), Andrew Ludlow (NIST), Helen Margolis (NPL), Gianna Panfilo (BIPM) and Marco Pizzocaro (INRIM). They were supported in this work by Roxanne Siadat (NPL).

### Proposals for new additions to the list

In the responses received to the questionnaire issued by the WGFS, two new transitions were proposed for inclusion in the list:

1) $^{176}Lu^+$ ($^1S_0 - {}^3D_1$) standard frequency at ~ 354 THz (defined as the hyperfine average of $^1S_0 - {}^3D_1$ transitions), measured with a fractional uncertainty of $9.2\times10^{-16}$ [1]
2) $^{138}Ba^+$ $^2S_{1/2} - {}^2D_{5/2}$ transition at ~ 170 THz, measured with a fractional uncertainty of $2.8\times10^{-15}$ [2]

Only a single absolute frequency measurement is available for either transition so far. Both systems were considered by the WGFS to be promising for the future, but they considered that the uncertainty associated with the $^{138}Ba^+$ $^2S_{1/2} - {}^2D_{5/2}$ transition did not yet meet the threshold for inclusion. Hence only the $^{176}Lu^+$ transition was added, with an adopted value based on the single available measurement, but with a standard uncertainty increased by a factor of three to reflect the scarcity of the input data.

### Global adjustment of clock comparison data

A global analysis of other measurements of frequency ratios published in peer-reviewed publications and available to the WGFS by the cut-off date of 30th June 2025 was carried out following the methods presented in [3–9]. This analysis, or least-squares adjustment, determined the frequency of 14 transitions which were either already adopted as secondary representations of the second (SRS) [9] or considered as candidates for SRS. This list of 14 transitions remains unchanged from the previous analysis in 2021, and the conventional numbering used then [9] is retained.

The input data file for the 2025 update to the recommended frequency values includes 153 measurements [10]. A full list of references for the published data considered is provided in a separate file [11]. However, 7 of these ($q_{51}$, $q_{117}$, $q_{127}$, $q_{142}$, $q_{147}$, $q_{151}$, $q_{153}$) were effectively excluded from the calculation by expanding the published uncertainties by a large factor.

In the case of $q_{51}$ this was because the same data was used to contribute to $q_{90}$ [9]. The others were excluded in consultation with the team responsible for operation of the PTB-Sr3 optical frequency standard, due to observed inconsistencies in its frequency over time.

Allowing for these exclusions, the 2025 analysis took into account 146 measurements, 56 of which were frequency ratios and 90 of which were absolute frequency measurements (i.e. ratios to the $^{133}Cs$ frequency).

As in the 2021 update [9], a few other adjustments were made to the input data, to improve its consistency, or to avoid unphysical values of correlation coefficients:

- The published uncertainty of outlier $q_1$ was multiplied by three, as in the 2021 update [9]
- The new data available for $^{88}Sr^+$ now indicates that $q_{31}$ is a significant outlier, and its published uncertainty was therefore multiplied by three.
- The published uncertainty of outlier $q_{52}$ was multiplied by six, as in the 2021 update [9]
- The uncertainty of $q_{73}$ was increased from $1.5\times10^{-16}$ to $1.65\times10^{-16}$ in order to avoid otherwise unphysical values of correlation coefficients (see [9] for further details)

- The uncertainty of $q_{90}$ was increased from $1.8\times10^{-16}$ to $4.4\times10^{-16}$, because recent investigations indicate that the lack of Doppler stabilization to the lattice end mirror may have caused a shift of up to $4\times10^{-16}$ in this measurement. NICT advice to apply an added uncertainty of $4\times10^{-16}$ to the reported result was therefore followed.
- The uncertainty of $q_{98}$ was increased from $1.3\times10^{-16}$ to $1.6\times10^{-16}$ in order to avoid otherwise unphysical values of correlation coefficients (see [9] for further details)

As in the previous update to the recommended frequencies [9], detailed attention was given to correlations between the input data. In the 2025 update, a total of 1168 correlation coefficients were estimated:

- 927 of these were due to the use of the same primary or secondary frequency standards to access the SI second. In this case the method of estimation used previously [9] was adapted to account for the increased number and weight of optical frequency standards contributing to TAI [12].
- The remaining 241 coefficients arose from a variety of other sources of correlation and were determined in a series of different analyses, following the guidelines [13].

The analysis provides crucial information about the consistency of the input data. An initial adjustment indicated that there were inconsistencies in the global body of published data. In the final adjustment, a random noise of approximately $5.3\times10^{-17}$ was added in quadrature to all fractional uncertainties of input ratios in order to recover a Birge ratio of unity [14]. This value is indicative of the level of inconsistencies in the input data.

The results of the adjustment consist of the values of recommended frequencies [15] and a full covariance matrix [16]. The recommended value for each transition is the direct result from the adjustment, rounded as deemed adequate with respect to the recommended uncertainty. The recommended standard uncertainty is 2.35 times the square root of the corresponding diagonal entry in the covariance matrix of the adjustment. This multiplication factor was recommended by the CCL-CCTF WGFS to allow for as-yet-unidentified sources of uncertainty or correlation, in keeping with the purpose of the recommended values [8], and is separate from any coverage factor used to calculate expanded uncertainties or confidence intervals, as described in [17].

The adjustment also provides optimized values for frequency ratios between all pairs of transitions in the input data set, which can be found in [15]. These ratio values are consistent with the recommended frequency values within rounding, but are reported with a higher number of digits. Their uncertainties should be multiplied by 2.35 for consistency with the recommended frequency values.

**References**


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[3] H. S. Margolis and P. Gill, Least-squares analysis of clock frequency comparison data to deduced optimized frequency and frequency ratio values, Metrologia 52, 628–34 (2015)

[4] L. Robertsson, On the evaluation of ultra-high-precision frequency ratio measuremets: examining closed loops in a graph theory framework, Metrologia 53, 1272–80 (2016)

[5] G. Panfilo, communication to the CCL-CCTF WGFS; A new implementation of [4] was realized in MatLab at the BIPM (2020)

[6] C. Oates, communication to the CCL-CCTF WGFS; An independent program was developed in Mathematica at NIST (2017)

[7] M. Pizzocaro, communication to the CCL-CCTF WGFS; An independent program was developed in Python at INRIM (2025) (available at https://github.com/INRIM/GaussMarkovSRS)

[8] F. Riehle, P. Gill, F. Arias, L. Robertsson, The CIPM list of recommended frequency standard values: guidelines and procedures, Metrologia 55, 188–200 (2018)

[9] H. S. Margolis, G. Panfilo, G. Petit, C. Oates, T. Ido and S. Bize, The CIPM list `Recommended values of standard frequencies': 2021 update, Metrologia 61, 035005 (2024)

[10] ClockInputData2025_mod_basis_n=153_TAI_05092025_ExpandUncV1_AdHocCorrV2_2.dat

[11] Reference_list_for_input_data_2025.pdf

[12] Guidelines_correlations_TAI.pdf

[13] ROCIT_guidelines_on_correlations.pdf

[14] The exact value of the random noise term used was $5.324\,065\,020\times10^{-17}$

[15] ClockOutputData2025_mod_basis_n=153_TAI_05092025_ExpandUncV1_AdHocCorrV2_2 (1).dat

[16] ClockOutputCovData2025_mod_basis_n=153_TAI_05092025_ExpandUncV1_AdHocCorrV2_2.dat

[17] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and IOML: Evaluation of measurement data – guide to the expression of uncertainty in measurement; Joint Committee for Guides in Metrology, JCGM vol. 100 (available at: https://www.bipm.org/en/publications/guides/gum.html)