CCL

Measuring by counting in length metrology

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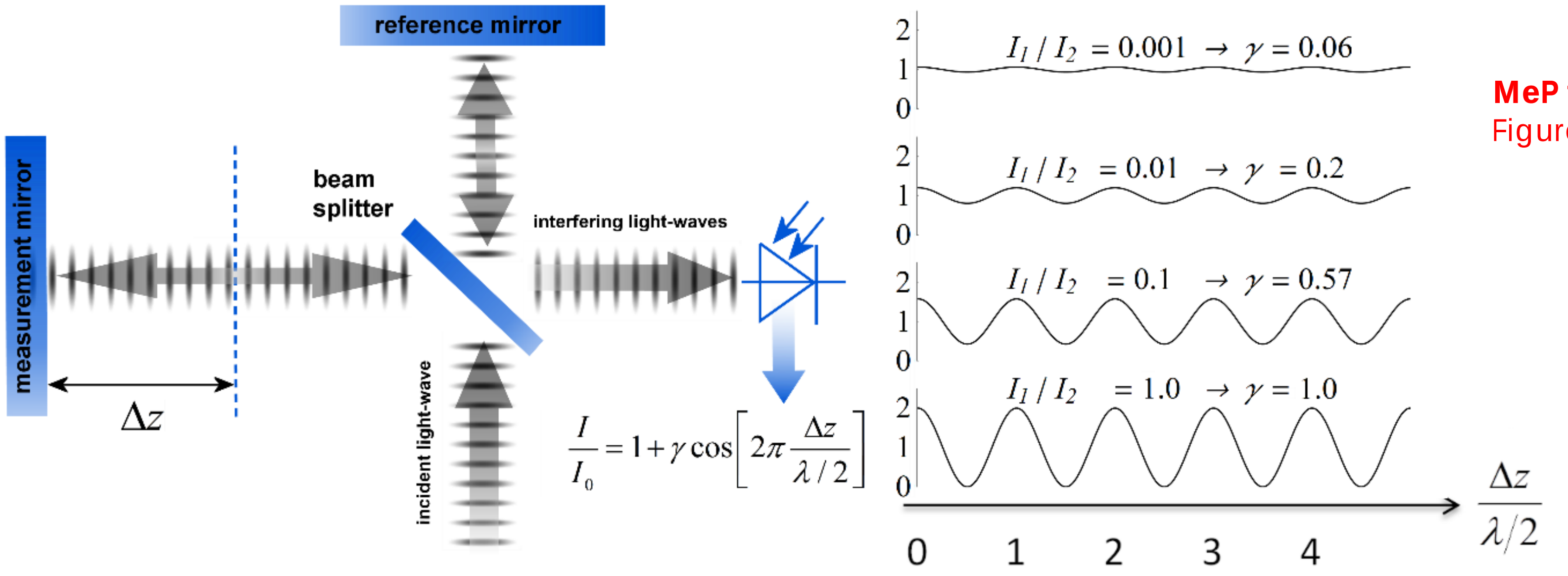
Outline

- Some examples of measurement by counting in length metrology
- A possible contribution to the general discussion: counting vs countable
- Conclusion

Interferometry

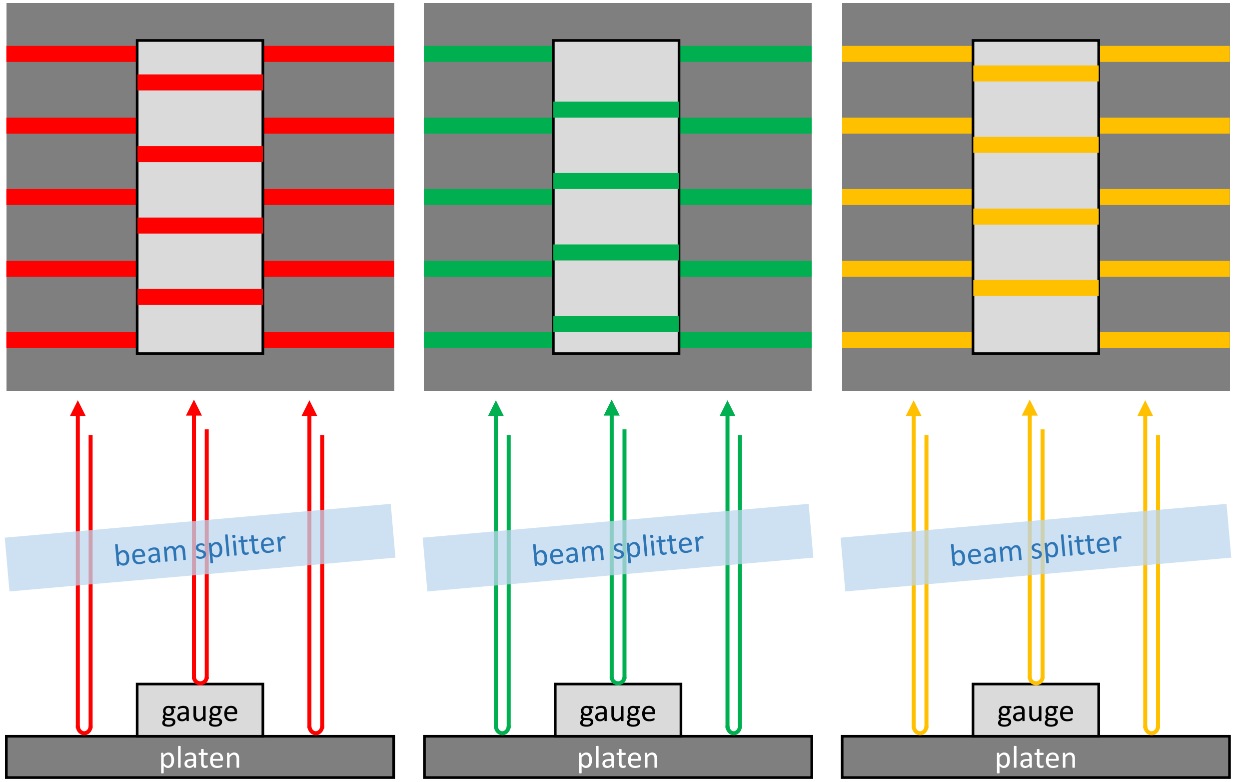


MeP for the metre
Figure 2

- Interferometry is the most widely technique used to link the metre definition – based on the speed of light – to a physical dimension
- When the measurement mirror moves, the light received from the reference and the measurement mirrors interfere giving rise to a periodic intensity signal whose period is half the light wavelength in the medium (usually air)
- The length is measured by *counting* the periods and interpolating the residual period fraction
- An alternative view is that the length is transformed one-to-one to the cumulated interferometric phase, which is measured as the integer numbers of cycles plus the intra-cycle phase

Counting can even be inferred

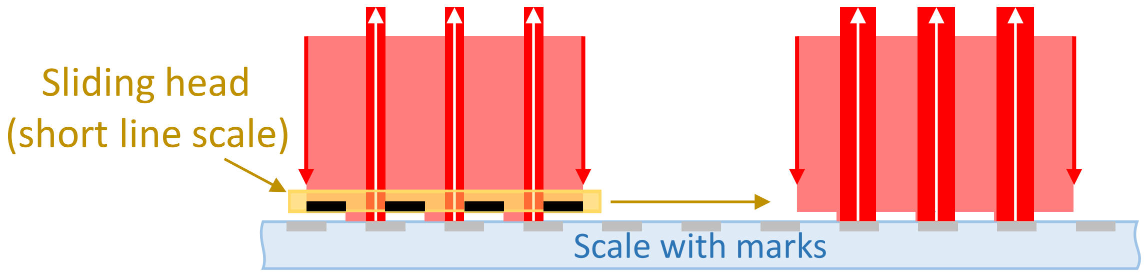
- A typical and widespread interferometric application is for calibration of gauge blocks with a Fizeau interferometer
- Interferograms are formed both on the platen and on the gauge (reflecting) surfaces
- The two interferograms are out of phase, the “fringe fraction” being the intra-cycle phase
- There is no counting, but the number of cycles (interference order) is needed
- The measurement is repeated with different wavelengths, resulting in different fringe fractions (intra-cycle phases)
- A length only of the gauge results in a number of cycles and an intra-cycle phase compatible with the observations for all wavelengths
- The estimation of the interference order is carried out with no actual counting
- Pre-knowledge of the gauge length is needed within an ambiguity interval

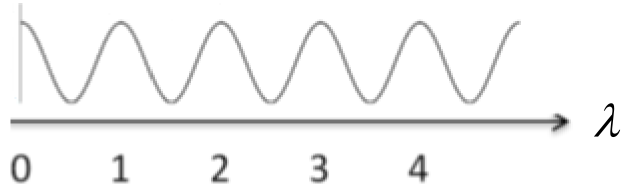


Linear and rotary encoders

- Encoders are widely used particularly in industrial metrology
- All linear and rotational axes of machines are equipped with encoders to measure their positions
- A head with a short scale slides on a still scale
- When illuminated, ...

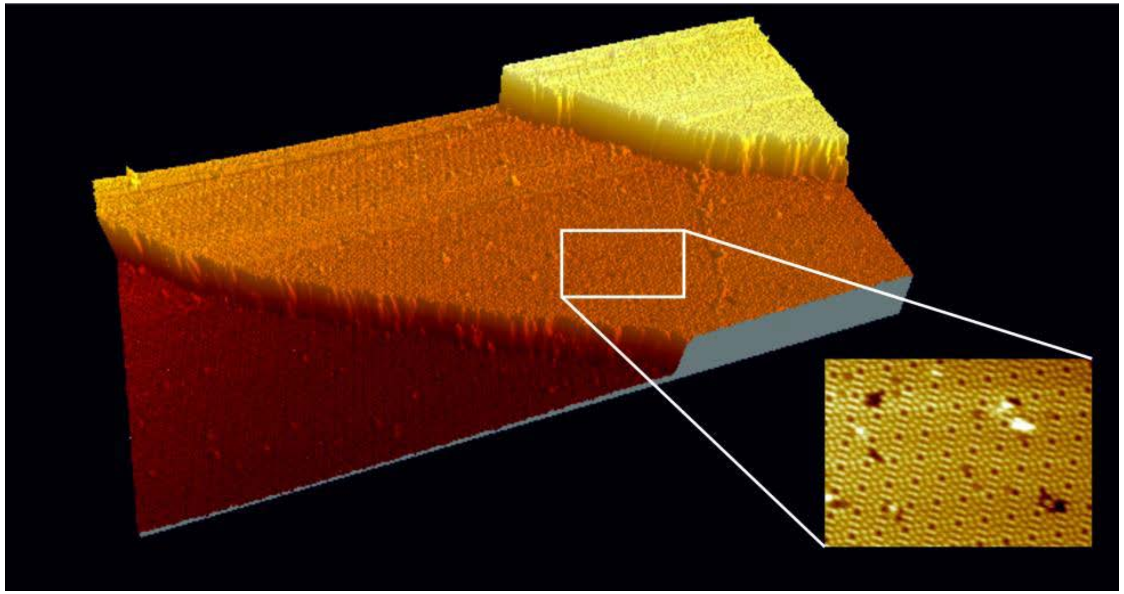
  ... the returned amount of light depends on the mutual positon
- A periodic signal is obtained, very much as in interferometry
- Cycles are counted and intra-cycle phase is measured





λ = pitch of the scale

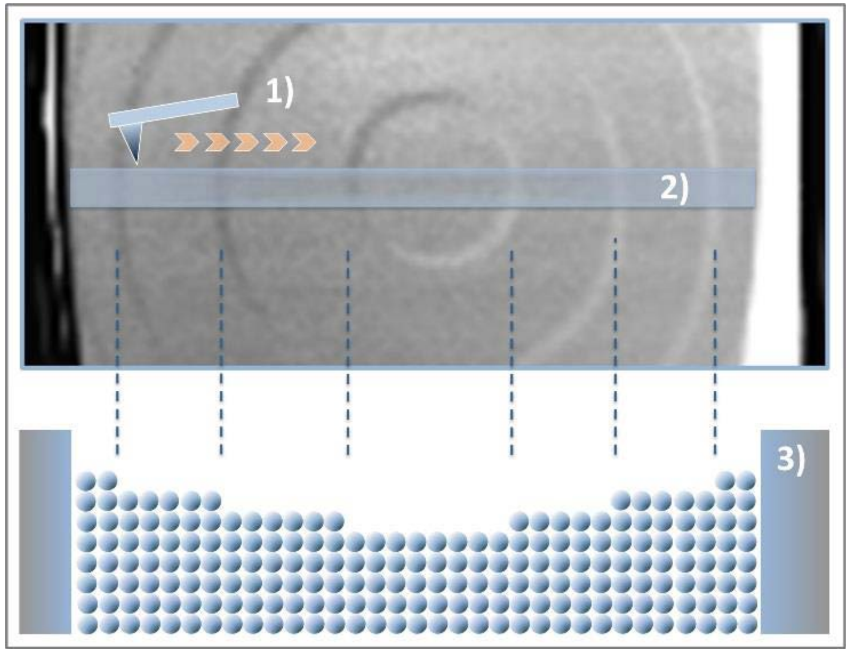
Counting atoms (atomic steps)



CCL-GD-MeP-3

Figure 3

Figure 5



- The very intense and long-standing work for the determination of the Avogadro and Plank constants underpinning the current definition of the mole and the kilogram resulted in an extremely accurate value of the natural Si lattice spacing, with a relative standard uncertainty of 1.7×10^{-8} (CODATA)
- The Si lattice spacing is a secondary realisation of the metre used in nanometrology
- Standards can be manufactured to realise monoatomic steps: a height can be measured by counting the steps
- Similarly, the Si lattice spacing can be used in X-ray interferometry (where the signal cycles are generated by a relative movement of two Si crystals, CCL-GD-MeP-1) and in electron microscopy (when Si atoms in the crystal are visible, CCL-GD-MeP-2)

Counting vs countable

- In the preparation of this workshop, the distinction between two quite different concepts was realised, *counting* and *countable*
- *Counting* is a measurement process, e.g.
  - *Counting* the interference fringes
  - *Counting* the cycles of a signal
  - *Counting* the atomic planes
- *Countable* is an attribute of a quantity, e.g.
  - The number of people attending this workshop is a *countable* quantity
  - The number of molecules in a box is a *countable* quantity
  - The number of atoms in a Si sphere is a *countable* quantity

Counting

- Counting requires to separate individual entities
- The separation is made based on thresholds
  - Counting the cycles of an (electronic) periodic signal requires thresholds on the signal level: when the signal crosses the threshold, a count occurs
  - Counting the people boarding an aircraft requires a threshold to separate the persons who board; the threshold is visual or through other means such as turnstiles
  - Counting the spots in an image requires a threshold on the image contrast
- The thresholding sensitivity may or may not be down to the unity
  - When it is, the uncertainty of counting may go down to zero (additional measurement accidents are possible)
  - When it is not, counting may be possible of clusters only

Countable

- Countable quantities are such *by nature* independently of how they are measured
  - The number of people who boarded an aircraft is a countable quantity independently of whether measured visually by the steward, or by a mechanical device at the entrance, or by a camera and vision algorithms
- Countable quantities have a finite cardinality, that is, can be related one-to-one to natural numbers; they are discrete by nature
  - Non-countable quantities have infinite cardinality instead
  - The size of physical objects might be thought of as ultimately discrete because objects are made up of individual atoms and molecules (if the size of an atom were defined and agreed upon)
  - Length as a quantity is not discrete in nature: sub-atomic lengths or lengths not exact multiples of atom “sizes” are possible, e.g. in vacuum
- There is no defintional uncertainty associated with countable quantities as such, but there may be some associated to the definition of the counted entities

All four combinations are possible

Examples	Countable quantity	Non-countable quantity
Measured by counting	Number of people who boarded an aircraft counted by the steward at the entrance	A length measured interferometrically by counting the fringes of an interferogram
Measured not by counting	Number of (nominally) identical items in a set, measured by weighting an individual item and the whole set	Usual case, not of interest in this workshop

Conclusion

- Length metrology is very much based on counting
- A fundamental distinction should be made between counting (a measurement process) and countable (an attribute of a quantity)
- Countable quantities suffer no definitional uncertainty as such but some may occur of the counted entities
- Counting suffers the uncertainty related to its thresholding capacity (of separating individuals), which may be null
- There is a lot of counting in length metrology but no countable quantities