National Institute of Standards and Technology
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Concepts of Continuous Quantities &

Countable Aggregates and Nomenclature

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Concepts of continuous quantities & countable aggregates and nomenclature

Main purposes of this presentation

- What do the VIM and 9th SI Brochure have to say about 'counting'?

- What don't the VIM and 9th SI Brochure have to say about 'counting', but could?

- What might it be a good idea for those documents to have added about counting in the future?

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Main purposes of this presentation

- Identify overlaps and differences between concepts of measurement and counting

- Highlight ambiguities associated with different concepts

- To present ideas for developing a rigorous concept system for counting

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What do the VIM and 9th SI Brochure have to say about 'counting'?

Not a lot!

- The VIM3 has a Note to 'measurement' that "Measurement implies comparison of quantities or counting of entities", which is elaborated somewhat in the VIM4 1CD.

- The 9th SI Brochure has one Section (2.3.3 "Dimensions of quantities") that discusses "counting quantities", but mostly in the context of 'quantity dimensions' and 'quantities of unit one'.

- Section 5.4.7 of 9th SI Brochure: "Stating quantity values being pure numbers" also addresses "Quantities relating to counting …"

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What do the VIM and 9th SI Brochure have to say about ‘counting’?

- Neither the VIM nor the SI Brochure define ‘counting’.

- Should they?

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ISO 704 Terminology Work -- Principles and Methods

Provides details on concept systems

ISO 10241-1 Terminological entries in standards
Part 1 General requirements and examples of presentation

Explains how to structure definitions in standards

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Side-by-side comparison of concepts of quantities and aggregates

- **(continuous/physical) quantity**	- **Countable aggregate / (counting) quantity**
- measurement unit	- counting unit (?)
- value of a (continuous) quantity	- value of a (counting) quantity
- measurement	- counting
- dimension of a (continuous) quantity	- dimension of a (counting) quantity (?)
- traceability of a measurement result to the SI	- traceability of a counting result to the SI (?)

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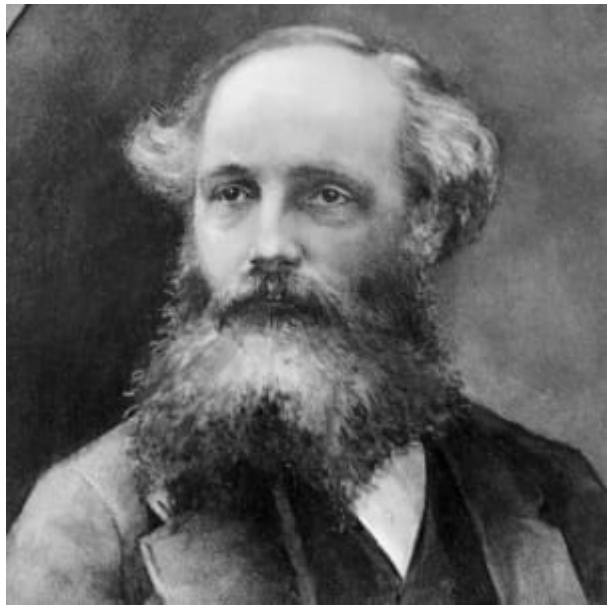
‘(value of a) quantity’ from J.C. Maxwell (1873) and J. DeBoer (1994/95)



James Clerk Maxwell

Photo credit: Wikipedia

“Every expression of a quantity consists of two factors or components.

One of these is the *name* of a certain known *quantity* of the same kind of the quantity to be expressed, which is taken as a *standard of reference*.

The other component is the *number* of times the standard is to be taken in order to make up the required quantity.

The standard quantity is technically called the *Unit* and the number is called the *Numerical Value* of the quantity.”

This may be expressed in the following notation:

physical quantity = numerical value x unit

$$Q = \{Q\}_{[Q]} \cdot [Q]$$

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Evolution of definition of ‘quantity’ in the VIM

VIM1 (1984): (measurable) quantity
An attribute of a phenomenon, body or substance, …

VIM2 (1993): (measurable) quantity
attribute of a phenomenon, body or substance …

VIM3 (2008): quantity
property of a phenomenon, body or substance, where the property has a magnitude that can be expressed as a number and a reference

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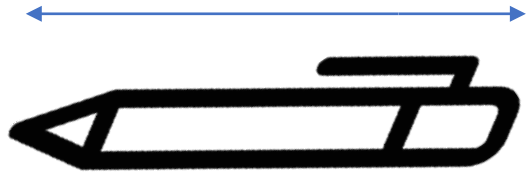
Some common properties of (physical) objects:

- Spatial extent or separation between points on the surface of the object;



- Heaviness of the object, or resistance to movement when being pushed on;

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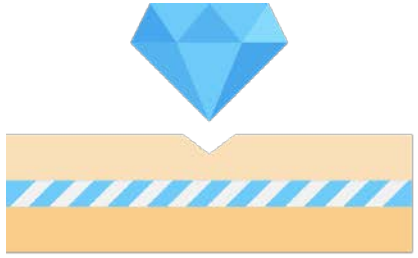
Some common properties of (physical) objects:

- Shape; well-characterized vs. amorphous;

- Hardness; Resistance to scratching;



- Etc.

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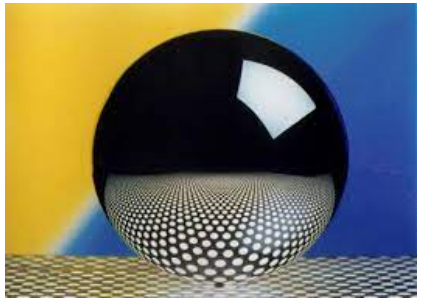
Some common properties of (physical) objects:

- Number of entities in an aggregate

aggregate of jellybeans

- Number of entities comprising an object/body



aggregate of silicon atoms

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‘measurement unit’ in the VIM

VIM4 1CD (2021): measurement unit
real scalar individual quantity, defined and adopted by convention, with which any other quantity of the same kind can be compared by ratio, resulting in a number

What is the purpose of the concept of ‘measurement unit’?

To provide an agreed upon (amount of) reference for a given kind of quantity, to be used as a means of comparison.

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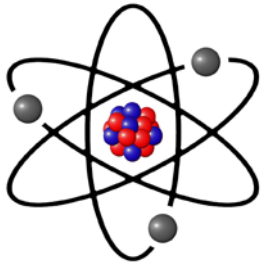
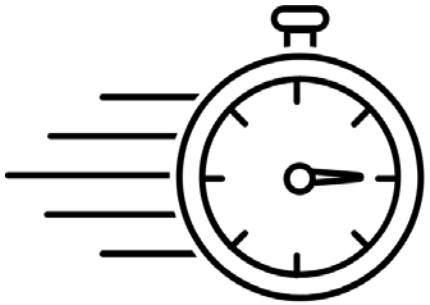
l_{meter}

Image courtesy of BIPM





Some basic concepts of measurement:

- "Object" to be measured
- Specified property of that object
- Measuring instrument/system
- Measurement unit
- Comparison
- [Measured] value
- [Measurement] uncertainty
- …

light moving a specified distance in a second …

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‘Counting unit’ in a concept system for counting?

What would be the purpose of the concept ‘counting unit’?

To serve as a reference, the same as for continuous quantities.

But is that really necessary? **Probably Not!**

aggregate of discrete entities

Count of entities in aggregate = {(whole?) number}

Saying that the counting unit is one is not necessary, and somewhat misleading and confusing!

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dimension of a (physical) quantity

VIM4 2CD:
relation of a **quantity** to the **base quantities** of a **system of quantities** as a product of the base quantities each raised to a power, omitting any proportionality factor

The dimension of a quantity *Q* is denoted by:

$$\dim Q = \mathsf{T}^{\alpha} \mathsf{L}^{\beta} \mathsf{M}^{\gamma} \mathsf{I}^{\delta} \Theta^{\varepsilon} \mathsf{N}^{\zeta} \mathsf{J}^{\eta}$$

where the exponents, named “dimensional exponents”, are positive or negative numbers, or zero.

The symbols T, L, M, I, Θ, N, and J represent the base quantities of the SI, not the base units.

The ‘equation’ for dim Q is not really a mathematical equation, and so perhaps should rather be written:

$$\dim Q \rightarrow \mathsf{T}^{\alpha} \mathsf{L}^{\beta} \mathsf{M}^{\gamma} \mathsf{I}^{\delta} \Theta^{\varepsilon} \mathsf{N}^{\zeta} \mathsf{J}^{\eta} \, .$$

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dimensionless quantity

When all of the dimensional exponents of the base quantities are zero, the dimension of the quantity Q is said to be equal to one, and the quantity is said to be “dimensionless”:

$$\dim Q \rightarrow \mathsf{T}^0\,\mathsf{L}^0\,\mathsf{M}^0\,\mathsf{I}^0\,\Theta^0\,\mathsf{N}^0\,\mathsf{J}^0 = 1$$

But the symbols T, L, M, I, Θ, N, and J represent the base [kind of] quantities of the SI, and not individual quantities that can have quantity values.

Therefore, raising such a symbol to the power 0 doesn’t make sense:

A dimension of a [kind of] quantity is not a numerical entity, and so cannot = 1.

Also, what does this have to do with measurement units or counting units for a given [kind of] quantity …?

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dimension of a quantity (including counting quantity)

The dimension of a quantity *Q* is denoted by: $\dim Q \rightarrow T^{\alpha} L^{\beta} M^{\gamma} I^{\delta} \Theta^{\varepsilon} N^{\zeta} J^{\eta}$

Could add the dimension of 'count' (symbol "C"):

$$\dim Q \rightarrow C^{\theta} T^{\alpha} L^{\beta} M^{\gamma} I^{\delta} \Theta^{\varepsilon} N^{\zeta} J^{\eta}$$

For example: Then the dimension of a count of pearls along a necklace could be:

$$\dim (\text{pearls along necklace chain}) \rightarrow C L^{-1}$$

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Count – number of discrete entities in an aggregate

value of a counting quantity: 100, or 100 · 1, or 100 entities?

$$C_{jellybeans} \stackrel{?}{=} \{100\}$$

$$C_{jellybeans} \stackrel{?}{=} \{100\} \cdot [1]$$

$$C_{jellybeans} \stackrel{?}{=} \{100\} \cdot [\text{entities (jellybeans)}]$$

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What are the basic concepts and complexities surrounding 'counting'?

- **How is 'counting' typically defined in dictionaries**? Examples:
  - Process of determining the number of elements of a finite set of objects;

- **How does the 3rd edition of the VIM define 'measurement'?**
  - Process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity

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Summary and Conclusions:

What might be some of the basic concepts of countable aggregates in a rigorous concept system for counting?

- countable aggregate / (counting) quantity
- counting unit (?)
- value of a counting quantity
- counting
- Dimension of a counting quantity (?)
- traceability of a count to the SI
- Etc.

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Summary and Conclusions:

- ‘dimension’, ‘(measurement) unit’ and ‘number’ are three very different concepts

- Should the number one (symbol “1”) be regarded as a (counting) unit?

- Should the term “unitless quantity” be used rather than “dimensionless quantity”, when there is no measurement unit?

Thank you for your attention!

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Questions?

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