CCL-CCTF-WGFS 2025 adjustment of standard frequencies: Accounting for correlations due to the access to the SI second

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1 Introduction

The CCL-CCTF-WGFS subgroup worked throughout 2024–2025 to perform the computation of the 2025 list of recommended frequencies (see `https://www.bipm.org/en/publications/mises-en-pratique/standard-frequencies-second`). The list of 14 transitions remains unchanged from the fit calculated in 2021, and the conventional numbering is retained.

nu1	$^{115}In^{+}$
nu2	^{1}H
nu3	^{199}Hg
nu4	$^{27}Al^{+}$
nu5	$^{199}Hg^{+}$
nu6	$^{171}Yb^{+}$(E2)
nu7	$^{171}Yb^{+}$(E3)
nu8	^{171}Yb
nu9	^{40}Ca
nu10	$^{88}Sr^{+}$
nu11	^{88}Sr
nu12	^{87}Sr
nu13	$^{40}Ca^{+}$
nu14	^{87}Rb

Measurements are associated either with one value nuI in the case of an absolute frequency measurement with respect to the SI second, or with two values nuI and nuJ in the case of measurements of a frequency ratio nuI / nuJ. The conventional identification of measurements is qX where X is an index varying from 1 to the total number of measurements (153 in the 2025 adjustment).

The WGFS decided to include correlations between the different measurements wherever possible, to ensure that the formal uncertainties obtained from the adjustment may be considered as realistic as possible. This memo describes the computation of correlations between absolute measurements when using the same Primary Frequency Standard (PFS) or Secondary Frequency Standard (SFS) to access the SI second. In particular, it is an extension and update of the existing note documenting the calculation details from 2021 [1]. It further expands the correlations to include cases where a species, used as a standard in the fit, is also involved in the TAI computation.

2 Assumptions

The aim is to estimate the correlation between absolute frequency measurements when they use the same PSFS to access the SI second, either directly or through d_{TAI}. To reach this goal, we assume that [2, 3]:

- The period from 01/2014 to 01/2025 is considered to read *etyy.mm* files (which can be found here https://webtai.bipm.org/ftp/pub/tai/ other-products/etoile/), where all the PSFS used to compute d_{TAI} for that specific month are listed.

- The standards which contributed with weights lower than 0.05% are not considered.

- Only the part of u_B that is correlated from month to month should be used in the calculation. This part can be indicated with u_{Bsta}. As five Primary and Secondary Frequency Standards (PSFS) make an important weight (at the beginning of the period on average more than 90%) of the monthly estimation of d_{TAI}, a detailed analysis has been carried out only for PTB-CsF1, PTB-CsF2, LNEOP-FO1, LNEOP-FO2, LNEOP-FORb [1]. For all other PSFS, the full u_B value has been used. Given the increasing diversity of PSFS contributing to TAI, it is likely that a different type of analysis will be required in the future.
  The uncertainty u_{Srep} is also taken into account in computing the final correlated part of the uncertainty.

  $\mathbf{U_{Bsta}}$ for PTB fountains
  PTB-CsF1:
  201401-201902: 3.1×10^{-16}
  201903-202306: 2.0×10^{-16}
  202307-202408: 3.0×10^{-16}
  202409-202501: 2.0×10^{-16}
  PTB-CsF2:
  201401-202501: $min(2.1 \times 10^{-16}, u_B)$

  $\mathbf{U_{Bsta}}$ for LNEOP fountains
  Among the main components of u_{Bsta}, the Zeeman, Blackbody and MW lensing shifts were constant but there were variations over the years for the collisional and gravitational shift uncertainty.u_{Bsta} computation was not necessary for LNEOP-FOM which has higher uncertainty and fewer

[1] LNE-SYRTE is now LNE-OP : Observatoire de Paris/Laboratoire Temps Espace. These are the updated names of the same standards previously designated SYRTE-FO1, SYRTE-FO2, SYRTE-FO2(Rb) , respectively.

evaluations. For the other fountains, the value and uncertainty of the gravitational shift was changed in March 2018.
LNEOP-FO1:
201401-201802: 2.5×10^{-16}
201803-201808: 2.3×10^{-16}
201809-202204: 1.7×10^{-16}
202205-202309: 1.2×10^{-16}
202310-202405: 1.3×10^{-16}
202406-202501: 1.7×10^{-16}
LNEOP-FO2:
201401-201509: 1.8×10^{-16}
201510-201802: 1.5×10^{-16}
201803-202012: 1.2×10^{-16}
202101-202203: 1.1×10^{-16}
202204-202501: 1.0×10^{-16}
LNEOP-FORb:
201401-201509: 2.3×10^{-16}
201510-201802: 2.1×10^{-16}
201803-202104: 1.8×10^{-16}
202105-202204: 1.7×10^{-16}
202205-202309: 1.2×10^{-16}
202310-202405: 1.3×10^{-16}
202406-202501: 1.7×10^{-16}

- 937 correlation coefficients computed in this analysis were found to be greater than 0.001. They were used in the adjustment of reference frequencies, with the exception of 10 where a specific calculation provided a more accurate estimation. These 10 are:
  r(q_{14}, q_{18}): [Godun2014], 171Yb+(E2)-171Yb+(E3)
  r(q_{46}, q_{47}): [LeTargat2013-Lodewyck2016], 87Sr
  r(q_{50}, q_{90}): [Hachisu2017-Nemitz2021], 87Sr
  r(q_{110}, q_{111}): [Tofful2024] 171Yb+(E3)-171Yb+(E3)
  r(q_{111}, q_{113}): [Tofful2024] 171Yb+(E3)-171Yb+(E3)
  r(q_{112}, q_{113}): [Tofful2024] 171Yb+(E3)-171Yb+(E3)
  r(q_{131}, q_{132}): [Kobayashi2022] 171Yb-171Yb
  r(q_{24}, q_{122}): [Pizzocaro2017-Goti2023] 171Yb-171Yb
  r(q_{89}, q_{132}): [Kobayashi2020-Kobayashi2022] 171Yb-171Yb
  r(q_{98}, q_{136}): [Lange2021-Hausser2025] 171Yb+(E3)-115In+
  A further 231 "ad-hoc" correlation coefficients were evaluated, bringing the total number of coefficients considered in the fit to 1168.

3 Evaluation

The idea is to evaluate the correlations due to TAI of the absolute frequency measurements in the list of input data. In 2021, the relation was found considering that Optical Frequency Standards (OFSs) were not included in TAI evaluation, but now some OFSs contributes to TAI with a relevant weight (for several months) so the relation used need to be expanded to take in to account this part of correlations. If we indicated with $q_x = OFS_x - TAI_x$ and $q_y = OFS_y - TAI_y$, we want to evaluate the correlations between q_x and q_y. We have indicated here with TAI_x the frequency of TAI over the period of the evaluation of OFS_x and with TAI_y the frequency of TAI over the period of the evaluation of OFS_y. We consider the monthly frequency of TAI calculated as the weighted average of the PSFS present over the last year and published in the file *etyy.mm*. We have however to take into account that q_x can be performed over different months of TAI calculation and the values of uncertainties of PSFS can change. So we need to index the month used in TAI calculation. We can report the relation in this way:

$$TAI_x = \sum_{i_x} w_{i_x} \sum_k w_{i_x,k}(TAI - PSFS_{i_x,k}) \tag{1}$$

and

$$TAI_y = \sum_{i_y} w_{i_y} \sum_k w_{i_y,k}(TAI - PSFS_{i_y,k}) \tag{2}$$

Explicitly reporting the expression for the calculation of the correlation between $q_x = OFS_x - TAI_x$ and $q_y = OFS_y - TAI_y$ we have:

$$\begin{aligned} COV(q_x, q_y) &= COV(OFS_x - TAI_x, OFS_y - TAI_y) \\ &= COV(OFS_x - \sum_{i_x} w_{i_x} \sum_k w_{i_x,k}(TAI - PSFS_{i_x,k}), \\ OFS_y &- \sum_{i_y} w_{i_y} \sum_k w_{i_y,k}(TAI - PSFS_{i_y,k})) \end{aligned} \tag{3}$$

From one of the properties of covariances we have:

$$\begin{aligned} COV(q_x, q_y) = {} & COV(OFS_x, OFS_y) + COV(OFS_x, \sum_{i_y} w_{i_y} \sum_k w_{i_y,k}(TAI - PSFS_{i_y,k})) + \\ & + COV(\sum_{i_x} w_{i_x} \sum_k w_{i_x,k}(TAI - PSFS_{i_x,k}), OFS_y) + \\ & + COV(\sum_{i_x} w_{i_x} \sum_k w_{i_x,k}(TAI - PSFS_{i_x,k}), \sum_{i_y} w_{i_y} \sum_k w_{i_y,k}(TAI - PSFS_{i_y,k})) \end{aligned} \tag{4}$$

In 2021 the only term considered different from zero is the last one and the result gives the following correlation coefficient:

$$r(q_x, q_y) = \frac{\sum_{i_x,i_y} w_{i_x} w_{i_y} \sum_k w_{i_x,k} u_{\mathrm{bS}_{i_x,k}} w_{i_y,k} u_{\mathrm{bS}_{i_y,k}}}{u_x u_y} \tag{5}$$

where i_x and i_y index the months used to access the SI second for measurements q_x and q_y and w_{i_x} and w_{i_y} are the weights of month i_x and i_y respectively. The PSFS are labeled with the index k.

For measurements accessing the SI second through d_{TAI} $w_{i,k}$ is the weight of standard k in the estimation of d_{TAI} for month i. The set of weights $w_{i,k}$ for all standards for each month i is collected from the monthly files *etyy.mm*, which provide the fractional frequency of the free atomic time scale EAL (Échelle Atomique Libre) as estimated from primary and secondary frequency standards. For measurements accessing the SI second through local standards, $w_{i,k}$ is the weight of each local PSFS used in the comparisons. The total uncertainties of the two measurements are u_x and u_y. However, when $i_x = i_y = i$ (two measurements made in a single common month), correlation is through the total uncertainty $u_{i,k}$ of the standard k in the estimation of d_{TAI} for this month, not through $u_{\mathrm{bS}_{i,k}}$. In all cases u_{bS} may vary with time so the month of operation needs to be specified. In the current situation, where it is possible to have OFS included in the list of PSFS used to obtain the frequency of TAI, the second and third terms can be significant.

The following approximation has been applied from 4 to 5:

$$COV(PSFS_{i_x,k}, PSFS_{i_y,k}) = u_{\mathrm{bS}_{i_x,k}} u_{\mathrm{bS}_{i_y,k}} \tag{6}$$

$$\begin{aligned} COV(OFS_x, \sum_{i_y} w_{i_y} \sum_{k} w_{i_y,k}(TAI - PSFS_{i_y,k})) = \\ = - \sum_{i_y} w_{i_y} \sum_{k} w_{i_y,k} COV(OFS_x, PSFS_{i_y,k}) \end{aligned} \tag{7}$$

when the $\mathrm{PSFS}_{i_y,k}$ is equal to OFS_x or of the same kind of clock (for example Yb) we have a correlation different from zero that can be approximated by:

$$\sum_{i_y} w_{i_y} \sum_{k(\in PSFS(OFS_x))} w_{i_y,k} u_{\mathrm{bS}_{i_y,k}} \tag{8}$$

uSrep is considered in the global calculation of correlations. The updated relation for the correlation coefficient should be:

$$\begin{aligned} r(q_x, q_y) = \Bigg(& \sum_{i_x,i_y} w_{i_x} w_{i_y} \sum_{k} w_{i_x,k} u_{\mathrm{bS}_{i_x,k}} w_{i_y,k} u_{\mathrm{bS}_{i_y,k}} + \\ & + \sum_{i_y} w_{i_y} \sum_{k \in PSFS(OFS_x)} w_{i_y,k} u_{\mathrm{bS}_{i_y,k}} + \\ & + \sum_{i_x} w_{i_x} \sum_{k \in PSFS(OFS_y)} w_{i_x,k} u_{\mathrm{bS}_{i_x,k}} \Bigg) / (u_x u_y) \end{aligned} \tag{9}$$

References

[1] https://webtai.bipm.org/ftp/pub/tai/publication/wgfs/TM288_WGFS-Correlations.pdf

[2] H S Margolis et al 2024 Metrologia 61 035005

[3] Margolis H S, Panfilo G, Petit G, Oates C, Ido T and Bize S 2023 Data associated with the 2021 adjustment of standard frequencies (available at: https://doi.org/10.59161/StdFreq202301)