2023 CCU/CCQM workshop on
The metrology of quantities which can be counted

Candela - by counting photons?

CCPR, Stefan Kück

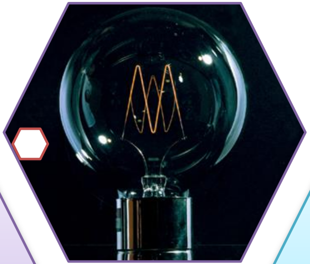
Bureau
International des
Poids et
Mesures

Overview

1 cd ≙ 4.09… × 10^{15} ph/(s sr)

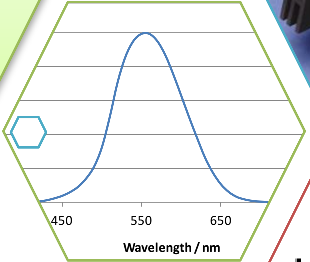
SI base unit: candela (cd)

The candela in numbers



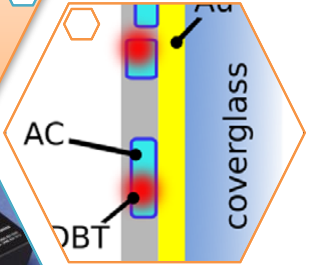
Photometric units vs. Radiometric units

How to measure single photons



How to generate single photons

152.83 µmol/s/m^2 (PPFD)



Photons and mole?

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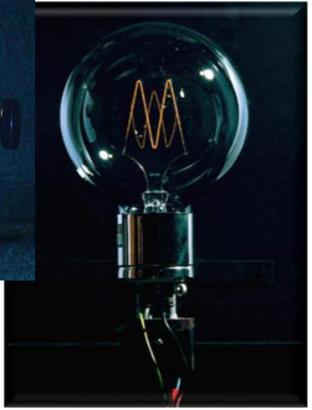
SI base unit: candela (cd)

The candela, symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency 540 x 10^{12} Hz, K_{cd}, to be 683 when expressed in the unit lm W^{-1}, which is equal to cd sr W^{-1}, or cd sr kg^{-1} m^{-2} s^{3}, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{Cs}$.

The candela is the ***luminous intensity***, in a given direction, of a source that emits monochromatic radiation of frequency 540 x 10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

⇒ The candela corresponds to a ***radiant intensity*** of 1/683 watt per steradian for monochromatic radiation of frequency 540 x 10^{12} hertz.



Photometric Units vs. Radiometric Units

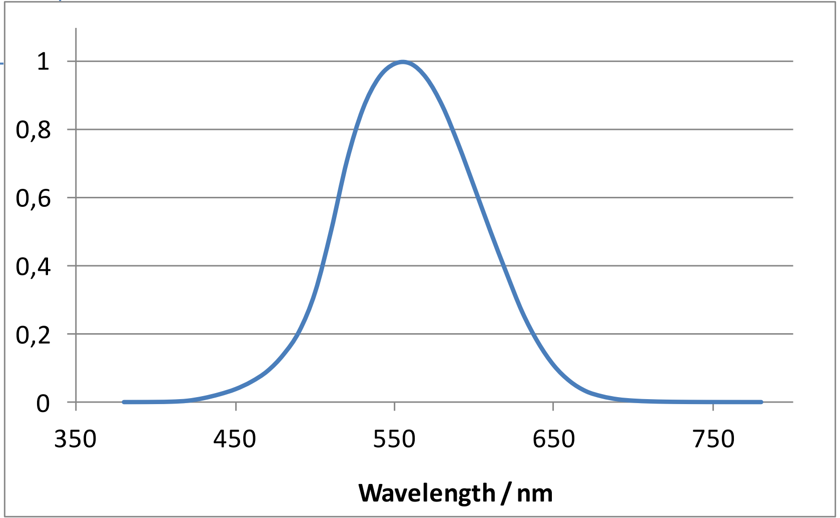
To consider:

Measured quantities in photometry are spectrally integrated quantities!

$$X_{\mathrm{v,x}} = \frac{K_{\mathrm{cd}}}{V_{\mathrm{x}}(\lambda_a)} \int_{\lambda} X_{\mathrm{e},\lambda}(\lambda) V_{\mathrm{x}}(\lambda) \mathrm{d}\lambda$$

The most important of these visual functions is the photopic luminous efficiency function for the light-adapted eye, $V(\lambda)$, which is defined by the CIE over the wavelength range 360 nm to 830 nm at 1 nm intervals.

CCPR-WG-SP-TG16:
Cone Fundamentals



Nonetheless: the candela (cd) in numbers

A radiant intensity of 1/683 W per steradian for photons with a frequency of 540×10^{12} Hz corresponds to 1/683 W/(hν) photons per second per steradian:

$\Rightarrow$ $N/s = 1/683\ W/(h\nu) = 1\ Js^{-1} / (683 \times 6.626\ 070\ 15 \times 10^{-34}\ Js \times 540 \times 10^{12}\ s^{-1})$

$\Rightarrow$ $N/s = 4.091942356\ldots \times 10^{15}\ s^{-1}$

I.e.,:

- **the candela corresponds to 4.091942356… × 10^{15} photons per second per steradian with photons at a frequency of 540 × 10^{12}.**
- **a nanocandela corresponds to 4.091942356… × 10^{6} photons per second per steradian with photons at a frequency of 540 × 10^{12}.**

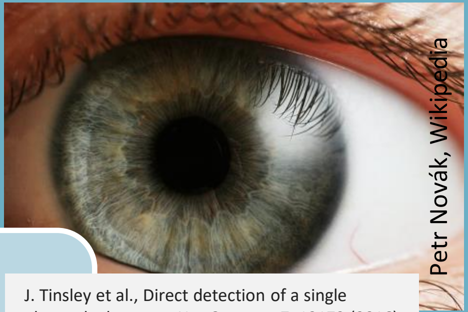
Measurable (countable) with single-photon detectors!

How to measure single-photons?

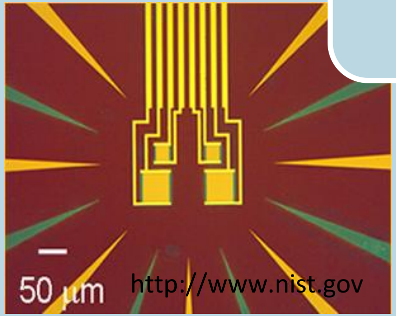
Single-photon detectors

Single Photon Avalanche Diode (SPAD)

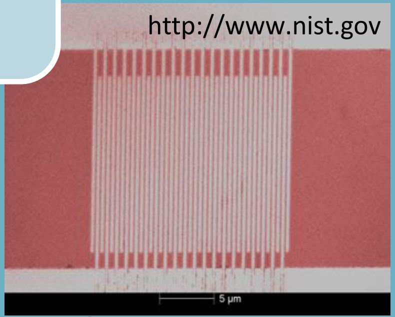
Human Eye!



J. Tinsley et al., Direct detection of a single photon by humans. *Nat Commun* **7**, 12172 (2016)



Transition Edge Sensor (TES)



Superconducting Nanowire Single-Photon Detector (SNSPD)

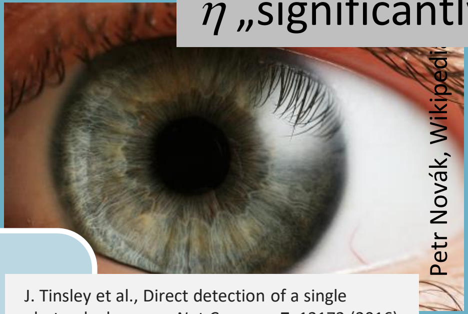
How to measure single-photons?

Single Photon Avalanche Diode (SPAD)

$\eta \approx 80\ \%$

Human Eye!

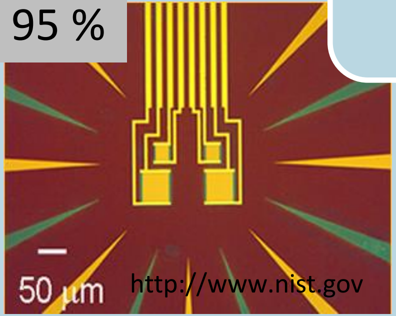
η „significantly above chance“



J. Tinsley et al., Direct detection of a single photon by humans. *Nat Commun* **7**, 12172 (2016)

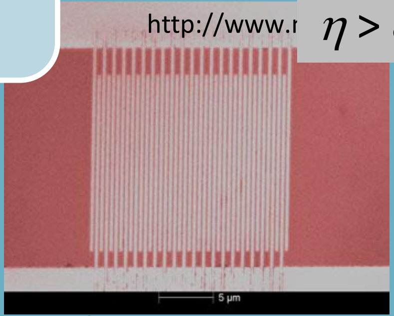
Single-photon detectors

$\eta > 95\ \%$



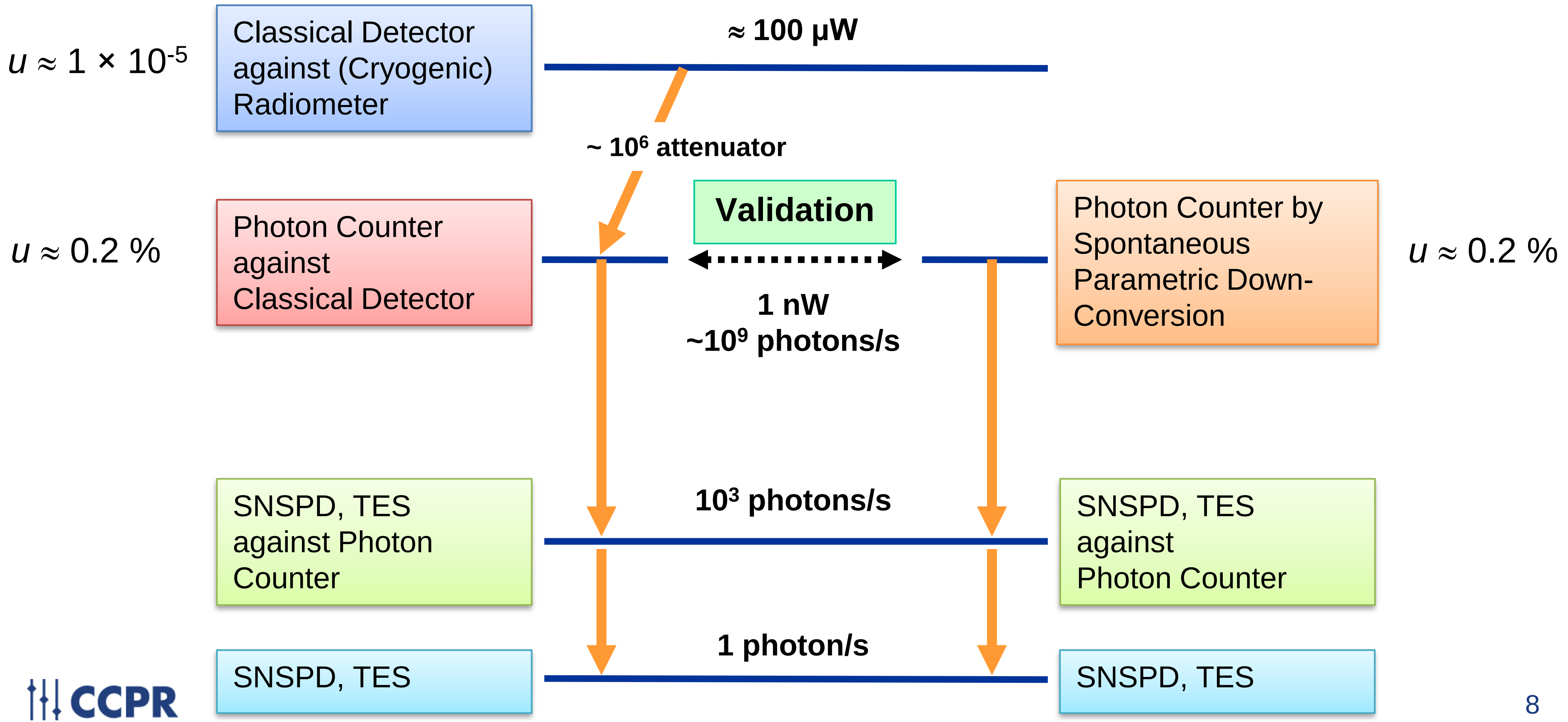
Transition Edge Sensor (TES)

$\eta > 85\ \%$



Superconducting Nanowire Single-Photon Detector (SNSPD)

Standard detector – Traceability



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Standard detector – Traceability

$u \approx 1 \times 10^{-5}$

Classical Detector against (Cryogenic) Radiometer

≈ 100 µW

~ 10^6

$u \approx 0.2$ %

Photon Counter against Classical De

Counter by ous tric Down- nversion

$u \approx 0.2$ %

Classical methods are much better than quantum methods for classical photometry and radiometry!

At least so far…

SNSPD, against Ph Counter

ons/s

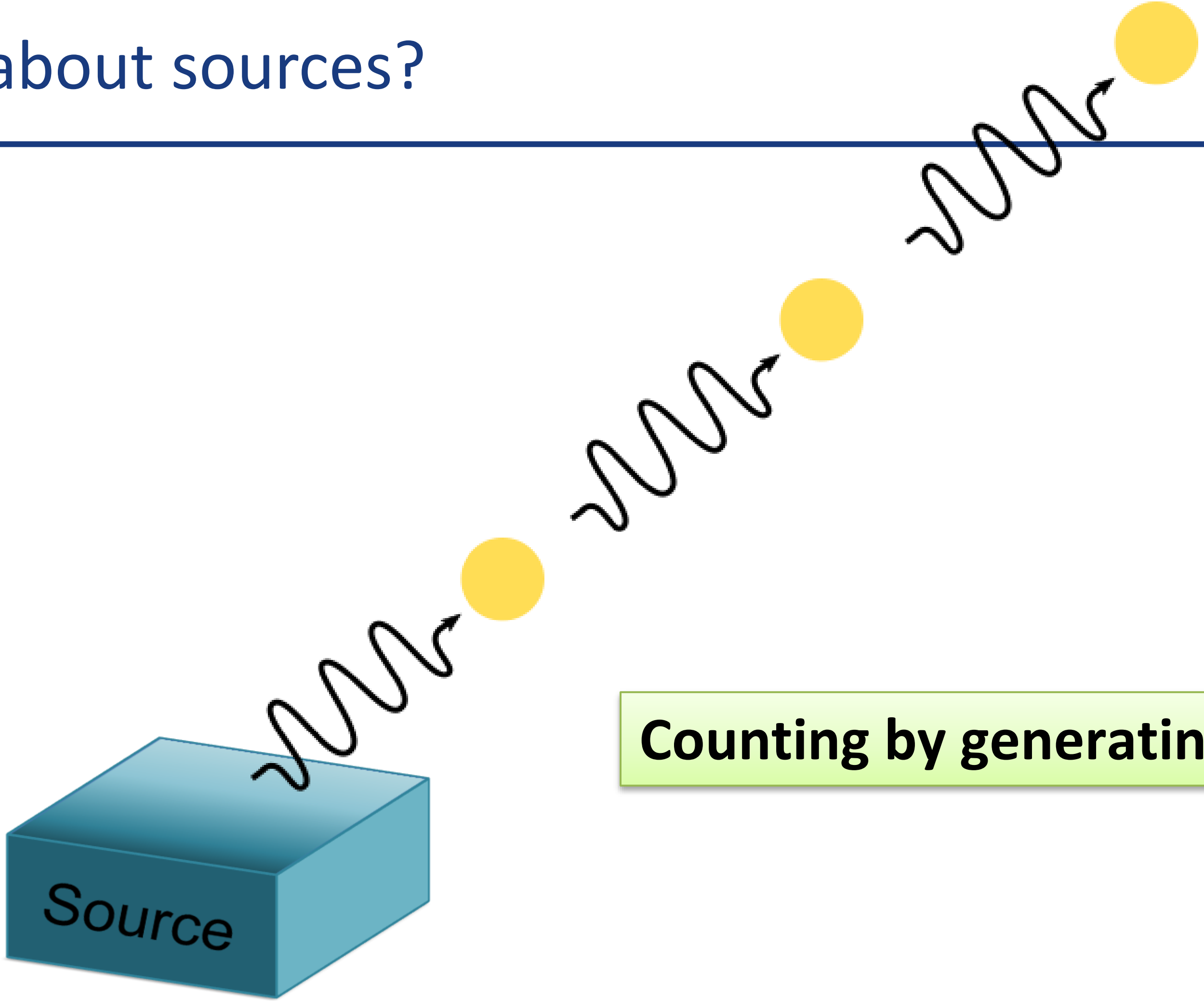
SNSPD, TES against Photon Counter

1 photon/s

SNSPD, TES

SNSPD, TES

What about sources?



Counting by generating!?

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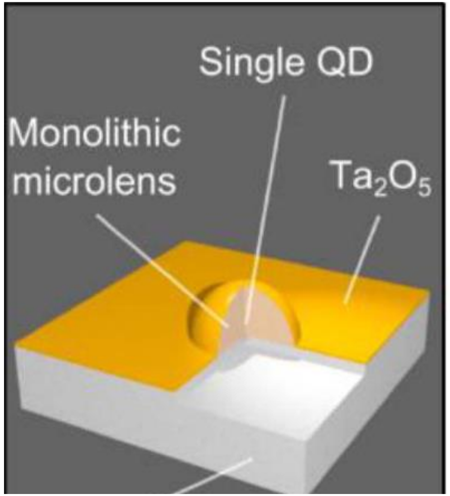
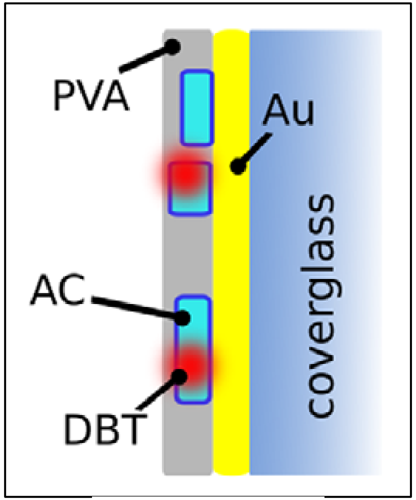
Single photon sources – how to?

Semiconductor quantum dots	Single molecules	Colour centres in diamond

Semiconductor quantum dots

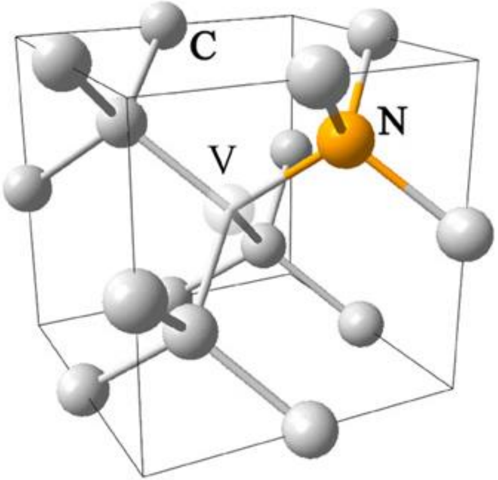


Single molecules



CNR-INO
ISTITUTO NAZIONALE DI OTTICA
CONSIGLIO NAZIONALE DELLE RICERCHE

Colour centres in diamond



I. Aharonovich et al.,
Rep. Prog. Phys. 74 076501 (2011)

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My dream…

Number of photons

Radiant Flux
Power

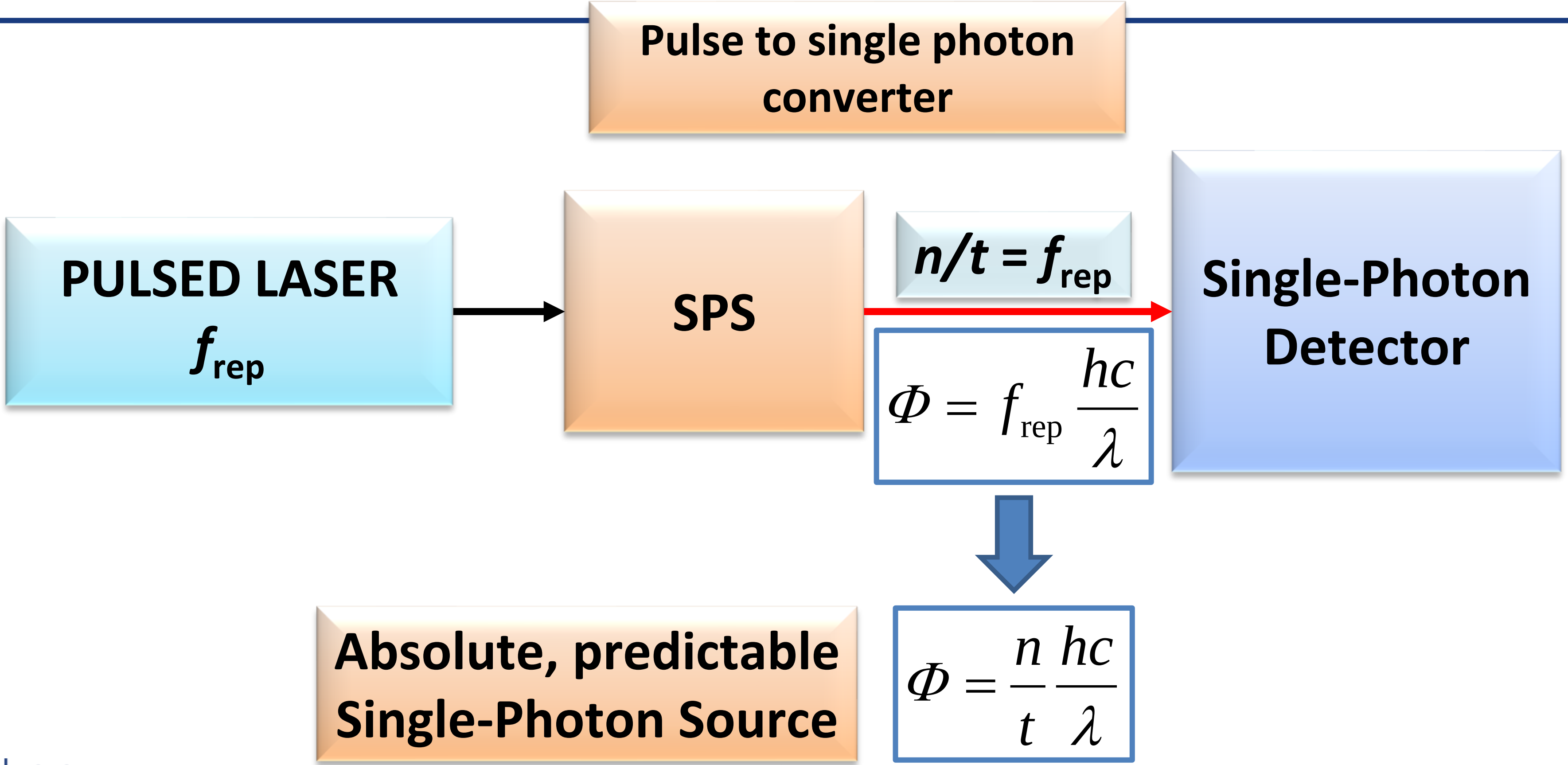
$$\Phi = \frac{n}{t}\frac{hc}{\lambda}$$

Time

Photon energy
- **Constants of nature**
- **Wavelength**

Note:
Candela is the unit for luminous intensity, thus involving the steradian

My dream… comes true!



Waking up is hard…!

Pulse to single photon converter

PULSED LASER
f_{rep}

SPS

$n/t = f_{rep}$

$$\Phi = f_{rep} \frac{hc}{\lambda}$$

Single-Photon Detector

Internal Quantum Efficiency < 100 %
Photon Collection Efficiency < 100 %

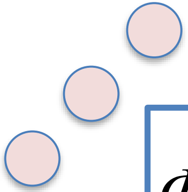
Absolute, predictable Single-Photon Source

$$\Phi = \frac{n}{t} \frac{hc}{\lambda}$$

Motivation for single-photon sources in metrology

Quantum Radiometry

- Reduction of measurement uncertainty
- Standard source
- Realization of photon-number-based candela



"N_{cd}"

$$\Phi = f_{\mathrm{rep}} \frac{hc}{\lambda}$$

Sub-shot noise metrology

Ideal SPS has no noise!
Noise-reduced measurements:

- e. g. transmission measurement

$$\frac{\Delta T_{\mathrm{SP}}^2}{\Delta T_{\mathrm{C}}^2} = 1 - 2\eta \frac{T}{1+T}$$

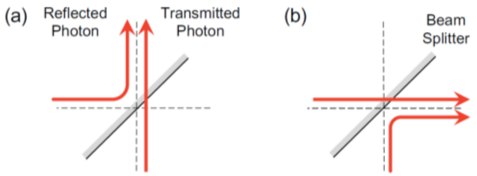
ΔT variance in transmission
T transmission
η total efficiency of setup

B. Lounis, M. Orrit, Rep. Prog. Phys. **68** 1129 (2004)

Photon-photon entanglement

Applications, e.g.:

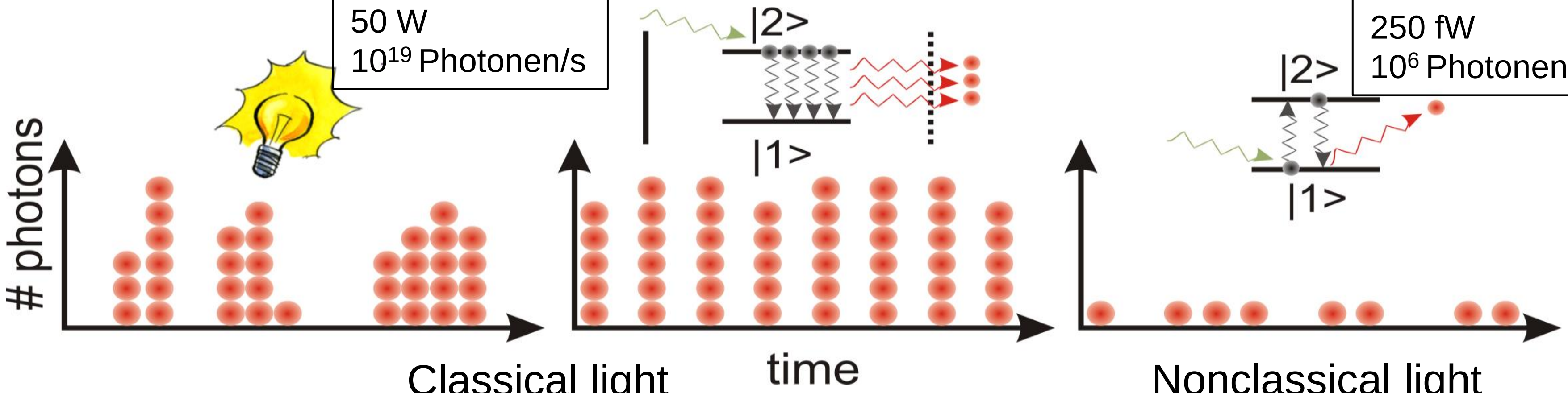
- quantum cryptography
- quantum repeater
- quantum computing



Photon statistics

50 W
10^{19} Photonen/s

250 fW
10^{6} Photonen/s



Classical light

Nonclassical light

Thermal light	Coherent (Laser) light	
Super-Poissonian-Distribution	Poissonian-Distribution	Sub-Poissonian-Distribution
bunching effect		anti-bunching effect

$$P_{h\nu}^{therm.}(n) = \frac{\langle n\rangle^n}{(\langle n\rangle + 1)^{n+1}}$$

$$P_{h\nu}^{Laser}(n) = \frac{\langle n\rangle^n e^{-\langle n\rangle}}{n!}$$

$$P_{h\nu}^{\text{Fock m}}(n) = \delta_{n,m}$$

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Photon statistics

50 W
10^{19} Photonen/s

250 fW
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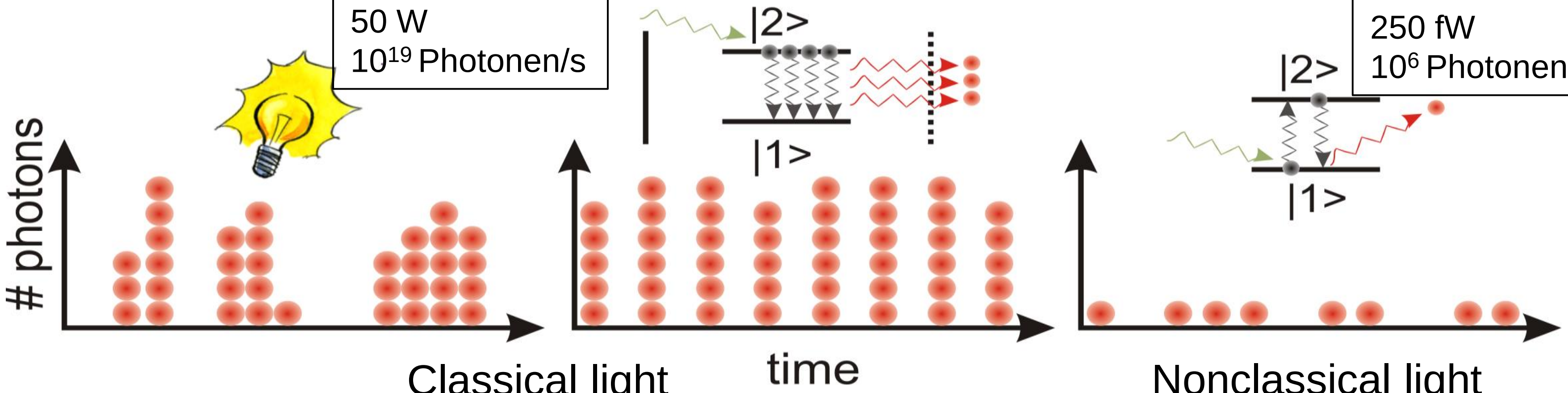


Classical light

Nonclassical light

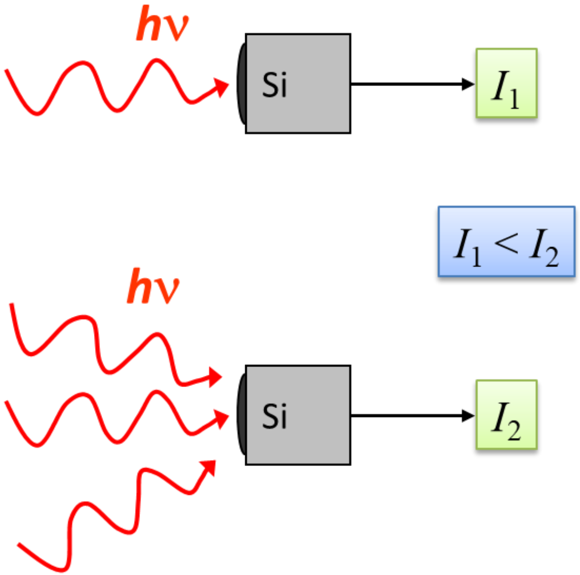
With thermal or with laser light, there will always be - with a specific probability - more than one photon within a time slot!

Sub-Poissonian-Distribution
anti-bunching effect

$$P_{h\nu}^{\text{Fock m}}(n) = \delta_{n,m}$$

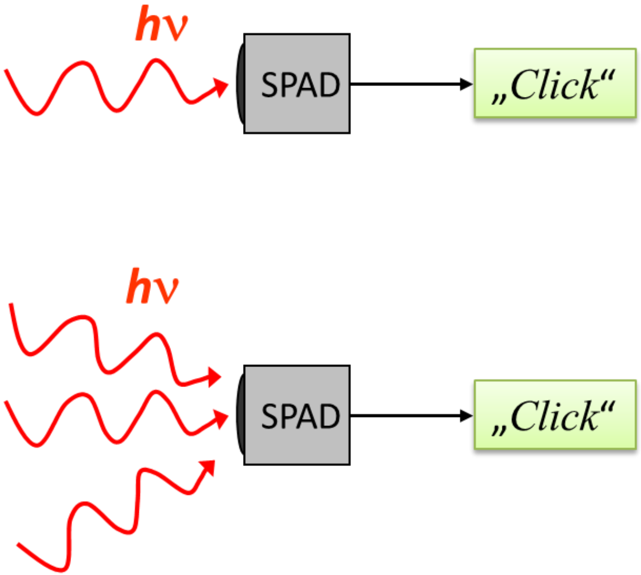
Motivation for quantum radiometry

Analogue detector



Signal ∝ Number of photons

Digital detector



Only one "click" per time

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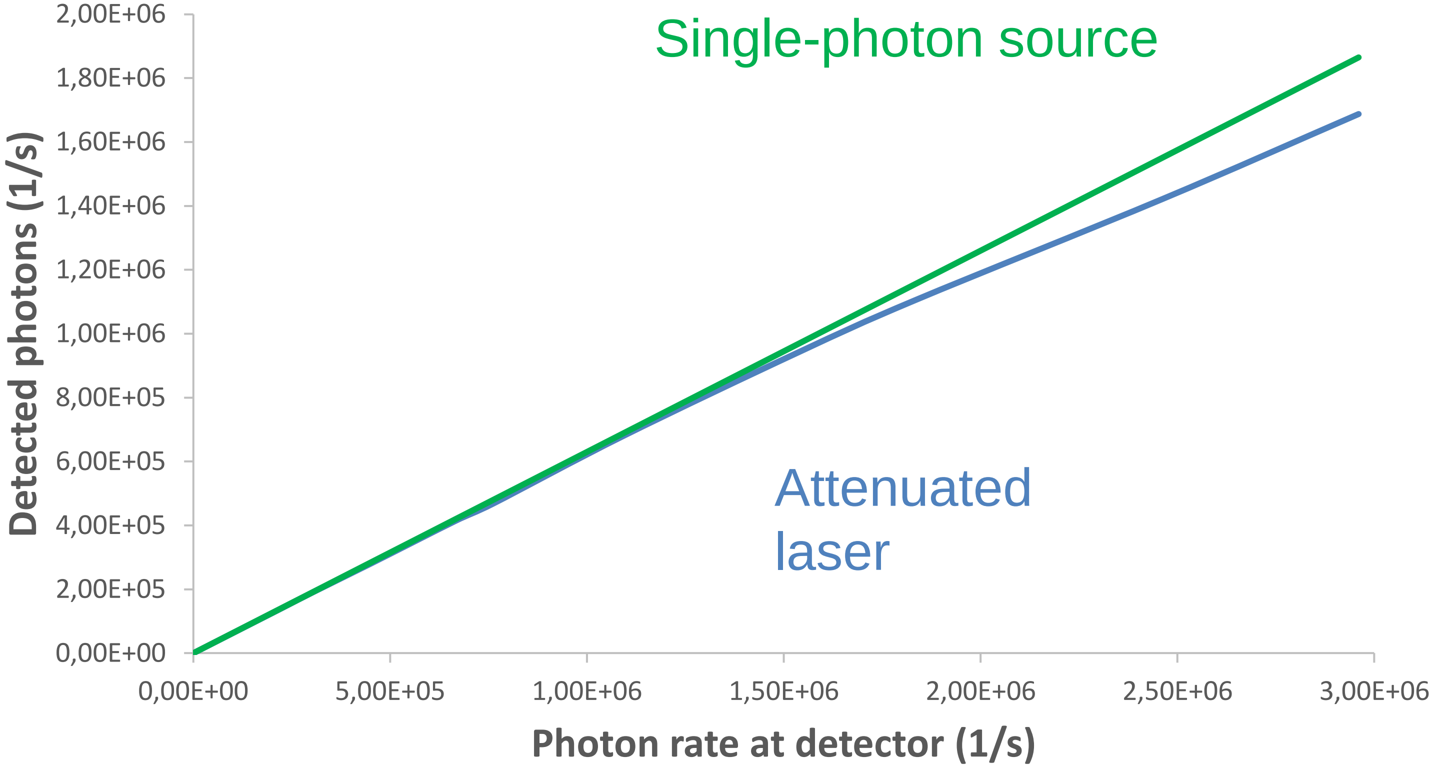
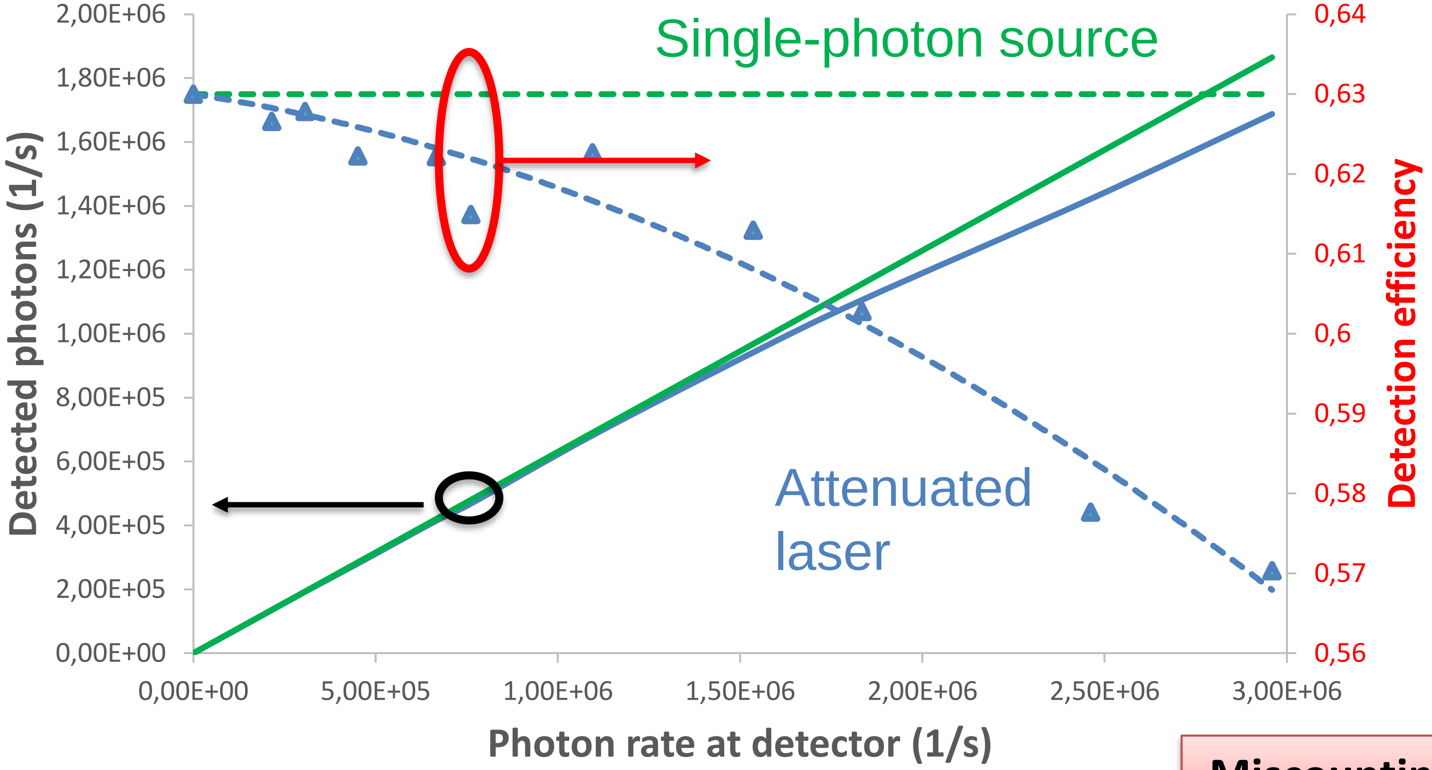
Influence on measurement



Influence on measurement



Miscounting!

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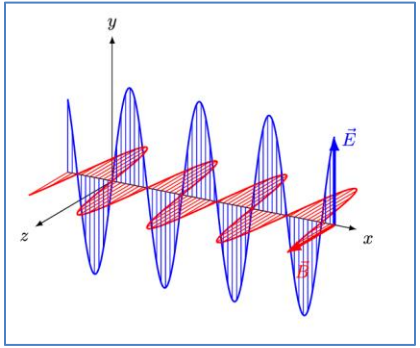
Finally: the candela and the mole?

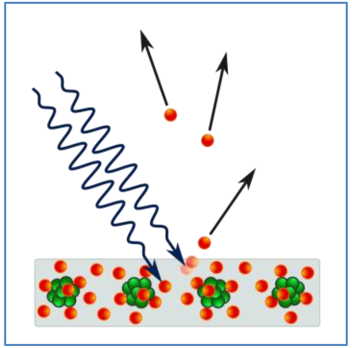
$$4.091942356\ldots \times 10^{15} \text{ photons/(s sr)}$$

$$=$$

$$6.794830142\ldots \times 10^{-9} \text{ mol/(s sr)}$$

Note:
The mole is the unit of amount of substance
Photons sometimes are / behave like particles





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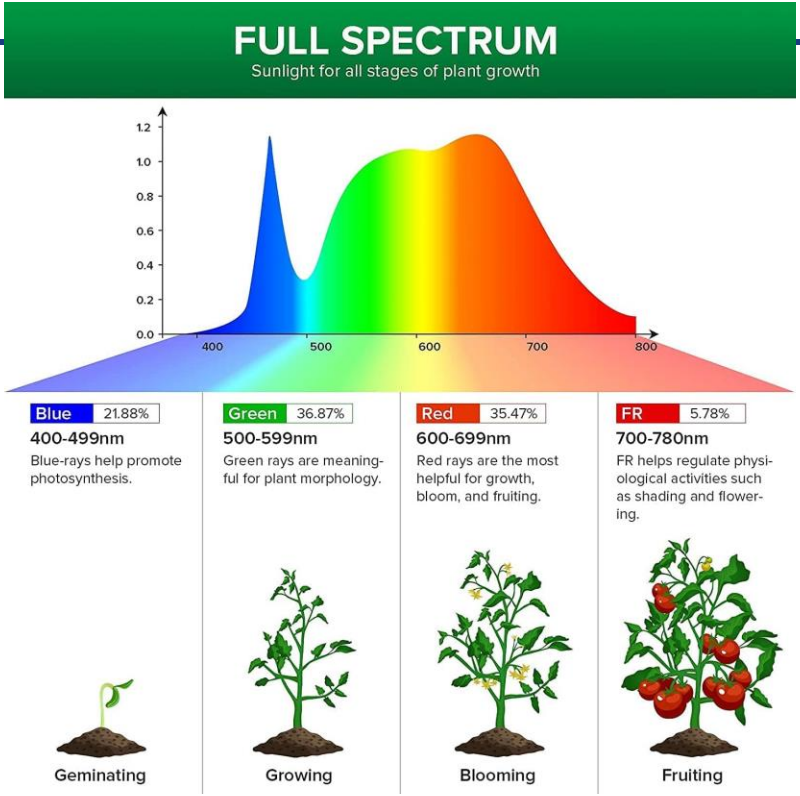
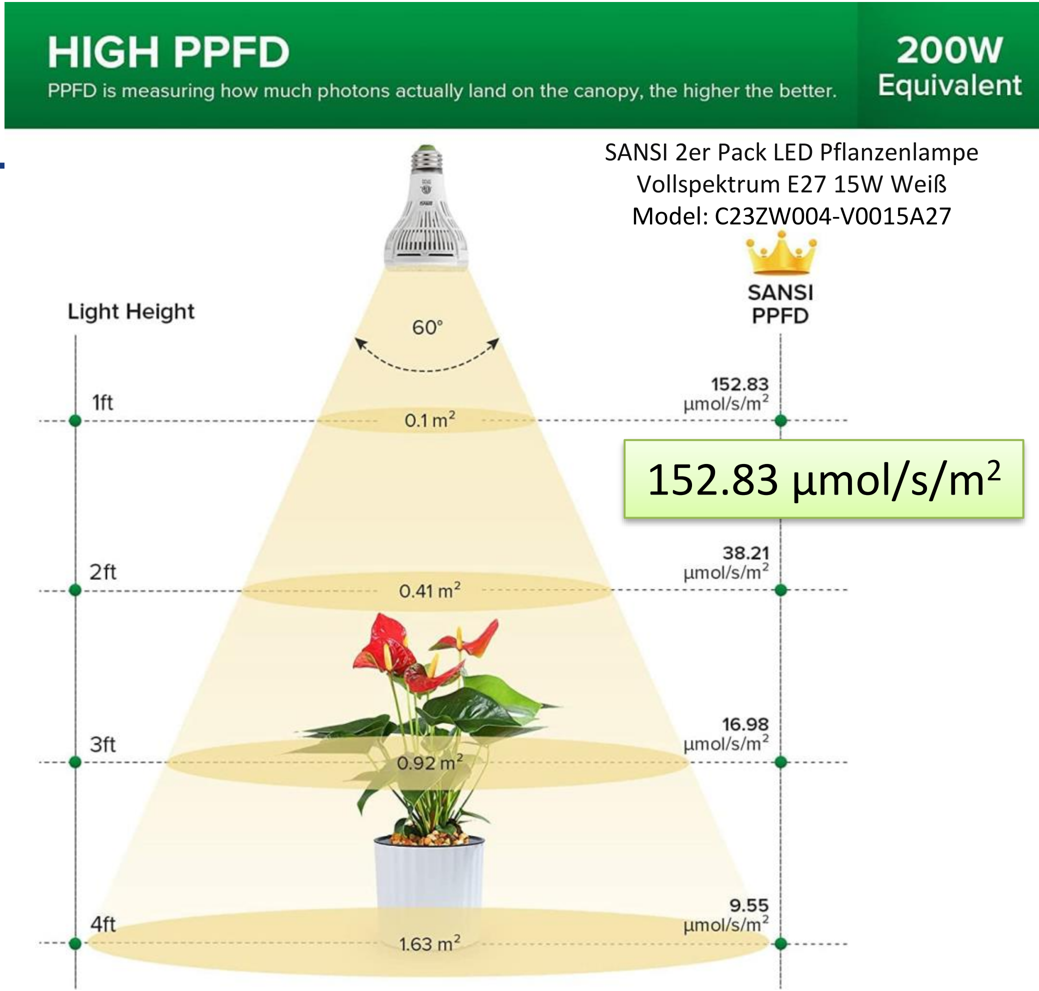
Photons and mole?



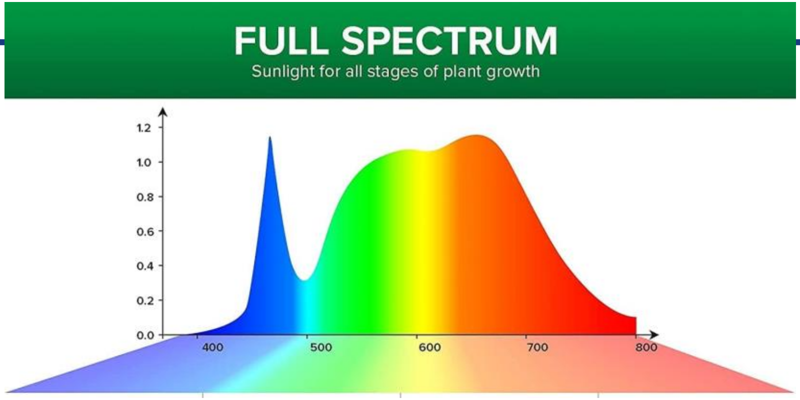


SANSI 2er Pack LED Pflanzenlampe Vollspektrum E27 15W Weiß
Model: C23ZW004-V0015A27

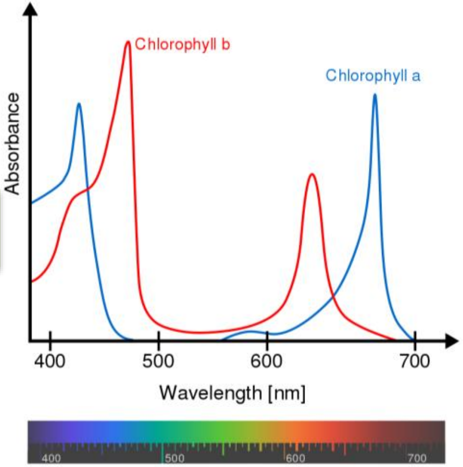
152.83 µmol/s/m^2

PPFD: Photosynthetic Photon Flux Density

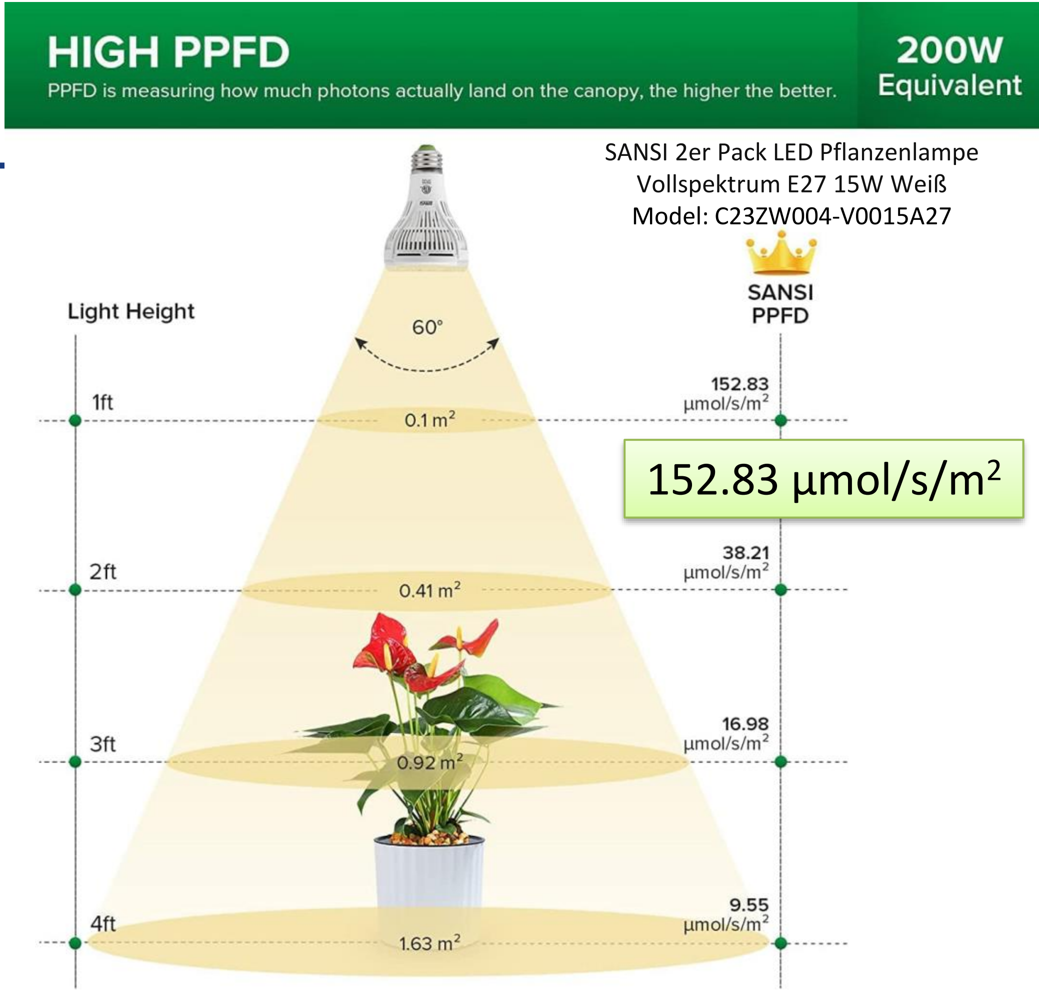
Photons and mole?



Action spectrum



By Original: Daniele Pugliesi
Vector: M0tty - This file was derived from: Chlorophyll ab spectra2.PNG:, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=20509583



SANSI 2er Pack LED Pflanzenlampe Vollspektrum E27 15W Weiß
Model: C23ZW004-V0015A27

152.83 µmol/s/m²

PPFD: Photosynthetic Photon Flux Density

Take home messages

Candela – by counting photons?

- No, at least not yet

Nonetheless, **counting photons** is useful for many applications, e.g.:

- Quantum communication
- Quantum computing
- Low flux radiometry / Quantum radiometry

Photons and mol:

- PPFD: µmol/s/m^2
- Are photons „entities“ or an „amount of substance“, i.e., are they like Ni-atoms or like fish?